

glucat  
0.12.0

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# Chapter 1

## Namespace Index

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## Chapter 5

# Namespace Documentation

### 5.1 cga3 Namespace Reference

Definitions for 3D Conformal Geometric Algebra [DL].

#### Functions

- `template<typename Multivector_T >`  
`Multivector_T cga3 (const Multivector_T &x)`  
*Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].*
- `template<typename Multivector_T >`  
`Multivector_T cga3std (const Multivector_T &X)`  
*Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].*
- `template<typename Multivector_T >`  
`Multivector_T agc3 (const Multivector_T &X)`  
*Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].*

#### 5.1.1 Detailed Description

Definitions for 3D Conformal Geometric Algebra [DL].

#### 5.1.2 Function Documentation

##### 5.1.2.1 agc3()

```
template<typename Multivector_T >
Multivector_T cga3::agc3 (
    const Multivector_T & X ) [inline]
```

Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].

Definition at line 126 of file [PyClical.h](#).

References [cga3std\(\)](#).

### 5.1.2.2 cga3()

```
template<typename Multivector_T >
Multivector_T cga3::cga3 (
    const Multivector_T & x ) [inline]
```

Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].

Definition at line 103 of file [PyClical.h](#).

### 5.1.2.3 cga3std()

```
template<typename Multivector_T >
Multivector_T cga3::cga3std (
    const Multivector_T & X ) [inline]
```

Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].

Definition at line 114 of file [PyClical.h](#).

Referenced by [agc3\(\)](#).

## 5.2 glucat Namespace Reference

### Namespaces

- namespace [gen](#)
- namespace [matrix](#)
- namespace [timing](#)

### Classes

- class [basis\\_table](#)  
*Table of basis elements used as a cache by basis\_element()*
- class [bool\\_to\\_type](#)  
*Bool to type.*
- class [clifford\\_algebra](#)  
*clifford\_algebra<> declares the operations of a Clifford algebra*
- class [compare\\_types](#)  
*Type comparison.*
- class [compare\\_types< T, T >](#)
- class [control\\_t](#)  
*Parameters to control tests.*
- struct [CTAssertion](#)  
*Compile time assertion.*
- struct [CTAssertion< true >](#)
- class [error](#)  
*Specific exception class.*
- class [framed\\_multi](#)  
*A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.*

- class [glucat\\_error](#)  
*Abstract exception class.*
- class [index\\_set](#)  
*Index set class based on `std::bitset<>` in Gnu standard C++ library.*
- class [index\\_set\\_hash](#)
- class [matrix\\_multi](#)  
*A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.*
- class [numeric\\_traits](#)  
*Extra traits which extend numeric limits.*
- class [random\\_generator](#)  
*Random number generator with single instance per `Scalar_T`.*
- class [sorted\\_range](#)  
*Sorted range for use with output.*
- class [sorted\\_range<Sorted\\_Map\\_T,Sorted\\_Map\\_T>](#)

## Typedefs

- using [index\\_t](#) = int  
*Size of `index_t` should be enough to represent LO, HI.*
- using [set\\_value\\_t](#) = unsigned long  
*Size of `set_value_t` should be enough to contain `index_set<LO,HI>`*
- typedef int(\* [intfn](#)) ()  
*For exception catching: pointer to function returning int.*
- typedef int(\* [intintfn](#)) (int)  
*For exception catching: pointer to function of int returning int.*
- using [tuning\\_slow](#)
- using [tuning\\_naive](#)
- using [tuning\\_fast](#)

## Functions

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of multivectors.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator!=](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> bool  
*Test for inequality of multivector and scalar.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [operator!=](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool  
*Test for inequality of scalar and multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [error\\_squared\\_tol](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T  
*Quadratic norm error tolerance relative to a specific multivector.*
- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [error\\_squared](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold) -> Scalar\_T

*Relative or absolute error using the quadratic norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`

*Test for approximate equality of multivectors.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`

*Test for approximate equality of multivectors.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric sum of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric sum of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric sum.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric difference of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator- (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric difference of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator- (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric difference.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator\* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Product of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator\* (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Product of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >  
auto operator\* (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`



*Geometric product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator^ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Outer product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator& (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator% (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto star (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of multivector and scalar.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Quotient of scalar and multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Geometric multiplicative inverse.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Integer power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Multivector power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_←  
_T, LO, HI, Tune_P >`

*Outer product power of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Real part: synonym for scalar part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Imaginary part: deprecated (always 0)*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Pure part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`

*Vector part of multivector, as a vector\_t with respect to frame()*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each {i} is replaced by -{i} in each term, eg. {1}\*{2} -> (-{2})\*(-{1})*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg. {1}\*{2} -> {2}\*{1}.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form == (rev(x)\*x)(0)*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [norm](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Scalar\_T norm == sum of norm of coordinates.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [abs](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Absolute value == sqrt(norm)*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [max\\_abs](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [complexifier](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [elliptic](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [sqrt](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [sqrt](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [clifford\\_exp](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Exponential of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [log](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [log](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [cos](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val, const Multivector< Scalar\_T, LO, HI, Tune\_P > &i, const bool prechecked=false) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Cosine of multivector with specified complexifier.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [cos](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >



*Inverse hyperbolic sine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto asinh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic sine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto tanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto atanh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`static void check\_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

*Check that i is a valid complexifier for val.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator\* (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Geometric product.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator^ (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Outer product.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator& (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator% (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto star (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator/ (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator| (const framed\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed\_multi< Scalar_T, LO, HI, Tune_P > &rhs) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator>> (std::istream &s, framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`  
*Read multivector from input.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator<< (std::ostream &os, const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI>`  
`auto operator<< (std::ostream &os, const std::pair< const index\_set< LO, HI >, Scalar_T > &term) -> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto exp (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Exponential of multivector.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI>`  
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI>`  
`auto operator\* (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index\_set< LO, HI >, Scalar_T >`  
*Product of terms.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto sqrt (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Square root of multivector with specified complexifier.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`  
*Natural logarithm of multivector with specified complexifier.*
- `template<typename Scalar_T , const index\_t LO, const index\_t HI>`  
`static auto crd\_of\_mult (const std::pair< const index\_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index\_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`

- Coordinate of product of terms.*

  - `_GLUCAT_CTAssert` (std::numeric\_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const `index_t` BITS\_PER\_CHAR

*If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.*

  - `_GLUCAT_CTAssert` (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const `index_t` DEFAULT\_LO

*Default lowest index in an index set.*

  - template<typename LHS\_T, typename RHS\_T >  
auto `pos_mod` (LHS\_T lhs, RHS\_T rhs) -> LHS\_T

*Modulo function which works reliably for lhs < 0.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator^` (const `index_set`< LO, HI > &lhs, const `index_set`< LO, HI > &rhs) -> const `index_set`< LO, HI >

*Symmetric set difference: exclusive or.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator&` (const `index_set`< LO, HI > &lhs, const `index_set`< LO, HI > &rhs) -> const `index_set`< LO, HI >

*Set intersection: and.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator|` (const `index_set`< LO, HI > &lhs, const `index_set`< LO, HI > &rhs) -> const `index_set`< LO, HI >

*Set union: or.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `compare` (const `index_set`< LO, HI > &a, const `index_set`< LO, HI > &b) -> int

*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*

  - `_GLUCAT_CTAssert` (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >), Default\_index\_set\_too\_big\_for\_value) template< const `index_t` LO

*Size of set\_value\_t should be enough to contain bitset<DEFAULT\_HI-DEFAULT\_LO>*

  - const `index_t` HI auto `operator<<` (std::ostream &os, const `index_set`< LO, HI > &ist) -> std::ostream &
  - template<const `index_t` LO, const `index_t` HI>  
auto `operator>>` (std::istream &s, `index_set`< LO, HI > &ist) -> std::istream &

*Read in index set.*

  - auto `sign_of_square` (`index_t` j) -> int

*Square of generator {j}.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `min_neg` (const `index_set`< LO, HI > &ist) -> `index_t`

*Minimum negative index, or 0 if none.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `max_pos` (const `index_set`< LO, HI > &ist) -> `index_t`

*Maximum positive index, or 0 if none.*

  - template<const `index_t` LO, const `index_t` HI>  
auto `operator<<` (std::ostream &os, const `index_set`< LO, HI > &ist) -> std::ostream &

*Write out index set.*

  - static auto `inverse_reversed_gray` (unsigned long x) -> unsigned long

*Inverse reversed Gray code.*

  - static auto `inverse_gray` (unsigned long x) -> unsigned long

*Inverse Gray code.*

  - template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `operator*` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >

*Geometric product.*

  - template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `operator^` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >



*Outer product.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator& (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Inner product.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator% (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Left contraction.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto star (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> Scalar_T`

*Hestenes scalar product.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator/ (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Geometric quotient.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator| (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Transformation via twisted adjoint action.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator>> (std::istream &s, matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream &`

*Read multivector from input.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto operator<< (std::ostream &os, const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::ostream`  
`&`

*Write multivector to output.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto reframe (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs, const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs, matrix\_multi< Scalar_T, LO, HI, Tune_P > &lhs_reframed, matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &rhs_reframed) -> const index\_set< LO, HI >`

*Find a common frame for operands of a binary operator.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto sqrt (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &i, bool prechecked) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto matrix\_sqrt (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, const index\_t level) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto log (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P > &i, bool prechecked) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto matrix\_log (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, const index\_t level) -> const matrix\_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto exp (const matrix\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix\_multi< Scalar_T, LO, HI,`  
`Tune_P >`

*Exponential of multivector.*

- `auto offset\_level (const index\_t p, const index\_t q) -> index\_t`



*Determine the log2 dim corresponding to signature p, q.*

- template<typename Matrix\_Index\_T, const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [folded\\_dim](#) (const [index\\_set](#)< LO, HI > &sub) -> Matrix\_Index\_T

*Determine the matrix dimension of the fold of a subalgebra.*

- template<typename Multivector\_T, typename Matrix\_T, typename Basis\_Matrix\_T >  
static auto [fast](#) (const Matrix\_T &X, [index\\_t](#) level) -> Multivector\_T

*Inverse generalized Fast Fourier Transform.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P, const size\_t Size>  
static auto [pade\\_approx](#) (const std::array< Scalar\_T, Size > &numer, const std::array< Scalar\_T, Size > &denom, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &X) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static void [db\\_step](#) ([matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &M, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &Y)

*Single step of product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [db\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 4)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [cr\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_Y\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 1)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Cyclic reduction square root iteration.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [pade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation of log.*

- template<typename Scalar\_T, const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [cascade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Incomplete square root cascade and Pade' approximation of log.*

- template<typename Scalar\_T >  
auto [log2](#) (const Scalar\_T &x) -> Scalar\_T

*Log base 2 of scalar.*

- template<typename Scalar\_T >  
auto [to\\_promote](#) (const Scalar\_T &val) -> typename [numeric\\_traits](#)< Scalar\_T >::promoted::type

*Cast to promote.*

- template<typename Scalar\_T >  
auto [to\\_demote](#) (const Scalar\_T &val) -> typename [numeric\\_traits](#)< Scalar\_T >::demoted::type

*Cast to demote.*

- int [try\\_catch](#) (intfn f)

*Exception catching for functions returning int.*

- int [try\\_catch](#) (intintfn f, int arg)

*Exception catching for functions of int returning int.*

## Variables

- const double [MS\\_PER\\_S](#) = 1000.0

*Timing constant: deprecated here - moved to [test/timing.h](#).*

- const [index\\_t](#) [BITS\\_PER\\_SET\\_VALUE](#) = std::numeric\_limits<[set\\_value\\_t](#)>::digits

*Number of bits in [set\\_value\\_t](#).*

- const [index\\_t](#) [DEFAULT\\_HI](#) = [index\\_t](#)([BITS\\_PER\\_SET\\_VALUE](#) / 2)  
*Default highest index in an index set.*
- static const long double [I\\_pi](#) = 3.1415926535897932384626433832795029L
- static const long double [I\\_ln2](#) = 0.6931471805599453094172321214581766L
- const unsigned int [Tuning\\_Int\\_Digits](#) = [std::numeric\\_limits<int>::digits](#)
- const unsigned int [Tuning\\_Max\\_Threshold](#) = 1 << [Tuning\\_Int\\_Digits](#)
- const unsigned int [Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 5.2.1 Typedef Documentation

### 5.2.1.1 [index\\_t](#)

using [glucat::index\\_t](#) = int

Size of [index\\_t](#) should be enough to represent LO, HI.

Definition at line 77 of file [global.h](#).

### 5.2.1.2 [intfn](#)

```
typedef int(* glucat::intfn) ()
```

For exception catching: pointer to function returning int.

Definition at line 37 of file [try\\_catch.h](#).

### 5.2.1.3 [intintfn](#)

```
typedef int(* glucat::intintfn) (int)
```

For exception catching: pointer to function of int returning int.

Definition at line 40 of file [try\\_catch.h](#).

#### 5.2.1.4 set\_value\_t

using `glucat::set_value_t` = unsigned long

Size of `set_value_t` should be enough to contain `index_set<LO,HI>`

Definition at line 79 of file [global.h](#).

#### 5.2.1.5 tuning\_fast

using `glucat::tuning_fast`

##### Initial value:

```
tuning
<
  Tuning_Fast_Mult_Matrix_Threshold,
  Tuning_Fast_Div_Max_Steps,
  Tuning_Fast_CR_Sqrt_Max_Steps,
  Tuning_Fast_DB_Sqrt_Max_Steps,
  Tuning_Fast_Log_Max_Outer_Steps,
  Tuning_Fast_Log_Max_Inner_Steps,
  Tuning_Fast_Basis_Max_Count,
  Tuning_Fast_Fast_Size_Threshold,
  Tuning_Fast_Inv_Fast_Dim_Threshold,
  Tuning_Fast_Products_Size_Threshold,
  Tuning_Default_Denom_Different_Bits,
  Tuning_Default_Extra_Different_Bits,
  Tuning_Default_Function_Precision
>
```

Definition at line 97 of file [tuning.h](#).

#### 5.2.1.6 tuning\_naive

using `glucat::tuning_naive`

##### Initial value:

```
tuning
<
  Tuning_Naive_Mult_Matrix_Threshold,
  Tuning_Default_Div_Max_Steps,
  Tuning_Default_CR_Sqrt_Max_Steps,
  Tuning_Default_DB_Sqrt_Max_Steps,
  Tuning_Default_Log_Max_Outer_Steps,
  Tuning_Default_Log_Max_Inner_Steps,
  Tuning_Naive_Basis_Max_Count,
  Tuning_Naive_Fast_Size_Threshold,
  Tuning_Naive_Inv_Fast_Dim_Threshold,
  Tuning_Default_Products_Size_Threshold,
  Tuning_Default_Denom_Different_Bits,
  Tuning_Default_Extra_Different_Bits,
  Tuning_Default_Function_Precision
>
```

Definition at line 69 of file [tuning.h](#).

### 5.2.1.7 tuning\_slow

using `glucat::tuning_slow`

#### Initial value:

```
tuning
<
    Tuning_Slow_Mult_Matrix_Threshold,
    Tuning_Default_Div_Max_Steps,
    Tuning_Default_CR_Sqrt_Max_Steps,
    Tuning_Default_DB_Sqrt_Max_Steps,
    Tuning_Default_Log_Max_Outer_Steps,
    Tuning_Default_Log_Max_Inner_Steps,
    Tuning_Slow_Basis_Max_Count,
    Tuning_Slow_Fast_Size_Threshold,
    Tuning_Slow_Inv_Fast_Dim_Threshold,
    Tuning_Slow_Products_Size_Threshold,
    Tuning_Default_Denom_Different_Bits,
    Tuning_Default_Extra_Different_Bits,
    Tuning_Default_Function_Precision
>
```

Definition at line 47 of file `tuning.h`.

## 5.2.2 Function Documentation

### 5.2.2.1 \_GLUCAT\_CTAssert() [1/3]

```
glucat::_GLUCAT_CTAssert (
    _GLUCAT_BITS_PER_ULONG  = BITS_PER_SET_VALUE,
    BitsPerUlongDoesNotMatchSetValueT ) const
```

Default lowest index in an index set.

### 5.2.2.2 \_GLUCAT\_CTAssert() [2/3]

```
glucat::_GLUCAT_CTAssert (
    sizeof(set_value_t) >=sizeof(std::bitset< DEFAULT_HI-DEFAULT_LO >) ,
    Default_index_set_too_big_for_value ) const
```

Size of `set_value_t` should be enough to contain `bitset<DEFAULT_HI-DEFAULT_LO>`

Write out index set

### 5.2.2.3 \_GLUCAT\_CTAssert() [3/3]

```
glucat::_GLUCAT_CTAssert (
    std::numeric_limits< unsigned char >::radix  = 2,
    CannotDetermineBitsPerChar ) const
```

If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.

Number of bits per char is used to determine number of bits in `set_value_t`

#### 5.2.2.4 abs()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Absolute value == sqrt(norm)

Definition at line 577 of file [clifford\\_algebra\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::sqrt\(\)](#).

Referenced by [PyClical.clifford::abs\(\)](#), [clifford\\_to\\_str\(\)](#), [matrix\\_log\(\)](#), and [matrix\\_sqrt\(\)](#).

#### 5.2.2.5 acos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse cosine of multivector.

Definition at line 903 of file [clifford\\_algebra\\_imp.h](#).

References [acos\(\)](#), and [complexifier\(\)](#).

#### 5.2.2.6 acos() [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::acos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse cosine of multivector with specified complexifier.

Definition at line 883 of file [clifford\\_algebra\\_imp.h](#).

References [acosh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [acos\(\)](#).

### 5.2.2.7 `acosh()` [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic cosine of multivector.

Definition at line 844 of file [clifford\\_algebra\\_imp.h](#).

References [acosh\(\)](#), and [complexifier\(\)](#).

### 5.2.2.8 `acosh()` [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::acosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse hyperbolic cosine of multivector with specified complexifier.

Definition at line 825 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [acos\(\)](#), and [acosh\(\)](#).

### 5.2.2.9 `approx_equal()` [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 169 of file [clifford\\_algebra\\_imp.h](#).

References [approx\\_equal\(\)](#), and [error\\_squared\\_tol\(\)](#).

**5.2.2.10 approx\_equal()** [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::approx_equal (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold,
    const Scalar_T tolerance ) -> bool [inline]
```

Test for approximate equality of multivectors.

Definition at line 154 of file [clifford\\_algebra\\_imp.h](#).

References [error\\_squared\(\)](#).

Referenced by [approx\\_equal\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.11 asin()** [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Inverse sine of multivector.

Definition at line 1008 of file [clifford\\_algebra\\_imp.h](#).

References [asin\(\)](#), and [complexifier\(\)](#).

**5.2.2.12 asin()** [2/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::asin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse sine of multivector with specified complexifier.

Definition at line 988 of file [clifford\\_algebra\\_imp.h](#).

References [asinh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [asin\(\)](#).

**5.2.2.13 asinh()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic sine of multivector.

Definition at line 949 of file [clifford\\_algebra\\_imp.h](#).

References [asinh\(\)](#), and [complexifier\(\)](#).

**5.2.2.14 asinh()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::asinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse hyperbolic sine of multivector with specified complexifier.

Definition at line 930 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), [norm\(\)](#), and [sqrt\(\)](#).

Referenced by [asin\(\)](#), and [asinh\(\)](#).

**5.2.2.15 atan()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Inverse tangent of multivector.

Definition at line 1108 of file [clifford\\_algebra\\_imp.h](#).

References [atan\(\)](#), and [complexifier\(\)](#).



**5.2.2.16 atan()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse tangent of multivector with specified complexifier.

Definition at line [1088](#) of file [clifford\\_algebra\\_imp.h](#).

References [atanh\(\)](#), and [check\\_complex\(\)](#).

Referenced by [atan\(\)](#).

**5.2.2.17 atanh()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Inverse hyperbolic tangent of multivector.

Definition at line [1052](#) of file [clifford\\_algebra\\_imp.h](#).

References [atanh\(\)](#), and [complexifier\(\)](#).

**5.2.2.18 atanh()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::atanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Inverse hyperbolic tangent of multivector with specified complexifier.

Definition at line [1035](#) of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [log\(\)](#), and [norm\(\)](#).

Referenced by [atan\(\)](#), and [atanh\(\)](#).

### 5.2.2.19 cascade\_log()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static auto glucat::cascade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const matrix\_multi<Scalar↵
_T,LO,HI,Tune_P>    [static]
```

Incomplete square root cascade and Pade' approximation of log.

Definition at line 1920 of file [matrix\\_multi\\_imp.h](#).

References [db\\_step\(\)](#), [epsilon](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), [norm\(\)](#), and [pade\\_log\(\)](#).

Referenced by [matrix\\_log\(\)](#).

### 5.2.2.20 check\_complex()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static void glucat::check_complex (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false )    [inline], [static]
```

Check that i is a valid complexifier for val.

Definition at line 652 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [cos\(\)](#), [log\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

### 5.2.2.21 clifford\_exp()

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::clifford_exp (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar↵
_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line 690 of file [clifford\\_algebra\\_imp.h](#).

References [log2\(\)](#).

Referenced by [exp\(\)](#), and [exp\(\)](#).

### 5.2.2.22 compare()

```
template<const index_t LO, const index_t HI>
auto glucat::compare (
    const index_set< LO, HI > & a,
    const index_set< LO, HI > & b ) -> int [inline]
```

"lexicographic compare" eg. {3,4,5} is less than {3,7,8}

Lexicographic ordering of two sets: -1 if  $a < b$ , +1 if  $a > b$ , 0 if  $a == b$ .

Definition at line 574 of file [index\\_set\\_imp.h](#).

### 5.2.2.23 complexifier()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::complexifier (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↔
_T,LO,HI,Tune_P>
```

Square root of -1 which commutes with all members of the frame of the given multivector.

Definition at line 592 of file [clifford\\_algebra\\_imp.h](#).

References [pos\\_mod\(\)](#).

Referenced by [acos\(\)](#), [acosh\(\)](#), [asin\(\)](#), [asinh\(\)](#), [atan\(\)](#), [atanh\(\)](#), [check\\_complex\(\)](#), [cos\(\)](#), [elliptic\(\)](#), [log\(\)](#), [sin\(\)](#), [sqrt\(\)](#), and [tan\(\)](#).

### 5.2.2.24 conj()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::conj (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↔
_T,LO,HI,Tune_P> [inline]
```

Conjugation, rev o invo == invo o rev.

Definition at line 553 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.25 cos() [1/2]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↔
_T,LO,HI,Tune_P> [inline]
```

Cosine of multivector.

Definition at line 874 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [cos\(\)](#).

### 5.2.2.26 `cos()` [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::cos (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Cosine of multivector with specified complexifier.

Definition at line 851 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), and [exp\(\)](#).

Referenced by [cos\(\)](#), and [tan\(\)](#).

### 5.2.2.27 `cosh()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::cosh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Hyperbolic cosine of multivector.

Definition at line 807 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#).

Referenced by [tanh\(\)](#).

### 5.2.2.28 `cr_sqrt()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static auto glucat::cr_sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_Y_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1) ) ->
const matrix\_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Cyclic reduction square root iteration.

Definition at line 1349 of file [matrix\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [norm\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.29 crd\_of\_mult()** [1/2]

```
template<typename Scalar_T , const index_t LO, const index_t HI>
static auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs ) -> Scalar_T
T [inline], [static]
```

Coordinate of product of terms.

Referenced by [operator%\(\)](#), [operator&\(\)](#), [operator\\*\(\)](#), and [operator^\(\)](#).

**5.2.2.30 crd\_of\_mult()** [2/2]

```
template<typename Scalar_T , const index_t LO, const index_t HI>
static auto glucat::crd_of_mult (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs ) -> Scalar_T
T [inline], [static]
```

Coordinate of product of terms.

Definition at line 1709 of file [framed\\_multi\\_imp.h](#).

**5.2.2.31 db\_sqrt()**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
static auto glucat::db_sqrt (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    Scalar_T norm_tol = std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4) ) ->
const matrix_multi<Scalar_T,LO,HI,Tune_P> [static]
```

Product form of Denman-Beavers square root iteration.

Definition at line 1320 of file [matrix\\_multi\\_imp.h](#).

References [db\\_step\(\)](#), and [norm\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#).

**5.2.2.32 db\_step()**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
static void glucat::db_step (
    matrix_multi< Scalar_T, LO, HI, Tune_P > & M,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & Y ) [inline], [static]
```

Single step of product form of Denman-Beavers square root iteration.

Definition at line 1308 of file [matrix\\_multi\\_imp.h](#).

References [inv\(\)](#).

Referenced by [cascade\\_log\(\)](#), and [db\\_sqrt\(\)](#).

### 5.2.2.33 `elliptic()`

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::elliptic (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Square root of -1 which commutes with all members of the frame of the given multivector The name "elliptic" is now deprecated: use "complexifier" instead.

Definition at line 643 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#).

### 5.2.2.34 `error_squared()`

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::error_squared (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs,
    const Scalar_T threshold ) -> Scalar_T [inline]
```

Relative or absolute error using the quadratic norm.

Definition at line 134 of file [clifford\\_algebra\\_imp.h](#).

References [norm\(\)](#).

Referenced by [approx\\_equal\(\)](#).

### 5.2.2.35 `error_squared_tol()`

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::error_squared_tol (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T
```

Quadratic norm error tolerance relative to a specific multivector.

Definition at line 112 of file [clifford\\_algebra\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::pow\(\)](#).

Referenced by [approx\\_equal\(\)](#).

**5.2.2.36 even()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::even (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Even part.

Definition at line 513 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.37 exp() [1/2]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::exp (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const framed\_multi<Scalar_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line 1750 of file [framed\\_multi\\_imp.h](#).

References [clifford\\_exp\(\)](#), [exp\(\)](#), and [scalar\(\)](#).

Referenced by [cos\(\)](#), [cosh\(\)](#), [exp\(\)](#), [matrix\\_log\(\)](#), [matrix\\_sqrt\(\)](#), [pow\(\)](#), [sin\(\)](#), and [sinh\(\)](#).

**5.2.2.38 exp() [2/2]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::exp (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Exponential of multivector.

Definition at line 2086 of file [matrix\\_multi\\_imp.h](#).

References [clifford\\_exp\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

**5.2.2.39 fast()**

```
template<typename Multivector_T , typename Matrix_T , typename Basis_Matrix_T >
static auto glucat::fast (
    const Matrix_T & X,
    index\_t level ) -> Multivector_T [static]
```

Inverse generalized Fast Fourier Transform.

Definition at line 1027 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix::signed\\_perm\\_nork\(\)](#).

#### 5.2.2.40 folded\_dim()

```
template<typename Matrix_Index_T , const index_t LO, const index_t HI>
static auto glucat::folded_dim (
    const index_set< LO, HI > & sub ) -> Matrix_Index_T    [inline], [static]
```

Determine the matrix dimension of the fold of a subalgebra.

Definition at line 101 of file [matrix\\_multi\\_imp.h](#).

References [offset\\_level\(\)](#).

#### 5.2.2.41 imag()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::imag (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T    [inline]
```

Imaginary part: deprecated (always 0)

Definition at line 497 of file [clifford\\_algebra\\_imp.h](#).

#### 5.2.2.42 inv()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::inv (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P>    [inline]
```

Geometric multiplicative inverse.

Definition at line 400 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [db\\_step\(\)](#), and [matrix\\_log\(\)](#).

#### 5.2.2.43 inverse\_gray()

```
static auto glucat::inverse_gray (
    unsigned long x ) -> unsigned long    [inline], [static]
```

Inverse Gray code.

Definition at line 863 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::sign\\_of\\_mult\(\)](#).



**5.2.2.44 inverse\_reversed\_gray()**

```
static auto glucat::inverse_reversed_gray (
    unsigned long x ) -> unsigned long    [inline], [static]
```

Inverse reversed Gray code.

Definition at line 846 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::sign\\_of\\_mult\(\)](#).

**5.2.2.45 involute()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::involute (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1}\*{2} -> (-{2})\*(-{1})

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

Definition at line 537 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.46 log() [1/4]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::log (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1800 of file [framed\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), and [log\(\)](#).

**5.2.2.47 log() [2/4]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::log (
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 2045 of file [matrix\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [matrix\\_log\(\)](#).

**5.2.2.48 log()** [3/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Natural logarithm of multivector.

Definition at line 799 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [log\(\)](#).

**5.2.2.49 log()** [4/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::log (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Natural logarithm of multivector with specified complexifier.

Definition at line 791 of file [clifford\\_algebra\\_imp.h](#).

References [log\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [log\(\)](#), [log\(\)](#), [log\(\)](#), and [pow\(\)](#).

**5.2.2.50 log2()**

```
template<typename Scalar_T >
auto glucat::log2 (
    const Scalar_T & x ) -> Scalar_T [inline]
```

Log base 2 of scalar.

Definition at line 303 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

Referenced by [clifford\\_exp\(\)](#).

### 5.2.2.51 `matrix_log()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index\_t level ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Natural logarithm of multivector with specified complexifier.

Definition at line 1967 of file [matrix\\_multi\\_imp.h](#).

References [abs\(\)](#), [cascade\\_log\(\)](#), [glucat::matrix::classify\\_eigenvalues\(\)](#), [exp\(\)](#), [inv\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, matrix\\_log\(\), norm\(\), and glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

Referenced by [log\(\)](#), and [matrix\\_log\(\)](#).

### 5.2.2.52 `matrix_sqrt()`

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix_sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    const index\_t level ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line 1571 of file [matrix\\_multi\\_imp.h](#).

References [abs\(\)](#), [approx\\_equal\(\)](#), [glucat::matrix::classify\\_eigenvalues\(\)](#), [cr\\_sqrt\(\)](#), [db\\_sqrt\(\)](#), [exp\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, matrix\\_sqrt\(\), norm\(\), pade\\_approx\(\), pow\(\), and glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::scalar\(\)](#).

Referenced by [matrix\\_sqrt\(\)](#), and [sqrt\(\)](#).

### 5.2.2.53 `max_abs()`

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::max_abs (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

Definition at line 585 of file [clifford\\_algebra\\_imp.h](#).

### 5.2.2.54 `max_pos()`

```
template<const index\_t LO, const index\_t HI>
auto glucat::max_pos (
    const index\_set< LO, HI > & ist ) -> index\_t [inline]
```

Maximum positive index, or 0 if none.

Definition at line 977 of file [index\\_set\\_imp.h](#).

### 5.2.2.55 min\_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::min_neg (
    const index_set< LO, HI > & ist ) -> index_t [inline]
```

Minimum negative index, or 0 if none.

Definition at line 970 of file [index\\_set\\_imp.h](#).

### 5.2.2.56 norm()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::norm (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Scalar\_T norm == sum of norm of coordinates.

Definition at line 569 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [atanh\(\)](#), [cascade\\_log\(\)](#), [cr\\_sqrt\(\)](#), [db\\_sqrt\(\)](#), [error\\_squared\(\)](#), [matrix\\_log\(\)](#), and [matrix\\_sqrt\(\)](#).

### 5.2.2.57 odd()

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::odd (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_←
_T,LO,HI,Tune_P> [inline]
```

Odd part.

Definition at line 521 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#).

### 5.2.2.58 offset\_level()

```
auto glucat::offset_level (
    const index_t p,
    const index_t q ) -> index_t [inline]
```

Determine the log2 dim corresponding to signature p, q.

Definition at line 86 of file [matrix\\_multi\\_imp.h](#).

References [pos\\_mod\(\)](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), and [folded\\_dim\(\)](#).

**5.2.2.59 operator"!="() [1/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator!=(
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> bool [inline]
```

Test for inequality of multivectors.

Definition at line 86 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.60 operator"!="() [2/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator!=(
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> bool [inline]
```

Test for inequality of multivector and scalar.

Definition at line 94 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.61 operator"!="() [3/3]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator!=(
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> bool [inline]
```

Test for inequality of scalar and multivector.

Definition at line 102 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.62 operator%() [1/3]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator%(
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed_multi<Scalar_
_T,LO,HI,Tune_P>
```

Left contraction.

Definition at line 597 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.63 operator%() [2/3]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator% (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Left contraction.

Definition at line 581 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.64 operator%() [3/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator% (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Left contraction.

Definition at line 322 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.65 operator&() [1/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator& (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Inner product.

Definition at line 495 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.66 operator&() [2/4]**

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator& (
    const index\_set< LO, HI > & lhs,
    const index\_set< LO, HI > & rhs ) -> const index\_set<LO,HI> [inline]
```

Set intersection: and.

Definition at line 186 of file [index\\_set\\_imp.h](#).

**5.2.2.67 operator&() [3/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator& (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Inner product.

Definition at line 562 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.68 operator&() [4/4]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator& (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Inner product.

Definition at line 307 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.69 operator\*() [1/6]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator* (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P>
```

Geometric product.

Definition at line 374 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#).

**5.2.2.70 operator\*() [2/6]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator* (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Geometric product.

Definition at line 502 of file [matrix\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#), and [reframe\(\)](#).

**5.2.2.71 operator\*() [3/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric product.

Definition at line 277 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.72 operator\*() [4/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Product of multivector and scalar.

Definition at line 251 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.73 operator\*() [5/6]**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator* (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Product of scalar and multivector.

Definition at line 262 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.74 operator\*() [6/6]**

```
template<typename Scalar_T , const index_t LO, const index_t HI>
auto glucat::operator* (
    const std::pair< const index_set< LO, HI >, Scalar_T > & lhs,
    const std::pair< const index_set< LO, HI >, Scalar_T > & rhs ) -> const std::pair<const index_set<LO,HI>, Scalar_T> [inline]
```

Product of terms.

Definition at line 1717 of file [framed\\_multi\\_imp.h](#).

References [ord\\_of\\_mult\(\)](#).



**5.2.2.75 operator+()** [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum.

Definition at line 206 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.76 operator+()** [2/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator+ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum of multivector and scalar.

Definition at line 181 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.77 operator+()** [3/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator+ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric sum of scalar and multivector.

Definition at line 192 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.78 operator-()** [1/3]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric difference.

Definition at line 240 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.79 operator-() [2/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator- (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric difference of multivector and scalar.

Definition at line 217 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.80 operator-() [3/3]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator- (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar↔
_T,LO,HI,Tune_P> [inline]
```

Geometric difference of scalar and multivector.

Definition at line 228 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.81 operator/() [1/5]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator/ (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar↔
_T,LO,HI,Tune_P> [inline]
```

Geometric quotient.

Definition at line 734 of file [framed\\_multi\\_imp.h](#).

**5.2.2.82 operator/() [2/5]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator/ (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar↔
_T,LO,HI,Tune_P>
```

Geometric quotient.

Definition at line 614 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix::isnan\(\)](#), and [reframe\(\)](#).

**5.2.2.83 operator/()** [3/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
template< typename, const index_t, const index_t, typename > class RHS, typename Scalar_T ,
const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Geometric quotient.

Definition at line 374 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.84 operator/()** [4/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator/ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const Scalar_T & scr ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Quotient of multivector and scalar.

Definition at line 348 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.85 operator/()** [5/5]

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator/ (
    const Scalar_T & scr,
    const Multivector< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Quotient of scalar and multivector.

Definition at line 359 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.86 operator<<()** [1/5]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator<< (
    std::ostream & os,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::ostream&
```

Write multivector to output.

Definition at line 1148 of file [framed\\_multi\\_imp.h](#).

References [scalar\(\)](#).

**5.2.2.87 operator<<() [2/5]**

```
const index_t HI auto glucat::operator<< (
    std::ostream & os,
    const index_set< LO, HI > & ist ) -> std::ostream &
```

**5.2.2.88 operator<<() [3/5]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const index_set< LO, HI > & ist ) -> std::ostream&
```

Write out index set.

Definition at line 611 of file [index\\_set\\_imp.h](#).

**5.2.2.89 operator<<() [4/5]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator<< (
    std::ostream & os,
    const matrix_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::ostream& [inline]
```

Write multivector to output.

Definition at line 955 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.90 operator<<() [5/5]**

```
template<typename Scalar_T , const index_t LO, const index_t HI>
auto glucat::operator<< (
    std::ostream & os,
    const std::pair< const index_set< LO, HI >, Scalar_T > & term ) -> std::ostream&
```

Write term to output.

Definition at line 1209 of file [framed\\_multi\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**5.2.2.91 operator>>() [1/3]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator>> (
    std::istream & s,
    framed_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::istream&
```

Read multivector from input.

Definition at line 1248 of file [framed\\_multi\\_imp.h](#).

**5.2.2.92 operator>>() [2/3]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator>> (
    std::istream & s,
    index_set< LO, HI > & ist ) -> std::istream&
```

Read in index set.

Definition at line 634 of file [index\\_set\\_imp.h](#).

**5.2.2.93 operator>>() [3/3]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator>> (
    std::istream & s,
    matrix_multi< Scalar_T, LO, HI, Tune_P > & val ) -> std::istream& [inline]
```

Read multivector from input.

Definition at line 966 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.94 operator^() [1/4]**

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::operator^ (
    const framed_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
```

Outer product.

Definition at line 416 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), and [crd\\_of\\_mult\(\)](#).

**5.2.2.95 operator^() [2/4]**

```
template<const index_t LO, const index_t HI>
auto glucat::operator^ (
    const index_set< LO, HI > & lhs,
    const index_set< LO, HI > & rhs ) -> const index_set<LO,HI> [inline]
```

Symmetric set difference: exclusive or.

Definition at line 161 of file [index\\_set\\_imp.h](#).

**5.2.2.96 operator^() [3/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator^ (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Outer product.

Definition at line 543 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.97 operator^() [4/4]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator^ (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Outer product.

Definition at line 292 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.98 operator" | () [1/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator| (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const framed\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 760 of file [framed\\_multi\\_imp.h](#).

**5.2.2.99 operator" | () [2/4]**

```
template<const index\_t LO, const index\_t HI>
auto glucat::operator| (
    const index\_set< LO, HI > & lhs,
    const index\_set< LO, HI > & rhs ) -> const index\_set<LO,HI> [inline]
```

Set union: or.

Definition at line 211 of file [index\\_set\\_imp.h](#).

**5.2.2.100 operator" | () [3/4]**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator| (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 717 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.101 operator" | () [4/4]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::operator| (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
T,LO,HI,Tune_P> [inline]
```

Transformation via twisted adjoint action.

Definition at line 389 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.102 outer\_pow()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::outer_pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Outer product power of multivector.

Definition at line 470 of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.103 pade\_approx()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P , const size_↵
_t Size>
static auto glucat::pade_approx (
    const std::array< Scalar_T, Size > & numer,
    const std::array< Scalar_T, Size > & denom,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & X ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [inline], [static]
```

Pade' approximation.

Definition at line 1245 of file [matrix\\_multi\\_imp.h](#).

Referenced by [matrix\\_sqrt\(\)](#), and [pade\\_log\(\)](#).

**5.2.2.104 pade\_log()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
static auto glucat::pade_log (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val ) -> const matrix\_multi<Scalar_↵
_T,LO,HI,Tune_P> [static]
```

Pade' approximation of log.

Definition at line 1900 of file [matrix\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [pade\\_approx\(\)](#).

Referenced by [cascade\\_log\(\)](#).

**5.2.2.105 pos\_mod()**

```
template<typename LHS_T , typename RHS_T >
auto glucat::pos_mod (
    LHS_T lhs,
    RHS_T rhs ) -> LHS_T [inline]
```

Modulo function which works reliably for lhs < 0.

Definition at line 117 of file [global.h](#).

Referenced by [complexifier\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_vector\(\)](#), [offset\\_level\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::operator\(\)\(\)](#).

**5.2.2.106 pow() [1/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Multivector power of multivector.

Definition at line 446 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#), and [log\(\)](#).

**5.2.2.107 pow() [2/2]**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::pow (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    int rhs ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Integer power of multivector.

Definition at line 407 of file [clifford\\_algebra\\_imp.h](#).

Referenced by [matrix\\_sqrt\(\)](#).



**5.2.2.108 pure()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::pure (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Pure part.

Definition at line [505](#) of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.109 quad()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::quad (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Scalar\_T quadratic form == (rev(x)\*x)(0)

Definition at line [561](#) of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.110 real()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::real (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Real part: synonym for scalar part.

Definition at line [486](#) of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.111 reframe()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::reframe (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs_reframed,
    matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs_reframed ) -> const index\_set<LO,HI>
[inline]
```

Find a common frame for operands of a binary operator.

Definition at line [345](#) of file [matrix\\_multi\\_imp.h](#).

Referenced by [operator\\*\(\)](#), and [operator/\(\)](#).

**5.2.2.112 reverse()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::reverse (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .

Definition at line [545](#) of file [clifford\\_algebra\\_imp.h](#).

**5.2.2.113 scalar()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::scalar (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> Scalar_T [inline]
```

Scalar part.

Definition at line [478](#) of file [clifford\\_algebra\\_imp.h](#).

Referenced by [exp\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#), and [operator<<\(\)](#).

**5.2.2.114 sign\_of\_square()**

```
auto glucat::sign_of_square (
    index\_t j ) -> int [inline]
```

Square of generator {j}.

Square of generator index j.

Definition at line [963](#) of file [index\\_set\\_imp.h](#).

**5.2.2.115 sin()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Sine of multivector.

Definition at line [979](#) of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [sin\(\)](#).

**5.2.2.116 sin()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sin (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
```

Sine of multivector with specified complexifier.

Definition at line 956 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), and [exp\(\)](#).

Referenced by [sin\(\)](#), and [tan\(\)](#).

**5.2.2.117 sinh()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sinh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Hyperbolic sine of multivector.

Definition at line 911 of file [clifford\\_algebra\\_imp.h](#).

References [exp\(\)](#).

Referenced by [tanh\(\)](#).

**5.2.2.118 sqrt()** [1/4]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const framed\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line 1727 of file [framed\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), and [sqrt\(\)](#).

**5.2.2.119 sqrt()** [2/4]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & val,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & i,
    bool prechecked ) -> const matrix\_multi<Scalar_T,LO,HI,Tune_P>
```

Square root of multivector with specified complexifier.

Definition at line 1667 of file [matrix\\_multi\\_imp.h](#).

References [check\\_complex\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::isnan\(\)](#), and [matrix\\_sqrt\(\)](#).

**5.2.2.120 sqrt()** [3/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_
_T,LO,HI,Tune_P> [inline]
```

Square root of multivector.

Definition at line 683 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [sqrt\(\)](#).

**5.2.2.121 sqrt()** [4/4]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::sqrt (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Square root of multivector with specified complexifier.

Definition at line 675 of file [clifford\\_algebra\\_imp.h](#).

References [sqrt\(\)](#).

Referenced by [acosh\(\)](#), [asinh\(\)](#), [sqrt\(\)](#), [sqrt\(\)](#), and [sqrt\(\)](#).

**5.2.2.122 star()** [1/3]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::star (
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const framed\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> Scalar_T
```

Hestenes scalar product.

Definition at line 684 of file [framed\\_multi\\_imp.h](#).

**5.2.2.123 star()** [2/3]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::star (
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & lhs,
    const matrix\_multi< Scalar_T, LO, HI, Tune_P > & rhs ) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line 600 of file [matrix\\_multi\\_imp.h](#).

**5.2.2.124 star()** [3/3]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T ,
const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::star (
    const Multivector< Scalar_T, LO, HI, Tune_P > & lhs,
    const RHS< Scalar_T, LO, HI, Tune_P > & rhs ) -> Scalar_T [inline]
```

Hestenes scalar product.

Definition at line 337 of file [clifford\\_algebra\\_imp.h](#).

References [star\(\)](#).

Referenced by [star\(\)](#).

**5.2.2.125 tan()** [1/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_↵
_T,LO,HI,Tune_P> [inline]
```

Tangent of multivector.

Definition at line 1079 of file [clifford\\_algebra\\_imp.h](#).

References [complexifier\(\)](#), and [tan\(\)](#).

**5.2.2.126 tan()** [2/2]

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::tan (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val,
    const Multivector< Scalar_T, LO, HI, Tune_P > & i,
    const bool prechecked = false ) -> const Multivector<Scalar_T,LO,HI,Tune_P>
[inline]
```

Tangent of multivector with specified complexifier.

Definition at line 1060 of file [clifford\\_algebra\\_imp.h](#).

References [check\\_complex\(\)](#), [cos\(\)](#), and [sin\(\)](#).

Referenced by [tan\(\)](#).

**5.2.2.127 tanh()**

```
template<template< typename, const index\_t, const index\_t, typename > class Multivector,
typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::tanh (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const Multivector<Scalar_T,LO,HI,Tune_P> [inline]
```

Hyperbolic tangent of multivector.

Definition at line [1016](#) of file [clifford\\_algebra\\_imp.h](#).

References [cosh\(\)](#), and [sinh\(\)](#).

**5.2.2.128 to\_demote()**

```
template<typename Scalar_T >
auto glucat::to_demote (
    const Scalar_T & val ) -> typename numeric\_traits<Scalar_T>::demoted::type [inline]
```

Cast to demote.

Definition at line [135](#) of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**5.2.2.129 to\_promote()**

```
template<typename Scalar_T >
auto glucat::to_promote (
    const Scalar_T & val ) -> typename numeric\_traits<Scalar_T>::promoted::type
[inline]
```

Cast to promote.

Definition at line [125](#) of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

**5.2.2.130 try\_catch()** [1/2]

```
int glucat::try_catch (
    intfn f )
```

Exception catching for functions returning int.

Definition at line [49](#) of file [try\\_catch.h](#).

Referenced by [glucat::control\\_t::call\(\)](#), and [glucat::control\\_t::call\(\)](#).

**5.2.2.131 try\_catch()** [2/2]

```
int glucat::try_catch (
    int (*f),
    int arg )
```

Exception catching for functions of int returning int.

Definition at line 64 of file [try\\_catch.h](#).

**5.2.2.132 vector\_part()**

```
template<template< typename, const index_t, const index_t, typename > class Multivector,
typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::vector_part (
    const Multivector< Scalar_T, LO, HI, Tune_P > & val ) -> const std::vector<Scalar_T> [inline]
```

Vector part of multivector, as a vector\_t with respect to frame()

Definition at line 529 of file [clifford\\_algebra\\_imp.h](#).

**5.2.3 Variable Documentation****5.2.3.1 BITS\_PER\_SET\_VALUE**

```
const index_t glucat::BITS_PER_SET_VALUE = std::numeric_limits<set_value_t>::digits
```

Number of bits in set\_value\_t.

Definition at line 103 of file [global.h](#).

**5.2.3.2 DEFAULT\_HI**

```
const index_t glucat::DEFAULT_HI = index_t(BITS_PER_SET_VALUE / 2)
```

Default highest index in an index set.

Definition at line 111 of file [global.h](#).

**5.2.3.3 l\_ln2**

```
const long double glucat::l_ln2 = 0.6931471805599453094172321214581766L [static]
```

Definition at line 44 of file [long\\_double.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::ln\\_2\(\)](#).

#### 5.2.3.4 l\_pi

```
const long double glucat::l_pi = 3.1415926535897932384626433832795029L [static]
```

Definition at line 43 of file [long\\_double.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::pi\(\)](#).

#### 5.2.3.5 MS\_PER\_S

```
const double glucat::MS_PER_S = 1000.0
```

Timing constant: deprecated here - moved to [test/timing.h](#).

Definition at line 83 of file [global.h](#).

#### 5.2.3.6 Tuning\_Fast\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Fast_Basis_Max_Count = 1
```

Definition at line 92 of file [tuning.h](#).

#### 5.2.3.7 Tuning\_Fast\_CR\_Sqrt\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_CR_Sqrt_Max_Steps = 256
```

Definition at line 88 of file [tuning.h](#).

#### 5.2.3.8 Tuning\_Fast\_DB\_Sqrt\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_DB_Sqrt_Max_Steps = 256
```

Definition at line 89 of file [tuning.h](#).

#### 5.2.3.9 Tuning\_Fast\_Div\_Max\_Steps

```
const unsigned int glucat::Tuning_Fast_Div_Max_Steps = 0
```

Definition at line 87 of file [tuning.h](#).

#### 5.2.3.10 Tuning\_Fast\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Fast_Fast_Size_Threshold = 0
```

Definition at line 93 of file [tuning.h](#).



#### 5.2.3.11 Tuning\_Fast\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Fast_Inv_Fast_Dim_Threshold = 0
```

Definition at line 94 of file [tuning.h](#).

#### 5.2.3.12 Tuning\_Fast\_Log\_Max\_Inner\_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Inner_Steps = 8
```

Definition at line 91 of file [tuning.h](#).

#### 5.2.3.13 Tuning\_Fast\_Log\_Max\_Outer\_Steps

```
const unsigned int glucat::Tuning_Fast_Log_Max_Outer_Steps = 16
```

Definition at line 90 of file [tuning.h](#).

#### 5.2.3.14 Tuning\_Fast\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Fast_Mult_Matrix_Threshold = 0
```

Definition at line 86 of file [tuning.h](#).

#### 5.2.3.15 Tuning\_Fast\_Products\_Size\_Threshold

```
const unsigned int glucat::Tuning_Fast_Products_Size_Threshold = 0
```

Definition at line 95 of file [tuning.h](#).

#### 5.2.3.16 Tuning\_Int\_Digits

```
const unsigned int glucat::Tuning_Int_Digits = std::numeric_limits<int>::digits
```

Definition at line 36 of file [tuning.h](#).

#### 5.2.3.17 Tuning\_Max\_Threshold

```
const unsigned int glucat::Tuning_Max_Threshold = 1 << Tuning\_Int\_Digits
```

Definition at line 37 of file [tuning.h](#).

#### 5.2.3.18 Tuning\_Naive\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Naive_Basis_Max_Count = Tuning\_Max\_Threshold
```

Definition at line 65 of file [tuning.h](#).

#### 5.2.3.19 Tuning\_Naive\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 66 of file [tuning.h](#).

#### 5.2.3.20 Tuning\_Naive\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 67 of file [tuning.h](#).

#### 5.2.3.21 Tuning\_Naive\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Naive_Mult_Matrix_Threshold = 0
```

Definition at line 64 of file [tuning.h](#).

#### 5.2.3.22 Tuning\_Slow\_Basis\_Max\_Count

```
const unsigned int glucat::Tuning_Slow_Basis_Max_Count = 0
```

Definition at line 42 of file [tuning.h](#).

#### 5.2.3.23 Tuning\_Slow\_Fast\_Size\_Threshold

```
const unsigned int glucat::Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 43 of file [tuning.h](#).

#### 5.2.3.24 Tuning\_Slow\_Inv\_Fast\_Dim\_Threshold

```
const unsigned int glucat::Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold
```

Definition at line 44 of file [tuning.h](#).

#### 5.2.3.25 Tuning\_Slow\_Mult\_Matrix\_Threshold

```
const unsigned int glucat::Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold
```

Definition at line 41 of file [tuning.h](#).

#### 5.2.3.26 Tuning\_Slow\_Products\_Size\_Threshold

```
const unsigned int glucat::Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold
```

Definition at line 45 of file [tuning.h](#).

## 5.3 glucat::gen Namespace Reference

### Classes

- class [generator\\_table](#)  
*Table of generators for specific signatures.*

### Typedefs

- using [signature\\_t](#) = std::pair<[index\\_t](#), [index\\_t](#)>  
*A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min()*

### Variables

- static const std::array< [index\\_t](#), 8 > [offset\\_to\\_super](#) = {0,-1, 0,-1,-2, 3, 2, 1}  
*Offsets between the current signature and that of the real superalgebra.*

### 5.3.1 Typedef Documentation

#### 5.3.1.1 signature\_t

```
using glucat::gen::signature_t = std::pair<index_t, index_t>
```

A signature is a pair of indices, p, q, with p == frame.max(), q == -frame.min()

Definition at line 48 of file [generation.h](#).

### 5.3.2 Variable Documentation

#### 5.3.2.1 offset\_to\_super

```
const std::array<index_t, 8> glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1} [static]
```

Offsets between the current signature and that of the real superalgebra.

Definition at line 86 of file [generation.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::operator\(\)](#).

## 5.4 glucat::matrix Namespace Reference

### Classes

- struct [eig\\_genus](#)  
*Structure containing classification of eigenvalues.*

## Typedefs

- using [eig\\_case\\_t](#)  
*Classification of eigenvalues of a matrix.*

## Functions

- template<typename LHS\_T , typename RHS\_T >  
auto [kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Kronecker tensor product of matrices - as per Matlab kron.*
- template<typename LHS\_T , typename RHS\_T >  
auto [mono\\_kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Sparse Kronecker tensor product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto [nork](#) (const LHS\_T &lhs, const RHS\_T &rhs, const bool mono=true) -> const RHS\_T  
*Left inverse of Kronecker product.*
- template<typename LHS\_T , typename RHS\_T >  
auto [signed\\_perm\\_nork](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T >  
auto [nnz](#) (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T >  
auto [isinf](#) (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T >  
auto [isnan](#) (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T >  
auto [unit](#) (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
- template<typename LHS\_T , typename RHS\_T >  
auto [mono\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto [sparse\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*
- template<typename LHS\_T , typename RHS\_T >  
auto [prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of matrices.*
- template<typename Scalar\_T , typename LHS\_T , typename RHS\_T >  
auto [inner](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.rows()$*
- template<typename Matrix\_T >  
auto [norm\\_frob2](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Square of Frobenius norm.*
- template<typename Matrix\_T >  
auto [trace](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Matrix trace.*

- `template<typename Matrix_T >`  
`auto eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >`  
*Eigenvalues of a matrix.*
- `template<typename Matrix_T >`  
`auto classify\_eigenvalues (const Matrix_T &val) -> eig\_genus< Matrix_T >`  
*Classify the eigenvalues of a matrix.*
- `template<typename LHS_T, typename RHS_T >`  
`void nork\_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const`  
`typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`  
*Utility routine for nork: calculate result for a range of indices.*
- `template<typename Matrix_T >`  
`static auto to\_lapack (const Matrix_T &val) -> ublas::matrix< double, ublas::column_major >`  
*Convert matrix to LAPACK format.*

## 5.4.1 Typedef Documentation

### 5.4.1.1 eig\_case\_t

using [glucat::matrix::eig\\_case\\_t](#)

**Initial value:**

```
enum {
    safe_eigs,
    neg_real_eigs,
    both_eigs}
```

Classification of eigenvalues of a matrix.

Definition at line 133 of file [matrix.h](#).

## 5.4.2 Function Documentation

### 5.4.2.1 classify\_eigenvalues()

```
template<typename Matrix_T >
auto glucat::matrix::classify_eigenvalues (
    const Matrix_T & val ) -> eig\_genus<Matrix_T>
```

Classify the eigenvalues of a matrix.

Definition at line 548 of file [matrix\\_imp.h](#).

References [eigenvalues\(\)](#), [epsilon](#), [glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_eig\\_case](#), [glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_glucat::matrix::eig\\_genus< Matrix\\_T >::m\\_safe\\_arg](#), and [glucat::numeric\\_traits< Scalar\\_T >::pi\(\)](#).

Referenced by [glucat::matrix\\_log\(\)](#), and [glucat::matrix\\_sqrt\(\)](#).

#### 5.4.2.2 eigenvalues()

```
template<typename Matrix_T >
auto glucat::matrix::eigenvalues (
    const Matrix_T & val ) -> std::vector< std::complex<double> >
```

Eigenvalues of a matrix.

Definition at line 500 of file [matrix\\_imp.h](#).

References [to\\_lapack\(\)](#).

Referenced by [classify\\_eigenvalues\(\)](#).

#### 5.4.2.3 inner()

```
template<typename Scalar_T , typename LHS_T , typename RHS_T >
auto glucat::matrix::inner (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> Scalar_T
```

Inner product:  $\text{sum}(x(i,j)*y(i,j))/x.\text{nrows}()$

Inner product:  $\text{sum}(lhs(i,j)*rhs(i,j))/lhs.\text{nrows}()$

Definition at line 373 of file [matrix\\_imp.h](#).

#### 5.4.2.4 isinf()

```
template<typename Matrix_T >
auto glucat::matrix::isinf (
    const Matrix_T & m ) -> bool
```

Infinite.

Definition at line 275 of file [matrix\\_imp.h](#).

#### 5.4.2.5 isnan()

```
template<typename Matrix_T >
auto glucat::matrix::isnan (
    const Matrix_T & m ) -> bool
```

Not a Number.

Definition at line 292 of file [matrix\\_imp.h](#).

Referenced by [glucat::operator/\(\)](#).

#### 5.4.2.6 kron()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::kron (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> const RHS_T
```

Kronecker tensor product of matrices - as per Matlab kron.

Definition at line 83 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\(\)](#).

#### 5.4.2.7 mono\_kron()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::mono_kron (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> const RHS_T
```

Sparse Kronecker tensor product of monomial matrices.

Definition at line 119 of file [matrix\\_imp.h](#).

Referenced by [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pm1\\_qm1\(\)](#).

#### 5.4.2.8 mono\_prod()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::mono_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs ) -> const typename RHS_T::expression←
_type
```

Product of monomial matrices.

Definition at line 320 of file [matrix\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_f](#), [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_pp4\\_qm4\(\)](#), and [glucat::gen::generator\\_table< Matrix\\_T >::gen\\_from\\_qp1\\_pm](#).

#### 5.4.2.9 nnz()

```
template<typename Matrix_T >
auto glucat::matrix::nnz (
    const Matrix_T & m ) -> typename Matrix_T::size_type
```

Number of non-zeros.

Definition at line 258 of file [matrix\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

#### 5.4.2.10 `nork()`

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::nork (
    const LHS_T & lhs,
    const RHS_T & rhs,
    const bool mono = true ) -> const RHS_T
```

Left inverse of Kronecker product.

Definition at line 182 of file [matrix\\_imp.h](#).

References [norm\\_frob2\(\)](#).

#### 5.4.2.11 `nork_range()`

```
template<typename LHS_T , typename RHS_T >
void glucat::matrix::nork_range (
    RHS_T & result,
    const typename LHS_T::const_iterator2 lhs_it2,
    const RHS_T & rhs,
    const typename RHS_T::size_type res_s1,
    const typename RHS_T::size_type res_s2 )
```

Utility routine for nork: calculate result for a range of indices.

Definition at line 152 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#).

#### 5.4.2.12 `norm_frob2()`

```
template<typename Matrix_T >
auto glucat::matrix::norm_frob2 (
    const Matrix_T & val ) -> typename Matrix_T::value_type
```

Square of Frobenius norm.

Definition at line 395 of file [matrix\\_imp.h](#).

Referenced by [nork\(\)](#).

#### 5.4.2.13 `prod()`

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs ) -> const typename RHS_T::expression←
_type [inline]
```

Product of matrices.

Definition at line 361 of file [matrix\\_imp.h](#).



#### 5.4.2.14 signed\_perm\_nork()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::signed_perm_nork (
    const LHS_T & lhs,
    const RHS_T & rhs ) -> const RHS_T
```

Left inverse of Kronecker product where lhs is a signed permutation matrix.

Definition at line 228 of file [matrix\\_imp.h](#).

Referenced by [glucat::fast\(\)](#).

#### 5.4.2.15 sparse\_prod()

```
template<typename LHS_T , typename RHS_T >
auto glucat::matrix::sparse_prod (
    const ublas::matrix_expression< LHS_T > & lhs,
    const ublas::matrix_expression< RHS_T > & rhs ) -> const typename RHS_T::expression←
_type [inline]
```

Product of sparse matrices.

Definition at line 350 of file [matrix\\_imp.h](#).

#### 5.4.2.16 to\_lapack()

```
template<typename Matrix_T >
static auto glucat::matrix::to_lapack (
    const Matrix_T & val ) -> ublas::matrix<double, ublas::column_major> [static]
```

Convert matrix to LAPACK format.

Definition at line 440 of file [matrix\\_imp.h](#).

Referenced by [eigenvalues\(\)](#).

#### 5.4.2.17 trace()

```
template<typename Matrix_T >
auto glucat::matrix::trace (
    const Matrix_T & val ) -> typename Matrix_T::value_type
```

Matrix trace.

Definition at line 416 of file [matrix\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::NaN\(\)](#).

#### 5.4.2.18 unit()

```
template<typename Matrix_T >
auto glucat::matrix::unit (
    const typename Matrix_T::size_type n ) -> const Matrix_T [inline]
```

Unit matrix - as per Matlab eye.

Definition at line 310 of file [matrix\\_imp.h](#).

## 5.5 glucat::timing Namespace Reference

### Functions

- static double [elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*

### Variables

- const double [MS\\_PER\\_SEC](#) = 1000.0  
*Timing constant: milliseconds per second.*
- const double [MS\\_PER\\_CLOCK](#) = [MS\\_PER\\_SEC](#) / double(CLOCKS\_PER\_SEC)  
*Timing constant: milliseconds per clock.*
- const int [EXTRA\\_TRIALS](#) = 2  
*Timing constant: trial expansion factor.*

### 5.5.1 Function Documentation

#### 5.5.1.1 elapsed()

```
static double glucat::timing::elapsed (
    clock_t cpu_time ) [inline], [static]
```

Elapsed time in milliseconds.

Definition at line 51 of file [timing.h](#).

References [MS\\_PER\\_CLOCK](#).

### 5.5.2 Variable Documentation

#### 5.5.2.1 EXTRA\_TRIALS

```
const int glucat::timing::EXTRA_TRIALS = 2
```

Timing constant: trial expansion factor.

Definition at line 45 of file [timing.h](#).

### 5.5.2.2 MS\_PER\_CLOCK

```
const double glucat::timing::MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC)
```

Timing constant: milliseconds per clock.

Definition at line 42 of file [timing.h](#).

Referenced by [elapsed\(\)](#).

### 5.5.2.3 MS\_PER\_SEC

```
const double glucat::timing::MS_PER_SEC = 1000.0
```

Timing constant: milliseconds per second.

Definition at line 39 of file [timing.h](#).

## 5.6 pade Namespace Reference

### Classes

- struct [pade\\_log\\_denom](#)

*Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*

- struct [pade\\_log\\_denom](#)< [dd\\_real](#) >
- struct [pade\\_log\\_denom](#)< [float](#) >
- struct [pade\\_log\\_denom](#)< [long double](#) >
- struct [pade\\_log\\_denom](#)< [qd\\_real](#) >
- struct [pade\\_log\\_numer](#)

*Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*

- struct [pade\\_log\\_numer](#)< [dd\\_real](#) >
- struct [pade\\_log\\_numer](#)< [float](#) >
- struct [pade\\_log\\_numer](#)< [long double](#) >
- struct [pade\\_log\\_numer](#)< [qd\\_real](#) >
- struct [pade\\_sqrt\\_denom](#)

*Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*

- struct [pade\\_sqrt\\_denom](#)< [dd\\_real](#) >
- struct [pade\\_sqrt\\_denom](#)< [float](#) >
- struct [pade\\_sqrt\\_denom](#)< [long double](#) >
- struct [pade\\_sqrt\\_denom](#)< [qd\\_real](#) >
- struct [pade\\_sqrt\\_numer](#)

*Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*

- struct [pade\\_sqrt\\_numer](#)< [dd\\_real](#) >
- struct [pade\\_sqrt\\_numer](#)< [float](#) >
- struct [pade\\_sqrt\\_numer](#)< [long double](#) >
- struct [pade\\_sqrt\\_numer](#)< [qd\\_real](#) >

## 5.7 PyClical Namespace Reference

### Classes

- class [clifford](#)
- class [index\\_set](#)

### Functions

- [index\\_set\\_hidden\\_doctests](#) ()
- [clifford\\_hidden\\_doctests](#) ()
- [e](#) (obj)
- [istpq](#) (p, q)
- [\\_test](#) ()

### Variables

- [\\_\\_version\\_\\_](#) = str([glucat\\_package\\_version](#), 'utf-8')
- [lhs](#)
- [rhs](#)
- [threshold](#) = error\_squared\_tol([rhs](#)) if threshold is [None](#) else threshold
- [None](#)
- [tol](#) = error\_squared\_tol([rhs](#)) if tol is [None](#) else tol
- [obj](#)
- [i](#)
- [ixt](#)
- [fill](#)
- [scalar\\_epsilon](#) = [epsilon](#)
- float [pi](#) = atan([clifford](#)(1.0)) \* 4.0
- float [tau](#) = atan([clifford](#)(1.0)) \* 8.0
- [cl](#) = [clifford](#)
- [ist](#) = [index\\_set](#)
- [ninf3](#) = [e](#)(4) + [e](#)(-1)
- [nbar3](#) = [e](#)(4) - [e](#)(-1)

### 5.7.1 Function Documentation

#### 5.7.1.1 [\\_test\(\)](#)

`PyClical._test ( )` [protected]

Definition at line [1962](#) of file [PyClical.pyx](#).

References [\\_test\(\)](#).

Referenced by [\\_test\(\)](#).

## 5.7.1.2 clifford\_hidden\_doctests()

```
PyClical.clifford_hidden_doctests ( )
```

Tests for functions that Doctest cannot see.

For clifford.\_\_cinit\_\_: Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2, index_set({1,2})))
2{1,2}
>>> print(clifford([2,3], index_set({1,2})))
2{1}+3{2}
>>> print(clifford([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'list'>.
>>> print(clifford(None))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from <class 'NoneType'>.
>>> print(clifford(None, [1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
>>> print(clifford([1,2], [1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
>>> print(clifford(""))
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string ''.
>>> print(clifford("{")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{'.
>>> print(clifford("{1")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1'.
>>> print(clifford("{1+")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1+'.
>>> print(clifford("{1+")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1+'.
>>> print(clifford("{1}+")
Traceback (most recent call last):
...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.

For clifford.__richcmp__: Compare objects of type clifford.

>>> clifford("{1}") == clifford("1{1}")
True
```

```
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 1253 of file [PyClicl.pyx](#).

### 5.7.1.3 e()

```
PyClicl.e (
    obj )
```

Abbreviation for `clifford(index_set(obj))`.

```
>>> print(e(1))
{1}
>>> print(e(-1))
{-1}
>>> print(e(0))
1
```

Definition at line 1936 of file [PyClicl.pyx](#).

### 5.7.1.4 index\_set\_hidden\_doctests()

```
PyClicl.index_set_hidden_doctests ( )
```

Tests for functions that Doctest cannot see.

For `index_set.__cinit__`: Construct `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set(""))
{}
>>> print(index_set("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1'.
>>> print(index_set("{1}")
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1'.
```

```
>>> print(index_set("{1,2,100}"))
Traceback (most recent call last):
...
ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
>>> print(index_set({1,2,100}))
Traceback (most recent call last):
...
IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
>>> print(index_set([1,2]))
Traceback (most recent call last):
...
TypeError: Cannot initialize index_set object from <class 'list'>.

For index_set.__richcmp__: Compare two objects of class index_set.
```

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
>>> None == index_set({1,2})
False
>>> None != index_set({1,2})
True
>>> None < index_set({1,2})
False
>>> None <= index_set({1,2})
False
>>> None > index_set({1,2})
False
>>> None >= index_set({1,2})
False
>>> index_set({1,2}) == None
False
>>> index_set({1,2}) != None
True
>>> index_set({1,2}) < None
False
>>> index_set({1,2}) <= None
False
>>> index_set({1,2}) > None
False
>>> index_set({1,2}) >= None
False
```

Definition at line 406 of file [PyClical.pyx](#).

#### 5.7.1.5 istpq()

```
PyClical.istpq (
    p,
    q )
```

Abbreviation for `index_set({-q,...p})`.

```
>>> print(istpq(2,3))
{-3,-2,-1,1,2}
```

Definition at line 1949 of file [PyClical.pyx](#).

## 5.7.2 Variable Documentation

### 5.7.2.1 `__version__`

```
PyClical.__version__ = str(glucat_package_version, 'utf-8') [private]
```

Definition at line 35 of file [PyClical.pyx](#).

### 5.7.2.2 `cl`

```
PyClical.cl = clifford
```

Definition at line 1910 of file [PyClical.pyx](#).

### 5.7.2.3 `fill`

```
PyClical.fill
```

Definition at line 1864 of file [PyClical.pyx](#).

### 5.7.2.4 `i`

```
PyClical.i
```

Definition at line 1591 of file [PyClical.pyx](#).

### 5.7.2.5 `ist`

```
PyClical.ist = index_set
```

Definition at line 1928 of file [PyClical.pyx](#).

### 5.7.2.6 `ixt`

```
PyClical.ixt
```

Definition at line 1864 of file [PyClical.pyx](#).

### 5.7.2.7 `lhs`

```
PyClical.lhs
```

Definition at line 1359 of file [PyClical.pyx](#).



#### 5.7.2.8 nbar3

```
PyClical.nbar3 = e(4) - e(-1)
```

Definition at line 1959 of file [PyClical.pyx](#).

#### 5.7.2.9 ninf3

```
PyClical.ninf3 = e(4) + e(-1)
```

Definition at line 1958 of file [PyClical.pyx](#).

#### 5.7.2.10 None

```
PyClical.None
```

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.11 obj

```
PyClical.obj
```

Definition at line 1591 of file [PyClical.pyx](#).

#### 5.7.2.12 pi

```
float PyClical.pi = atan(clifford(1.0)) * 4.0
```

Definition at line 1907 of file [PyClical.pyx](#).

#### 5.7.2.13 rhs

```
PyClical.rhs
```

Definition at line 1359 of file [PyClical.pyx](#).

#### 5.7.2.14 scalar\_epsilon

```
PyClical.scalar_epsilon = epsilon
```

Definition at line 1905 of file [PyClical.pyx](#).

#### 5.7.2.15 tau

```
float PyClical.tau = atan(clifford(1.0)) * 8.0
```

Definition at line 1908 of file [PyClical.pyx](#).

### 5.7.2.16 threshold

```
PyClical.threshold = error_squared_tol(rhs) if threshold is None else threshold
```

Definition at line 1359 of file [PyClical.pyx](#).

### 5.7.2.17 tol

```
PyClical.tol = error_squared_tol(rhs) if tol is None else tol
```

Definition at line 1359 of file [PyClical.pyx](#).

## 5.8 std Namespace Reference

### Classes

- struct [numeric\\_limits](#)< [glucat::framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >  
*Numeric limits for framed\_multi inherit limits for the corresponding scalar type.*
- struct [numeric\\_limits](#)< [glucat::matrix\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) > >  
*Numeric limits for matrix\_multi inherit limits for the corresponding scalar type.*

## Chapter 6

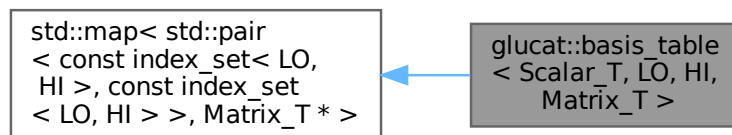
# Class Documentation

### 6.1 `glucat::basis_table< Scalar_T, LO, HI, Matrix_T >` Class Template Reference

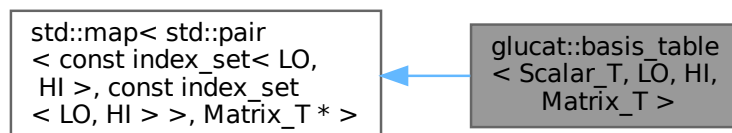
Table of basis elements used as a cache by `basis_element()`

```
#include <matrix_multi_imp.h>
```

Inheritance diagram for `glucat::basis_table< Scalar_T, LO, HI, Matrix_T >`:



Collaboration diagram for `glucat::basis_table< Scalar_T, LO, HI, Matrix_T >`:



## Public Member Functions

- [basis\\_table](#) (const [basis\\_table](#) &)=delete
- auto [operator=](#) (const [basis\\_table](#) &) -> [basis\\_table](#) &=delete

## Static Public Member Functions

- static auto [basis](#) () -> [basis\\_table](#) &  
*Single instance of basis table.*

## Private Member Functions

- [basis\\_table](#) ()=default
- [~basis\\_table](#) ()=default

## Friends

- class [friend\\_for\\_private\\_destructor](#)

## 6.1.1 Detailed Description

```
template<typename Scalar_T, const index\_t LO, const index\_t HI, typename Matrix_T>
class glucat::basis_table< Scalar_T, LO, HI, Matrix_T >
```

Table of basis elements used as a cache by [basis\\_element](#)()

Definition at line [1162](#) of file [matrix\\_multi\\_imp.h](#).

## 6.1.2 Constructor & Destructor Documentation

### 6.1.2.1 [basis\\_table](#)() [1/2]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis_table ( ) [private], [default]
```

### 6.1.2.2 [~basis\\_table](#)()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::~~basis\_table ( ) [private], [default]
```

### 6.1.2.3 [basis\\_table](#)() [2/2]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Matrix_T >
glucat::basis\_table< Scalar_T, LO, HI, Matrix_T >::basis_table (
    const basis\_table< Scalar_T, LO, HI, Matrix_T > & ) [delete]
```

### 6.1.3 Member Function Documentation

#### 6.1.3.1 basis()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Matrix_T >
static auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::basis ( ) -> basis_table&
[inline], [static]
```

Single instance of basis table.

Definition at line 1168 of file [matrix\\_multi\\_imp.h](#).

#### 6.1.3.2 operator=()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Matrix_T >
auto glucat::basis_table< Scalar_T, LO, HI, Matrix_T >::operator= (
    const basis_table< Scalar_T, LO, HI, Matrix_T > & ) -> basis_table &=delete
[delete]
```

### 6.1.4 Friends And Related Symbol Documentation

#### 6.1.4.1 friend\_for\_private\_destructor

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Matrix_T >
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 1173 of file [matrix\\_multi\\_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.2 glucat::bool\_to\_type< truth\_value > Class Template Reference

Bool to type.

```
#include <global.h>
```

#### Private Types

- enum { [value](#) = truth\_value }

### 6.2.1 Detailed Description

```
template<bool truth_value>
class glucat::bool_to_type< truth_value >
```

Bool to type.

Definition at line 69 of file [global.h](#).

### 6.2.2 Member Enumeration Documentation

#### 6.2.2.1 anonymous enum

```
template<bool truth_value>
anonymous enum [private]
```

Enumerator

value	
-------	--

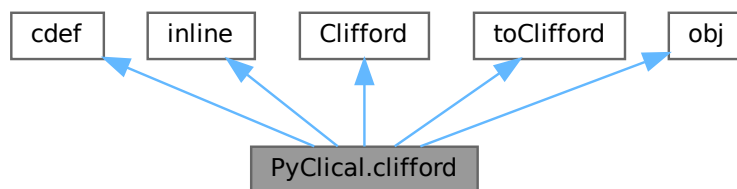
Definition at line 72 of file [global.h](#).

The documentation for this class was generated from the following file:

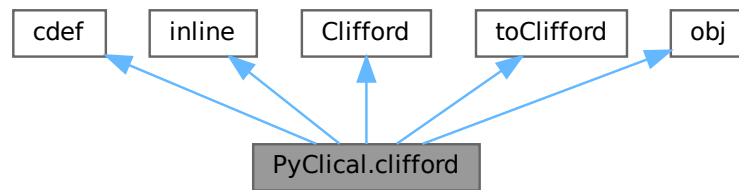
- [glucat/global.h](#)

## 6.3 PyClical.clifford Class Reference

Inheritance diagram for PyClical.clifford:



Collaboration diagram for PyClical.clifford:



### Public Member Functions

- `__cinit__` (self, other=0, `ixt=None`)
- `__dealloc__` (self)
- `__contains__` (self, x)
- `__iter__` (self)
- `reframe` (self, `ixt`)
- `__richcmp__` (lhs, rhs, int, op)
- `__getitem__` (self, `ixt`)
- `__neg__` (self)
- `__pos__` (self)
- `__add__` (lhs, rhs)
- `__iadd__` (self, rhs)
- `__sub__` (lhs, rhs)
- `__isub__` (self, rhs)
- `__mul__` (lhs, rhs)
- `__imul__` (self, rhs)
- `__mod__` (lhs, rhs)
- `__imod__` (self, rhs)
- `__and__` (lhs, rhs)
- `__iand__` (self, rhs)
- `__xor__` (lhs, rhs)
- `__ixor__` (self, rhs)
- `__truediv__` (lhs, rhs)
- `__idiv__` (self, rhs)
- `inv` (self)
- `__or__` (lhs, rhs)
- `__ior__` (self, rhs)
- `__pow__` (self, m, dummy)
- `pow` (self, m)
- `outer_pow` (self, m)
- `__call__` (self, grade)
- `scalar` (self)
- `pure` (self)
- `even` (self)
- `odd` (self)
- `vector_part` (self, frm=`None`)
- `involute` (self)
- `reverse` (self)

- [conj](#) (self)
- [quad](#) (self)
- [norm](#) (self)
- [abs](#) (self)
- [max\\_abs](#) (self)
- [truncated](#) (self, limit)
- [isinf](#) (self)
- [isnan](#) (self)
- [frame](#) (self)
- [\\_\\_repr\\_\\_](#) (self)
- [\\_\\_str\\_\\_](#) (self)

## Public Attributes

- [instance](#)

### 6.3.1 Detailed Description

Python class `clifford` wraps C++ class `Clifford`.

Definition at line 532 of file [PyClical.pyx](#).

### 6.3.2 Member Function Documentation

#### 6.3.2.1 `__add__()`

```
PyClical.clifford.__add__ (
    lhs,
    rhs )
```

Geometric sum.

```
>>> print(clifford(1) + clifford("{2}"))
1+{2}
>>> print(clifford("{1}") + clifford("{2}"))
{1}+{2}
```

Definition at line 740 of file [PyClical.pyx](#).

#### 6.3.2.2 `__and__()`

```
PyClical.clifford.__and__ (
    lhs,
    rhs )
```

Inner product.

```
>>> print(clifford("{1}") & clifford("{2}"))
0
>>> print(clifford(2) & clifford("{2}"))
0
>>> print(clifford("{1}") & clifford("{1}"))
1
>>> print(clifford("{1}") & clifford("{1,2}"))
{2}
```

Definition at line 836 of file [PyClical.pyx](#).



### 6.3.2.3 `__call__()`

```
PyClical.clifford.__call__ (
    self,
    grade )
```

Pure grade-vector part.

```
>>> print(clifford("{1}") (1))
{1}
>>> print(clifford("{1}") (0))
0
>>> print(clifford("1+{1}+{1,2}") (0))
1
>>> print(clifford("1+{1}+{1,2}") (1))
{1}
>>> print(clifford("1+{1}+{1,2}") (2))
{1,2}
>>> print(clifford("1+{1}+{1,2}") (3))
0
```

Definition at line 1020 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.3.2.4 `__cinit__()`

```
PyClical.clifford.__cinit__ (
    self,
    other = 0,
    ixt = None )
```

Construct an object of type clifford.

```
>>> print(clifford(2))
2
>>> print(clifford(2.0))
2
>>> print(clifford(1.0e-1))
0.1
>>> print(clifford("2"))
2
>>> print(clifford("2{1,2,3}"))
2{1,2,3}
>>> print(clifford(clifford("2{1,2,3}")))
2{1,2,3}
>>> print(clifford("-{1}"))
-{1}
>>> print(clifford(2, index_set({1,2})))
2{1,2}
>>> print(clifford([2,3], index_set({1,2})))
2{1}+3{2}
```

Definition at line 565 of file [PyClical.pyx](#).

### 6.3.2.5 `__contains__()`

```
PyClical.clifford.__contains__ (
    self,
    x )
```

Not applicable.

```
>>> x=clifford(index_set({-3,4,7})); -3 in x
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 627 of file [PyClical.pyx](#).

### 6.3.2.6 `__dealloc__()`

```
PyClical.clifford.__dealloc__ (
    self )
```

Clean up by deallocating the instance of C++ class Clifford.

Definition at line 621 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.3.2.7 `__getitem__()`

```
PyClical.clifford.__getitem__ (
    self,
    ixt )
```

Subscripting: map from index set to scalar coordinate.

```
>>> clifford("{1}")[index_set(1)]
1.0
>>> clifford("{1}")[index_set({1})]
1.0
>>> clifford("{1}")[index_set({1,2})]
0.0
>>> clifford("2{1,2}")[index_set({1,2})]
2.0
```

Definition at line 707 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

**6.3.2.8 `__iadd__()`**

```
PyClical.clifford.__iadd__ (
    self,
    rhs )
```

Geometric sum.

```
>>> x = clifford(1); x += clifford("{2}"); print(x)
1+{2}
```

Definition at line 751 of file [PyClical.pyx](#).

**6.3.2.9 `__iand__()`**

```
PyClical.clifford.__iand__ (
    self,
    rhs )
```

Inner product.

```
>>> x = clifford("{1}"); x &= clifford("{2}"); print(x)
0
>>> x = clifford(2); x &= clifford("{2}"); print(x)
0
>>> x = clifford("{1}"); x &= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
{2}
```

Definition at line 851 of file [PyClical.pyx](#).

**6.3.2.10 `__idiv__()`**

```
PyClical.clifford.__idiv__ (
    self,
    rhs )
```

Geometric quotient.

```
>>> x = clifford("{1}"); x /= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x /= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x /= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
-{2}
```

Definition at line 911 of file [PyClical.pyx](#).

**6.3.2.11 `__imod__()`**

```
PyClical.clifford.__imod__ (
    self,
    rhs )
```

Contraction.

```
>>> x = clifford("{1}"); x %= clifford("{2}"); print(x)
0
>>> x = clifford(2); x %= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x %= clifford("{1}"); print(x)
1
>>> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
{2}
```

Definition at line 821 of file [PyClical.pyx](#).

**6.3.2.12 `__imul__()`**

```
PyClical.clifford.__imul__ (
    self,
    rhs )
```

Geometric product.

```
>>> x = clifford(2); x *= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x *= clifford("{2}"); print(x)
{1,2}
>>> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
{2}
```

Definition at line 793 of file [PyClical.pyx](#).

**6.3.2.13 `__ior__()`**

```
PyClical.clifford.__ior__ (
    self,
    rhs )
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
-{1}
```

Definition at line 950 of file [PyClical.pyx](#).

**6.3.2.14** `__isub__()`

```
PyClical.clifford.__isub__ (
    self,
    rhs )
```

Geometric difference.

```
>>> x = clifford(1); x -= clifford("{2}"); print(x)
1-{2}
```

Definition at line 771 of file [PyClical.pyx](#).

**6.3.2.15** `__iter__()`

```
PyClical.clifford.__iter__ (
    self )
```

Not applicable.

```
>>> for a in clifford(index_set({-3,4,7})):print(a, end=",")
Traceback (most recent call last):
...
TypeError: Not applicable.
```

Definition at line 638 of file [PyClical.pyx](#).

**6.3.2.16** `__ixor__()`

```
PyClical.clifford.__ixor__ (
    self,
    rhs )
```

Outer product.

```
>>> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
{1,2}
>>> x = clifford(2); x ^= clifford("{2}"); print(x)
2{2}
>>> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
0
>>> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
0
```

Definition at line 881 of file [PyClical.pyx](#).

**6.3.2.17 `__mod__()`**

```
PyClical.clifford.__mod__ (
    lhs,
    rhs )
```

Contraction.

```
>>> print(clifford("{1}") % clifford("{2}"))
0
>>> print(clifford(2) % clifford("{2}"))
2{2}
>>> print(clifford("{1}") % clifford("{1}"))
1
>>> print(clifford("{1}") % clifford("{1,2}"))
{2}
```

Definition at line 806 of file [PyClical.pyx](#).

**6.3.2.18 `__mul__()`**

```
PyClical.clifford.__mul__ (
    lhs,
    rhs )
```

Geometric product.

```
>>> print(clifford("{1}") * clifford("{2}"))
{1,2}
>>> print(clifford(2) * clifford("{2}"))
2{2}
>>> print(clifford("{1}") * clifford("{1,2}"))
{2}
```

Definition at line 780 of file [PyClical.pyx](#).

**6.3.2.19 `__neg__()`**

```
PyClical.clifford.__neg__ (
    self )
```

Unary `-`.

```
>>> print(-clifford("{1}"))
-{1}
```

Definition at line 722 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

**6.3.2.20 `__or__()`**

```
PyClical.clifford.__or__ (
    lhs,
    rhs )
```

Transform left hand side, using right hand side as a transformation.

```
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
-{1}
>>> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
-{1}
```

Definition at line 939 of file [PyClical.pyx](#).

**6.3.2.21 `__pos__()`**

```
PyClical.clifford.__pos__ (
    self )
```

Unary +.

```
>>> print(+clifford("{1}"))
{1}
```

Definition at line 731 of file [PyClical.pyx](#).

**6.3.2.22 `__pow__()`**

```
PyClical.clifford.__pow__ (
    self,
    m,
    dummy )
```

Power: self to the m.

```
>>> x=clifford("{1}"); print(x ** 2)
1
>>> x=clifford("2"); print(x ** 2)
4
>>> x=clifford("2+{1}"); print(x ** 0)
1
>>> x=clifford("2+{1}"); print(x ** 1)
2+{1}
>>> x=clifford("2+{1}"); print(x ** 2)
5+4{1}
>>> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
1
```

Definition at line 961 of file [PyClical.pyx](#).

References [PyClical.clifford.pow\(\)](#).

**6.3.2.23 `__repr__()`**

```
PyClical.clifford.__repr__ (
    self )
```

The "official" string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
```

Definition at line 1235 of file [PyClical.pyx](#).

References [clifford\\_to\\_repr\(\)](#).

**6.3.2.24 `__richcmp__()`**

```
PyClical.clifford.__richcmp__ (
    lhs,
    rhs,
    int,
    op )
```

Compare objects of type clifford.

```
>>> clifford("{1}") == clifford("1{1}")
True
>>> clifford("{1}") != clifford("1.0{1}")
False
>>> clifford("{1}") != clifford("1.0")
True
>>> clifford("{1,2}") == None
False
>>> clifford("{1,2}") != None
True
>>> None == clifford("{1,2}")
False
>>> None != clifford("{1,2}")
True
```

Definition at line 672 of file [PyClical.pyx](#).

**6.3.2.25 `__str__()`**

```
PyClical.clifford.__str__ (
    self )
```

The "informal" string representation of self.

```
>>> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
'1+3{-1}+2{1,2}+4{-2,7}'
```

Definition at line 1244 of file [PyClical.pyx](#).

References [clifford\\_to\\_str\(\)](#).



**6.3.2.26 `__sub__()`**

```
PyClical.clifford.__sub__ (
    lhs,
    rhs )
```

Geometric difference.

```
>>> print(clifford(1) - clifford("{2}"))
1-{2}
>>> print(clifford("{1}") - clifford("{2}"))
{1}-{2}
```

Definition at line 760 of file [PyClical.pyx](#).

**6.3.2.27 `__truediv__()`**

```
PyClical.clifford.__truediv__ (
    lhs,
    rhs )
```

Geometric quotient.

```
>>> print(clifford("{1}") / clifford("{2}"))
{1,2}
>>> print(clifford(2) / clifford("{2}"))
2{2}
>>> print(clifford("{1}") / clifford("{1}"))
1
>>> print(clifford("{1}") / clifford("{1,2}"))
-{2}
```

Definition at line 896 of file [PyClical.pyx](#).

**6.3.2.28 `__xor__()`**

```
PyClical.clifford.__xor__ (
    lhs,
    rhs )
```

Outer product.

```
>>> print(clifford("{1}") ^ clifford("{2}"))
{1,2}
>>> print(clifford(2) ^ clifford("{2}"))
2{2}
>>> print(clifford("{1}") ^ clifford("{1}"))
0
>>> print(clifford("{1}") ^ clifford("{1,2}"))
0
```

Definition at line 866 of file [PyClical.pyx](#).

### 6.3.2.29 abs()

```
PyClical.clifford.abs (
    self )
```

Absolute value: square root of norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
2.0
```

Definition at line 1175 of file [PyClical.pyx](#).

References [glucat.abs\(\)](#).

### 6.3.2.30 conj()

```
PyClical.clifford.conj (
    self )
```

Conjugation, reverse o involute == involute o reverse.

```
>>> print((clifford("{1}")).conj())
-1
>>> print((clifford("{2}") * clifford("{1}")).conj())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).conj())
-1,2
>>> print(clifford("1+{1}+{1,2}").conj())
1-{1}-{1,2}
```

Definition at line 1138 of file [PyClical.pyx](#).

References [PyClical.clifford.conj\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.clifford.conj\(\)](#).

### 6.3.2.31 even()

```
PyClical.clifford.even (
    self )
```

Even part of multivector, sum of even grade terms.

```
>>> print(clifford("1+{1}+{1,2}").even())
1+{1,2}
```

Definition at line 1061 of file [PyClical.pyx](#).

References [PyClical.clifford.even\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.clifford.even\(\)](#).

**6.3.2.32 frame()**

```
PyClical.clifford.frame (
    self )
```

Subalgebra generated by all generators of terms of given multivector.

```
>>> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
{-2,-1,1,2,7}
>>> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
<class 'PyClical.index_set'>
```

Definition at line 1224 of file [PyClical.pyx](#).

References [PyClical.clifford.frame\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.clifford.frame\(\)](#).

**6.3.2.33 inv()**

```
PyClical.clifford.inv (
    self )
```

Geometric multiplicative inverse.

```
>>> x = clifford("{1}"); print(x.inv())
{1}
>>> x = clifford(2); print(x.inv())
0.5
>>> x = clifford("{1,2}"); print(x.inv())
-1,2}
```

Definition at line 926 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.inv\(\)](#).

Referenced by [PyClical.clifford.inv\(\)](#).

**6.3.2.34 involute()**

```
PyClical.clifford.involute (
    self )
```

Main involution, each {i} is replaced by -{i} in each term,  
eg. `clifford("{1}") -> -clifford("{1}")`.

```
>>> print(clifford("{1}").involute())
-1}
>>> print((clifford("{2}") * clifford("{1}")).involute())
-1,2}
>>> print((clifford("{1}") * clifford("{2}")).involute())
1,2}
>>> print(clifford("1+{1}+{1,2}").involute())
1-1}+1,2}
```

Definition at line 1107 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.involute\(\)](#).

Referenced by [PyClical.clifford.involute\(\)](#).

### 6.3.2.35 isinf()

```
PyClical.clifford.isinf (
    self )
```

Check if a multivector contains any infinite values.

```
>>> clifford().isinf()
False
```

Definition at line 1206 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.isnan\(\)](#).

### 6.3.2.36 isnan()

```
PyClical.clifford.isnan (
    self )
```

Check if a multivector contains any IEEE NaN values.

```
>>> clifford().isnan()
False
```

Definition at line 1215 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.isnan\(\)](#).

Referenced by [PyClical.clifford.isinf\(\)](#), and [PyClical.clifford.isnan\(\)](#).

### 6.3.2.37 max\_abs()

```
PyClical.clifford.max_abs (
    self )
```

Maximum of absolute values of components of multivector: multivector infinity norm.

```
>>> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
1.0
>>> clifford("3+2{1}+{1,2}").max_abs()
3.0
```

Definition at line 1184 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.max\\_abs\(\)](#).

Referenced by [PyClical.clifford.max\\_abs\(\)](#).

**6.3.2.38 norm()**

```
PyClical.clifford.norm (
    self )
```

Norm == sum of squares of coordinates.

```
>>> clifford("1+{1}+{1,2}").norm()
3.0
>>> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
4.0
```

Definition at line 1164 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.norm\(\)](#).

Referenced by [PyClical.clifford.norm\(\)](#).

**6.3.2.39 odd()**

```
PyClical.clifford.odd (
    self )
```

Odd part of multivector, sum of odd grade terms.

```
>>> print(clifford("1+{1}+{1,2}").odd())
{1}
```

Definition at line 1070 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.odd\(\)](#).

Referenced by [PyClical.clifford.odd\(\)](#).

**6.3.2.40 outer\_pow()**

```
PyClical.clifford.outer_pow (
    self,
    m )
```

Outer product power.

```
>>> x=clifford("2+{1}"); print(x.outer_pow(0))
1
>>> x=clifford("2+{1}"); print(x.outer_pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.outer_pow(2))
4+4{1}
>>> print(clifford("1+{1}+{1,2}").outer_pow(3))
1+3{1}+3{1,2}
```

Definition at line 1004 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.outer\\_pow\(\)](#).

Referenced by [PyClical.clifford.outer\\_pow\(\)](#).

### 6.3.2.41 pow()

```

PyClical.clifford.pow (
    self,
    m )

Power: self to the m.

>>> x=clifford("{1}"); print(x.pow(2))
1
>>> x=clifford("2"); print(x.pow(2))
4
>>> x=clifford("2+{1}"); print(x.pow(0))
1
>>> x=clifford("2+{1}"); print(x.pow(1))
2+{1}
>>> x=clifford("2+{1}"); print(x.pow(2))
5+4{1}
>>> print(clifford("1+{1}+{1,2}").pow(3))
1+3{1}+3{1,2}
>>> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
1

```

Definition at line 980 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.pow\(\)](#).

Referenced by [PyClical.clifford.\\_\\_pow\\_\\_\(\)](#), and [PyClical.clifford.pow\(\)](#).

### 6.3.2.42 pure()

```

PyClical.clifford.pure (
    self )

Pure part.

>>> print(clifford("1+{1}+{1,2}").pure())
{1}+{1,2}
>>> print(clifford("{1,2}").pure())
{1,2}

```

Definition at line 1050 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.pure\(\)](#).

Referenced by [PyClical.clifford.pure\(\)](#).

### 6.3.2.43 quad()

```

PyClical.clifford.quad (
    self )

Quadratic form == (rev(x)*x)(0).

>>> print(clifford("1+{1}+{1,2}").quad())
3.0
>>> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
2.0

```

Definition at line 1153 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.quad\(\)](#).

Referenced by [PyClical.clifford.quad\(\)](#).

**6.3.2.44 reframe()**

```
PyClical.clifford.reframe (
    self,
    ixt )
```

Put self into a larger frame, containing the union of self.frame() and index set ixt. This can be used to make multiplication faster, by multiplying within a common frame.

```
>>> clifford("2+3{1}").reframe(index_set({1,2,3}))
clifford("2+3{1}")
>>> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);
True
```

Definition at line 649 of file [PyClical.pyx](#).

**6.3.2.45 reverse()**

```
PyClical.clifford.reverse (
    self )
```

Reversion, eg. `clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}")`.

```
>>> print(clifford("{1}").reverse())
{1}
>>> print((clifford("{2}") * clifford("{1}")).reverse())
{1,2}
>>> print((clifford("{1}") * clifford("{2}")).reverse())
-{1,2}
>>> print(clifford("1+{1}+{1,2}").reverse())
1+{1}-{1,2}
```

Definition at line 1123 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.reverse\(\)](#).

Referenced by [PyClical.clifford.reverse\(\)](#).

**6.3.2.46 scalar()**

```
PyClical.clifford.scalar (
    self )
```

Scalar part.

```
>>> clifford("1+{1}+{1,2}").scalar()
1.0
>>> clifford("{1,2}").scalar()
0.0
```

Definition at line 1039 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.scalar\(\)](#).

Referenced by [PyClical.clifford.scalar\(\)](#).

### 6.3.2.47 truncated()

```
PyClical.clifford.truncated (
    self,
    limit )
```

Remove all terms of self with relative size smaller than limit.

```
>>> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
clifford("100000000")
>>> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
clifford("10000+{1}")
```

Definition at line 1195 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.truncated\(\)](#).

Referenced by [PyClical.clifford.truncated\(\)](#).

### 6.3.2.48 vector\_part()

```
PyClical.clifford.vector_part (
    self,
    frm = None )
```

Vector part of multivector, as a Python list, with respect to frm.

```
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
[2.0, 3.0]
>>> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set((-1,1,2))))
[0.0, 2.0, 3.0]
```

Definition at line 1079 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.clifford.vector\\_part\(\)](#).

Referenced by [PyClical.clifford.vector\\_part\(\)](#).

## 6.3.3 Member Data Documentation

### 6.3.3.1 instance

[PyClical.clifford.instance](#)

Definition at line 592 of file [PyClical.pyx](#).

Referenced by [PyClical.clifford.\\_\\_call\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_contains\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.clifford.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_getitem\\_\\_\(\)](#), [PyClical.clifford.\\_\\_getitem\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_invert\\_\\_\(\)](#), [PyClical.clifford.\\_\\_neg\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_setitem\\_\\_\(\)](#), [PyClical.clifford.conj\(\)](#), [PyClical.index\\_set.count\(\)](#), [PyClical.index\\_set.count\\_neg\(\)](#), [PyClical.index\\_set.count\\_pos\(\)](#), [PyClical.clifford.even\(\)](#), [PyClical.clifford.frame\(\)](#), [PyClical.index\\_set.hash\\_fn\(\)](#), [PyClical.clifford.inv\(\)](#), [PyClical.clifford.involute\(\)](#), [PyClical.clifford.isinf\(\)](#), [PyClical.clifford.isnan\(\)](#), [PyClical.index\\_set.max\(\)](#), [PyClical.clifford.max\\_abs\(\)](#), [PyClical.index\\_set.min\(\)](#), [PyClical.clifford.norm\(\)](#), [PyClical.clifford.odd\(\)](#), [PyClical.clifford.outer\\_pow\(\)](#), [PyClical.clifford.pow\(\)](#), [PyClical.clifford.pure\(\)](#), [PyClical.clifford.quad\(\)](#), [PyClical.clifford.reverse\(\)](#), [PyClical.clifford.scalar\(\)](#), [PyClical.index\\_set.sign\\_of\\_mult\(\)](#), [PyClical.index\\_set.sign\\_of\\_square\(\)](#), [PyClical.clifford.truncated\(\)](#), and [PyClical.clifford.vector\\_part\(\)](#).

The documentation for this class was generated from the following file:

- [pyclical/PyClical.pyx](#)

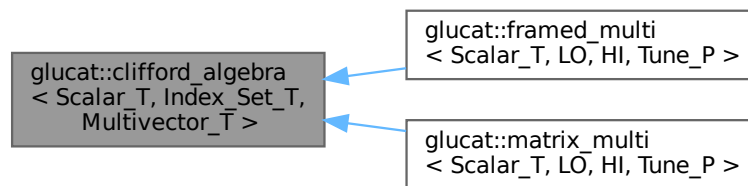


## 6.4 glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T > Class Template Reference

clifford\_algebra<> declares the operations of a Clifford algebra

```
#include <clifford_algebra.h>
```

Inheritance diagram for glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >:



### Public Types

- using `scalar_t` = `Scalar_T`
- using `index_set_t` = `Index_Set_T`
- using `multivector_t` = `Multivector_T`
- using `pair_t` = `std::pair<const index_set_t, Scalar_T>`
- using `vector_t` = `std::vector<Scalar_T>`

### Public Member Functions

- virtual `~clifford_algebra()` = default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0  
*Test for equality of multivectors.*
- virtual auto `operator==` (const `Scalar_T` &scr) const -> bool=0  
*Test for equality of multivector and scalar.*
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric sum.*
- virtual auto `operator+=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Geometric sum of multivector and scalar.*
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric difference.*
- virtual auto `operator-=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Geometric difference of multivector and scalar.*
- virtual auto `operator-` () const -> const `multivector_t`=0  
*Unary -.*
- virtual auto `operator*=` (const `Scalar_T` &scr) -> `multivector_t` &=0  
*Product of multivector and scalar.*
- virtual auto `operator*=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric product.*

- virtual auto `operator%=(const multivector_t &rhs) -> multivector_t &=0`  
*Contraction.*
- virtual auto `operator&=(const multivector_t &rhs) -> multivector_t &=0`  
*Inner product.*
- virtual auto `operator^=(const multivector_t &rhs) -> multivector_t &=0`  
*Outer product.*
- virtual auto `operator/=(const Scalar_T &scr) -> multivector_t &=0`  
*Quotient of multivector and scalar.*
- virtual auto `operator/=(const multivector_t &rhs) -> multivector_t &=0`  
*Geometric quotient.*
- virtual auto `operator|=(const multivector_t &rhs) -> multivector_t &=0`  
*Transformation via twisted adjoint action.*
- virtual auto `inv () const -> const multivector_t=0`  
*Geometric multiplicative inverse.*
- virtual auto `pow (int m) const -> const multivector_t=0`  
*\*this to the m*
- virtual auto `outer_pow (int m) const -> const multivector_t=0`  
*Outer product power.*
- virtual auto `frame () const -> const index_set_t=0`  
*Subalgebra generated by all generators of terms of given multivector.*
- virtual auto `grade () const -> index_t=0`  
*Maximum of the grades of each term.*
- virtual auto `operator[] (const index_set_t ist) const -> Scalar_T=0`  
*Subscripting: map from index set to scalar coordinate.*
- virtual auto `operator() (index_t grade) const -> const multivector_t=0`  
*Pure grade-vector part.*
- virtual auto `scalar () const -> Scalar_T=0`  
*Scalar part.*
- virtual auto `pure () const -> const multivector_t=0`  
*Pure part.*
- virtual auto `even () const -> const multivector_t=0`  
*Even part of multivector, sum of even grade terms.*
- virtual auto `odd () const -> const multivector_t=0`  
*Odd part of multivector, sum of odd grade terms.*
- virtual auto `vector_part () const -> const vector_t=0`  
*Vector part of multivector, as a vector\_t with respect to frame()*
- virtual auto `vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t=0`  
*Vector part of multivector, as a vector\_t with respect to frm.*
- virtual auto `involute () const -> const multivector_t=0`  
*Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*
- virtual auto `reverse () const -> const multivector_t=0`  
*Reversion, eg. {1}\*{2} -> {2}\*{1}.*
- virtual auto `conj () const -> const multivector_t=0`  
*Conjugation, reverse o involute == involute o reverse.*
- virtual auto `quad () const -> Scalar_T=0`  
*Scalar\_T quadratic form == (rev(x)\*x)(0)*
- virtual auto `norm () const -> Scalar_T=0`  
*Scalar\_T norm == sum of norm of coordinates.*
- virtual auto `max_abs () const -> Scalar_T=0`  
*Maximum of absolute values of components of multivector: multivector infinity norm.*
- virtual auto `truncated (const Scalar_T &limit=default_truncation) const -> const multivector_t=0`

- *Remove all terms with relative size smaller than limit.*  
virtual auto [isinf](#) () const -> bool=0
- *Check if a multivector contains any infinite values.*  
virtual auto [isnan](#) () const -> bool=0
- *Check if a multivector contains any IEEE NaN values.*  
virtual void [write](#) (const std::string &msg="") const =0
- *Write formatted multivector to output.*  
virtual void [write](#) (std::ofstream &ofile, const std::string &msg="") const =0
- *Write formatted multivector to file.*

### Static Public Member Functions

- static auto [classname](#) () -> const std::string

### Static Public Attributes

- static const [index\\_t v\\_lo](#) = index\_set\_t::v\_lo
- static const [index\\_t v\\_hi](#) = index\_set\_t::v\_hi
- static const [Scalar\\_T default\\_truncation](#) = std::numeric\_limits<Scalar\_T>::epsilon()  
*Default for truncation.*

## 6.4.1 Detailed Description

**template<typename Scalar\_T, typename Index\_Set\_T, typename Multivector\_T>**  
**class glucat::clifford\_algebra< Scalar\_T, Index\_Set\_T, Multivector\_T >**

clifford\_algebra<> declares the operations of a Clifford algebra

Definition at line 45 of file [clifford\\_algebra.h](#).

## 6.4.2 Member Typedef Documentation

### 6.4.2.1 index\_set\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::index_set_t = Index_↔
Set_T
```

Definition at line 49 of file [clifford\\_algebra.h](#).

### 6.4.2.2 multivector\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::multivector_t = Multivector_↔
_T
```

Definition at line 52 of file [clifford\\_algebra.h](#).

### 6.4.2.3 pair\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pair_t = std::pair<const
index_set_t, Scalar_T>
```

Definition at line 53 of file [clifford\\_algebra.h](#).

### 6.4.2.4 scalar\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar_t = Scalar_T
```

Definition at line 48 of file [clifford\\_algebra.h](#).

### 6.4.2.5 vector\_t

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
using glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_t = std::vector<Scalar_
_T>
```

Definition at line 54 of file [clifford\\_algebra.h](#).

## 6.4.3 Constructor & Destructor Documentation

### 6.4.3.1 ~clifford\_algebra()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::~~clifford_algebra (
) [virtual], [default]
```

## 6.4.4 Member Function Documentation

### 6.4.4.1 classname()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::classname ( ) -> const
std::string [static]
```

Definition at line 66 of file [clifford\\_algebra\\_imp.h](#).

### 6.4.4.2 conj()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::conj ( ) const
-> const multivector_t [pure virtual]
```

Conjugation, reverse o involute == involute o reverse.

#### 6.4.4.3 even()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::even ( ) const
-> const multivector_t [pure virtual]
```

Even part of multivector, sum of even grade terms.

#### 6.4.4.4 frame()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::frame ( ) const
-> const index_set_t [pure virtual]
```

Subalgebra generated by all generators of terms of given multivector.

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#) and [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

#### 6.4.4.5 grade()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::grade ( ) const
-> index_t [pure virtual]
```

Maximum of the grades of each term.

#### 6.4.4.6 inv()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::inv ( ) const
-> const multivector_t [pure virtual]
```

Geometric multiplicative inverse.

#### 6.4.4.7 involute()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::involute ( )
const -> const multivector_t [pure virtual]
```

Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.

#### 6.4.4.8 isinf()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::isinf ( ) const
-> bool [pure virtual]
```

Check if a multivector contains any infinite values.

#### 6.4.4.9 isnan()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::isnan ( ) const
-> bool [pure virtual]
```

Check if a multivector contains any IEEE NaN values.

Referenced by [glucat::cascade\\_log\(\)](#), [glucat::exp\(\)](#), [glucat::log\(\)](#), [glucat::matrix\\_log\(\)](#), [glucat::matrix\\_sqrt\(\)](#), [glucat::pade\\_log\(\)](#), and [glucat::sqrt\(\)](#).

#### 6.4.4.10 max\_abs()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::max_abs ( )
const -> Scalar_T [pure virtual]
```

Maximum of absolute values of components of multivector: multivector infinity norm.

#### 6.4.4.11 norm()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::norm ( ) const
-> Scalar_T [pure virtual]
```

Scalar\_T norm == sum of norm of coordinates.

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

#### 6.4.4.12 odd()

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::odd ( ) const
-> const multivector\_t [pure virtual]
```

Odd part of multivector, sum of odd grade terms.

#### 6.4.4.13 operator%=( )

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator%= (
    const multivector\_t & rhs ) -> multivector\_t & [pure virtual]
```

Contraction.

#### 6.4.4.14 operator&=( )

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator&= (
    const multivector\_t & rhs ) -> multivector\_t & [pure virtual]
```

Inner product.

**6.4.4.15 operator()()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator() (
    index_t grade ) const -> const multivector_t [pure virtual]
```

Pure grade-vector part.

**6.4.4.16 operator\*=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Geometric product.

**6.4.4.17 operator\*=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator*= (
    const Scalar_T & scr ) -> multivector_t & [pure virtual]
```

Product of multivector and scalar.

**6.4.4.18 operator+=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Geometric sum.

**6.4.4.19 operator+=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator+= (
    const Scalar_T & scr ) -> multivector_t & [pure virtual]
```

Geometric sum of multivector and scalar.

**6.4.4.20 operator-( )**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator- ( )
const -> const multivector_t [pure virtual]
```

Unary -.

**6.4.4.21 operator-=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const multivector\_t & rhs ) -> multivector\_t & [pure virtual]
```

Geometric difference.

**6.4.4.22 operator-=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator-= (
    const Scalar_T & scr ) -> multivector\_t & [pure virtual]
```

Geometric difference of multivector and scalar.

**6.4.4.23 operator/=( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const multivector\_t & rhs ) -> multivector\_t & [pure virtual]
```

Geometric quotient.

**6.4.4.24 operator/=( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator/= (
    const Scalar_T & scr ) -> multivector\_t & [pure virtual]
```

Quotient of multivector and scalar.

**6.4.4.25 operator==( ) [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator==(
    const multivector\_t & val ) const -> bool [pure virtual]
```

Test for equality of multivectors.

**6.4.4.26 operator==( ) [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator==(
    const Scalar_T & scr ) const -> bool [pure virtual]
```

Test for equality of multivector and scalar.



**6.4.4.27 operator[]()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator[] (
    const index_set_t ist ) const -> Scalar_T [pure virtual]
```

Subscripting: map from index set to scalar coordinate.

**6.4.4.28 operator^=()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator^= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Outer product.

**6.4.4.29 operator" |=()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::operator|= (
    const multivector_t & rhs ) -> multivector_t & [pure virtual]
```

Transformation via twisted adjoint action.

**6.4.4.30 outer\_pow()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::outer_pow (
    int m ) const -> const multivector_t [pure virtual]
```

Outer product power.

**6.4.4.31 pow()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pow (
    int m ) const -> const multivector_t [pure virtual]
```

\*this to the m

**6.4.4.32 pure()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::pure ( ) const
-> const multivector_t [pure virtual]
```

Pure part.

**6.4.4.33 quad()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::quad ( ) const
-> Scalar_T [pure virtual]
```

Scalar\_T quadratic form == (rev(x)\*x)(0)

**6.4.4.34 reverse()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::reverse ( )
const -> const multivector\_t [pure virtual]
```

Reversion, eg. {1}\*{2} -> {2}\*{1}.

**6.4.4.35 scalar()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::scalar ( )
const -> Scalar_T [pure virtual]
```

Scalar part.

Referenced by [glucat::exp\(\)](#), [glucat::matrix\\_log\(\)](#), and [glucat::matrix\\_sqrt\(\)](#).

**6.4.4.36 truncated()**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::truncated (
    const Scalar_T & limit = default\_truncation ) const -> const multivector\_t [pure
virtual]
```

Remove all terms with relative size smaller than limit.

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

**6.4.4.37 vector\_part()** [1/2]

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part ( )
const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector\\_t](#) with respect to [frame\(\)](#)

**6.4.4.38 vector\_part()** [2/2]

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual auto glucat::clifford\_algebra< Scalar_T, Index_Set_T, Multivector_T >::vector_part (
    const index\_set\_t frm,
    const bool prechecked ) const -> const vector\_t [pure virtual]
```

Vector part of multivector, as a [vector\\_t](#) with respect to frm.

**6.4.4.39 write() [1/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual void glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    const std::string & msg = "" ) const [pure virtual]
```

Write formatted multivector to output.

**6.4.4.40 write() [2/2]**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
virtual void glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::write (
    std::ostream & ofile,
    const std::string & msg = "" ) const [pure virtual]
```

Write formatted multivector to file.

**6.4.5 Member Data Documentation****6.4.5.1 default\_truncation**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
const Scalar_T glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::default_↵
truncation = std::numeric_limits<Scalar_T>::epsilon() [static]
```

Default for truncation.

Definition at line 59 of file [clifford\\_algebra.h](#).

**6.4.5.2 v\_hi**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_hi = index↵
_set_t::v_hi [static]
```

Definition at line 51 of file [clifford\\_algebra.h](#).

**6.4.5.3 v\_lo**

```
template<typename Scalar_T , typename Index_Set_T , typename Multivector_T >
const index_t glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >::v_lo = index↵
_set_t::v_lo [static]
```

Definition at line 50 of file [clifford\\_algebra.h](#).

The documentation for this class was generated from the following files:

- [glucat/clifford\\_algebra.h](#)
- [glucat/clifford\\_algebra\\_imp.h](#)

## 6.5 glucat::compare\_types< LHS\_T, RHS\_T > Class Template Reference

Type comparison.

```
#include <global.h>
```

### Public Types

- enum { [are\\_same](#) = false }

### 6.5.1 Detailed Description

```
template<typename LHS_T, typename RHS_T>
class glucat::compare_types< LHS_T, RHS_T >
```

Type comparison.

Definition at line 54 of file [global.h](#).

### 6.5.2 Member Enumeration Documentation

#### 6.5.2.1 anonymous enum

```
template<typename LHS_T , typename RHS_T >
anonymous enum
```

#### Enumerator

<a href="#">are_same</a>	
--------------------------	--

Definition at line 57 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

## 6.6 glucat::compare\_types< T, T > Class Template Reference

```
#include <global.h>
```

### Public Types

- enum { [are\\_same](#) = true }

## 6.6.1 Detailed Description

```
template<typename T>
class glucat::compare_types< T, T >
```

Definition at line 60 of file [global.h](#).

## 6.6.2 Member Enumeration Documentation

### 6.6.2.1 anonymous enum

```
template<typename T >
anonymous enum
```

#### Enumerator

are_same	
----------	--

Definition at line 63 of file [global.h](#).

The documentation for this class was generated from the following file:

- [glucat/global.h](#)

## 6.7 glucat::control\_t Class Reference

Parameters to control tests.

```
#include <control.h>
```

### Public Member Functions

- int [call](#) ([intfn](#) f) const  
*Call a function that returns int.*
- int [call](#) ([intintfn](#) f, int arg) const  
*Call a function of int that returns int.*

### Static Public Member Functions

- static const [control\\_t](#) & [control](#) (int argc, char \*\*argv)
- static bool [verbose](#) ()  
*Produce more detailed output from tests.*

### Private Member Functions

- bool [valid](#) () const
- bool [catch\\_exceptions](#) () const
- [control\\_t](#) (int argc, char \*\*argv)  
*Constructor from program arguments.*
- [control\\_t](#) ()=default
- [~control\\_t](#) ()=default
- [control\\_t](#) (const [control\\_t](#) &)=delete
- [control\\_t](#) & [operator=](#) (const [control\\_t](#) &)=delete

### Private Attributes

- bool [m\\_valid](#)  
*Test parameters are valid.*
- bool [m\\_catch\\_exceptions](#)  
*Catch exceptions.*

### Static Private Attributes

- static bool [m\\_verbose\\_output](#) = false  
*Produce more detailed output from tests.*

### Friends

- class [friend\\_for\\_private\\_destructor](#)

## 6.7.1 Detailed Description

Parameters to control tests.

Definition at line 39 of file [control.h](#).

## 6.7.2 Constructor & Destructor Documentation

### 6.7.2.1 [control\\_t](#)() [1/3]

```
glucat::control_t::control_t (
    int argc,
    char ** argv ) [private]
```

Constructor from program arguments.

Test control constructor from program arguments.

Definition at line 88 of file [control.h](#).

References [GLUCAT\\_PACKAGE\\_NAME](#), [GLUCAT\\_VERSION](#), [m\\_catch\\_exceptions](#), [m\\_valid](#), [m\\_verbose\\_output](#), and [valid\(\)](#).

**6.7.2.2 control\_t()** [2/3]

```
glucat::control_t::control_t ( ) [private], [default]
```

**6.7.2.3 ~control\_t()**

```
glucat::control_t::~~control_t ( ) [private], [default]
```

**6.7.2.4 control\_t()** [3/3]

```
glucat::control_t::control_t (
    const control\_t & ) [private], [delete]
```

**6.7.3 Member Function Documentation****6.7.3.1 call()** [1/2]

```
int glucat::control_t::call (
    intfn f ) const [inline]
```

Call a function that returns int.

Definition at line [136](#) of file [control.h](#).

References [catch\\_exceptions\(\)](#), [glucat::try\\_catch\(\)](#), and [valid\(\)](#).

**6.7.3.2 call()** [2/2]

```
int glucat::control_t::call (
    intintfn f,
    int arg ) const [inline]
```

Call a function of int that returns int.

Definition at line [150](#) of file [control.h](#).

References [catch\\_exceptions\(\)](#), [glucat::try\\_catch\(\)](#), and [valid\(\)](#).

**6.7.3.3 catch\_exceptions()**

```
bool glucat::control_t::catch_exceptions ( ) const [inline], [private]
```

Definition at line [49](#) of file [control.h](#).

References [m\\_catch\\_exceptions](#).

Referenced by [call\(\)](#), and [call\(\)](#).

#### 6.7.3.4 control()

```
static const control\_t & glucat::control_t::control (
    int argc,
    char ** argv ) [inline], [static]
```

Single instance Ref: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.

Definition at line 71 of file [control.h](#).

#### 6.7.3.5 operator=()

```
control\_t & glucat::control_t::operator= (
    const control\_t & ) [private], [delete]
```

#### 6.7.3.6 valid()

```
bool glucat::control_t::valid ( ) const [inline], [private]
```

Definition at line 44 of file [control.h](#).

References [m\\_valid](#).

Referenced by [call\(\)](#), [call\(\)](#), and [control\\_t\(\)](#).

#### 6.7.3.7 verbose()

```
static bool glucat::control_t::verbose ( ) [inline], [static]
```

Produce more detailed output from tests.

Definition at line 80 of file [control.h](#).

References [m\\_verbose\\_output](#).

### 6.7.4 Friends And Related Symbol Documentation

#### 6.7.4.1 friend\_for\_private\_destructor

```
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 67 of file [control.h](#).



## 6.7.5 Member Data Documentation

### 6.7.5.1 m\_catch\_exceptions

```
bool glucat::control_t::m_catch_exceptions [private]
```

Catch exceptions.

Definition at line 48 of file [control.h](#).

Referenced by [catch\\_exceptions\(\)](#), and [control\\_t\(\)](#).

### 6.7.5.2 m\_valid

```
bool glucat::control_t::m_valid [private]
```

Test parameters are valid.

Definition at line 43 of file [control.h](#).

Referenced by [control\\_t\(\)](#), and [valid\(\)](#).

### 6.7.5.3 m\_verbose\_output

```
bool glucat::control_t::m_verbose_output = false [static], [private]
```

Produce more detailed output from tests.

Definition at line 53 of file [control.h](#).

Referenced by [control\\_t\(\)](#), and [verbose\(\)](#).

The documentation for this class was generated from the following file:

- test/[control.h](#)

## 6.8 glucat::CTAssertion< bool > Struct Template Reference

Compile time assertion.

### 6.8.1 Detailed Description

```
template<bool>
struct glucat::CTAssertion< bool >
```

Compile time assertion.

Definition at line 46 of file [global.h](#).

The documentation for this struct was generated from the following file:

- glucat/[global.h](#)

## 6.9 `glucat::CTAssertion< true >` Struct Reference

```
#include <global.h>
```

### 6.9.1 Detailed Description

Definition at line 47 of file [global.h](#).

The documentation for this struct was generated from the following file:

- [glucat/global.h](#)

## 6.10 `glucat::numeric_traits< Scalar_T >::demoted` Struct Reference

Demoted type for long double.

```
#include <promotion.h>
```

### Public Types

- using `type` = float
- using `type` = float

### 6.10.1 Detailed Description

```
template<typename Scalar_T>
struct glucat::numeric_traits< Scalar_T >::demoted
```

Demoted type for long double.

Demoted type.

Definition at line 76 of file [promotion.h](#).

### 6.10.2 Member Typedef Documentation

#### 6.10.2.1 `type` [1/2]

```
template<typename Scalar_T >
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 78 of file [promotion.h](#).

### 6.10.2.2 type [2/2]

```
template<typename Scalar_T >
using glucat::numeric_traits< Scalar_T >::demoted::type = float
```

Definition at line 148 of file [scalar.h](#).

The documentation for this struct was generated from the following files:

- [glucat/promotion.h](#)
- [glucat/scalar.h](#)

## 6.11 glucat::matrix::eig\_genus< Matrix\_T > Struct Template Reference

Structure containing classification of eigenvalues.

```
#include <matrix.h>
```

### Public Types

- using [Scalar\\_T](#) = typename Matrix\_T::value\_type

### Public Attributes

- bool [m\\_is\\_singular](#) = false  
*Is the matrix singular?*
- [eig\\_case\\_t](#) [m\\_eig\\_case](#) = safe\_eigs  
*What kind of eigenvalues does the matrix contain?*
- [Scalar\\_T](#) [m\\_safe\\_arg](#) = [Scalar\\_T](#)(0)  
*Argument such that  $\exp(\pi i m\_safe\_arg)$  lies between arguments of eigenvalues.*

### 6.11.1 Detailed Description

```
template<typename Matrix_T>
struct glucat::matrix::eig_genus< Matrix_T >
```

Structure containing classification of eigenvalues.

Definition at line 140 of file [matrix.h](#).

### 6.11.2 Member Typedef Documentation

#### 6.11.2.1 Scalar\_T

```
template<typename Matrix_T >
using glucat::matrix::eig_genus< Matrix_T >::Scalar_T = typename Matrix_T::value_type
```

Definition at line 142 of file [matrix.h](#).

### 6.11.3 Member Data Documentation

#### 6.11.3.1 m\_eig\_case

```
template<typename Matrix_T >
eig_case_t glucat::matrix::eig_genus< Matrix_T >::m_eig_case = safe_eigs
```

What kind of eigenvalues does the matrix contain?

Definition at line 146 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 6.11.3.2 m\_is\_singular

```
template<typename Matrix_T >
bool glucat::matrix::eig_genus< Matrix_T >::m_is_singular = false
```

Is the matrix singular?

Definition at line 144 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 6.11.3.3 m\_safe\_arg

```
template<typename Matrix_T >
Scalar_T glucat::matrix::eig_genus< Matrix_T >::m_safe_arg = Scalar_T(0)
```

Argument such that  $\exp(\pi \cdot m\_safe\_arg)$  lies between arguments of eigenvalues.

Definition at line 148 of file [matrix.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

The documentation for this struct was generated from the following file:

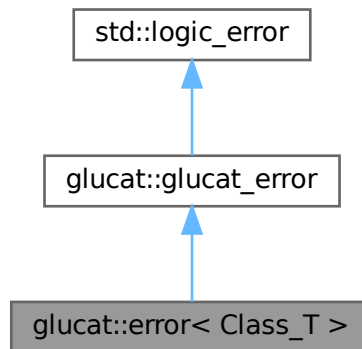
- [glucat/matrix.h](#)

## 6.12 glucat::error< Class\_T > Class Template Reference

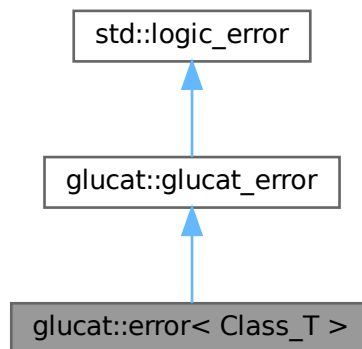
Specific exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::error< Class\_T >:



Collaboration diagram for glucat::error< Class\_T >:



### Public Member Functions

- [error](#) (const std::string &msg)  
*Specific exception class.*
- [error](#) (const std::string &context, const std::string &msg)
- auto [heading](#) () const noexcept -> const std::string override
- auto [classname](#) () const noexcept -> const std::string override
- void [print\\_error\\_msg](#) () const override

## Public Member Functions inherited from [glucat::glucat\\_error](#)

- [glucat\\_error](#) (const std::string &context, const std::string &msg)
- [~glucat\\_error](#) () noexcept override=default

## Additional Inherited Members

## Public Attributes inherited from [glucat::glucat\\_error](#)

- std::string [name](#)

### 6.12.1 Detailed Description

```
template<class Class_T>
class glucat::error< Class_T >
```

Specific exception class.

Definition at line 56 of file [errors.h](#).

### 6.12.2 Constructor & Destructor Documentation

#### 6.12.2.1 error() [1/2]

```
template<class Class_T >
glucat::error< Class_T >::error (
    const std::string & msg )
```

Specific exception class.

Definition at line 44 of file [errors\\_imp.h](#).

#### 6.12.2.2 error() [2/2]

```
template<class Class_T >
glucat::error< Class_T >::error (
    const std::string & context,
    const std::string & msg )
```

Definition at line 50 of file [errors\\_imp.h](#).

### 6.12.3 Member Function Documentation

#### 6.12.3.1 classname()

```
template<class Class_T >
auto glucat::error< Class_T >::classname ( ) const -> const std::string [override], [virtual],
[noexcept]
```

Implements [glucat::glucat\\_error](#).

Definition at line 63 of file [errors\\_imp.h](#).

6.12.3.2 `heading()`

```
template<class Class_T >
auto glucat::error< Class_T >::heading ( ) const -> const std::string [override], [virtual],
[noexcept]
```

Implements `glucat::glucat_error`.

Definition at line 57 of file `errors_imp.h`.

6.12.3.3 `print_error_msg()`

```
template<class Class_T >
void glucat::error< Class_T >::print_error_msg ( ) const [override], [virtual]
```

Implements `glucat::glucat_error`.

Definition at line 69 of file `errors_imp.h`.

The documentation for this class was generated from the following files:

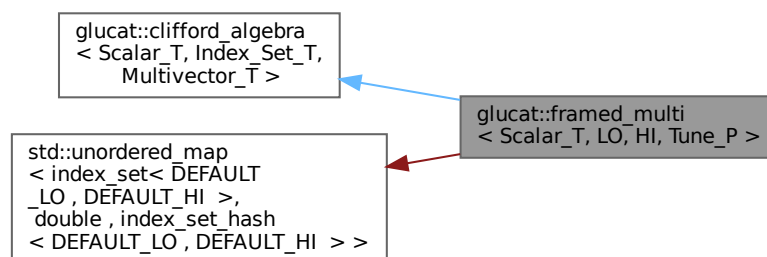
- `glucat/errors.h`
- `glucat/errors_imp.h`

## 6.13 `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >` Class Template Reference

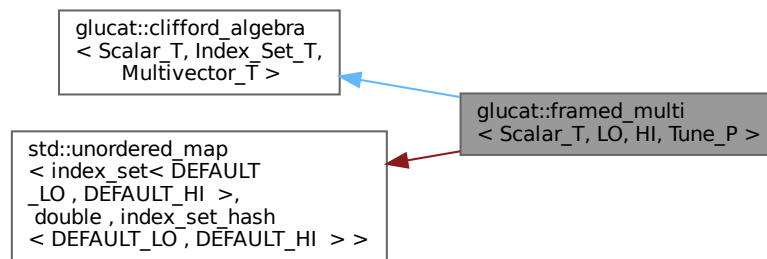
A `framed_multi<Scalar_T,LO,HI,Tune_P>` is a framed approximation to a multivector.

```
#include <framed_multi.h>
```

Inheritance diagram for `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`:



Collaboration diagram for `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`:



## Classes

- class `hash_size_t`
- class `var_term`

*Variable term.*

## Public Types

- using `multivector_t` = `framed_multi`
- using `framed_multi_t` = `multivector_t`
- using `scalar_t` = `Scalar_T`
- using `tune_p` = `Tune_P`
- using `index_set_t` = `index_set<LO, HI>`
- using `term_t` = `std::pair<const index_set_t, Scalar_T>`
- using `vector_t` = `std::vector<Scalar_T>`
- using `error_t` = `error<multivector_t>`
- using `matrix_multi_t` = `matrix_multi<Scalar_T,LO,HI,Tune_P >`

## Public Types inherited from

`glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`

- using `scalar_t` = `Scalar_T`
- using `index_set_t` = `Index_Set_T`
- using `multivector_t` = `Multivector_T`
- using `pair_t` = `std::pair<const index_set_t, Scalar_T>`
- using `vector_t` = `std::vector<Scalar_T>`



## Public Member Functions

- [~framed\\_multi](#) () override=default  
*Destructor.*
- [framed\\_multi](#) ()  
*Default constructor.*
- template<typename Other\_Scalar\_T >  
[framed\\_multi](#) (const [framed\\_multi](#)< Other\_Scalar\_T, LO, HI, Tune\_P > &val)  
*Construct a multivector from a multivector with a different scalar type.*
- template<typename Other\_Scalar\_T >  
[framed\\_multi](#) (const [framed\\_multi](#)< Other\_Scalar\_T, LO, HI, Tune\_P > &val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- [framed\\_multi](#) (const [framed\\_multi\\_t](#) &val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- [framed\\_multi](#) (const [index\\_set\\_t](#) ist, const Scalar\_T &crd=Scalar\_T(1))  
*Construct a multivector from an index set and a scalar coordinate.*
- [framed\\_multi](#) (const [index\\_set\\_t](#) ist, const Scalar\_T &crd, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from an index set and a scalar coordinate.*
- [framed\\_multi](#) (const Scalar\_T &scr, const [index\\_set\\_t](#) frm=[index\\_set\\_t](#)())  
*Construct a multivector from a scalar (within a frame, if given)*
- [framed\\_multi](#) (const int scr, const [index\\_set\\_t](#) frm=[index\\_set\\_t](#)())  
*Construct a multivector from an int (within a frame, if given)*
- [framed\\_multi](#) (const [vector\\_t](#) &vec, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given vector.*
- [framed\\_multi](#) (const std::string &str)  
*Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- [framed\\_multi](#) (const std::string &str, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- [framed\\_multi](#) (const char \*str)  
*Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*
- [framed\\_multi](#) (const char \*str, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".*
- template<typename Other\_Scalar\_T >  
[framed\\_multi](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, LO, HI, Tune\_P > &val)  
*Construct a multivector from a matrix\_multi\_t.*
- template<typename Other\_Scalar\_T >  
auto [fast\\_matrix\\_multi](#) (const [index\\_set\\_t](#) frm) const -> const [matrix\\_multi](#)< Other\_Scalar\_T, LO, HI, Tune\_P >  
*Use generalized FFT to construct a matrix\_multi\_t.*
- auto [fast\\_framed\\_multi](#) () const -> const [framed\\_multi\\_t](#)  
*Use inverse generalized FFT to construct a framed\_multi\_t.*
- [\\_GLUCAT\\_CLIFFORD\\_ALGEBRA\\_OPERATIONS](#) auto [nbr\\_terms](#) () const -> unsigned long  
*Number of terms.*
- auto [operator+=](#) (const [term\\_t](#) &term) -> [multivector\\_t](#) &  
*Add a term, if non-zero.*

## Public Member Functions inherited from [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >](#)

- virtual [~clifford\\_algebra](#) ()=default
- virtual auto [operator==](#) (const [multivector\\_t](#) &val) const -> bool=0  
*Test for equality of multivectors.*
- virtual auto [operator==](#) (const [Scalar\\_T](#) &scr) const -> bool=0  
*Test for equality of multivector and scalar.*
- virtual auto [operator+=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Geometric sum.*
- virtual auto [operator+=](#) (const [Scalar\\_T](#) &scr) -> [multivector\\_t](#) &=0  
*Geometric sum of multivector and scalar.*
- virtual auto [operator-=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Geometric difference.*
- virtual auto [operator-=](#) (const [Scalar\\_T](#) &scr) -> [multivector\\_t](#) &=0  
*Geometric difference of multivector and scalar.*
- virtual auto [operator-](#) () const -> const [multivector\\_t](#)=0  
*Unary -.*
- virtual auto [operator\\*=](#) (const [Scalar\\_T](#) &scr) -> [multivector\\_t](#) &=0  
*Product of multivector and scalar.*
- virtual auto [operator\\*=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Geometric product.*
- virtual auto [operator%=>](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Contraction.*
- virtual auto [operator&=>](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Inner product.*
- virtual auto [operator^=>](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Outer product.*
- virtual auto [operator/=](#) (const [Scalar\\_T](#) &scr) -> [multivector\\_t](#) &=0  
*Quotient of multivector and scalar.*
- virtual auto [operator/=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Geometric quotient.*
- virtual auto [operator|=](#) (const [multivector\\_t](#) &rhs) -> [multivector\\_t](#) &=0  
*Transformation via twisted adjoint action.*
- virtual auto [inv](#) () const -> const [multivector\\_t](#)=0  
*Geometric multiplicative inverse.*
- virtual auto [pow](#) (int m) const -> const [multivector\\_t](#)=0  
*\*this to the m*
- virtual auto [outer\\_pow](#) (int m) const -> const [multivector\\_t](#)=0  
*Outer product power.*
- virtual auto [frame](#) () const -> const [index\\_set\\_t](#)=0  
*Subalgebra generated by all generators of terms of given multivector.*
- virtual auto [grade](#) () const -> [index\\_t](#)=0  
*Maximum of the grades of each term.*
- virtual auto [operator\[\]](#) (const [index\\_set\\_t](#) ist) const -> [Scalar\\_T](#)=0  
*Subscripting: map from index set to scalar coordinate.*
- virtual auto [operator\(\)](#) ([index\\_t](#) grade) const -> const [multivector\\_t](#)=0  
*Pure grade-vector part.*
- virtual auto [scalar](#) () const -> [Scalar\\_T](#)=0  
*Scalar part.*
- virtual auto [pure](#) () const -> const [multivector\\_t](#)=0

- *Pure part.*  
virtual auto [even](#) () const -> const [multivector\\_t](#)=0
- *Even part of multivector, sum of even grade terms.*  
virtual auto [odd](#) () const -> const [multivector\\_t](#)=0
- *Odd part of multivector, sum of odd grade terms.*  
virtual auto [vector\\_part](#) () const -> const [vector\\_t](#)=0
- *Vector part of multivector, as a vector\_t with respect to [frame\(\)](#)*  
virtual auto [vector\\_part](#) (const [index\\_set\\_t](#) frm, const bool prechecked) const -> const [vector\\_t](#)=0
- *Vector part of multivector, as a vector\_t with respect to frm.*  
virtual auto [involute](#) () const -> const [multivector\\_t](#)=0
- *Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*  
virtual auto [reverse](#) () const -> const [multivector\\_t](#)=0
- *Reversion, eg. {1}\*{2} -> {2}\*{1}.*  
virtual auto [conj](#) () const -> const [multivector\\_t](#)=0
- *Conjugation, reverse o involute == involute o reverse.*  
virtual auto [quad](#) () const -> [Scalar\\_T](#)=0
- *Scalar\_T quadratic form == (rev(x)\*x)(0)*  
virtual auto [norm](#) () const -> [Scalar\\_T](#)=0
- *Scalar\_T norm == sum of norm of coordinates.*  
virtual auto [max\\_abs](#) () const -> [Scalar\\_T](#)=0
- *Maximum of absolute values of components of multivector: multivector infinity norm.*  
virtual auto [truncated](#) (const [Scalar\\_T](#) &limit=[default\\_truncation](#)) const -> const [multivector\\_t](#)=0
- *Remove all terms with relative size smaller than limit.*  
virtual auto [isinf](#) () const -> bool=0
- *Check if a multivector contains any infinite values.*  
virtual auto [isnan](#) () const -> bool=0
- *Check if a multivector contains any IEEE NaN values.*  
virtual void [write](#) (const std::string &msg="") const =0
- *Write formatted multivector to output.*  
virtual void [write](#) (std::ofstream &ofile, const std::string &msg="") const =0
- *Write formatted multivector to file.*

### Static Public Member Functions

- static auto [classname](#) () -> const std::string  
*Class name used in messages.*
- static auto [random](#) (const [index\\_set\\_t](#) frm, [Scalar\\_T](#) fill=[Scalar\\_T](#)(1)) -> const [multivector\\_t](#)  
*Random multivector within a frame.*

### Static Public Member Functions inherited from [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >](#)

- static auto [classname](#) () -> const std::string

## Private Types

- using `var_term_t` = class `var_term`
- using `matrix_t` = typename `matrix_multi_t::matrix_t`
- using `sorted_map_t` = `std::map< index_set_t, Scalar_T, std::less<const index_set_t> >`
- using `map_t` = `std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>>`
- using `framed_pair_t` = `std::pair<const multivector_t, const multivector_t>`
- using `size_type` = typename `map_t::size_type`
- using `iterator` = typename `map_t::iterator`
- using `const_iterator` = typename `map_t::const_iterator`

## Private Member Functions

- `framed_multi` (const `hash_size_t` &hash\_size)  
*Private constructor using hash\_size.*
- auto `fold` (const `index_set_t` frm) const -> `multivector_t`  
*Subalgebra isomorphism: fold each term within the given frame.*
- auto `unfold` (const `index_set_t` frm) const -> `multivector_t`  
*Subalgebra isomorphism: unfold each term within the given frame.*
- auto `centre_pm4_qp4` (`index_t` &p, `index_t` &q) -> `multivector_t` &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p-4,q+4\}}$ .*
- auto `centre_pp4_qm4` (`index_t` &p, `index_t` &q) -> `multivector_t` &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p+4,q-4\}}$ .*
- auto `centre_qp1_pm1` (`index_t` &p, `index_t` &q) -> `multivector_t` &  
*Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{q+1,p-1\}}$ .*
- auto `divide` (const `index_set_t` ist) const -> const `framed_pair_t`  
*Divide multivector into part divisible by `index_set` and remainder.*
- auto `fast` (const `index_t` level, const bool odd) const -> const `matrix_t`  
*Generalized FFT from `multivector_t` to `matrix_t`.*

## Friends

- template<typename Other\_Scalar\_T, const `index_t` Other\_LO, const `index_t` Other\_HI, typename Other\_Tune\_P >  
class `matrix_multi`
- template<typename Other\_Scalar\_T, const `index_t` Other\_LO, const `index_t` Other\_HI, typename Other\_Tune\_P >  
class `framed_multi`
- auto `operator*` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator^` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator&` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator%` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `star` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> `Scalar_T`
- auto `operator/` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator|` (const `multivector_t` &lhs, const `multivector_t` &rhs) -> const `multivector_t`
- auto `operator>>` (std::istream &s, `multivector_t` &val) -> std::istream &
- auto `operator<<` (std::ostream &os, const `multivector_t` &val) -> std::ostream &
- auto `operator<<` (std::ostream &os, const `term_t` &term) -> std::ostream &
- auto `exp` (const `multivector_t` &val) -> const `multivector_t`

## Additional Inherited Members

### Static Public Attributes inherited from

[glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >](#)

- static const [index\\_t v\\_lo](#) = index\_set\_t::v\_lo
- static const [index\\_t v\\_hi](#) = index\_set\_t::v\_hi
- static const [Scalar\\_T default\\_truncation](#) = std::numeric\_limits<Scalar\_T>::epsilon()

*Default for truncation.*

## 6.13.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar\_T, LO, HI, Tune\_P >
```

A framed\_multi<Scalar\_T,LO,HI,Tune\_P> is a framed approximation to a multivector.

Definition at line 126 of file [framed\\_multi.h](#).

## 6.13.2 Member Typedef Documentation

### 6.13.2.1 const\_iterator

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::const_iterator = typename map_t<
::const_iterator [private]
```

Definition at line 167 of file [framed\\_multi.h](#).

### 6.13.2.2 error\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::error_t = error<multivector\_t>
```

Definition at line 138 of file [framed\\_multi.h](#).

### 6.13.2.3 framed\_multi\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::framed_multi_t = multivector\_t
```

Definition at line 132 of file [framed\\_multi.h](#).

#### 6.13.2.4 framed\_pair\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_pair_t = std::pair<const multivector_t,
const multivector_t> [private]
```

Definition at line 164 of file [framed\\_multi.h](#).

#### 6.13.2.5 index\_set\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 135 of file [framed\\_multi.h](#).

#### 6.13.2.6 iterator

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::iterator = typename map_t::iterator
[private]
```

Definition at line 166 of file [framed\\_multi.h](#).

#### 6.13.2.7 map\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::map_t = std::unordered_map<index_set_t,
Scalar_T, index_set_hash<LO, HI>> [private]
```

Definition at line 150 of file [framed\\_multi.h](#).

#### 6.13.2.8 matrix\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = matrix_multi<Scalar_T, LO, HI, Tune_P >
```

Definition at line 139 of file [framed\\_multi.h](#).

#### 6.13.2.9 matrix\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = typename matrix_multi_t::matrix_t
[private]
```

Definition at line 148 of file [framed\\_multi.h](#).

### 6.13.2.10 multivector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = framed_multi
```

Definition at line 131 of file [framed\\_multi.h](#).

### 6.13.2.11 scalar\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 133 of file [framed\\_multi.h](#).

### 6.13.2.12 size\_type

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::size_type = typename map_t::size_type
[private]
```

Definition at line 165 of file [framed\\_multi.h](#).

### 6.13.2.13 sorted\_map\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::sorted_map_t = std::map< index_set_t,
Scalar_T, std::less<const index_set_t> > [private]
```

Definition at line 149 of file [framed\\_multi.h](#).

### 6.13.2.14 term\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 136 of file [framed\\_multi.h](#).

### 6.13.2.15 tune\_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 134 of file [framed\\_multi.h](#).

### 6.13.2.16 var\_term\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term_t = class var_term [private]
```

Definition at line 147 of file [framed\\_multi.h](#).

### 6.13.2.17 vector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 137 of file [framed\\_multi.h](#).

## 6.13.3 Constructor & Destructor Documentation

### 6.13.3.1 ~framed\_multi()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::~~framed_multi ( ) [override], [default]
```

Destructor.

### 6.13.3.2 framed\_multi() [1/15]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi ( )
```

Default constructor.

Definition at line 59 of file [framed\\_multi\\_imp.h](#).

### 6.13.3.3 framed\_multi() [2/15]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const hash_size_t & hash_size ) [private]
```

Private constructor using hash\_size.

Definition at line 66 of file [framed\\_multi\\_imp.h](#).



**6.13.3.4 framed\_multi()** [3/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 74 of file [framed\\_multi\\_imp.h](#).

**6.13.3.5 framed\_multi()** [4/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 85 of file [framed\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [Index\\_Set\\_T](#), [Multivector\\_T](#) >::frame().

**6.13.3.6 framed\_multi()** [5/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const framed\_multi\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 98 of file [framed\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [Index\\_Set\\_T](#), [Multivector\\_T](#) >::frame().

**6.13.3.7 framed\_multi()** [6/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1) )
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 111 of file [framed\\_multi\\_imp.h](#).

**6.13.3.8 framed\_multi()** [7/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 121 of file [framed\\_multi\\_imp.h](#).

**6.13.3.9 framed\_multi()** [8/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from a scalar (within a frame, if given)

Definition at line 134 of file [framed\\_multi\\_imp.h](#).

**6.13.3.10 framed\_multi()** [9/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from an int (within a frame, if given)

Definition at line 144 of file [framed\\_multi\\_imp.h](#).

**6.13.3.11 framed\_multi()** [10/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 154 of file [framed\\_multi\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::index\\_set< LO, HI >::max\(\)](#), and [glucat::index\\_set< LO, HI >::min\(\)](#).

**6.13.3.12 framed\_multi()** [11/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str )
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 176 of file [framed\\_multi\\_imp.h](#).

**6.13.3.13 framed\_multi()** [12/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 192 of file [framed\\_multi\\_imp.h](#).

**6.13.3.14 framed\_multi()** [13/15]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str ) [inline]
```

Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 209 of file [framed\\_multi.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi](#).

**6.13.3.15 framed\_multi()** [14/15]

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const char * str,
    const index\_set\_t frm,
    const bool prechecked = false ) [inline]
```

Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 212 of file [framed\\_multi.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi](#).

### 6.13.3.16 framed\_multi() [15/15]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::framed_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a [matrix\\_multi\\_t](#).

Definition at line 205 of file [framed\\_multi\\_imp.h](#).

References [\\_GLUCAT\\_HASH\\_SIZE\\_T](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::frame\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_ma](#), [glucat::matrix::nnz\(\)](#), [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::norm\(\)](#), and [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::norm\(\)](#).

## 6.13.4 Member Function Documentation

### 6.13.4.1 centre\_pm4\_qp4()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::centre_pm4_qp4 (
    index\_t & p,
    index\_t & q ) -> multivector\_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p-4,q+4\}}$ .

Definition at line 1469 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

### 6.13.4.2 centre\_pp4\_qm4()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::centre_pp4_qm4 (
    index\_t & p,
    index\_t & q ) -> multivector\_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{p+4,q-4\}}$ .

Definition at line 1511 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

### 6.13.4.3 centre\_qp1\_pm1()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::centre_qp1_pm1 (
    index\_t & p,
    index\_t & q ) -> multivector\_t& [private]
```

Subalgebra isomorphism:  $R_{\{p,q\}}$  to  $R_{\{q+1,p-1\}}$ .

Definition at line 1553 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

#### 6.13.4.4 classname()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::classname ( ) -> const std::string
[static]
```

Class name used in messages.

Definition at line 50 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.5 divide()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::divide (
    const index\_set\_t ist ) const -> const framed\_pair\_t [private]
```

Divide multivector into part divisible by [index\\_set](#) and remainder.

Divide multivector into quotient with terms divisible by index set, and remainder.

Definition at line 1586 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.6 fast()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fast (
    const index\_t level,
    const bool odd ) const -> const matrix\_t [private]
```

Generalized FFT from multivector\_t to matrix\_t.

Definition at line 1602 of file [framed\\_multi\\_imp.h](#).

References [glucat::matrix::kron\(\)](#), [glucat::odd\(\)](#), and [glucat::scalar\(\)](#).

#### 6.13.4.7 fast\_framed\_multi()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi ( ) const -> const
framed\_multi\_t [inline]
```

Use inverse generalized FFT to construct a framed\_multi\_t.

Definition at line 1700 of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.8 fast\_matrix\_multi()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index\_set\_t frm ) const -> const matrix\_multi<Other_Scalar_T,LO,HI,Tune_P
>
```

Use generalized FFT to construct a [matrix\\_multi\\_t](#).

Definition at line [1668](#) of file [framed\\_multi\\_imp.h](#).

References [glucat::gen::offset\\_to\\_super](#), and [glucat::pos\\_mod\(\)](#).

#### 6.13.4.9 fold()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::fold (
    const index\_set\_t frm ) const -> multivector\_t [private]
```

Subalgebra isomorphism: fold each term within the given frame.

Definition at line [1434](#) of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.10 nbr\_terms()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::nbr_terms ( ) const -> unsigned long
```

Number of terms.

Definition at line [1356](#) of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.11 operator+=(())

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term\_t & term ) -> multivector\_t& [inline]
```

Add a term, if non-zero.

Insert a term into a multivector, add terms with same index set.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line [295](#) of file [framed\\_multi\\_imp.h](#).

#### 6.13.4.12 random()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index_set_t frm,
    Scalar_T fill = Scalar_T(1) ) -> const multivector_t [static]
```

Random multivector within a frame.

Definition at line 1058 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::random\(\)](#).

#### 6.13.4.13 unfold()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::unfold (
    const index_set_t frm ) const -> multivector_t [private]
```

Subalgebra isomorphism: unfold each term within the given frame.

Definition at line 1451 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::fast\\_framed\\_multi\(\)](#).

### 6.13.5 Friends And Related Symbol Documentation

#### 6.13.5.1 exp

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
auto exp (
    const multivector_t & val ) -> const multivector_t [friend]
```

#### 6.13.5.2 framed\_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
friend class framed_multi [friend]
```

Definition at line 143 of file [framed\\_multi.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

### 6.13.5.3 matrix\_multi

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >
friend class matrix_multi [friend]
```

Definition at line 141 of file [framed\\_multi.h](#).

### 6.13.5.4 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

### 6.13.5.5 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

### 6.13.5.6 operator\*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

### 6.13.5.7 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

### 6.13.5.8 operator<< [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val ) -> std::ostream & [friend]
```



**6.13.5.9 operator<< [2/2]**

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const term_t & term ) -> std::ostream & [friend]
```

**6.13.5.10 operator>>**

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector_t & val ) -> std::istream & [friend]
```

**6.13.5.11 operator^**

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

**6.13.5.12 operator"|**

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> const multivector_t [friend]
```

**6.13.5.13 star**

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const multivector_t & lhs,
    const multivector_t & rhs ) -> Scalar_T [friend]
```

The documentation for this class was generated from the following files:

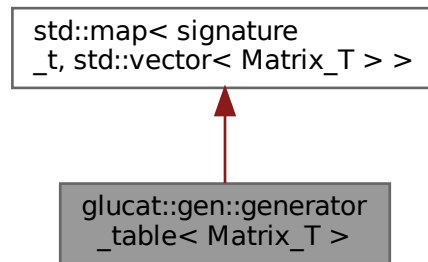
- [glucat/framed\\_multi.h](#)
- [glucat/framed\\_multi\\_imp.h](#)

## 6.14 glucat::gen::generator\_table< Matrix\_T > Class Template Reference

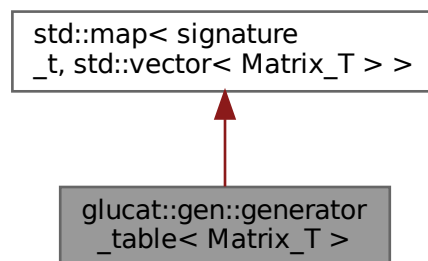
Table of generators for specific signatures.

```
#include <generation.h>
```

Inheritance diagram for glucat::gen::generator\_table< Matrix\_T >:



Collaboration diagram for glucat::gen::generator\_table< Matrix\_T >:



### Public Member Functions

- auto `operator()` (const `index_t` p, const `index_t` q) -> const `Matrix_T` \*  
*Pointer to generators for a specific signature.*
- `generator_table` (const `generator_table` &)=delete
- auto `operator=` (const `generator_table` &) -> `generator_table` &=delete

### Static Public Member Functions

- static auto `generator` () -> `generator_table`< `Matrix_T` > &  
*Single instance of generator table.*

### Private Member Functions

- auto [gen\\_vector](#) (const [index\\_t](#) p, const [index\\_t](#) q) -> const std::vector< Matrix\_T > &  
*Construct a vector of generators for a specific signature.*
- void [gen\\_from\\_pm1\\_qm1](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p-1,q-1.*
- void [gen\\_from\\_pm4\\_qp4](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p-4,q+4.*
- void [gen\\_from\\_pp4\\_qm4](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for p+4,q-4.*
- void [gen\\_from\\_qp1\\_pm1](#) (const std::vector< Matrix\_T > &old, const [signature\\_t](#) sig)  
*Construct generators for p,q given generators for q+1,p-1.*
- [generator\\_table](#) ()=default
- [~generator\\_table](#) ()=default

### Friends

- class [friend\\_for\\_private\\_destructor](#)

## 6.14.1 Detailed Description

```
template<class Matrix_T>
class glucat::gen::generator_table< Matrix_T >
```

Table of generators for specific signatures.

Definition at line 52 of file [generation.h](#).

## 6.14.2 Constructor & Destructor Documentation

### 6.14.2.1 generator\_table() [1/2]

```
template<class Matrix_T >
glucat::gen::generator_table< Matrix_T >::generator_table ( ) [private], [default]
```

### 6.14.2.2 ~generator\_table()

```
template<class Matrix_T >
glucat::gen::generator_table< Matrix_T >::~~generator_table ( ) [private], [default]
```

### 6.14.2.3 generator\_table() [2/2]

```
template<class Matrix_T >
glucat::gen::generator_table< Matrix_T >::generator_table (
    const generator\_table< Matrix_T > & ) [delete]
```

### 6.14.3 Member Function Documentation

#### 6.14.3.1 `gen_from_pm1_qm1()`

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_pm1_qm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for p-1,q-1.

Definition at line 127 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_kron\(\)](#).

#### 6.14.3.2 `gen_from_pm4_qp4()`

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_pm4_qp4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for p-4,q+4.

Definition at line 165 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

#### 6.14.3.3 `gen_from_pp4_qm4()`

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_pp4_qm4 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for p+4,q-4.

Definition at line 198 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

#### 6.14.3.4 `gen_from_qp1_pm1()`

```
template<class Matrix_T >
void glucat::gen::generator_table< Matrix_T >::gen_from_qp1_pm1 (
    const std::vector< Matrix_T > & old,
    const signature_t sig ) [private]
```

Construct generators for p,q given generators for q+1,p-1.

Definition at line 231 of file [generation\\_imp.h](#).

References [glucat::matrix::mono\\_prod\(\)](#).

### 6.14.3.5 gen\_vector()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::gen_vector (
    const index_t p,
    const index_t q ) -> const std::vector<Matrix_T>& [private]
```

Construct a vector of generators for a specific signature.

Definition at line 79 of file [generation\\_imp.h](#).

References [glucat::pos\\_mod\(\)](#).

### 6.14.3.6 generator()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::generator ( ) -> generator_table<Matrix_T>&
[static]
```

Single instance of generator table.

Definition at line 49 of file [generation\\_imp.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::basis\\_element\(\)](#).

### 6.14.3.7 operator()()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::operator() (
    const index_t p,
    const index_t q ) -> const Matrix_T* [inline]
```

Pointer to generators for a specific signature.

Definition at line 58 of file [generation\\_imp.h](#).

References [glucat::gen::offset\\_to\\_super](#), and [glucat::pos\\_mod\(\)](#).

### 6.14.3.8 operator=()

```
template<class Matrix_T >
auto glucat::gen::generator_table< Matrix_T >::operator= (
    const generator_table< Matrix_T > & ) -> generator_table &=delete [delete]
```

## 6.14.4 Friends And Related Symbol Documentation

### 6.14.4.1 friend\_for\_private\_destructor

```
template<class Matrix_T >
friend class friend_for_private_destructor [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 75 of file [generation.h](#).

The documentation for this class was generated from the following files:

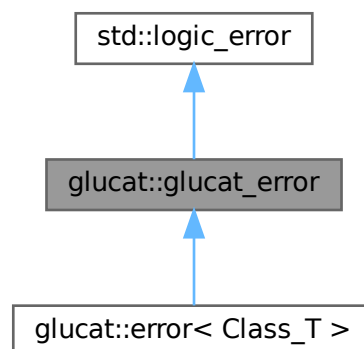
- [glucat/generation.h](#)
- [glucat/generation\\_imp.h](#)

## 6.15 glucat::glucat\_error Class Reference

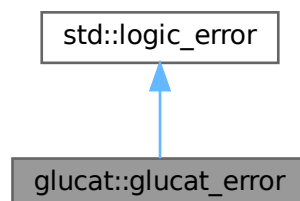
Abstract exception class.

```
#include <errors.h>
```

Inheritance diagram for glucat::glucat\_error:



Collaboration diagram for glucat::glucat\_error:



## Public Member Functions

- [glucat\\_error](#) (const std::string &context, const std::string &msg)
- [~glucat\\_error](#) () noexcept override=default
- virtual auto [heading](#) () const noexcept -> const std::string=0
- virtual auto [classname](#) () const noexcept -> const std::string=0
- virtual void [print\\_error\\_msg](#) () const =0

## Public Attributes

- std::string [name](#)

### 6.15.1 Detailed Description

Abstract exception class.

Definition at line 41 of file [errors.h](#).

### 6.15.2 Constructor & Destructor Documentation

#### 6.15.2.1 glucat\_error()

```
glucat::glucat_error::glucat_error (
    const std::string & context,
    const std::string & msg ) [inline]
```

Definition at line 44 of file [errors.h](#).

#### 6.15.2.2 ~glucat\_error()

```
glucat::glucat_error::~~glucat_error ( ) [override], [default], [noexcept]
```

### 6.15.3 Member Function Documentation

#### 6.15.3.1 classname()

```
virtual auto glucat::glucat_error::classname ( ) const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class\\_T >](#).

#### 6.15.3.2 heading()

```
virtual auto glucat::glucat_error::heading ( ) const -> const std::string [pure virtual],
[noexcept]
```

Implemented in [glucat::error< Class\\_T >](#).

### 6.15.3.3 print\_error\_msg()

```
virtual void glucat::glucat_error::print_error_msg ( ) const [pure virtual]
```

Implemented in [glucat::error< Class\\_T >](#).

## 6.15.4 Member Data Documentation

### 6.15.4.1 name

```
std::string glucat::glucat_error::name
```

Definition at line 51 of file [errors.h](#).

The documentation for this class was generated from the following file:

- [glucat/errors.h](#)

## 6.16 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::hash\_size\_t Class Reference

### Public Member Functions

- [hash\\_size\\_t](#) (size\_t hash\_size)
- auto [operator\(\)](#) () const -> size\_t

### Private Attributes

- size\_t [n](#)

### 6.16.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t
```

Definition at line 152 of file [framed\\_multi.h](#).

### 6.16.2 Constructor & Destructor Documentation

#### 6.16.2.1 hash\_size\_t()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
glucat::framed\_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::hash_size_t (
    size_t hash_size ) [inline]
```

Definition at line 155 of file [framed\\_multi.h](#).



### 6.16.3 Member Function Documentation

#### 6.16.3.1 operator()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::operator() ( ) const ->
size_t [inline]
```

Definition at line 158 of file [framed\\_multi.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::hash\\_size\\_t::n](#).

### 6.16.4 Member Data Documentation

#### 6.16.4.1 n

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
size_t glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t::n [private]
```

Definition at line 161 of file [framed\\_multi.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::hash\\_size\\_t::operator\(\)](#).

The documentation for this class was generated from the following file:

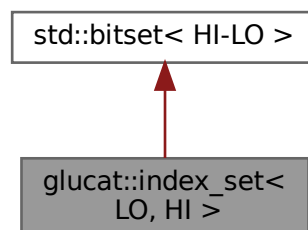
- [glucat/framed\\_multi.h](#)

## 6.17 glucat::index\_set< LO, HI > Class Template Reference

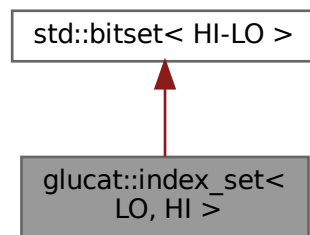
Index set class based on `std::bitset<>` in Gnu standard C++ library.

```
#include <index_set.h>
```

Inheritance diagram for `glucat::index_set< LO, HI >`:



Collaboration diagram for `glucat::index_set< LO, HI >`:



## Classes

- class [reference](#)  
*Index set member reference.*

## Public Types

- using [index\\_set\\_t](#) = [index\\_set](#)
- using [index\\_pair\\_t](#) = `std::pair<index\_t, index\_t>`

## Public Member Functions

- [index\\_set](#) ()=default  
*Default constructor creates an empty set.*
- [index\\_set](#) (const [bitset\\_t](#) bst)  
*Constructor from [bitset\\_t](#).*
- [index\\_set](#) (const [index\\_t](#) idx)  
*Constructor from [index](#).*
- [index\\_set](#) (const [set\\_value\\_t](#) folded\_val, const [index\\_set\\_t](#) frm, const bool prechecked=false)  
*Constructor from [set value](#) of an [index set](#) folded within the given frame.*
- [index\\_set](#) (const [index\\_pair\\_t](#) &range, const bool prechecked=false)  
*Constructor from range of indices from [range.first](#) to [range.second](#).*
- [index\\_set](#) (const std::string &str)  
*Constructor from [string](#).*
- auto [operator==](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Equality.*
- auto [operator!=](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Inequality.*
- auto [operator~](#) () const -> [index\\_set\\_t](#)  
*Set complement: [not](#).*
- auto [operator^](#) = (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &  
*Symmetric set difference: [exclusive or](#).*
- auto [operator&=](#) (const [index\\_set\\_t](#) rhs) -> [index\\_set\\_t](#) &

- Set intersection: and.*

  - auto `operator|=` (const `index_set_t` rhs) -> `index_set_t` &
- Set union: or.*

  - auto `operator[]` (const `index_t` idx) const -> bool
- Subscripting: Test idx for membership: test value of bit idx.*

  - auto `test` (const `index_t` idx) const -> bool
- Test idx for membership: test value of bit idx.*

  - auto `set` () -> `index_set_t` &
- Include all indices except 0: set all bits except 0.*

  - auto `set` (const `index_t` idx) -> `index_set_t` &
- Include idx: Set bit at idx if idx != 0.*

  - auto `set` (const `index_t` idx, const int val) -> `index_set_t` &
- Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.*

  - auto `reset` () -> `index_set_t` &
- Make set empty: Set all bits to 0.*

  - auto `reset` (const `index_t` idx) -> `index_set_t` &
- Exclude idx: Set bit at idx to 0.*

  - auto `flip` () -> `index_set_t` &
- Set complement, except 0: flip all bits, except 0.*

  - auto `flip` (const `index_t` idx) -> `index_set_t` &
- Complement membership of idx if idx != 0: flip bit at idx if idx != 0.*

  - auto `count` () const -> `index_t`
- Cardinality: Number of indices included in set.*

  - auto `count_neg` () const -> `index_t`
- Number of negative indices included in set.*

  - auto `count_pos` () const -> `index_t`
- Number of positive indices included in set.*

  - auto `min` () const -> `index_t`
- Minimum member.*

  - auto `max` () const -> `index_t`
- Maximum member.*

  - auto `operator<` (const `index_set_t` rhs) const -> bool
- Less than operator used for comparisons, map, etc.*

  - auto `is_contiguous` () const -> bool
- Determine if the index set is contiguous, ie. has no gaps.*

  - auto `fold` () const -> const `index_set_t`
- Fold this index set within itself as a frame.*

  - auto `fold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`
- Fold this index set within the given frame.*

  - auto `unfold` (const `index_set_t` frm, const bool prechecked=false) const -> const `index_set_t`
- Unfold this index set within the given frame.*

  - auto `value_of_fold` (const `index_set_t` frm) const -> `set_value_t`
- The set value of the fold of this index set within the given frame.*

  - auto `sign_of_mult` (const `index_set_t` ist) const -> int
- Sign of geometric product of two Clifford basis elements.*

  - auto `sign_of_square` () const -> int
- Sign of geometric square of a Clifford basis element.*

  - auto `hash_fn` () const -> `size_t`
- Hash function.*

  - auto `operator[]` (`index_t` idx) -> [reference](#)
- Subscripting: Element access.*

### Static Public Member Functions

- static auto [classname](#) () -> const std::string

### Static Public Attributes

- static const [index\\_t v\\_lo](#) = LO
- static const [index\\_t v\\_hi](#) = HI

### Private Types

- using [bitset\\_t](#) = std::bitset<HI - LO>
- using [error\\_t](#) = [error](#)<[index\\_set](#)>

### Private Member Functions

- [BOOST\\_STATIC\\_ASSERT](#) ((LO<=0) &&(0<=HI) &&(LO< HI) &&(-LO< \_GLUCAT\_BITS\_PER\_ULONG) &&(HI< \_GLUCAT\_BITS\_PER\_ULONG) &&(HI-LO<=\_GLUCAT\_BITS\_PER\_ULONG))
- auto [lex\\_less\\_than](#) (const [index\\_set\\_t](#) rhs) const -> bool  
*Lexicographic ordering of two sets: \*this < rhs.*

### Friends

- class [reference](#)
- auto [operator^](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [operator&](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [operator|](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> const [index\\_set\\_t](#)
- auto [compare](#) (const [index\\_set\\_t](#) &lhs, const [index\\_set\\_t](#) &rhs) -> int

## 6.17.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index\_set< LO, HI >
```

Index set class based on std::bitset<> in Gnu standard C++ library.

Definition at line 73 of file [index\\_set.h](#).

## 6.17.2 Member Typedef Documentation

### 6.17.2.1 [bitset\\_t](#)

```
template<const index\_t LO, const index\_t HI>
using glucat::index\_set< LO, HI >::bitset\_t = std::bitset<HI - LO> [private]
```

Definition at line 81 of file [index\\_set.h](#).

### 6.17.2.2 error\_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::error_t = error<index_set> [private]
```

Definition at line 82 of file [index\\_set.h](#).

### 6.17.2.3 index\_pair\_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::index_pair_t = std::pair<index_t, index_t>
```

Definition at line 85 of file [index\\_set.h](#).

### 6.17.2.4 index\_set\_t

```
template<const index_t LO, const index_t HI>
using glucat::index_set< LO, HI >::index_set_t = index_set
```

Definition at line 84 of file [index\\_set.h](#).

## 6.17.3 Constructor & Destructor Documentation

### 6.17.3.1 index\_set() [1/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set ( ) [default]
```

Default constructor creates an empty set.

### 6.17.3.2 index\_set() [2/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const bitset_t bst )
```

Constructor from bitset\_t.

Definition at line 61 of file [index\\_set\\_imp.h](#).

### 6.17.3.3 index\_set() [3/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const index_t idx )
```

Constructor from index.

Constructor from index value.

Definition at line 55 of file [index\\_set\\_imp.h](#).

**6.17.3.4 index\_set()** [4/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const set_value_t folded_val,
    const index_set_t frm,
    const bool prechecked = false )
```

Constructor from set value of an index set folded within the given frame.

Definition at line 68 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::index\\_set< LO, HI >::fold\(\)](#), [glucat::index\\_set< LO, HI >::min\(\)](#), and [glucat::index\\_set< LO, HI >::unfold\(\)](#).

**6.17.3.5 index\_set()** [5/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const index_pair_t & range,
    const bool prechecked = false )
```

Constructor from range of indices from range.first to range.second.

Definition at line 82 of file [index\\_set\\_imp.h](#).

**6.17.3.6 index\_set()** [6/6]

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::index_set (
    const std::string & str )
```

Constructor from string.

Definition at line 102 of file [index\\_set\\_imp.h](#).

**6.17.4 Member Function Documentation****6.17.4.1 BOOST\_STATIC\_ASSERT()**

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::BOOST_STATIC_ASSERT (
    (LO<=0) && (0<=HI) && (LO< HI) && (-LO< _GLUCAT_BITS_PER_ULONGLONG) && (HI< _GLUCAT_↵
    BITS_PER_ULONGLONG) && (HI-LO<=_GLUCAT_BITS_PER_ULONGLONG) ) [private]
```

**6.17.4.2 classname()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::classname ( ) -> const std::string [inline], [static]
```

Definition at line 49 of file [index\\_set\\_imp.h](#).

### 6.17.4.3 count()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count ( ) const -> index_t [inline]
```

Cardinality: Number of indices included in set.

Definition at line 344 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::count\\_neg\(\)](#), [glucat::index\\_set< LO, HI >::count\\_pos\(\)](#), [glucat::framed\\_multi< Scalar\\_T, index\\_set\\_t, LO, HI >::index\\_set\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

### 6.17.4.4 count\_neg()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_neg ( ) const -> index_t [inline]
```

Number of negative indices included in set.

Definition at line 364 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#).

### 6.17.4.5 count\_pos()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::count_pos ( ) const -> index_t [inline]
```

Number of positive indices included in set.

Definition at line 376 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#).

### 6.17.4.6 flip() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip ( ) -> index_set_t& [inline]
```

Set complement, except 0: flip all bits, except 0.

Definition at line 319 of file [index\\_set\\_imp.h](#).

### 6.17.4.7 flip() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::flip (
    const index_t idx ) -> index_set_t& [inline]
```

Complement membership of idx if idx != 0: flip bit at idx if idx != 0.

Definition at line 330 of file [index\\_set\\_imp.h](#).

**6.17.4.8 fold()** [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold ( ) const -> const index_set_t [inline]
```

Fold this index set within itself as a frame.

Definition at line 747 of file [index\\_set\\_imp.h](#).

Referenced by [glucat::index\\_set< LO, HI >::index\\_set\(\)](#).

**6.17.4.9 fold()** [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::fold (
    const index_set_t frm,
    const bool prechecked = false ) const -> const index_set_t
```

Fold this index set within the given frame.

Definition at line 755 of file [index\\_set\\_imp.h](#).

**6.17.4.10 hash\_fn()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::hash_fn ( ) const -> size_t [inline]
```

Hash function.

Definition at line 950 of file [index\\_set\\_imp.h](#).

**6.17.4.11 is\_contiguous()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::is_contiguous ( ) const -> bool [inline]
```

Determine if the index set is contiguous, ie. has no gaps.

Determine if the index set is contiguous, ie. has no gaps when 0 is included.

Definition at line 732 of file [index\\_set\\_imp.h](#).

**6.17.4.12 lex\_less\_than()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::lex_less_than (
    const index_set_t rhs ) const -> bool [inline], [private]
```

Lexicographic ordering of two sets: \*this < rhs.

Definition at line 588 of file [index\\_set\\_imp.h](#).



**6.17.4.13 max()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::max ( ) const -> index_t
```

Maximum member.

Maximum member, or 0 if none.

Definition at line 550 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical.index\\_set::\\_\\_iter\\_\\_\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

**6.17.4.14 min()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::min ( ) const -> index_t
```

Minimum member.

Minimum member, or 0 if none.

Definition at line 461 of file [index\\_set\\_imp.h](#).

Referenced by [PyClical.index\\_set::\\_\\_iter\\_\\_\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::index\\_set< LO, HI >::index\\_set\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), and [glucat::index\\_set< LO, HI >::unfold\(\)](#).

**6.17.4.15 operator"!="()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator!= (
    const index_set_t rhs ) const -> bool [inline]
```

Inequality.

Definition at line 130 of file [index\\_set\\_imp.h](#).

**6.17.4.16 operator&=()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator&= (
    const index_set_t rhs ) -> index_set_t& [inline]
```

Set intersection: and.

Definition at line 174 of file [index\\_set\\_imp.h](#).

**6.17.4.17 operator<()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator< (
    const index_set_t rhs ) const -> bool [inline]
```

Less than operator used for comparisons, map, etc.

Definition at line 596 of file [index\\_set\\_imp.h](#).

**6.17.4.18 operator==( )**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator==( (
    const index_set_t rhs ) const -> bool [inline]
```

Equality.

Definition at line 119 of file [index\\_set\\_imp.h](#).

**6.17.4.19 operator[]() [1/2]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    const index_t idx ) const -> bool [inline]
```

Subscripting: Test idx for membership: test value of bit idx.

Definition at line 232 of file [index\\_set\\_imp.h](#).

**6.17.4.20 operator[]() [2/2]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator[] (
    index_t idx ) -> reference [inline]
```

Subscripting: Element access.

Definition at line 224 of file [index\\_set\\_imp.h](#).

**6.17.4.21 operator^=( )**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator^= (
    const index_set_t rhs ) -> index_set_t& [inline]
```

Symmetric set difference: exclusive or.

Definition at line 149 of file [index\\_set\\_imp.h](#).

**6.17.4.22 operator" |=()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator|= (
    const index_set_t rhs ) -> index_set_t& [inline]
```

Set union: or.

Definition at line 199 of file [index\\_set\\_imp.h](#).

**6.17.4.23 operator~()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::operator~ ( ) const -> index_set_t [inline]
```

Set complement: not.

Definition at line 141 of file [index\\_set\\_imp.h](#).

**6.17.4.24 reset() [1/2]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset ( ) -> index_set_t& [inline]
```

Make set empty: Set all bits to 0.

Definition at line 294 of file [index\\_set\\_imp.h](#).

**6.17.4.25 reset() [2/2]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reset (
    const index_t idx ) -> index_set_t& [inline]
```

Exclude idx: Set bit at idx to 0.

Definition at line 305 of file [index\\_set\\_imp.h](#).

**6.17.4.26 set() [1/3]**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set ( ) -> index_set_t& [inline]
```

Include all indices except 0: set all bits except 0.

Definition at line 255 of file [index\\_set\\_imp.h](#).

**6.17.4.27 set()** [2/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx ) -> index_set_t& [inline]
```

Include idx: Set bit at idx if idx != 0.

Definition at line 266 of file [index\\_set\\_imp.h](#).

**6.17.4.28 set()** [3/3]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::set (
    const index_t idx,
    const int val ) -> index_set_t& [inline]
```

Set membership of idx to val if idx != 0: Set bit at idx to val if idx != 0.

Definition at line 280 of file [index\\_set\\_imp.h](#).

**6.17.4.29 sign\_of\_mult()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_mult (
    const index_set_t ist ) const -> int
```

Sign of geometric product of two Clifford basis elements.

Definition at line 880 of file [index\\_set\\_imp.h](#).

References [glucat::inverse\\_gray\(\)](#), and [glucat::inverse\\_reversed\\_gray\(\)](#).

**6.17.4.30 sign\_of\_square()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::sign_of_square ( ) const -> int [inline]
```

Sign of geometric square of a Clifford basis element.

Definition at line 930 of file [index\\_set\\_imp.h](#).

**6.17.4.31 test()**

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::test (
    const index_t idx ) const -> bool [inline]
```

Test idx for membership: test value of bit idx.

Definition at line 240 of file [index\\_set\\_imp.h](#).

### 6.17.4.32 unfold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::unfold (
    const index_set_t frm,
    const bool prechecked = false ) const -> const index_set_t
```

Unfold this index set within the given frame.

Definition at line 794 of file [index\\_set\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::min\(\)](#).

Referenced by [glucat::index\\_set< LO, HI >::index\\_set\(\)](#).

### 6.17.4.33 value\_of\_fold()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::value_of_fold (
    const index_set_t frm ) const -> set_value_t [inline]
```

The set value of the fold of this index set within the given frame.

Definition at line 829 of file [index\\_set\\_imp.h](#).

## 6.17.5 Friends And Related Symbol Documentation

### 6.17.5.1 compare

```
template<const index_t LO, const index_t HI>
auto compare (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> int [friend]
```

### 6.17.5.2 operator&

```
template<const index_t LO, const index_t HI>
auto operator& (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> const index_set_t [friend]
```

### 6.17.5.3 operator^

```
template<const index_t LO, const index_t HI>
auto operator^ (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> const index_set_t [friend]
```

#### 6.17.5.4 operator" |

```
template<const index_t LO, const index_t HI>
auto operator| (
    const index_set_t & lhs,
    const index_set_t & rhs ) -> const index_set_t [friend]
```

#### 6.17.5.5 reference

```
template<const index_t LO, const index_t HI>
friend class reference [friend]
```

Definition at line 174 of file [index\\_set.h](#).

### 6.17.6 Member Data Documentation

#### 6.17.6.1 v\_hi

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_hi = HI [static]
```

Definition at line 88 of file [index\\_set.h](#).

#### 6.17.6.2 v\_lo

```
template<const index_t LO, const index_t HI>
const index_t glucat::index_set< LO, HI >::v_lo = LO [static]
```

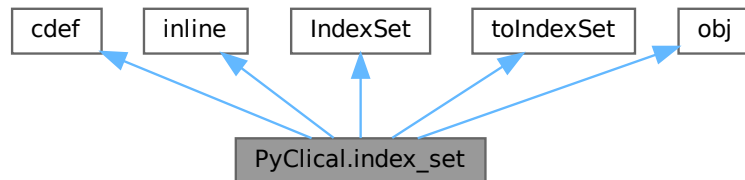
Definition at line 87 of file [index\\_set.h](#).

The documentation for this class was generated from the following files:

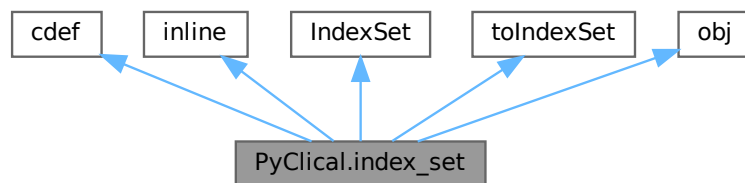
- [glucat/index\\_set.h](#)
- [glucat/index\\_set\\_imp.h](#)

## 6.18 PyClical.index\_set Class Reference

Inheritance diagram for PyClical.index\_set:



Collaboration diagram for PyClical.index\_set:



### Public Member Functions

- [\\_\\_cinit\\_\\_](#) (self, other=0)
- [\\_\\_dealloc\\_\\_](#) (self)
- [\\_\\_richcmp\\_\\_](#) (lhs, rhs, int, op)
- [\\_\\_setitem\\_\\_](#) (self, idx, val)
- [\\_\\_getitem\\_\\_](#) (self, idx)
- [\\_\\_contains\\_\\_](#) (self, idx)
- [\\_\\_iter\\_\\_](#) (self)
- [\\_\\_invert\\_\\_](#) (self)
- [\\_\\_xor\\_\\_](#) (lhs, rhs)
- [\\_\\_ixor\\_\\_](#) (self, rhs)
- [\\_\\_and\\_\\_](#) (lhs, rhs)
- [\\_\\_iand\\_\\_](#) (self, rhs)
- [\\_\\_or\\_\\_](#) (lhs, rhs)
- [\\_\\_ior\\_\\_](#) (self, rhs)
- [count](#) (self)
- [count\\_neg](#) (self)
- [count\\_pos](#) (self)
- [min](#) (self)
- [max](#) (self)

- [hash\\_fn](#) (self)
- [sign\\_of\\_mult](#) (self, rhs)
- [sign\\_of\\_square](#) (self)
- [\\_\\_repr\\_\\_](#) (self)
- [\\_\\_str\\_\\_](#) (self)

## Public Attributes

- [instance](#)

## 6.18.1 Detailed Description

Return the C++ IndexSet instance wrapped by `index_set(obj)`.

Python class `index_set` wraps C++ class `IndexSet`.

Definition at line 38 of file [PyClical.pyx](#).

## 6.18.2 Member Function Documentation

### 6.18.2.1 `__and__()`

```
PyClical.index_set.__and__ (
    lhs,
    rhs )
```

Set intersection: `and`.

```
>>> print(index_set({1}) & index_set({2}))
{}
>>> print(index_set({1,2}) & index_set({2}))
{2}
```

Definition at line 271 of file [PyClical.pyx](#).

### 6.18.2.2 `__cinit__()`

```
PyClical.index_set.__cinit__ (
    self,
    other = 0 )
```

Construct an object of type `index_set`.

```
>>> print(index_set(1))
{1}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set(index_set({1,2})))
{1,2}
>>> print(index_set({1,2}))
{1,2}
>>> print(index_set({1,2,1}))
{1,2}
>>> print(index_set("{1,2,1}"))
{1,2}
>>> print(index_set(""))
{}
```

Definition at line 74 of file [PyClical.pyx](#).



### 6.18.2.3 `__contains__()`

```
PyClical.index_set.__contains__ (
    self,
    idx )
```

Check that an `index_set` object contains the index `idx`: `idx in self`.

```
>>> 1 in index_set({1})
True
>>> 2 in index_set({1})
False
>>> -1 in index_set({2})
False
>>> 1 in index_set({2})
False
>>> 2 in index_set({2})
True
>>> 33 in index_set({2})
False
```

Definition at line 210 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.18.2.4 `__dealloc__()`

```
PyClical.index_set.__dealloc__ (
    self )
```

Clean up by deallocating the instance of C++ class `IndexSet`.

Definition at line 116 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.18.2.5 `__getitem__()`

```
PyClical.index_set.__getitem__ (
    self,
    idx )
```

Get the value of an `index_set` object at an index.

```
>>> index_set({1})[1]
True
>>> index_set({1})[2]
False
>>> index_set({2})[-1]
False
>>> index_set({2})[1]
False
>>> index_set({2})[2]
True
>>> index_set({2})[33]
False
```

Definition at line 191 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.18.2.6 `__iand__()`

```
PyCliclcal.index_set.__iand__ (
    self,
    rhs )
```

Set intersection: and.

```
>>> x = index_set({1}); x &= index_set({2}); print(x)
{}
>>> x = index_set({1,2}); x &= index_set({2}); print(x)
{2}
```

Definition at line 282 of file [PyCliclcal.pyx](#).

### 6.18.2.7 `__invert__()`

```
PyCliclcal.index_set.__invert__ (
    self )
```

Set complement: not.

```
>>> print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,
```

Definition at line 240 of file [PyCliclcal.pyx](#).

References [PyCliclcal.index\\_set.instance](#), and [PyCliclcal.clifford.instance](#).

### 6.18.2.8 `__ior__()`

```
PyCliclcal.index_set.__ior__ (
    self,
    rhs )
```

Set union: or.

```
>>> x = index_set({1}); x |= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x |= index_set({2}); print(x)
{1,2}
```

Definition at line 304 of file [PyCliclcal.pyx](#).

### 6.18.2.9 `__iter__()`

```
PyCliclcal.index_set.__iter__ (
    self )
```

Iterate over the indices of an index\_set.

```
>>> for i in index_set({-3,4,7}):print(i, end=",")
-3,4,7,
```

Definition at line 229 of file [PyCliclcal.pyx](#).

References [glucat::index\\_set< DEFAULT\\_LO, DEFAULT\\_HI >.max\(\)](#), [glucat::index\\_set< LO, HI >.max\(\)](#), [PyCliclcal.index\\_set.max\(\)](#), [glucat::index\\_set< DEFAULT\\_LO, DEFAULT\\_HI >.min\(\)](#), [glucat::index\\_set< LO, HI >.min\(\)](#), and [PyCliclcal.index\\_set.min\(\)](#).

### 6.18.2.10 `__ixor__()`

```
PyClical.index_set.__ixor__ (
    self,
    rhs )
```

Symmetric set difference: exclusive or.

```
>>> x = index_set({1}); x ^= index_set({2}); print(x)
{1,2}
>>> x = index_set({1,2}); x ^= index_set({2}); print(x)
{1}
```

Definition at line 260 of file [PyClical.pyx](#).

### 6.18.2.11 `__or__()`

```
PyClical.index_set.__or__ (
    lhs,
    rhs )
```

Set union: or.

```
>>> print(index_set({1}) | index_set({2}))
{1,2}
>>> print(index_set({1,2}) | index_set({2}))
{1,2}
```

Definition at line 293 of file [PyClical.pyx](#).

### 6.18.2.12 `__repr__()`

```
PyClical.index_set.__repr__ (
    self )
```

The “official” string representation of self.

```
>>> index_set({1,2}).__repr__()
'index_set({1,2})'
>>> repr(index_set({1,2}))
'index_set({1,2})'
```

Definition at line 384 of file [PyClical.pyx](#).

References [index\\_set\\_to\\_repr\(\)](#).

### 6.18.2.13 `__richcmp__()`

```
PyClical.index_set.__richcmp__ (
    lhs,
    rhs,
    int,
    op )
```

Compare two objects of class `index_set`.

```
>>> index_set(1) == index_set({1})
True
>>> index_set({1}) != index_set({1})
False
>>> index_set({1}) != index_set({2})
True
>>> index_set({1}) == index_set({2})
False
>>> index_set({1}) < index_set({2})
True
>>> index_set({1}) <= index_set({2})
True
>>> index_set({1}) > index_set({2})
False
>>> index_set({1}) >= index_set({2})
False
```

Definition at line 122 of file [PyClical.pyx](#).

### 6.18.2.14 `__setitem__()`

```
PyClical.index_set.__setitem__ (
    self,
    idx,
    val )
```

Set the value of an `index_set` object at index `idx` to value `val`.

```
>>> s=index_set({1}); s[2] = True; print(s)
{1,2}
>>> s=index_set({1,2}); s[1] = False; print(s)
{2}
```

Definition at line 179 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

### 6.18.2.15 `__str__()`

```
PyClical.index_set.__str__ (
    self )
```

The “informal” string representation of `self`.

```
>>> index_set({1,2}).__str__()
'{1,2}'
>>> str(index_set({1,2}))
'{1,2}'
```

Definition at line 395 of file [PyClical.pyx](#).

References [index\\_set\\_to\\_str\(\)](#).

### 6.18.2.16 `__xor__()`

```
PyClical.index_set.__xor__ (
    lhs,
    rhs )
```

Symmetric set difference: exclusive or.

```
>>> print(index_set({1}) ^ index_set({2}))
{1,2}
>>> print(index_set({1,2}) ^ index_set({2}))
{1}
```

Definition at line 249 of file [PyClical.pyx](#).

### 6.18.2.17 `count()`

```
PyClical.index_set.count (
    self )
```

Cardinality: Number of indices included in set.

```
>>> index_set({-1,1,2}).count()
3
```

Definition at line 315 of file [PyClical.pyx](#).

References [PyClical.index\\_set.count\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.count\(\)](#).

### 6.18.2.18 `count_neg()`

```
PyClical.index_set.count_neg (
    self )
```

Number of negative indices included in set.

```
>>> index_set({-1,1,2}).count_neg()
1
```

Definition at line 324 of file [PyClical.pyx](#).

References [PyClical.index\\_set.count\\_neg\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.count\\_neg\(\)](#).

### 6.18.2.19 count\_pos()

```
PyClical.index_set.count_pos (  
    self )
```

Number of positive indices included in set.

```
>>> index_set({-1,1,2}).count_pos()  
2
```

Definition at line 333 of file [PyClical.pyx](#).

References [PyClical.index\\_set.count\\_pos\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.count\\_pos\(\)](#).

### 6.18.2.20 hash\_fn()

```
PyClical.index_set.hash_fn (  
    self )
```

Hash function.

Definition at line 360 of file [PyClical.pyx](#).

References [PyClical.index\\_set.hash\\_fn\(\)](#), [PyClical.index\\_set.instance](#), and [PyClical.clifford.instance](#).

Referenced by [PyClical.index\\_set.hash\\_fn\(\)](#).

### 6.18.2.21 max()

```
PyClical.index_set.max (  
    self )
```

Maximum member.

```
>>> index_set({-1,1,2}).max()  
2
```

Definition at line 351 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.max\(\)](#).

Referenced by [PyClical.index\\_set.\\_\\_iter\\_\\_\(\)](#), and [PyClical.index\\_set.max\(\)](#).

### 6.18.2.22 min()

```
PyClical.index_set.min (
    self )
```

Minimum member.

```
>>> index_set({-1,1,2}).min()
-1
```

Definition at line 342 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.min\(\)](#).

Referenced by [PyClical.index\\_set.\\_\\_iter\\_\\_\(\)](#), and [PyClical.index\\_set.min\(\)](#).

### 6.18.2.23 sign\_of\_mult()

```
PyClical.index_set.sign_of_mult (
    self,
    rhs )
```

Sign of geometric product of two Clifford basis elements.

```
>>> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
1
```

Definition at line 366 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.sign\\_of\\_mult\(\)](#).

Referenced by [PyClical.index\\_set.sign\\_of\\_mult\(\)](#).

### 6.18.2.24 sign\_of\_square()

```
PyClical.index_set.sign_of_square (
    self )
```

Sign of geometric square of a Clifford basis element.

```
>>> s = index_set({1,2}); s.sign_of_square()
-1
```

Definition at line 375 of file [PyClical.pyx](#).

References [PyClical.index\\_set.instance](#), [PyClical.clifford.instance](#), and [PyClical.index\\_set.sign\\_of\\_square\(\)](#).

Referenced by [PyClical.index\\_set.sign\\_of\\_square\(\)](#).

### 6.18.3 Member Data Documentation

#### 6.18.3.1 instance

`PyClical.index_set.instance`

Definition at line 95 of file [PyClical.pyx](#).

Referenced by [PyClical.clifford.\\_\\_call\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_contains\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.clifford.\\_\\_dealloc\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_getitem\\_\\_\(\)](#), [PyClical.clifford.\\_\\_getitem\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_invert\\_\\_\(\)](#), [PyClical.clifford.\\_\\_neg\\_\\_\(\)](#), [PyClical.index\\_set.\\_\\_setitem\\_\\_\(\)](#), [PyClical.clifford.conj\(\)](#), [PyClical.index\\_set.count\(\)](#), [PyClical.index\\_set.count\\_neg\(\)](#), [PyClical.index\\_set.count\\_pos\(\)](#), [PyClical.clifford.even\(\)](#), [PyClical.clifford.frame\(\)](#), [PyClical.index\\_set.hash\\_fn\(\)](#), [PyClical.clifford.inv\(\)](#), [PyClical.clifford.involute\(\)](#), [PyClical.clifford.isinf\(\)](#), [PyClical.clifford.isnan\(\)](#), [PyClical.index\\_set.max\(\)](#), [PyClical.clifford.max\\_abs\(\)](#), [PyClical.index\\_set.min\(\)](#), [PyClical.clifford.norm\(\)](#), [PyClical.clifford.odd\(\)](#), [PyClical.clifford.outer\\_pow\(\)](#), [PyClical.clifford.pow\(\)](#), [PyClical.clifford.pure\(\)](#), [PyClical.clifford.quad\(\)](#), [PyClical.clifford.reverse\(\)](#), [PyClical.clifford.scalar\(\)](#), [PyClical.index\\_set.sign\\_of\\_mult\(\)](#), [PyClical.index\\_set.sign\\_of\\_square\(\)](#), [PyClical.clifford.truncated\(\)](#), and [PyClical.clifford.vector\\_part\(\)](#).

The documentation for this class was generated from the following file:

- [pyclical/PyClical.pyx](#)

## 6.19 glucat::index\_set\_hash< LO, HI > Class Template Reference

```
#include <framed_multi.h>
```

### Public Types

- using `index_set_t` = `index_set<LO, HI>`

### Public Member Functions

- auto `operator()` (`index_set_t` val) const -> `size_t`

#### 6.19.1 Detailed Description

```
template<const index\_t LO, const index\_t HI>
class glucat::index_set_hash< LO, HI >
```

Definition at line 117 of file [framed\\_multi.h](#).

### 6.19.2 Member Typedef Documentation

#### 6.19.2.1 index\_set\_t

```
template<const index\_t LO, const index\_t HI>
using glucat::index_set_hash< LO, HI >::index_set_t = index\_set<LO, HI>
```

Definition at line 120 of file [framed\\_multi.h](#).



### 6.19.3 Member Function Documentation

#### 6.19.3.1 operator()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set_hash< LO, HI >::operator() (
    index_set_t val ) const -> size_t    [inline]
```

Definition at line 121 of file [framed\\_multi.h](#).

The documentation for this class was generated from the following file:

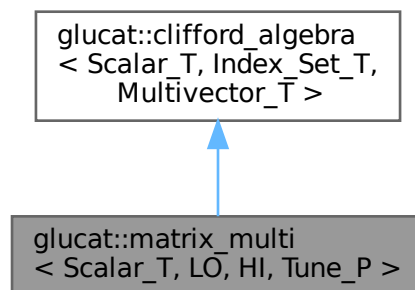
- [glucat/framed\\_multi.h](#)

## 6.20 glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P > Class Template Reference

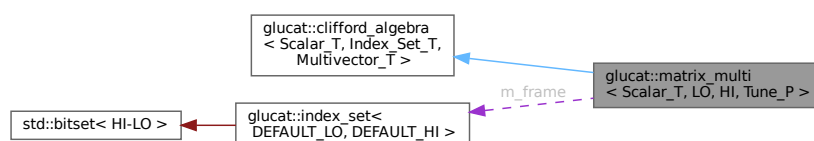
A matrix\_multi<Scalar\_T,LO,HI,Tune\_P> is a matrix approximation to a multivector.

```
#include <matrix_multi.h>
```

Inheritance diagram for glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >:



Collaboration diagram for glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >:



## Public Types

- using `multivector_t` = `matrix_multi`
- using `matrix_multi_t` = `multivector_t`
- using `scalar_t` = `Scalar_T`
- using `tune_p` = `Tune_P`
- using `index_set_t` = `index_set`<LO, HI>
- using `term_t` = `std::pair`<const `index_set_t`, `Scalar_T`>
- using `vector_t` = `std::vector`<`Scalar_T`>
- using `error_t` = `error`<`multivector_t`>
- using `framed_multi_t` = `framed_multi`<`Scalar_T`,LO,HI,`Tune_P`>

## Public Types inherited from

`glucat::clifford_algebra`< `Scalar_T`, `Index_Set_T`, `Multivector_T` >

- using `scalar_t` = `Scalar_T`
- using `index_set_t` = `Index_Set_T`
- using `multivector_t` = `Multivector_T`
- using `pair_t` = `std::pair`<const `index_set_t`, `Scalar_T`>
- using `vector_t` = `std::vector`<`Scalar_T`>

## Public Member Functions

- `~matrix_multi` () override=default  
*Destructor.*
- `matrix_multi` ()  
*Default constructor.*
- `template<typename Other_Scalar_T >`  
`matrix_multi` (const `matrix_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val)  
*Construct a multivector from a multivector with a different scalar type.*
- `template<typename Other_Scalar_T >`  
`matrix_multi` (const `matrix_multi`< `Other_Scalar_T`, LO, HI, `Tune_P` > &val, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- `matrix_multi` (const `multivector_t` &val, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given multivector.*
- `matrix_multi` (const `index_set_t` ist, const `Scalar_T` &crd=`Scalar_T`(1))  
*Construct a multivector from an index set and a scalar coordinate.*
- `matrix_multi` (const `index_set_t` ist, const `Scalar_T` &crd, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from an index set and a scalar coordinate.*
- `matrix_multi` (const `Scalar_T` &scr, const `index_set_t` frm=`index_set_t`())  
*Construct a multivector from a scalar (within a frame, if given)*
- `matrix_multi` (const int scr, const `index_set_t` frm=`index_set_t`())  
*Construct a multivector from an int (within a frame, if given)*
- `matrix_multi` (const `vector_t` &vec, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a given vector.*
- `matrix_multi` (const `std::string` &str)  
*Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `matrix_multi` (const `std::string` &str, const `index_set_t` frm, const bool prechecked=false)  
*Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".*
- `matrix_multi` (const char \*str)

- Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".
- `matrix_multi` (const char \*str, const `index_set_t` frm, const bool prechecked=false)
  - Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".
- template<typename Other\_Scalar\_T >
 `matrix_multi` (const `framed_multi`< Other\_Scalar\_T, LO, HI, Tune\_P > &val)
  - Construct a multivector from a `framed_multi_t`.
- template<typename Other\_Scalar\_T >
 `matrix_multi` (const `framed_multi`< Other\_Scalar\_T, LO, HI, Tune\_P > &val, const `index_set_t` frm, const bool prechecked=false)
  - Construct a multivector, within a given frame, from a `framed_multi_t`.
- auto `fast_matrix_multi` (const `index_set_t` frm) const -> const `matrix_multi_t`
  - Use generalized FFT to construct a `matrix_multi_t`.
- template<typename Other\_Scalar\_T >
 auto `fast_framed_multi` () const -> const `framed_multi`< Other\_Scalar\_T, LO, HI, Tune\_P >
  - Use inverse generalized FFT to construct a `framed_multi_t`.
- `_GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS` auto `operator=` (const `multivector_t` &rhs) -> `multivector_t` &
  - Assignment operator.
- auto `operator+=` (const `term_t` &rhs) -> `multivector_t` &
  - Add a term, if non-zero.

## Public Member Functions inherited from `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`

- virtual `~clifford_algebra` ()=default
- virtual auto `operator==` (const `multivector_t` &val) const -> bool=0
  - Test for equality of multivectors.
- virtual auto `operator==` (const `Scalar_T` &scr) const -> bool=0
  - Test for equality of multivector and scalar.
- virtual auto `operator+=` (const `multivector_t` &rhs) -> `multivector_t` &=0
  - Geometric sum.
- virtual auto `operator+=` (const `Scalar_T` &scr) -> `multivector_t` &=0
  - Geometric sum of multivector and scalar.
- virtual auto `operator-=` (const `multivector_t` &rhs) -> `multivector_t` &=0
  - Geometric difference.
- virtual auto `operator-=` (const `Scalar_T` &scr) -> `multivector_t` &=0
  - Geometric difference of multivector and scalar.
- virtual auto `operator-` () const -> const `multivector_t`=0
  - Unary -.
- virtual auto `operator*=` (const `Scalar_T` &scr) -> `multivector_t` &=0
  - Product of multivector and scalar.
- virtual auto `operator*=` (const `multivector_t` &rhs) -> `multivector_t` &=0
  - Geometric product.
- virtual auto `operator%=>` (const `multivector_t` &rhs) -> `multivector_t` &=0
  - Contraction.
- virtual auto `operator&=>` (const `multivector_t` &rhs) -> `multivector_t` &=0
  - Inner product.
- virtual auto `operator^=>` (const `multivector_t` &rhs) -> `multivector_t` &=0
  - Outer product.
- virtual auto `operator/=` (const `Scalar_T` &scr) -> `multivector_t` &=0
  - Quotient of multivector and scalar.

- virtual auto `operator/=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Geometric quotient.*
- virtual auto `operator|=` (const `multivector_t` &rhs) -> `multivector_t` &=0  
*Transformation via twisted adjoint action.*
- virtual auto `inv` () const -> const `multivector_t`=0  
*Geometric multiplicative inverse.*
- virtual auto `pow` (int m) const -> const `multivector_t`=0  
*\*this to the m*
- virtual auto `outer_pow` (int m) const -> const `multivector_t`=0  
*Outer product power.*
- virtual auto `frame` () const -> const `index_set_t`=0  
*Subalgebra generated by all generators of terms of given multivector.*
- virtual auto `grade` () const -> `index_t`=0  
*Maximum of the grades of each term.*
- virtual auto `operator[]` (const `index_set_t` ist) const -> `Scalar_T`=0  
*Subscripting: map from index set to scalar coordinate.*
- virtual auto `operator()` (`index_t` grade) const -> const `multivector_t`=0  
*Pure grade-vector part.*
- virtual auto `scalar` () const -> `Scalar_T`=0  
*Scalar part.*
- virtual auto `pure` () const -> const `multivector_t`=0  
*Pure part.*
- virtual auto `even` () const -> const `multivector_t`=0  
*Even part of multivector, sum of even grade terms.*
- virtual auto `odd` () const -> const `multivector_t`=0  
*Odd part of multivector, sum of odd grade terms.*
- virtual auto `vector_part` () const -> const `vector_t`=0  
*Vector part of multivector, as a vector\_t with respect to frame()*
- virtual auto `vector_part` (const `index_set_t` frm, const bool prechecked) const -> const `vector_t`=0  
*Vector part of multivector, as a vector\_t with respect to frm.*
- virtual auto `involute` () const -> const `multivector_t`=0  
*Main involution, each {i} is replaced by -{i} in each term, eg. {1} -> -{1}.*
- virtual auto `reverse` () const -> const `multivector_t`=0  
*Reversion, eg. {1}\*{2} -> {2}\*{1}.*
- virtual auto `conj` () const -> const `multivector_t`=0  
*Conjugation, reverse o involute == involute o reverse.*
- virtual auto `quad` () const -> `Scalar_T`=0  
*Scalar\_T quadratic form == (rev(x)\*x)(0)*
- virtual auto `norm` () const -> `Scalar_T`=0  
*Scalar\_T norm == sum of norm of coordinates.*
- virtual auto `max_abs` () const -> `Scalar_T`=0  
*Maximum of absolute values of components of multivector: multivector infinity norm.*
- virtual auto `truncated` (const `Scalar_T` &limit=`default_truncation`) const -> const `multivector_t`=0  
*Remove all terms with relative size smaller than limit.*
- virtual auto `isinf` () const -> bool=0  
*Check if a multivector contains any infinite values.*
- virtual auto `isnan` () const -> bool=0  
*Check if a multivector contains any IEEE NaN values.*
- virtual void `write` (const std::string &msg="") const =0  
*Write formatted multivector to output.*
- virtual void `write` (std::ofstream &ofile, const std::string &msg="") const =0  
*Write formatted multivector to file.*

## Static Public Member Functions

- static auto [classname](#) () -> const std::string  
*Class name used in messages.*
- static auto [random](#) (const [index\\_set\\_t](#) frm, Scalar\_T fill=Scalar\_T(1)) -> const [matrix\\_multi\\_t](#)  
*Random multivector within a frame.*

## Static Public Member Functions inherited from [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >](#)

- static auto [classname](#) () -> const std::string

## Private Types

- using [orientation\\_t](#) = ublas::row\_major
- using [basis\\_matrix\\_t](#) = ublas::compressed\_matrix<int, [orientation\\_t](#)>
- using [matrix\\_t](#) = ublas::matrix<Scalar\_T, [orientation\\_t](#)>
- using [matrix\\_index\\_t](#) = typename matrix\_t::size\_type

## Private Member Functions

- template<typename Matrix\_T >  
[matrix\\_multi](#) (const Matrix\_T &mtx, const [index\\_set\\_t](#) frm)  
*Construct a multivector within a given frame from a given matrix.*
- [matrix\\_multi](#) (const [matrix\\_t](#) &mtx, const [index\\_set\\_t](#) frm)  
*Construct a multivector within a given frame from a given matrix.*
- auto [basis\\_element](#) (const [index\\_set](#)< LO, HI > &ist) const -> const [basis\\_matrix\\_t](#)  
*Create a basis element matrix within the current frame.*

## Private Attributes

- [index\\_set\\_t m\\_frame](#)  
*Index set representing the frame for the subalgebra which contains the multivector.*
- [matrix\\_t m\\_matrix](#)  
*Matrix value representing the multivector within the folded frame.*

## Friends

- template<typename Other\_Scalar\_T , const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P >  
class [framed\\_multi](#)
- template<typename Other\_Scalar\_T , const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P >  
class [matrix\\_multi](#)
- auto [operator\\*](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator^](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator&](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator%](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [star](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> Scalar\_T
- auto [operator/](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)
- auto [operator|](#) (const [matrix\\_multi\\_t](#) &lhs, const [matrix\\_multi\\_t](#) &rhs) -> const [matrix\\_multi\\_t](#)

- auto [operator>>](#) (std::istream &s, [multivector\\_t](#) &val) -> std::istream &
- auto [operator<<](#) (std::ostream &os, const [multivector\\_t](#) &val) -> std::ostream &
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P >  
auto [reframe](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &lhs, const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &rhs, [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &lhs\_reframed, [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &rhs\_reframed) -> const [index\\_set](#)< Other\_LO, Other\_HI >
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P >  
auto [matrix\\_sqrt](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &val, const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P >
- template<typename Other\_Scalar\_T, const [index\\_t](#) Other\_LO, const [index\\_t](#) Other\_HI, typename Other\_Tune\_P >  
auto [matrix\\_log](#) (const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &val, const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Other\_Scalar\_T, Other\_LO, Other\_HI, Other\_Tune\_P >

## Additional Inherited Members

### Static Public Attributes inherited from

[glucat::clifford\\_algebra](#)< [Scalar\\_T](#), [Index\\_Set\\_T](#), [Multivector\\_T](#) >

- static const [index\\_t v\\_lo](#) = [index\\_set\\_t::v\\_lo](#)
- static const [index\\_t v\\_hi](#) = [index\\_set\\_t::v\\_hi](#)
- static const [Scalar\\_T default\\_truncation](#) = std::numeric\_limits<[Scalar\\_T](#)>::epsilon()

*Default for truncation.*

## 6.20.1 Detailed Description

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
```

```
class glucat::matrix_multi< Scalar\_T, LO, HI, Tune\_P >
```

A [matrix\\_multi](#)<[Scalar\\_T](#),LO,HI,[Tune\\_P](#)> is a matrix approximation to a multivector.

Definition at line 137 of file [matrix\\_multi.h](#).

## 6.20.2 Member Typedef Documentation

### 6.20.2.1 basis\_matrix\_t

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
using glucat::matrix\_multi< Scalar\_T, LO, HI, Tune\_P >::basis_matrix_t = ublas::compressed_
matrix<int, orientation\_t> [private]
```

Definition at line 157 of file [matrix\\_multi.h](#).

### 6.20.2.2 error\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::error_t = error<multivector_t>
```

Definition at line 148 of file [matrix\\_multi.h](#).

### 6.20.2.3 framed\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::framed_multi_t = framed_multi<Scalar_T, LO, HI, Tune_P>
```

Definition at line 149 of file [matrix\\_multi.h](#).

### 6.20.2.4 index\_set\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::index_set_t = index_set<LO, HI>
```

Definition at line 145 of file [matrix\\_multi.h](#).

### 6.20.2.5 matrix\_index\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_index_t = typename matrix_t<Scalar_T, LO, HI, Tune_P>::size_type [private]
```

Definition at line 159 of file [matrix\\_multi.h](#).

### 6.20.2.6 matrix\_multi\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi_t = multivector_t
```

Definition at line 142 of file [matrix\\_multi.h](#).

### 6.20.2.7 matrix\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_t = ublas::matrix<Scalar_T, LO, HI, Tune_P, orientation_t> [private]
```

Definition at line 158 of file [matrix\\_multi.h](#).

### 6.20.2.8 multivector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::multivector_t = matrix_multi
```

Definition at line 141 of file [matrix\\_multi.h](#).

### 6.20.2.9 orientation\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::orientation_t = ublas::row_major
[private]
```

Definition at line 156 of file [matrix\\_multi.h](#).

### 6.20.2.10 scalar\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::scalar_t = Scalar_T
```

Definition at line 143 of file [matrix\\_multi.h](#).

### 6.20.2.11 term\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::term_t = std::pair<const index_set_t,
Scalar_T>
```

Definition at line 146 of file [matrix\\_multi.h](#).

### 6.20.2.12 tune\_p

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::tune_p = Tune_P
```

Definition at line 144 of file [matrix\\_multi.h](#).

### 6.20.2.13 vector\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::vector_t = std::vector<Scalar_T>
```

Definition at line 147 of file [matrix\\_multi.h](#).



## 6.20.3 Constructor & Destructor Documentation

### 6.20.3.1 ~matrix\_multi()

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::~matrix\_multi ( ) [override], [default]
```

Destructor.

### 6.20.3.2 matrix\_multi() [1/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi ( )
```

Default constructor.

Definition at line 106 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

### 6.20.3.3 matrix\_multi() [2/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a multivector with a different scalar type.

Definition at line 115 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

### 6.20.3.4 matrix\_multi() [3/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 134 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_](#)

**6.20.3.5 matrix\_multi()** [4/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const multivector\_t & val,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given multivector.

Definition at line 159 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m](#)

**6.20.3.6 matrix\_multi()** [5/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd = Scalar_T(1) )
```

Construct a multivector from an index set and a scalar coordinate.

Definition at line 171 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m](#)

**6.20.3.7 matrix\_multi()** [6/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const index\_set\_t ist,
    const Scalar_T & crd,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from an index set and a scalar coordinate.

Definition at line 183 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

**6.20.3.8 matrix\_multi()** [7/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Scalar_T & scr,
    const index\_set\_t frm = index_set_t() )
```

Construct a multivector from a scalar (within a frame, if given)

Definition at line 197 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

**6.20.3.9 matrix\_multi()** [8/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const int scr,
    const index\_set\_t frm = index\_set\_t() )
```

Construct a multivector from an int (within a frame, if given)

Definition at line 209 of file [matrix\\_multi\\_imp.h](#).

**6.20.3.10 matrix\_multi()** [9/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const vector\_t & vec,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a given vector.

Definition at line 215 of file [matrix\\_multi\\_imp.h](#).

References [glucat::index\\_set< LO, HI >::count\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#), [glucat::index\\_set< LO, HI >::max\(\)](#), and [glucat::index\\_set< LO, HI >::min\(\)](#).

**6.20.3.11 matrix\_multi()** [10/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str )
```

Construct a multivector from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 240 of file [matrix\\_multi\\_imp.h](#).

**6.20.3.12 matrix\_multi()** [11/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const std::string & str,
    const index\_set\_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a string: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 246 of file [matrix\\_multi\\_imp.h](#).

**6.20.3.13 matrix\_multi()** [12/17]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str ) [inline]
```

Construct a multivector from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 196 of file [matrix\\_multi.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi](#).

**6.20.3.14 matrix\_multi()** [13/17]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const char * str,
    const index_set_t frm,
    const bool prechecked = false ) [inline]
```

Construct a multivector, within a given frame, from a char\*: eg: "3+2{1,2}-6.1e-2{2,3}".

Definition at line 199 of file [matrix\\_multi.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi](#).

**6.20.3.15 matrix\_multi()** [14/17]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val )
```

Construct a multivector from a framed\_multi\_t.

Definition at line 253 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_frame](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m](#)

**6.20.3.16 matrix\_multi()** [15/17]

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const framed_multi< Other_Scalar_T, LO, HI, Tune_P > & val,
    const index_set_t frm,
    const bool prechecked = false )
```

Construct a multivector, within a given frame, from a framed\_multi\_t.

Definition at line 277 of file [matrix\\_multi\\_imp.h](#).

References [glucat::clifford\\_algebra< Scalar\\_T, Index\\_Set\\_T, Multivector\\_T >::truncated\(\)](#).

**6.20.3.17 matrix\_multi()** [16/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
template<typename Matrix_T >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const Matrix_T & mtx,
    const index\_set\_t frm ) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 303 of file [matrix\\_multi\\_imp.h](#).

References [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::m\\_matrix](#).

**6.20.3.18 matrix\_multi()** [17/17]

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::matrix_multi (
    const matrix\_t & mtx,
    const index\_set\_t frm ) [private]
```

Construct a multivector within a given frame from a given matrix.

Definition at line 322 of file [matrix\\_multi\\_imp.h](#).

**6.20.4 Member Function Documentation****6.20.4.1 basis\_element()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::basis_element (
    const index\_set< LO, HI > & ist ) const -> const basis\_matrix\_t [private]
```

Create a basis element matrix within the current frame.

Definition at line 1186 of file [matrix\\_multi\\_imp.h](#).

References [glucat::gen::generator\\_table< Matrix\\_T >::generator\(\)](#), [glucat::matrix::mono\\_prod\(\)](#), and [glucat::offset\\_level\(\)](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#).

**6.20.4.2 classname()**

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::classname ( ) -> const std::string
[static]
```

Class name used in messages.

Definition at line 78 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.3 fast\_framed\_multi()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
template<typename Other_Scalar_T >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_framed_multi ( ) const -> const
framed_multi<Other_Scalar_T,LO,HI,Tune_P>
```

Use inverse generalized FFT to construct a framed\_multi\_t.

Definition at line 1109 of file [matrix\\_multi\\_imp.h](#).

References [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::centre\\_pm4\\_qp4\(\)](#), [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::centre\\_qp1\\_pm1\(\)](#), [glucat::gen::offset\\_to\\_super](#), [glucat::pos\\_mod\(\)](#), and [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::unfold\(\)](#).

#### 6.20.4.4 fast\_matrix\_multi()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::fast_matrix_multi (
    const index_set_t frm ) const -> const matrix_multi_t [inline]
```

Use generalized FFT to construct a matrix\_multi\_t.

Definition at line 1096 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.5 operator+=()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator+= (
    const term_t & rhs ) -> multivector_t& [inline]
```

Add a term, if non-zero.

Geometric sum.

Geometric sum of multivector and scalar.

Definition at line 416 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.6 operator=()

```
template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >
auto glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::operator= (
    const multivector_t & rhs ) -> multivector_t&
```

Assignment operator.

Definition at line 330 of file [matrix\\_multi\\_imp.h](#).

#### 6.20.4.7 random()

```
template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >
auto glucat::matrix\_multi< Scalar_T, LO, HI, Tune_P >::random (
    const index\_set\_t frm,
    Scalar_T fill = Scalar_T(1) ) -> const matrix\_multi\_t [static]
```

Random multivector within a frame.

Definition at line 926 of file [matrix\\_multi\\_imp.h](#).

References [glucat::framed\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) >::random().

### 6.20.5 Friends And Related Symbol Documentation

#### 6.20.5.1 framed\_multi

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
friend class framed\_multi [friend]
```

Definition at line 151 of file [matrix\\_multi.h](#).

#### 6.20.5.2 matrix\_log

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
auto matrix_log (
    const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index\_t level ) -> const matrix\_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > [friend]
```

#### 6.20.5.3 matrix\_multi

```
template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index\_t Other_LO, const index\_t Other_HI, typename Other_Tune_P >
friend class matrix\_multi [friend]
```

Definition at line 153 of file [matrix\\_multi.h](#).

Referenced by [glucat::matrix\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) >::matrix\_multi(), and [glucat::matrix\\_multi](#)< [Scalar\\_T](#), [LO](#), [HI](#), [Tune\\_P](#) >::matrix\_multi().

#### 6.20.5.4 matrix\_sqrt

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >
auto matrix_sqrt (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & val,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & i,
    const index_t level ) -> const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > [friend]
```

#### 6.20.5.5 operator%

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator% (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

#### 6.20.5.6 operator&

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator& (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

#### 6.20.5.7 operator\*

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator* (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

#### 6.20.5.8 operator/

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator/ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

#### 6.20.5.9 operator<<

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator<< (
    std::ostream & os,
    const multivector_t & val ) -> std::ostream & [friend]
```



### 6.20.5.10 operator>>

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator>> (
    std::istream & s,
    multivector_t & val ) -> std::istream & [friend]
```

### 6.20.5.11 operator^

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator^ (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.12 operator"|

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto operator| (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> const matrix_multi_t [friend]
```

### 6.20.5.13 reframe

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
template<typename Other_Scalar_T , const index_t Other_LO, const index_t Other_HI, typename Other_Tune_P >
auto reframe (
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs,
    const matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs,
    matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & lhs_reframed,
    matrix_multi< Other_Scalar_T, Other_LO, Other_HI, Other_Tune_P > & rhs_reframed )
-> const index_set< Other_LO, Other_HI > [friend]
```

### 6.20.5.14 star

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto star (
    const matrix_multi_t & lhs,
    const matrix_multi_t & rhs ) -> Scalar_T [friend]
```

## 6.20.6 Member Data Documentation

### 6.20.6.1 m\_frame

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
index_set_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_frame [private]
```

Index set representing the frame for the subalgebra which contains the multivector.

Definition at line 278 of file [matrix\\_multi.h](#).

Referenced by [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

### 6.20.6.2 m\_matrix

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
matrix_t glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >::m_matrix [private]
```

Matrix value representing the multivector within the folded frame.

Definition at line 280 of file [matrix\\_multi.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#), and [glucat::matrix\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::matrix\\_multi\(\)](#).

The documentation for this class was generated from the following files:

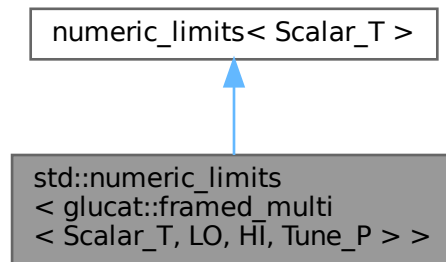
- [glucat/framed\\_multi.h](#)
- [glucat/matrix\\_multi.h](#)
- [glucat/matrix\\_multi\\_imp.h](#)

## 6.21 std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > > Struct Template Reference

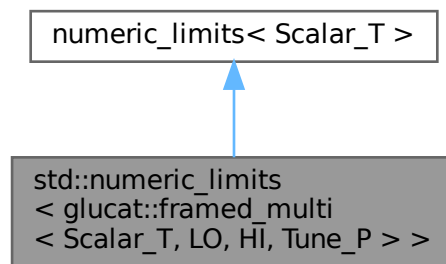
Numeric limits for framed\_multi inherit limits for the corresponding scalar type.

```
#include <framed_multi.h>
```

Inheritance diagram for std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >:



Collaboration diagram for std::numeric\_limits< glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P > >:



### 6.21.1 Detailed Description

```
template<typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P>
struct std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for framed\_multi inherit limits for the corresponding scalar type.

Definition at line 345 of file [framed\\_multi.h](#).

The documentation for this struct was generated from the following file:

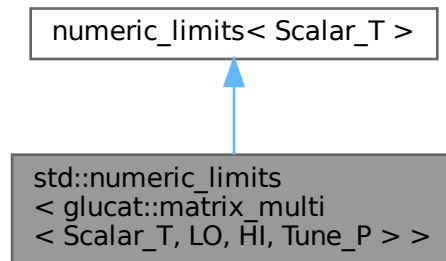
- [glucat/framed\\_multi.h](#)

## 6.22 `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >` Struct Template Reference

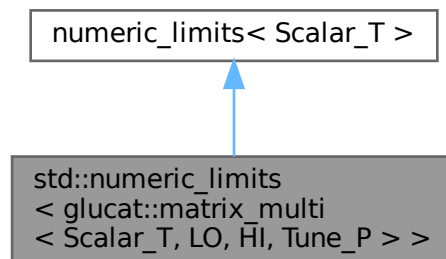
Numeric limits for `matrix_multi` inherit limits for the corresponding scalar type.

```
#include <matrix_multi.h>
```

Inheritance diagram for `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >`:



Collaboration diagram for `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >`:



### 6.22.1 Detailed Description

```
template<typename Scalar_T, const glucat::index\_t LO, const glucat::index\_t HI, typename Tune_P>
struct std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >
```

Numeric limits for `matrix_multi` inherit limits for the corresponding scalar type.

Definition at line [296](#) of file [matrix\\_multi.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi.h](#)

## 6.23 `glucat::numeric_traits< Scalar_T >` Class Template Reference

Extra traits which extend numeric limits.

```
#include <scalar.h>
```

### Classes

- struct `demoted`  
*Demoted type for long double.*
- struct `promoted`  
*Extra traits which extend numeric limits.*

### Public Member Functions

- auto `pi` () -> long double  
*Pi for long double.*
- auto `ln_2` () -> long double  
*log(2) for long double*
- auto `to_scalar_t` (const Other\_Scalar\_T &val) -> float  
*Extra traits which extend numeric limits.*
- auto `to_scalar_t` (const Other\_Scalar\_T &val) -> double  
*Cast to double.*
- auto `to_scalar_t` (const dd\_real &val) -> long double  
*Cast to long double.*
- auto `to_scalar_t` (const qd\_real &val) -> long double  
*Cast to long double.*
- auto `to_scalar_t` (const long double &val) -> dd\_real  
*Cast to dd\_real.*
- auto `to_scalar_t` (const qd\_real &val) -> dd\_real  
*Cast to dd\_real.*
- auto `to_scalar_t` (const long double &val) -> qd\_real  
*Cast to qd\_real.*
- auto `to_scalar_t` (const dd\_real &val) -> qd\_real  
*Cast to qd\_real.*

### Static Public Member Functions

- static auto `isInf` (const Scalar\_T &val) -> bool  
*Smart isinf.*
- static auto `isNaN` (const Scalar\_T &val) -> bool  
*Smart isnan.*
- static auto `isNaN_or_isInf` (const Scalar\_T &val) -> bool  
*Smart isnan or isinf.*
- static auto `NaN` () -> Scalar\_T  
*Smart NaN.*
- static auto `to_int` (const Scalar\_T &val) -> int  
*Cast to int.*
- static auto `to_double` (const Scalar\_T &val) -> double

*Cast to double.*

- `template<typename Other_Scalar_T >`  
`static auto to\_scalar\_t (const Other_Scalar_T &val) -> Scalar_T`

*Cast to Scalar\_T.*

- `static auto fmod (const Scalar_T &lhs, const Scalar_T &rhs) -> Scalar_T`

*Modulo function for scalar.*

- `static auto conj (const Scalar_T &val) -> Scalar_T`

*Complex conjugate of scalar.*

- `static auto real (const Scalar_T &val) -> Scalar_T`

*Real part of scalar.*

- `static auto imag (const Scalar_T &val) -> Scalar_T`

*Imaginary part of scalar.*

- `static auto abs (const Scalar_T &val) -> Scalar_T`

*Absolute value of scalar.*

- `static auto pi () -> Scalar_T`

*Pi.*

- `static auto ln\_2 () -> Scalar_T`

*log(2)*

- `static auto pow (const Scalar_T &val, int n) -> Scalar_T`

*Integer power.*

- `static auto sqrt (const Scalar_T &val) -> Scalar_T`

*Square root of scalar.*

- `static auto exp (const Scalar_T &val) -> Scalar_T`

*Exponential.*

- `static auto log (const Scalar_T &val) -> Scalar_T`

*Logarithm of scalar.*

- `static auto log2 (const Scalar_T &val) -> Scalar_T`

*Log base 2.*

- `static auto cos (const Scalar_T &val) -> Scalar_T`

*Cosine of scalar.*

- `static auto acos (const Scalar_T &val) -> Scalar_T`

*Inverse cosine of scalar.*

- `static auto cosh (const Scalar_T &val) -> Scalar_T`

*Hyperbolic cosine of scalar.*

- `static auto sin (const Scalar_T &val) -> Scalar_T`

*Sine of scalar.*

- `static auto asin (const Scalar_T &val) -> Scalar_T`

*Inverse sine of scalar.*

- `static auto sinh (const Scalar_T &val) -> Scalar_T`

*Hyperbolic sine of scalar.*

- `static auto tan (const Scalar_T &val) -> Scalar_T`

*Tangent of scalar.*

- `static auto atan (const Scalar_T &val) -> Scalar_T`

*Inverse tangent of scalar.*

- `static auto tanh (const Scalar_T &val) -> Scalar_T`

*Hyperbolic tangent of scalar.*

## Static Private Member Functions

- static auto [isInf](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< false >) -> bool  
*Smart isinf specialised for Scalar\_T without infinity.*
- static auto [isInf](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< true >) -> bool  
*Smart isinf specialised for Scalar\_T with infinity.*
- static auto [isNaN](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< false >) -> bool  
*Smart isnan specialised for Scalar\_T without quiet NaN.*
- static auto [isNaN](#) (const Scalar\_T &val, [bool\\_to\\_type](#)< true >) -> bool  
*Smart isnan specialised for Scalar\_T with quiet NaN.*

### 6.23.1 Detailed Description

**template<typename Scalar\_T>**  
**class glucat::numeric\_traits< Scalar\_T >**

Extra traits which extend numeric limits.

Definition at line 47 of file [scalar.h](#).

### 6.23.2 Member Function Documentation

#### 6.23.2.1 abs()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::abs (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Absolute value of scalar.

Definition at line 182 of file [scalar.h](#).

#### 6.23.2.2 acos()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::acos (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Inverse cosine of scalar.

Definition at line 245 of file [scalar.h](#).

#### 6.23.2.3 asin()

```
template<typename Scalar_T >
static auto glucat::numeric\_traits< Scalar_T >::asin (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Inverse sine of scalar.

Definition at line 266 of file [scalar.h](#).

#### 6.23.2.4 atan()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::atan (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Inverse tangent of scalar.

Definition at line 287 of file [scalar.h](#).

#### 6.23.2.5 conj()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::conj (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Complex conjugate of scalar.

Definition at line 161 of file [scalar.h](#).

#### 6.23.2.6 cos()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::cos (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Cosine of scalar.

Definition at line 238 of file [scalar.h](#).

#### 6.23.2.7 cosh()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::cosh (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Hyperbolic cosine of scalar.

Definition at line 252 of file [scalar.h](#).

#### 6.23.2.8 exp()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::exp (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Exponential.

Definition at line 217 of file [scalar.h](#).



### 6.23.2.9 `fmod()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::fmod (
    const Scalar_T & lhs,
    const Scalar_T & rhs ) -> Scalar_T    [inline], [static]
```

Modulo function for scalar.

Definition at line 154 of file [scalar.h](#).

### 6.23.2.10 `imag()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::imag (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Imaginary part of scalar.

Definition at line 175 of file [scalar.h](#).

### 6.23.2.11 `isInf()` [1/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val ) -> bool    [inline], [static]
```

Smart isinf.

Definition at line 83 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isInf\(\)](#).

### 6.23.2.12 `isInf()` [2/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool    [inline], [static], [private]
```

Smart isinf specialised for `Scalar_T` without infinity.

Definition at line 54 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::isInf\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::isNaN\\_or\\_isInf\(\)](#).

**6.23.2.13 isInf()** [3/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isInf (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool    [inline], [static], [private]
```

Smart isinf specialised for Scalar\_T with infinity.

Definition at line 61 of file [scalar.h](#).

References [\\_GLUCAT\\_ISINF](#).

**6.23.2.14 isNaN()** [1/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val ) -> bool    [inline], [static]
```

Smart isnan.

Definition at line 93 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#).

**6.23.2.15 isNaN()** [2/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< false > ) -> bool    [inline], [static], [private]
```

Smart isnan specialised for Scalar\_T without quiet NaN.

Definition at line 68 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::isNaN\\_or\\_isInf\(\)](#).

**6.23.2.16 isNaN()** [3/3]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN (
    const Scalar_T & val,
    bool_to_type< true > ) -> bool    [inline], [static], [private]
```

Smart isnan specialised for Scalar\_T with quiet NaN.

Definition at line 75 of file [scalar.h](#).

References [\\_GLUCAT\\_ISNAN](#).

### 6.23.2.17 `isNaN_or_isInf()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::isNaN_or_isInf (
    const Scalar_T & val ) -> bool    [inline], [static]
```

Smart isnan or isinf.

Definition at line 103 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::isInf\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::isNaN\(\)](#).

### 6.23.2.18 `ln_2()` [1/2]

```
auto glucat::numeric_traits< longdouble >::ln_2 ( ) -> long double    [inline]
```

$\log(2)$  for long double

Definition at line 59 of file [long\\_double.h](#).

References [glucat::l\\_ln2](#).

### 6.23.2.19 `ln_2()` [2/2]

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::ln_2 ( ) -> Scalar_T    [inline], [static]
```

$\log(2)$

Definition at line 196 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

### 6.23.2.20 `log()`

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::log (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Logarithm of scalar.

Definition at line 224 of file [scalar.h](#).

Referenced by [glucat::numeric\\_traits< Scalar\\_T >::log2\(\)](#).

**6.23.2.21 log2()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::log2 (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Log base 2.

Definition at line 231 of file [scalar.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::ln\\_2\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::log\(\)](#).

Referenced by [glucat::log2\(\)](#).

**6.23.2.22 NaN()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::NaN ( ) -> Scalar_T    [inline], [static]
```

Smart NaN.

Definition at line 115 of file [scalar.h](#).

Referenced by [glucat::cr\\_sqrt\(\)](#), [glucat::operator\\*\(\)](#), and [glucat::matrix::trace\(\)](#).

**6.23.2.23 pi() [1/2]**

```
auto glucat::numeric_traits< longdouble >::pi ( ) -> long double    [inline]
```

Pi for long double.

Definition at line 51 of file [long\\_double.h](#).

References [glucat::\\_pi](#).

**6.23.2.24 pi() [2/2]**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::pi ( ) -> Scalar_T    [inline], [static]
```

Pi.

Definition at line 189 of file [scalar.h](#).

Referenced by [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 6.23.2.25 pow()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::pow (
    const Scalar_T & val,
    int n ) -> Scalar_T    [inline], [static]
```

Integer power.

Definition at line 203 of file [scalar.h](#).

Referenced by [glucat::error\\_squared\\_tol\(\)](#).

#### 6.23.2.26 real()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::real (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Real part of scalar.

Definition at line 168 of file [scalar.h](#).

#### 6.23.2.27 sin()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::sin (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Sine of scalar.

Definition at line 259 of file [scalar.h](#).

#### 6.23.2.28 sinh()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::sinh (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Hyperbolic sine of scalar.

Definition at line 273 of file [scalar.h](#).

#### 6.23.2.29 sqrt()

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::sqrt (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Square root of scalar.

Definition at line 210 of file [scalar.h](#).

Referenced by [glucat::abs\(\)](#).

**6.23.2.30 tan()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::tan (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Tangent of scalar.

Definition at line 280 of file [scalar.h](#).

**6.23.2.31 tanh()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::tanh (
    const Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Hyperbolic tangent of scalar.

Definition at line 294 of file [scalar.h](#).

**6.23.2.32 to\_double()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::to_double (
    const Scalar_T & val ) -> double    [inline], [static]
```

Cast to double.

Definition at line 133 of file [scalar.h](#).

Referenced by [glucat::operator<<\(\)](#), [glucat::numeric\\_traits< Scalar\\_T >::to\\_scalar\\_t\(\)](#), and [glucat::numeric\\_traits< Scalar\\_T >::to\\_](#)

**6.23.2.33 to\_int()**

```
template<typename Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::to_int (
    const Scalar_T & val ) -> int    [inline], [static]
```

Cast to int.

Definition at line 126 of file [scalar.h](#).

**6.23.2.34 to\_scalar\_t() [1/9]**

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const dd_real & val ) -> long double    [inline]
```

Cast to long double.

Definition at line 71 of file [scalar\\_imp.h](#).

**6.23.2.35 to\_scalar\_t()** [2/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const dd_real & val ) -> qd_real    [inline]
```

Cast to qd\_real.

Definition at line 116 of file [scalar\\_imp.h](#).

**6.23.2.36 to\_scalar\_t()** [3/9]

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const long double & val ) -> dd_real    [inline]
```

Cast to dd\_real.

Definition at line 89 of file [scalar\\_imp.h](#).

**6.23.2.37 to\_scalar\_t()** [4/9]

```
auto glucat::numeric_traits< qd_real >::to_scalar_t (
    const long double & val ) -> qd_real    [inline]
```

Cast to qd\_real.

Definition at line 107 of file [scalar\\_imp.h](#).

**6.23.2.38 to\_scalar\_t()** [5/9]

```
auto glucat::numeric_traits< double >::to_scalar_t (
    const Other_Scalar_T & val ) -> double    [inline]
```

Cast to double.

Definition at line 61 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**6.23.2.39 to\_scalar\_t()** [6/9]

```
auto glucat::numeric_traits< float >::to_scalar_t (
    const Other_Scalar_T & val ) -> float    [inline]
```

Extra traits which extend numeric limits.

Cast to float

Definition at line 52 of file [scalar\\_imp.h](#).

References [glucat::numeric\\_traits< Scalar\\_T >::to\\_double\(\)](#).

**6.23.2.40 to\_scalar\_t() [7/9]**

```
template<typename Scalar_T >
template<typename Other_Scalar_T >
static auto glucat::numeric_traits< Scalar_T >::to_scalar_t (
    const Other_Scalar_T & val ) -> Scalar_T    [inline], [static]
```

Cast to Scalar\_T.

Definition at line 141 of file [scalar.h](#).

Referenced by [glucat::matrix::nork\\_range\(\)](#), [glucat::to\\_demote\(\)](#), and [glucat::to\\_promote\(\)](#).

**6.23.2.41 to\_scalar\_t() [8/9]**

```
auto glucat::numeric_traits< dd_real >::to_scalar_t (
    const qd_real & val ) -> dd_real    [inline]
```

Cast to dd\_real.

Definition at line 98 of file [scalar\\_imp.h](#).

**6.23.2.42 to\_scalar\_t() [9/9]**

```
auto glucat::numeric_traits< longdouble >::to_scalar_t (
    const qd_real & val ) -> long double    [inline]
```

Cast to long double.

Definition at line 80 of file [scalar\\_imp.h](#).

The documentation for this class was generated from the following file:

- [glucat/scalar.h](#)

**6.24 pade::pade\_log\_denom< Scalar\_T > Struct Template Reference**

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

**Public Types**

- [using array](#) = std::array<Scalar\_T, 14>

**Static Public Attributes**

- [static const array denom](#)



### 6.24.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

Definition at line 1731 of file [matrix\\_multi\\_imp.h](#).

### 6.24.2 Member Typedef Documentation

#### 6.24.2.1 array

```
template<typename Scalar_T >
using pade::pade_log_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1733 of file [matrix\\_multi\\_imp.h](#).

### 6.24.3 Member Data Documentation

#### 6.24.3.1 denom

```
template<typename Scalar_T >
const pade_log_denom< longdouble >::array pade::pade_log_denom< Scalar_T >::denom [static]
```

**Initial value:**

```
=
{
    1.0,                13.0/2.0,          468.0/25.0,      1573.0/50.0,
    1573.0/46.0,        11583.0/460.0,     10296.0/805.0,     2574.0/575.0,
    11583.0/10925.0,    143.0/874.0,        572.0/37145.0,    117.0/148580.0,
    13.0/742900.0,     1.0/10400600.0
}
```

Definition at line 1734 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.25 pade::pade\_log\_denom< dd\_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<dd_real, 22>`

## Static Public Attributes

- [static const array denom](#)

### 6.25.1 Detailed Description

Definition at line 1820 of file [matrix\\_multi\\_imp.h](#).

### 6.25.2 Member Typedef Documentation

#### 6.25.2.1 array

```
using pade::pade_log_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1822 of file [matrix\\_multi\\_imp.h](#).

### 6.25.3 Member Data Documentation

#### 6.25.3.1 denom

```
const pade_log_denom< dd_real >::array pade::pade_log_denom< dd_real >::denom [static]
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("2100")/dd_real("41"),
    dd_real("341145")/dd_real("1066"),
    dd_real("11069856")/dd_real("19721"),
    dd_real("6918660")/dd_real("19721"),
    dd_real("1410864")/dd_real("16687"),
    dd_real("734825")/dd_real("94054"),
    dd_real("348840")/dd_real("1363783"),
    dd_real("6783")/dd_real("2727566"),
    dd_real("266")/dd_real("53187537"),
    dd_real("7")/dd_real("8155422340"),
    dd_real("21")/dd_real("2"),
    dd_real("12635")/dd_real("82"),
    dd_real("1037799")/dd_real("2132"),
    dd_real("9883800")/dd_real("19721"),
    dd_real("293930")/dd_real("1517"),
    dd_real("88179")/dd_real("3034"),
    dd_real("305235")/dd_real("188108"),
    dd_real("40698")/dd_real("1363783"),
    dd_real("9975")/dd_real("70916716"),
    dd_real("7")/dd_real("70916716"),
    dd_real("1")/dd_real("538257874440")
}
```

Definition at line 1823 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.26 pade::pade\_log\_denom< float > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- [using array](#) = std::array<[float](#), 10>

**Static Public Attributes**

- [static const array denom](#)

**6.26.1 Detailed Description**

Definition at line 1758 of file [matrix\\_multi\\_imp.h](#).

**6.26.2 Member Typedef Documentation****6.26.2.1 array**

```
using pade::pade_log_denom< float >::array = std::array<float, 10>
```

Definition at line 1760 of file [matrix\\_multi\\_imp.h](#).

**6.26.3 Member Data Documentation****6.26.3.1 denom**

```
const pade_log_denom< float >::array pade::pade_log_denom< float >::denom [static]
```

**Initial value:**

```
=
{
    1.0,          9.0/2.0,      144.0/17.0,   147.0/17.0,
    441.0/85.0,   63.0/34.0,    84.0/221.0,   9.0/221.0,
    9.0/4862.0,   1.0/48620.0
}
```

Definition at line 1761 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

**6.27 `pade::pade_log_denom< long double >` Struct Reference**

```
#include <matrix_multi_imp.h>
```

**Public Types**

- `using array = std::array<long double, 18>`

**Static Public Attributes**

- [static const array denom](#)

### 6.27.1 Detailed Description

Definition at line 1785 of file [matrix\\_multi\\_imp.h](#).

### 6.27.2 Member Typedef Documentation

#### 6.27.2.1 array

```
using pade::pade_log_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1787 of file [matrix\\_multi\\_imp.h](#).

### 6.27.3 Member Data Documentation

#### 6.27.3.1 denom

```
const array pade::pade_log_denom< long double >::denom [static]
```

Definition at line 1788 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.28 pade::pade\_log\_denom< qd\_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- [using array](#) = std::array<[qd\\_real](#), 34>

### Static Public Attributes

- [static const array denom](#)

### 6.28.1 Detailed Description

Definition at line 1867 of file [matrix\\_multi\\_imp.h](#).

## 6.28.2 Member Typedef Documentation

### 6.28.2.1 array

```
using pade::pade_log_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1869 of file [matrix\\_multi\\_imp.h](#).

## 6.28.3 Member Data Documentation

### 6.28.3.1 denom

```
const pade_log_denom< qd_real >::array pade::pade_log_denom< qd_real >::denom [static]
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("33")/qd_real("2"),
    qd_real("8448")/qd_real("65"),
    qd_real("42284")/qd_real("65"),
    qd_real("211420")/qd_real("91"),
    qd_real("573562")/qd_real("91"),
    qd_real("32119472")/qd_real("2379"),
    qd_real("92917044")/qd_real("3965"),
    qd_real("603960786")/qd_real("17995"),
    qd_real("144626625")/qd_real("3599"),
    qd_real("2776831200")/qd_real("68381"),
    qd_real("16692542100")/qd_real("478667"),
    qd_real("12241197540")/qd_real("478667"),
    qd_real("1098569010")/qd_real("68381"),
    qd_real("31387686000")/qd_real("3624193"),
    qd_real("9939433900")/qd_real("2479711"),
    qd_real("67091178825")/qd_real("42155087"),
    qd_real("2683647153")/qd_real("4959422"),
    qd_real("19083713088")/qd_real("121505839"),
    qd_real("4708152900")/qd_real("121505839"),
    qd_real("941630580")/qd_real("116546417"),
    qd_real("88704330")/qd_real("62755763"),
    qd_real("12902448")/qd_real("62755763"),
    qd_real("1542684")/qd_real("62755763"),
    qd_real("6427850")/qd_real("2698497809"),
    qd_real("3471039")/qd_real("18889484663"),
    qd_real("8544096")/qd_real("774468871183"),
    qd_real("39556")/qd_real("79027435835"),
    qd_real("118668")/qd_real("7191496660985"),
    qd_real("10230")/qd_real("27327687311743"),
    qd_real("5456")/qd_real("1011124430534491"),
    qd_real("44")/qd_real("1011124430534491"),
    qd_real("11")/qd_real("70778710137414370"),
    qd_real("1")/qd_real("7219428434016265740")
}
```

Definition at line 1870 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.29 pade::pade\_log\_numer< Scalar\_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

## Public Types

- `using array = std::array<Scalar_T, 14>`

## Static Public Attributes

- `static const array number`

### 6.29.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_log_numer< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)

Definition at line 1714 of file [matrix\\_multi\\_imp.h](#).

### 6.29.2 Member Typedef Documentation

#### 6.29.2.1 array

```
template<typename Scalar_T >
using pade::pade_log_numer< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1716 of file [matrix\\_multi\\_imp.h](#).

### 6.29.3 Member Data Documentation

#### 6.29.3.1 number

```
template<typename Scalar_T >
const pade_log_numer< longdouble >::array pade::pade_log_numer< Scalar_T >::number [static]
```

#### Initial value:

```
=
{
    0.0,          1.0,          6.0,          4741.0/300.0,
    1441.0/60.0,   107091.0/4600.0,   8638.0/575.0,   263111.0/40250.0,
    153081.0/80500.0,   395243.0/1101240.0,   28549.0/688275.0,   605453.0/228813200.0,
    785633.0/10296594000.0,   1145993.0/1873980108000.0
}
```

Definition at line 1717 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.30 `pade::pade_log_numer< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<dd_real, 22>`

### Static Public Attributes

- `static const array number`

### 6.30.1 Detailed Description

Definition at line 1800 of file `matrix_multi_imp.h`.

### 6.30.2 Member Typedef Documentation

#### 6.30.2.1 `array`

```
using pade::pade_log_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1802 of file `matrix_multi_imp.h`.

### 6.30.3 Member Data Documentation

#### 6.30.3.1 `number`

```
const pade_log_numer< dd_real >::array pade::pade_log_numer< dd_real >::number [static]
```

Initial value:

```
=
{
    dd_real("0"),
    dd_real("10"),
    dd_real("21603")/dd_real("164"),
    dd_real("978724")/dd_real("2665"),
    dd_real("12874933")/dd_real("39442"),
    dd_real("2406734")/dd_real("22755"),
    dd_real("30653165")/dd_real("2402928"),
    dd_real("25346331")/dd_real("47074027"),
    dd_real("105689791")/dd_real("15601677520"),
    dd_real("969715")/dd_real("53502994116"),
    dd_real("118999")/dd_real("26204577562592"),
    dd_real("1"),
    dd_real("22781")/dd_real("492"),
    dd_real("5492649")/dd_real("21320"),
    dd_real("4191605")/dd_real("10619"),
    dd_real("11473457")/dd_real("54612"),
    dd_real("166770367")/dd_real("4004880"),
    dd_real("647746389")/dd_real("215195552"),
    dd_real("278270613")/dd_real("3900419380"),
    dd_real("606046475")/dd_real("1379188292768"),
    dd_real("11098301")/dd_real("26204577562592"),
    dd_real("18858053")/dd_real("1392249205900512960")
}
```

Definition at line 1803 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.31 `pade::pade_log_numer< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<float, 10>`

### Static Public Attributes

- `static const array numer`

### 6.31.1 Detailed Description

Definition at line 1746 of file `matrix_multi_imp.h`.

### 6.31.2 Member Typedef Documentation

#### 6.31.2.1 `array`

```
using pade::pade_log_numer< float >::array = std::array<float, 10>
```

Definition at line 1748 of file `matrix_multi_imp.h`.

### 6.31.3 Member Data Documentation

#### 6.31.3.1 `numer`

```
const pade_log_numer< float >::array pade::pade_log_numer< float >::numer [static]
```

#### Initial value:

```
=
{
    0.0,          1.0,          4.0,          1337.0/204.0,
    385.0/68.0,   1879.0/680.0,   193.0/255.0,   197.0/1820.0,
    419.0/61880.0, 7129.0/61261200.0
}
```

Definition at line 1749 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.32 `pade::pade_log_numer< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```



### Public Types

- `using array = std::array<long double, 18>`

### Static Public Attributes

- `static const array numer`

## 6.32.1 Detailed Description

Definition at line 1771 of file `matrix_multi_imp.h`.

## 6.32.2 Member Typedef Documentation

### 6.32.2.1 array

```
using pade::pade_log_number< long double >::array = std::array<long double, 18>
```

Definition at line 1773 of file `matrix_multi_imp.h`.

## 6.32.3 Member Data Documentation

### 6.32.3.1 numer

```
const array pade::pade_log_number< long double >::numer [static]
```

Definition at line 1774 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.33 `pade::pade_log_number< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<qd_real, 34>`

### Static Public Attributes

- `static const array numer`

### 6.33.1 Detailed Description

Definition at line 1841 of file [matrix\\_multi\\_imp.h](#).

### 6.33.2 Member Typedef Documentation

#### 6.33.2.1 array

```
using pade::pade_log_number< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1843 of file [matrix\\_multi\\_imp.h](#).

### 6.33.3 Member Data Documentation

#### 6.33.3.1 number

```
const pade_log_number< qd_real >::array pade::pade_log_number< qd_real >::number [static]
```

Initial value:

```
=
{
    qd_real("0"),
    qd_real("16"),
    qd_real("95201")/qd_real("780"),
    qd_real("30721")/qd_real("52"),
    qd_real("7416257")/qd_real("3640"),
    qd_real("1039099")/qd_real("195"),
    qd_real("6097772319")/qd_real("555100"),
    qd_real("1564058073")/qd_real("85400"),
    qd_real("30404640205")/qd_real("1209264"),
    qd_real("725351278")/qd_real("25193"),
    qd_real("4092322670789")/qd_real("147429436"),
    qd_real("4559713849589")/qd_real("201040140"),
    qd_real("5049361751189")/qd_real("320023080"),
    qd_real("74979677195")/qd_real("8000577"),
    qd_real("16569850691873")/qd_real("3481514244"),
    qd_real("1065906022369")/qd_real("515779888"),
    qd_real("335956770855841")/qd_real("438412904800"),
    qd_real("1462444287585964")/qd_real("6041877844275"),
    qd_real("397242326339851")/qd_real("6122436215532"),
    qd_real("64211291334131")/qd_real("4373168725380"),
    qd_real("142322343550859")/qd_real("51080680851480"),
    qd_real("154355972958659")/qd_real("351179680853925"),
    qd_real("167483568676259")/qd_real("2937139148960100"),
    qd_real("4230788929433")/qd_real("704913395750424"),
    qd_real("197968763176019")/qd_real("392923948371995600"),
    qd_real("10537522306718")/qd_real("319250708052246425"),
    qd_real("236648286272519")/qd_real("144249197475035425500"),
    qd_real("260715545088119")/qd_real("4375558990076074573500"),
    qd_real("289596255666839")/qd_real("192874640282553367199880"),
    qd_real("8802625510547")/qd_real("361639950529787563499775"),
    qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
    qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
    qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
}
```

Definition at line 1844 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.34 pade::pade\_sqrt\_denom< Scalar\_T > Struct Template Reference

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<Scalar_T, 14>`

### Static Public Attributes

- `static const array denom`

### 6.34.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_denom< Scalar_T >
```

Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

Definition at line 1401 of file [matrix\\_multi\\_imp.h](#).

### 6.34.2 Member Typedef Documentation

#### 6.34.2.1 array

```
template<typename Scalar_T >
using pade::pade_sqrt_denom< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1403 of file [matrix\\_multi\\_imp.h](#).

### 6.34.3 Member Data Documentation

#### 6.34.3.1 denom

```
template<typename Scalar_T >
const pade_sqrt_denom< longdouble >::array pade::pade_sqrt_denom< Scalar_T >::denom [static]
```

Initial value:

```
=
{
    1.0,          25.0/4.0,          69.0/4.0,          1771.0/64.0,
    7315.0/256.0,  20349.0/1024.0,   4845.0/512.0,   12597.0/4096.0,
    21879.0/32768.0, 12155.0/131072.0, 1001.0/131072.0, 1365.0/4194304.0,
    91.0/16777216.0, 1.0/67108864.0
}
```

Definition at line 1404 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.35 `pade::pade_sqrt_denom< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<dd_real, 22>`

### Static Public Attributes

- `static const array denom`

### 6.35.1 Detailed Description

Definition at line 1491 of file `matrix_multi_imp.h`.

### 6.35.2 Member Typedef Documentation

#### 6.35.2.1 `array`

```
using pade::pade_sqrt_denom< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1493 of file `matrix_multi_imp.h`.

### 6.35.3 Member Data Documentation

#### 6.35.3.1 `denom`

```
const pade_sqrt_denom< dd_real >::array pade::pade_sqrt_denom< dd_real >::denom [static]
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("195")/dd_real("4"),
    dd_real("73815")/dd_real("256"),
    dd_real("121737")/dd_real("256"),
    dd_real("4539051")/dd_real("16384"),
    dd_real("4032015")/dd_real("65536"),
    dd_real("86493225")/dd_real("16777216"),
    dd_real("5014575")/dd_real("33554432"),
    dd_real("5311735")/dd_real("4294967296"),
    dd_real("33649")/dd_real("17179869184"),
    dd_real("231")/dd_real("1099511627776"),
    dd_real("41")/dd_real("4"),
    dd_real("9139")/dd_real("64"),
    dd_real("435897")/dd_real("1024"),
    dd_real("840565")/dd_real("2048"),
    dd_real("9641775")/dd_real("65536"),
    dd_real("84672315")/dd_real("4194304"),
    dd_real("67863915")/dd_real("67108864"),
    dd_real("4345965")/dd_real("268435456"),
    dd_real("1081575")/dd_real("17179869184"),
    dd_real("8855")/dd_real("274877906944"),
    dd_real("1")/dd_real("4398046511104")
}
```

Definition at line 1494 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.36 `pade::pade_sqrt_denom< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<float, 10>`

### Static Public Attributes

- `static const array denom`

### 6.36.1 Detailed Description

Definition at line 1428 of file `matrix_multi_imp.h`.

### 6.36.2 Member Typedef Documentation

#### 6.36.2.1 `array`

```
using pade::pade_sqrt_denom< float >::array = std::array<float, 10>
```

Definition at line 1430 of file `matrix_multi_imp.h`.

### 6.36.3 Member Data Documentation

#### 6.36.3.1 `denom`

```
const pade_sqrt_denom< float >::array pade::pade_sqrt_denom< float >::denom [static]
```

#### Initial value:

```
=
{
    1.0,          17.0/4.0,      15.0/2.0,      455.0/64.0,
    1001.0/256.0,  1287.0/1024.0,  231.0/1024.0,  165.0/8192.0,
    45.0/65536,   1.0/262144.0
}
```

Definition at line 1431 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.37 `pade::pade_sqrt_denom< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

## Public Types

- `using array = std::array<long double, 18>`

## Static Public Attributes

- `static const array denom`

### 6.37.1 Detailed Description

Definition at line 1455 of file `matrix_multi_imp.h`.

### 6.37.2 Member Typedef Documentation

#### 6.37.2.1 array

```
using pade::pade_sqrt_denom< long double >::array = std::array<long double, 18>
```

Definition at line 1457 of file `matrix_multi_imp.h`.

### 6.37.3 Member Data Documentation

#### 6.37.3.1 denom

```
const array pade::pade_sqrt_denom< long double >::denom [static]
```

Definition at line 1458 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.38 pade::pade\_sqrt\_denom< qd\_real > Struct Reference

```
#include <matrix_multi_imp.h>
```

## Public Types

- `using array = std::array<qd_real, 34>`

## Static Public Attributes

- `static const array denom`

### 6.38.1 Detailed Description

Definition at line 1538 of file [matrix\\_multi\\_imp.h](#).

### 6.38.2 Member Typedef Documentation

#### 6.38.2.1 array

```
using pade::pade_sqrt_denom< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1540 of file [matrix\\_multi\\_imp.h](#).

### 6.38.3 Member Data Documentation

#### 6.38.3.1 denom

```
const pade_sqrt_denom< qd_real >::array pade::pade_sqrt_denom< qd_real >::denom [static]
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("126"),
    qd_real("557845")/qd_real("256"),
    qd_real("12515965")/qd_real("1024"),
    qd_real("1916797311")/qd_real("65536"),
    qd_real("4450881435")/qd_real("131072"),
    qd_real("171503444385")/qd_real("8388608"),
    qd_real("221120793075")/qd_real("33554432"),
    qd_real("4923689695575")/qd_real("4294967296"),
    qd_real("456864812569")/qd_real("4294967296"),
    qd_real("3486599885395")/qd_real("137438953472"),
    qd_real("2804116503573")/qd_real("549755813888"),
    qd_real("1886827875075")/qd_real("2199023255552"),
    qd_real("263012370465")/qd_real("2199023255552"),
    qd_real("240141729555")/qd_real("17592186044416"),
    qd_real("176848560525")/qd_real("140737488355328"),
    qd_real("51538723353")/qd_real("562949953421312"),
    qd_real("1450433115")/qd_real("281474976710656"),
    qd_real("977699359")/qd_real("4503599627370496"),
    qd_real("118183439")/qd_real("18014398509481984"),
    qd_real("9652005")/qd_real("72057594037927936"),
    qd_real("121737")/qd_real("72057594037927936"),
    qd_real("6545")/qd_real("576460752303423488"),
    qd_real("561")/qd_real("18446744073709551616"),
    qd_real("1")/qd_real("73786976294838206464")
}
```

Definition at line 1541 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.39 pade::pade\_sqrt\_numer< Scalar\_T > Struct Template Reference

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

```
#include <matrix_multi_imp.h>
```

## Public Types

- `using array = std::array<Scalar_T, 14>`

## Static Public Attributes

- `static const array number`

### 6.39.1 Detailed Description

```
template<typename Scalar_T>
struct pade::pade_sqrt_number< Scalar_T >
```

Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)

Definition at line 1384 of file [matrix\\_multi\\_imp.h](#).

### 6.39.2 Member Typedef Documentation

#### 6.39.2.1 array

```
template<typename Scalar_T >
using pade::pade_sqrt_number< Scalar_T >::array = std::array<Scalar_T, 14>
```

Definition at line 1386 of file [matrix\\_multi\\_imp.h](#).

### 6.39.3 Member Data Documentation

#### 6.39.3.1 numer

```
template<typename Scalar_T >
const pade_sqrt_number< longdouble >::array pade::pade_sqrt_number< Scalar_T >::numer [static]
```

#### Initial value:

```
=
{
    1.0,                27.0/4.0,        81.0/4.0,        2277.0/64.0,
    10395.0/256.0,       32319.0/1024.0,   8721.0/512.0,    26163.0/4096.0,
    53703.0/32768.0,     36465.0/131072.0,  3861.0/131072.0,   7371.0/4194304.0,
    819.0/16777216.0,    27.0/67108864.0
}
```

Definition at line 1387 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)



## 6.40 `pade::pade_sqrt_numer< dd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<dd_real, 22>`

### Static Public Attributes

- `static const array number`

### 6.40.1 Detailed Description

Definition at line 1471 of file `matrix_multi_imp.h`.

### 6.40.2 Member Typedef Documentation

#### 6.40.2.1 `array`

```
using pade::pade_sqrt_numer< dd_real >::array = std::array<dd_real, 22>
```

Definition at line 1473 of file `matrix_multi_imp.h`.

### 6.40.3 Member Data Documentation

#### 6.40.3.1 `number`

```
const pade_sqrt_numer< dd_real >::array pade::pade_sqrt_numer< dd_real >::number [static]
```

Initial value:

```
=
{
    dd_real("1"),
    dd_real("215")/dd_real("4"),
    dd_real("90687")/dd_real("256"),
    dd_real("168861")/dd_real("256"),
    dd_real("7228859")/dd_real("16384"),
    dd_real("7538115")/dd_real("65536"),
    dd_real("195747825")/dd_real("16777216"),
    dd_real("14375115")/dd_real("33554432"),
    dd_real("20764055")/dd_real("4294967296"),
    dd_real("206701")/dd_real("17179869184"),
    dd_real("3311")/dd_real("1099511627776"),
    dd_real("43")/dd_real("4"),
    dd_real("10621")/dd_real("64"),
    dd_real("567987")/dd_real("1024"),
    dd_real("1246355")/dd_real("2048"),
    dd_real("16583853")/dd_real("65536"),
    dd_real("173376645")/dd_real("4194304"),
    dd_real("171655785")/dd_real("67108864"),
    dd_real("14375115")/dd_real("268435456"),
    dd_real("5167525")/dd_real("17179869184"),
    dd_real("76153")/dd_real("274877906944"),
    dd_real("43")/dd_real("4398046511104")
}
```

Definition at line 1474 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.41 `pade::pade_sqrt_numer< float >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<float, 10>`

### Static Public Attributes

- `static const array numer`

### 6.41.1 Detailed Description

Definition at line 1416 of file `matrix_multi_imp.h`.

### 6.41.2 Member Typedef Documentation

#### 6.41.2.1 `array`

```
using pade::pade_sqrt_numer< float >::array = std::array<float, 10>
```

Definition at line 1418 of file `matrix_multi_imp.h`.

### 6.41.3 Member Data Documentation

#### 6.41.3.1 `numer`

```
const pade_sqrt_numer< float >::array pade::pade_sqrt_numer< float >::numer [static]
```

#### Initial value:

```
=
{
    1.0,          19.0/4.0,      19.0/2.0,      665.0/64.0,
    1729.0/256.0,  2717.0/1024.0,  627.0/1024.0,  627.0/8192.0,
    285.0/65536.0, 19.0/262144.0
}
```

Definition at line 1419 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.42 `pade::pade_sqrt_numer< long double >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<long double, 18>`

### Static Public Attributes

- `static const array numer`

## 6.42.1 Detailed Description

Definition at line 1441 of file `matrix_multi_imp.h`.

## 6.42.2 Member Typedef Documentation

### 6.42.2.1 array

```
using pade::pade_sqrt_numer< long double >::array = std::array<long double, 18>
```

Definition at line 1443 of file `matrix_multi_imp.h`.

## 6.42.3 Member Data Documentation

### 6.42.3.1 numer

```
const array pade::pade_sqrt_numer< long double >::numer [static]
```

Definition at line 1444 of file `matrix_multi_imp.h`.

The documentation for this struct was generated from the following file:

- `glucat/matrix_multi_imp.h`

## 6.43 `pade::pade_sqrt_numer< qd_real >` Struct Reference

```
#include <matrix_multi_imp.h>
```

### Public Types

- `using array = std::array<qd_real, 34>`

### Static Public Attributes

- `static const array numer`

### 6.43.1 Detailed Description

Definition at line 1512 of file [matrix\\_multi\\_imp.h](#).

### 6.43.2 Member Typedef Documentation

#### 6.43.2.1 array

```
using pade::pade_sqrt_number< qd_real >::array = std::array<qd_real, 34>
```

Definition at line 1514 of file [matrix\\_multi\\_imp.h](#).

### 6.43.3 Member Data Documentation

#### 6.43.3.1 numer

```
const pade_sqrt_number< qd_real >::array pade::pade_sqrt_number< qd_real >::numer [static]
```

Initial value:

```
=
{
    qd_real("1"),
    qd_real("134"),
    qd_real("633485")/qd_real("256"),
    qd_real("15246721")/qd_real("1024"),
    qd_real("2518145487")/qd_real("65536"),
    qd_real("6344873535")/qd_real("131072"),
    qd_real("267226297065")/qd_real("8388608"),
    qd_real("379874182975")/qd_real("33554432"),
    qd_real("9425348845815")/qd_real("4294967296"),
    qd_real("987417498133")/qd_real("4294967296"),
    qd_real("8055248011085")/qd_real("137438953472"),
    qd_real("6958363175533")/qd_real("549755813888"),
    qd_real("5056698705201")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("2199023255552"),
    qd_real("766166470485")/qd_real("17592186044416"),
    qd_real("623623871325")/qd_real("140737488355328"),
    qd_real("203123203803")/qd_real("562949953421312"),
    qd_real("6478601247")/qd_real("281474976710656"),
    qd_real("5038912081")/qd_real("4503599627370496"),
    qd_real("719844583")/qd_real("18014398509481984"),
    qd_real("71853815")/qd_real("72057594037927936"),
    qd_real("1165197")/qd_real("72057594037927936"),
    qd_real("87703")/qd_real("576460752303423488"),
    qd_real("12529")/qd_real("18446744073709551616"),
    qd_real("67")/qd_real("73786976294838206464")
}
```

Definition at line 1515 of file [matrix\\_multi\\_imp.h](#).

The documentation for this struct was generated from the following file:

- [glucat/matrix\\_multi\\_imp.h](#)

## 6.44 glucat::numeric\_traits< Scalar\_T >::promoted Struct Reference

Extra traits which extend numeric limits.

```
#include <promotion.h>
```

## Public Types

- using [type](#) = double
- using [type](#) = long double
- using [type](#) = double

### 6.44.1 Detailed Description

```
template<typename Scalar_T>
struct glucat::numeric_traits< Scalar_T >::promoted
```

Extra traits which extend numeric limits.

Promoted type.

Promoted type for long double.

Promoted type for double

Definition at line 70 of file [promotion.h](#).

### 6.44.2 Member Typedef Documentation

#### 6.44.2.1 [type](#) [1/3]

```
template<typename Scalar_T >
using glucat::numeric\_traits< Scalar_T >::promoted::type = double
```

Definition at line 72 of file [promotion.h](#).

#### 6.44.2.2 [type](#) [2/3]

```
template<typename Scalar_T >
using glucat::numeric\_traits< Scalar_T >::promoted::type = long double
```

Definition at line 86 of file [promotion.h](#).

#### 6.44.2.3 [type](#) [3/3]

```
template<typename Scalar_T >
using glucat::numeric\_traits< Scalar_T >::promoted::type = double
```

Definition at line 145 of file [scalar.h](#).

The documentation for this struct was generated from the following files:

- [glucat/promotion.h](#)
- [glucat/scalar.h](#)

## 6.45 `glucat::random_generator< Scalar_T >` Class Template Reference

Random number generator with single instance per `Scalar_T`.

```
#include <random.h>
```

### Public Member Functions

- `random_generator` (const `random_generator` &)=delete
- auto `operator=` (const `random_generator` &) -> `random_generator` &=delete
- auto `uniform` () -> `Scalar_T`
- auto `normal` () -> `Scalar_T`

### Static Public Member Functions

- static auto `generator` () -> `random_generator` &  
*Single instance of Random number generator.*

### Private Member Functions

- `random_generator` ()
- `~random_generator` ()=default

### Private Attributes

- `std::mt19937` `uint_gen`
- `std::uniform_real_distribution< double >` `uniform_dist`
- `std::normal_distribution< double >` `normal_dist`

### Static Private Attributes

- static const unsigned long `seed` = 19590921UL

### Friends

- class `friend_for_private_destructor`

### 6.45.1 Detailed Description

```
template<typename Scalar_T>
class glucat::random_generator< Scalar_T >
```

Random number generator with single instance per `Scalar_T`.

Definition at line 42 of file `random.h`.

## 6.45.2 Constructor & Destructor Documentation

### 6.45.2.1 random\_generator() [1/2]

```
template<typename Scalar_T >
glucat::random_generator< Scalar_T >::random_generator (
    const random_generator< Scalar_T > & ) [delete]
```

### 6.45.2.2 random\_generator() [2/2]

```
template<typename Scalar_T >
glucat::random_generator< Scalar_T >::random_generator ( ) [inline], [private]
```

Definition at line 61 of file [random.h](#).

References [glucat::random\\_generator< Scalar\\_T >::seed](#).

### 6.45.2.3 ~random\_generator()

```
template<typename Scalar_T >
glucat::random_generator< Scalar_T >::~~random_generator ( ) [private], [default]
```

## 6.45.3 Member Function Documentation

### 6.45.3.1 generator()

```
template<typename Scalar_T >
static auto glucat::random_generator< Scalar_T >::generator ( ) -> random_generator& [inline],
[static]
```

Single instance of Random number generator.

Definition at line 51 of file [random.h](#).

### 6.45.3.2 normal()

```
template<typename Scalar_T >
auto glucat::random_generator< Scalar_T >::normal ( ) -> Scalar_T [inline]
```

Definition at line 70 of file [random.h](#).

References [glucat::random\\_generator< Scalar\\_T >::normal\\_dist](#).

### 6.45.3.3 operator=()

```
template<typename Scalar_T >
auto glucat::random_generator< Scalar_T >::operator= (
    const random_generator< Scalar_T > & ) -> random_generator &=delete [delete]
```

#### 6.45.3.4 uniform()

```
template<typename Scalar_T >
auto glucat::random_generator< Scalar_T >::uniform ( ) -> Scalar_T    [inline]
```

Definition at line 68 of file [random.h](#).

References [glucat::random\\_generator< Scalar\\_T >::uniform\\_dist](#).

### 6.45.4 Friends And Related Symbol Documentation

#### 6.45.4.1 friend\_for\_private\_destructor

```
template<typename Scalar_T >
friend class friend_for_private_destructor    [friend]
```

Friend declaration to avoid compiler warning: "... only defines a private destructor and has no friends" Ref: Carlos O'Ryan, ACE <http://doc.ece.uci.edu>

Definition at line 48 of file [random.h](#).

### 6.45.5 Member Data Documentation

#### 6.45.5.1 normal\_dist

```
template<typename Scalar_T >
std::normal_distribution<double> glucat::random_generator< Scalar_T >::normal_dist    [private]
```

Definition at line 59 of file [random.h](#).

Referenced by [glucat::random\\_generator< Scalar\\_T >::normal\(\)](#).

#### 6.45.5.2 seed

```
template<typename Scalar_T >
const unsigned long glucat::random_generator< Scalar_T >::seed = 19590921UL    [static], [private]
```

Definition at line 55 of file [random.h](#).

Referenced by [glucat::random\\_generator< Scalar\\_T >::random\\_generator\(\)](#).

#### 6.45.5.3 uint\_gen

```
template<typename Scalar_T >
std::mt19937 glucat::random_generator< Scalar_T >::uint_gen    [private]
```

Definition at line 57 of file [random.h](#).



## 6.45.5.4 uniform\_dist

```
template<typename Scalar_T >
std::uniform_real_distribution<double> glucat::random_generator< Scalar_T >::uniform_dist
[private]
```

Definition at line 58 of file [random.h](#).

Referenced by [glucat::random\\_generator< Scalar\\_T >::uniform\(\)](#).

The documentation for this class was generated from the following file:

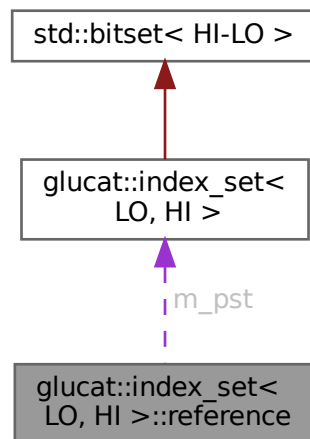
- [glucat/random.h](#)

## 6.46 glucat::index\_set&lt; LO, HI &gt;::reference Class Reference

Index set member reference.

```
#include <index_set.h>
```

Collaboration diagram for glucat::index\_set< LO, HI >::reference:



## Public Member Functions

- [reference](#) ()=delete  
*Default constructor is deleted.*
- [reference](#) ([index\\_set\\_t](#) &ist, [index\\_t](#) idx)  
*index\_set reference*
- [~reference](#) ()=default
- auto [operator==](#) (const [reference](#) &c\_j) const -> bool

- ```

for b[i] == c[j];
• auto operator= (const bool x) -> reference &
    for b[i] = x;
• auto operator= (const reference &c_j) -> reference &
    for b[i] = c[j];
• auto operator~ () const -> bool
    Flips a bit.
• operator bool () const
    for x = b[i];
• auto flip () -> reference &
    for b[i].flip();

```

### Private Attributes

- [index\\_set\\_t](#) \* m\_pst
- [index\\_t](#) m\_idx

### Friends

- class [index\\_set](#)

## 6.46.1 Detailed Description

template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
class [glucat::index\\_set](#)< LO, HI >::reference

Index set member reference.

Definition at line 177 of file [index\\_set.h](#).

## 6.46.2 Constructor & Destructor Documentation

### 6.46.2.1 reference() [1/2]

```

template<const index\_t LO, const index\_t HI>
glucat::index\_set< LO, HI >::reference::reference ( ) [delete]

```

Default constructor is deleted.

### 6.46.2.2 reference() [2/2]

```

template<const index\_t LO, const index\_t HI>
glucat::index\_set< LO, HI >::reference::reference (
    index\_set\_t & ist,
    index\_t idx ) [inline]

```

[index\\_set](#) reference

Definition at line 985 of file [index\\_set\\_imp.h](#).

### 6.46.2.3 ~reference()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::~reference ( ) [default]
```

## 6.46.3 Member Function Documentation

### 6.46.3.1 flip()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::flip ( ) -> reference& [inline]
```

for b[i].flip();

Definition at line 1049 of file index\_set\_imp.h.

References glucat::index\_set< LO, HI >::reference::flip().

Referenced by glucat::index\_set< LO, HI >::reference::flip().

### 6.46.3.2 operator bool()

```
template<const index_t LO, const index_t HI>
glucat::index_set< LO, HI >::reference::operator bool ( ) const [inline]
```

for x = b[i];

Definition at line 1041 of file index\_set\_imp.h.

### 6.46.3.3 operator=() [1/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const bool x ) -> reference& [inline]
```

for b[i] = x;

Definition at line 1003 of file index\_set\_imp.h.

### 6.46.3.4 operator=() [2/2]

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator= (
    const reference & c_j ) -> reference& [inline]
```

for b[i] = c[j];

Definition at line 1017 of file index\_set\_imp.h.

### 6.46.3.5 operator==()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator== (
    const reference & c_j ) const -> bool [inline]
```

for b[i] == c[j];

Definition at line 995 of file [index\\_set\\_imp.h](#).

### 6.46.3.6 operator~()

```
template<const index_t LO, const index_t HI>
auto glucat::index_set< LO, HI >::reference::operator~ ( ) const -> bool [inline]
```

Flips a bit.

flips the bit

Definition at line 1034 of file [index\\_set\\_imp.h](#).

## 6.46.4 Friends And Related Symbol Documentation

### 6.46.4.1 index\_set

```
template<const index_t LO, const index_t HI>
friend class index_set [friend]
```

Definition at line 178 of file [index\\_set.h](#).

## 6.46.5 Member Data Documentation

### 6.46.5.1 m\_idx

```
template<const index_t LO, const index_t HI>
index_t glucat::index_set< LO, HI >::reference::m_idx [private]
```

Definition at line 200 of file [index\\_set.h](#).

### 6.46.5.2 m\_pst

```
template<const index_t LO, const index_t HI>
index_set_t* glucat::index_set< LO, HI >::reference::m_pst [private]
```

Definition at line 199 of file [index\\_set.h](#).

The documentation for this class was generated from the following files:

- [glucat/index\\_set.h](#)
- [glucat/index\\_set\\_imp.h](#)

## 6.47 glucat::sorted\_range< Map\_T, Sorted\_Map\_T > Class Template Reference

Sorted range for use with output.

```
#include <framed_multi_imp.h>
```

### Public Types

- using [map\\_t](#) = Map\_T
- using [sorted\\_map\\_t](#) = Sorted\_Map\_T
- using [sorted\\_iterator](#) = typename Sorted\_Map\_T::const\_iterator

### Public Member Functions

- [sorted\\_range](#) (Sorted\_Map\_T &sorted\_val, const Map\_T &val)

### Public Attributes

- [sorted\\_iterator](#) [sorted\\_begin](#)
- [sorted\\_iterator](#) [sorted\\_end](#)

### 6.47.1 Detailed Description

```
template<typename Map_T, typename Sorted_Map_T>
class glucat::sorted_range< Map_T, Sorted_Map_T >
```

Sorted range for use with output.

Definition at line 1112 of file [framed\\_multi\\_imp.h](#).

### 6.47.2 Member Typedef Documentation

#### 6.47.2.1 map\_t

```
template<typename Map_T , typename Sorted_Map_T >
using glucat::sorted_range< Map_T, Sorted_Map_T >::map_t = Map_T
```

Definition at line 1115 of file [framed\\_multi\\_imp.h](#).

#### 6.47.2.2 sorted\_iterator

```
template<typename Map_T , typename Sorted_Map_T >
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T↔
::const_iterator
```

Definition at line 1117 of file [framed\\_multi\\_imp.h](#).

### 6.47.2.3 sorted\_map\_t

```
template<typename Map_T , typename Sorted_Map_T >
using glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1116 of file [framed\\_multi\\_imp.h](#).

## 6.47.3 Constructor & Destructor Documentation

### 6.47.3.1 sorted\_range()

```
template<typename Map_T , typename Sorted_Map_T >
glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Map_T & val ) [inline]
```

Definition at line 1119 of file [framed\\_multi\\_imp.h](#).

References [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::sorted\\_begin](#), and [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::s](#)

## 6.47.4 Member Data Documentation

### 6.47.4.1 sorted\_begin

```
template<typename Map_T , typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1126 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::sorted\\_range\(\)](#).

### 6.47.4.2 sorted\_end

```
template<typename Map_T , typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1127 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::sorted\\_range< Map\\_T, Sorted\\_Map\\_T >::sorted\\_range\(\)](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi\\_imp.h](#)

## 6.48 glucat::sorted\_range< Sorted\_Map\_T, Sorted\_Map\_T > Class Template Reference

```
#include <framed_multi_imp.h>
```

## Public Types

- using [map\\_t](#) = Sorted\_Map\_T
- using [sorted\\_map\\_t](#) = Sorted\_Map\_T
- using [sorted\\_iterator](#) = typename Sorted\_Map\_T::const\_iterator

## Public Member Functions

- [sorted\\_range](#) (Sorted\_Map\_T &sorted\_val, const Sorted\_Map\_T &val)

## Public Attributes

- [sorted\\_iterator](#) [sorted\\_begin](#)
- [sorted\\_iterator](#) [sorted\\_end](#)

### 6.48.1 Detailed Description

```
template<typename Sorted_Map_T>
class glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >
```

Definition at line 1131 of file [framed\\_multi\\_imp.h](#).

### 6.48.2 Member Typedef Documentation

#### 6.48.2.1 map\_t

```
template<typename Sorted_Map_T >
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::map_t = Sorted_Map_T
```

Definition at line 1134 of file [framed\\_multi\\_imp.h](#).

#### 6.48.2.2 sorted\_iterator

```
template<typename Sorted_Map_T >
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_iterator = typename Sorted_Map_T::const_iterator
```

Definition at line 1136 of file [framed\\_multi\\_imp.h](#).

#### 6.48.2.3 sorted\_map\_t

```
template<typename Sorted_Map_T >
using glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_map_t = Sorted_Map_T
```

Definition at line 1135 of file [framed\\_multi\\_imp.h](#).

### 6.48.3 Constructor & Destructor Documentation

#### 6.48.3.1 sorted\_range()

```
template<typename Sorted_Map_T >
glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_range (
    Sorted_Map_T & sorted_val,
    const Sorted_Map_T & val ) [inline]
```

Definition at line 1138 of file [framed\\_multi\\_imp.h](#).

### 6.48.4 Member Data Documentation

#### 6.48.4.1 sorted\_begin

```
template<typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_begin
```

Definition at line 1142 of file [framed\\_multi\\_imp.h](#).

#### 6.48.4.2 sorted\_end

```
template<typename Sorted_Map_T >
sorted_iterator glucat::sorted_range< Sorted_Map_T, Sorted_Map_T >::sorted_end
```

Definition at line 1143 of file [framed\\_multi\\_imp.h](#).

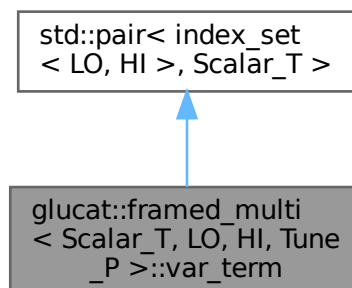
The documentation for this class was generated from the following file:

- [glucat/framed\\_multi\\_imp.h](#)

## 6.49 glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term Class Reference

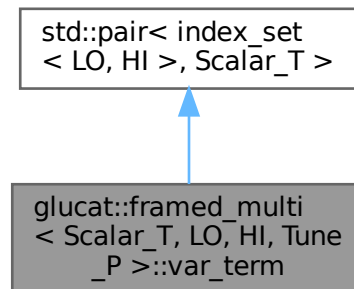
Variable term.

Inheritance diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term:





Collaboration diagram for glucat::framed\_multi< Scalar\_T, LO, HI, Tune\_P >::var\_term:



## Public Types

- using [var\\_pair\\_t](#) = std::pair<[index\\_set](#)<LO, HI>, Scalar\_T>

## Public Member Functions

- [~var\\_term](#) ()=default  
*Destructor.*
- [var\\_term](#) ()  
*Default constructor.*
- [var\\_term](#) (const [index\\_set\\_t](#) ist, const Scalar\_T &crd=Scalar\_T(1))  
*Construct a variable term from an index set and a scalar coordinate.*
- auto [operator\\*=](#) (const [term\\_t](#) &rhs) -> [var\\_term\\_t](#) &  
*Product of variable term and term.*

## Static Public Member Functions

- static auto [classname](#) () -> const std::string  
*Class name used in messages.*

### 6.49.1 Detailed Description

```

template<typename Scalar_T = double, const index\_t LO = DEFAULT_LO, const index\_t HI = DEFAULT_HI,
typename Tune_P = tuning<>>
class glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term

```

Variable term.

Definition at line 279 of file [framed\\_multi.h](#).

## 6.49.2 Member Typedef Documentation

### 6.49.2.1 var\_pair\_t

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
using glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>
```

Definition at line 283 of file [framed\\_multi.h](#).

## 6.49.3 Constructor & Destructor Documentation

### 6.49.3.1 ~var\_term()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::~~var_term ( ) [default]
```

Destructor.

### 6.49.3.2 var\_term() [1/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term ( ) [inline]
```

Default constructor.

Definition at line 291 of file [framed\\_multi.h](#).

### 6.49.3.3 var\_term() [2/2]

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::var_term (
    const index_set_t ist,
    const Scalar_T & crd = Scalar_T(1) ) [inline]
```

Construct a variable term from an index set and a scalar coordinate.

Definition at line 295 of file [framed\\_multi.h](#).

## 6.49.4 Member Function Documentation

### 6.49.4.1 classname()

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
static auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::classname ( ) ->
const std::string [inline], [static]
```

Class name used in messages.

Definition at line 286 of file [framed\\_multi.h](#).

#### 6.49.4.2 operator\*=( )

```
template<typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI, typename Tune_P = tuning<>>
auto glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term::operator*= (
    const term_t & rhs ) -> var_term_t&    [inline]
```

Product of variable term and term.

Definition at line 299 of file [framed\\_multi.h](#).

The documentation for this class was generated from the following file:

- [glucat/framed\\_multi.h](#)



## Chapter 7

# File Documentation

### 7.1 glucat/clifford\_algebra.h File Reference

```
#include "glucat/global.h"
```

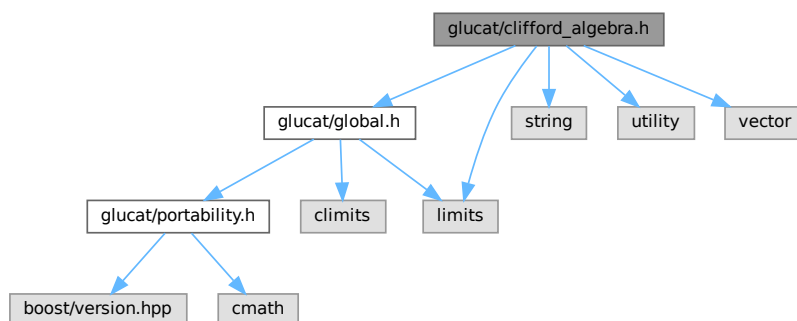
```
#include <limits>
```

```
#include <string>
```

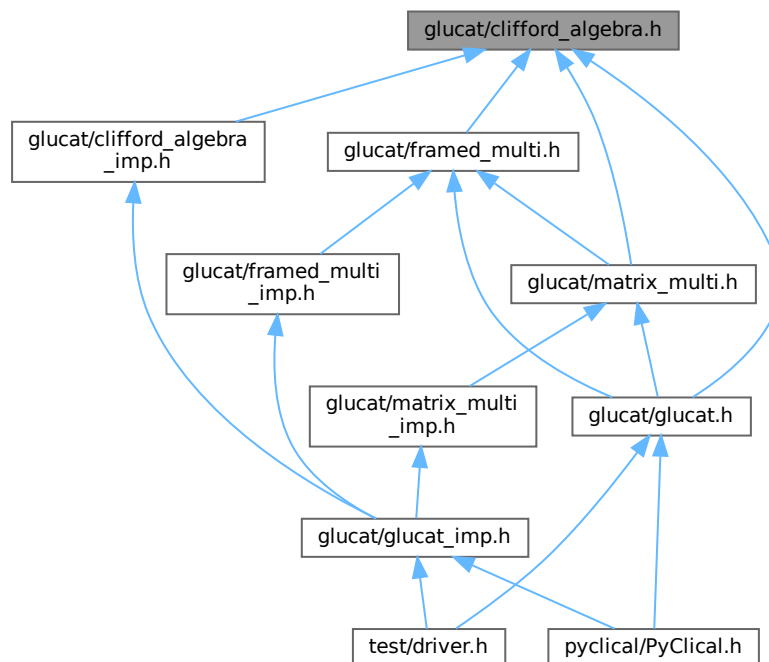
```
#include <utility>
```

```
#include <vector>
```

Include dependency graph for clifford\_algebra.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::clifford_algebra< Scalar_T, Index_Set_T, Multivector_T >`  
*clifford\_algebra<> declares the operations of a Clifford algebra*

## Namespaces

- namespace `glucat`

## Macros

- `#define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS`

## Functions

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool`

*Test for inequality of multivector and scalar.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator!=](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool

*Test for inequality of scalar and multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::error\\_squared\\_tol](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &val) -> Scalar\_T

*Quadratic norm error tolerance relative to a specific multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::error\\_squared](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold) -> Scalar\_T

*Relative or absolute error using the quadratic norm.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::approx\\_equal](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs, const Scalar\_T threshold, const Scalar\_T tolerance) -> bool

*Test for approximate equality of multivectors.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::approx\\_equal](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> bool

*Test for approximate equality of multivectors.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator+](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric sum of multivector and scalar.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator+](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric sum of scalar and multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator+](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric sum.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator-](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const Scalar\_T &scr) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric difference of multivector and scalar.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator-](#) (const Scalar\_T &scr, const Multivector< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric difference of scalar and multivector.*

- template<template< typename, const [index\\_t](#), const [index\\_t](#), typename > class Multivector, template< typename, const [index\\_t](#), const [index\\_t](#), typename > class RHS, typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator-](#) (const Multivector< Scalar\_T, LO, HI, Tune\_P > &lhs, const RHS< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const Multivector< Scalar\_T, LO, HI, Tune\_P >

*Geometric difference.*





- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric multiplicative inverse.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Integer power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Multivector power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Outer product power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Real part: synonym for scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Imaginary part: deprecated (always 0)*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Pure part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Even part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Odd part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_T >`  
*Vector part of multivector, as a vector\_t with respect to frame()*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-\{i\}$  in each term, eg.  $\{1\}*\{2\} \rightarrow (-\{2\})*(-\{1\})$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation, rev o invo == invo o rev.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form ==  $(rev(x)*x)(0)$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm == sum of norm of coordinates.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value ==  $\sqrt{\text{norm}}$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`





## 7.1.1 Macro Definition Documentation

### 7.1.1.1 \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

#define \_GLUCAT\_CLIFFORD\_ALGEBRA\_OPERATIONS

Definition at line 145 of file clifford\_algebra.h.

## 7.2 clifford\_algebra.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   clifford_algebra.h : Declare the operations of a Clifford algebra
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright             : (C) 2001-2021 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   See also Arvind Raja's original header comments in glucat.h
00032   *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <limits>
00037 #include <string>
00038 #include <utility>
00039 #include <vector>
00040
00041 namespace glucat
00042 {
00043     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00044     class clifford_algebra
00045     {
00046     public:
00047         using scalar_t = Scalar_T;
00048         using index_set_t = Index_Set_T;
00049         static const index_t v_lo = index_set_t::v_lo;
00050         static const index_t v_hi = index_set_t::v_hi;
00051         using multivector_t = Multivector_T;
00052         using pair_t = std::pair<const index_set_t, Scalar_T>;
00053         using vector_t = std::vector<Scalar_T>;
00054
00055         static auto classname() -> const std::string;
00056
00057         static const Scalar_T default_truncation;
00058
00059         virtual ~clifford_algebra() = default;
00060
00061         // clifford_algebra operations
00062         virtual auto operator== (const multivector_t& val) const -> bool = 0;
00063         virtual auto operator== (const Scalar_T& scr) const -> bool = 0;
00064         virtual auto operator+= (const multivector_t& rhs) -> multivector_t& = 0;
00065         virtual auto operator+= (const Scalar_T& scr) -> multivector_t& = 0;
00066         virtual auto operator-= (const multivector_t& rhs) -> multivector_t& = 0;

```

```

00075     virtual auto operator== (const Scalar_T& scr) -> multivector_t& = 0;
00077     virtual auto operator- (const Scalar_T& scr) -> multivector_t = 0;
00079     virtual auto operator+= (const Scalar_T& scr) -> multivector_t& = 0;
00081     virtual auto operator*= (const multivector_t& rhs) -> multivector_t& = 0;
00083     virtual auto operator%= (const multivector_t& rhs) -> multivector_t& = 0;
00085     virtual auto operator&= (const multivector_t& rhs) -> multivector_t& = 0;
00087     virtual auto operator^= (const multivector_t& rhs) -> multivector_t& = 0;
00089     virtual auto operator/= (const Scalar_T& scr) -> multivector_t = 0;
00091     virtual auto operator/= (const multivector_t& rhs) -> multivector_t& = 0;
00093     virtual auto operator|= (const multivector_t& rhs) -> multivector_t& = 0;
00095     virtual auto inv (const multivector_t& rhs) -> multivector_t = 0;
00097     virtual auto pow (int m) const -> const multivector_t = 0;
00099     virtual auto outer_pow (int m) const -> const multivector_t = 0;
00101     virtual auto frame (const multivector_t& rhs) -> multivector_t = 0;
00103     virtual auto grade (const multivector_t& rhs) -> index_t = 0;
00105     virtual auto operator[] (const index_set_t ist) const -> Scalar_T = 0;
00107     virtual auto operator() (const index_t grade) const -> const multivector_t = 0;
00109     virtual auto scalar (const multivector_t& rhs) -> Scalar_T = 0;
00111     virtual auto pure (const multivector_t& rhs) -> multivector_t = 0;
00113     virtual auto even (const multivector_t& rhs) -> multivector_t = 0;
00115     virtual auto odd (const multivector_t& rhs) -> multivector_t = 0;
00117     virtual auto vector_part (const multivector_t& rhs) -> vector_t = 0;
00119     virtual auto vector_part (const index_set_t frm, const bool prechecked) const -> const vector_t =
0;
00121     virtual auto involute (const multivector_t& rhs) -> multivector_t = 0;
00123     virtual auto reverse (const multivector_t& rhs) -> multivector_t = 0;
00125     virtual auto conj (const multivector_t& rhs) -> multivector_t = 0;
00127     virtual auto quad (const multivector_t& rhs) -> Scalar_T = 0;
00129     virtual auto norm (const multivector_t& rhs) -> Scalar_T = 0;
00131     virtual auto max_abs (const multivector_t& rhs) -> Scalar_T = 0;
00133     virtual auto truncated (const Scalar_T& limit = default_truncation) const -> const multivector_t
= 0;
00135     virtual auto isinf (const multivector_t& rhs) -> bool = 0;
00137     virtual auto isnan (const multivector_t& rhs) -> bool = 0;
00139     virtual void write (const std::string& msg="") const = 0;
00141     virtual void write (std::ofstream& ofile, const std::string& msg="") const = 0;
00142 };
00143
00144 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00145 #define _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS \
00146     auto operator== (const multivector_t& val) const -> bool override;\
00147     auto operator== (const Scalar_T& scr) const -> bool override;\
00148     auto operator+= (const multivector_t& rhs) -> multivector_t& override;\
00149     auto operator+= (const Scalar_T& scr) -> multivector_t& override;\
00150     auto operator-= (const multivector_t& rhs) -> multivector_t& override;\
00151     auto operator-= (const Scalar_T& scr) -> multivector_t& override;\
00152     auto operator- (const multivector_t& rhs) -> const multivector_t override;\
00153     auto operator*= (const Scalar_T& scr) -> multivector_t& override;\
00154     auto operator*= (const multivector_t& rhs) -> multivector_t& override;\
00155     auto operator%= (const multivector_t& rhs) -> multivector_t& override;\
00156     auto operator&= (const multivector_t& rhs) -> multivector_t& override;\
00157     auto operator^= (const multivector_t& rhs) -> multivector_t& override;\
00158     auto operator/= (const Scalar_T& scr) -> multivector_t& override;\
00159     auto operator/= (const multivector_t& rhs) -> multivector_t& override;\
00160     auto operator|= (const multivector_t& rhs) -> multivector_t& override;\
00161     auto inv (const multivector_t& rhs) -> const multivector_t override;\
00162     auto pow (int m) const -> const multivector_t override;\
00163     auto outer_pow (int m) const -> const multivector_t override;\
00164     auto frame (const multivector_t& rhs) -> const index_set_t override;\
00165     auto grade (const multivector_t& rhs) -> index_t override;\
00166     auto operator[] (const index_set_t ist) const -> Scalar_T override;\
00167     auto operator() (const index_t grade) const -> const multivector_t override;\
00168     auto scalar (const multivector_t& rhs) -> Scalar_T override;\
00169     auto pure (const multivector_t& rhs) -> const multivector_t override;\
00170     auto even (const multivector_t& rhs) -> const multivector_t override;\
00171     auto odd (const multivector_t& rhs) -> const multivector_t override;\
00172     auto vector_part (const multivector_t& rhs) -> const vector_t override;\
00173     auto vector_part (const index_set_t frm, const bool prechecked = false) const \
00174     -> const vector_t override;\
00175     auto involute (const multivector_t& rhs) -> const multivector_t override;\
00176     auto reverse (const multivector_t& rhs) -> const multivector_t override;\
00177     auto conj (const multivector_t& rhs) -> const multivector_t override;\
00178     auto quad (const multivector_t& rhs) -> Scalar_T override;\
00179     auto norm (const multivector_t& rhs) -> Scalar_T override;\
00180     auto max_abs (const multivector_t& rhs) -> Scalar_T override;\
00181     auto truncated (const Scalar_T& limit = multivector_t::default_truncation) const \
00182     -> const multivector_t override;\
00183     auto isinf (const multivector_t& rhs) -> bool override;\
00184     auto isnan (const multivector_t& rhs) -> bool override;\
00185     void write (const std::string& msg="") const override;\
00186     void write (std::ofstream& ofile, const std::string& msg="") const override;\
00187 #endif // _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00188
00189 template
00190 <
00191     template<typename, const index_t, const index_t, typename> class Multivector,
00192     template<typename, const index_t, const index_t, typename> class RHS,

```

```

00194     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00195 >
00196     auto
00197     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
bool;
00198
00200     template
00201     <
00202         template<typename, const index_t, const index_t, typename> class Multivector,
00203         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00204     >
00205     auto
00206     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool;
00207
00209     template
00210     <
00211         template<typename, const index_t, const index_t, typename> class Multivector,
00212         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00213     >
00214     auto
00215     operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00216
00218     template
00219     <
00220         template<typename, const index_t, const index_t, typename> class Multivector,
00221         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00222     >
00223     auto
00224     error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00225
00227     template
00228     <
00229         template<typename, const index_t, const index_t, typename> class Multivector,
00230         template<typename, const index_t, const index_t, typename> class RHS,
00231         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00232     >
00233     auto
00234     error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00235                  const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00236                  const Scalar_T threshold) -> Scalar_T;
00237
00239     template
00240     <
00241         template<typename, const index_t, const index_t, typename> class Multivector,
00242         template<typename, const index_t, const index_t, typename> class RHS,
00243         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00244     >
00245     auto
00246     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00247                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00248                 const Scalar_T threshold,
00249                 const Scalar_T tolerance) -> bool;
00250
00252     template
00253     <
00254         template<typename, const index_t, const index_t, typename> class Multivector,
00255         template<typename, const index_t, const index_t, typename> class RHS,
00256         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00257     >
00258     auto
00259     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00260                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool;
00261
00263     template
00264     <
00265         template<typename, const index_t, const index_t, typename> class Multivector,
00266         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00267     >
00268     auto
00269     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00270
00272     template
00273     <
00274         template<typename, const index_t, const index_t, typename> class Multivector,
00275         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00276     >
00277     auto
00278     operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00279
00281     template
00282     <
00283         template<typename, const index_t, const index_t, typename> class Multivector,
00284         template<typename, const index_t, const index_t, typename> class RHS,
00285         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00286     >

```



```

00287     auto
00288     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00289
00291     template
00292     <
00293         template<typename, const index_t, const index_t, typename> class Multivector,
00294         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00295     >
00296     auto
00297     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00298
00300     template
00301     <
00302         template<typename, const index_t, const index_t, typename> class Multivector,
00303         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304     >
00305     auto
00306     operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00307
00309     template
00310     <
00311         template<typename, const index_t, const index_t, typename> class Multivector,
00312         template<typename, const index_t, const index_t, typename> class RHS,
00313         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00314     >
00315     auto
00316     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00317
00319     template
00320     <
00321         template<typename, const index_t, const index_t, typename> class Multivector,
00322         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00323     >
00324     auto
00325     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00326
00328     template
00329     <
00330         template<typename, const index_t, const index_t, typename> class Multivector,
00331         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00332     >
00333     auto
00334     operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00335
00337     template
00338     <
00339         template<typename, const index_t, const index_t, typename> class Multivector,
00340         template<typename, const index_t, const index_t, typename> class RHS,
00341         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00342     >
00343     auto
00344     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00345
00347     template
00348     <
00349         template<typename, const index_t, const index_t, typename> class Multivector,
00350         template<typename, const index_t, const index_t, typename> class RHS,
00351         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00352     >
00353     auto
00354     operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00355
00357     template
00358     <
00359         template<typename, const index_t, const index_t, typename> class Multivector,
00360         template<typename, const index_t, const index_t, typename> class RHS,
00361         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00362     >
00363     auto
00364     operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00365
00367     template
00368     <
00369         template<typename, const index_t, const index_t, typename> class Multivector,
00370         template<typename, const index_t, const index_t, typename> class RHS,
00371         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00372     >
00373     auto

```



```

00374     operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00375
00376     template
00377     <
00378         template<typename, const index_t, const index_t, typename> class Multivector,
00379         template<typename, const index_t, const index_t, typename> class RHS,
00380         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00381     >
00382     auto
00383     star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
Scalar_T;
00384
00385     template
00386     <
00387         template<typename, const index_t, const index_t, typename> class Multivector,
00388         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00389     >
00390     auto
00391     operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00392
00393     template
00394     <
00395         template<typename, const index_t, const index_t, typename> class Multivector,
00396         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00397     >
00398     auto
00399     operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00400
00401     template
00402     <
00403         template<typename, const index_t, const index_t, typename> class Multivector,
00404         template<typename, const index_t, const index_t, typename> class RHS,
00405         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00406     >
00407     auto
00408     operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00409
00410     template
00411     <
00412         template<typename, const index_t, const index_t, typename> class Multivector,
00413         template<typename, const index_t, const index_t, typename> class RHS,
00414         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00415     >
00416     auto
00417     operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>;
00418
00419     template
00420     <
00421         template<typename, const index_t, const index_t, typename> class Multivector,
00422         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00423     >
00424     auto
00425     inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00426
00427     template
00428     <
00429         template<typename, const index_t, const index_t, typename> class Multivector,
00430         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00431     >
00432     auto
00433     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00434
00435     template
00436     <
00437         template<typename, const index_t, const index_t, typename> class Multivector,
00438         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00439     >
00440     auto
00441     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00442
00443     template< template<typename, const index_t, const index_t, typename> class Multivector,
00444         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00445     auto
00446     outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>;
00447
00448     template
00449     <
00450         template<typename, const index_t, const index_t, typename> class Multivector,

```

```

00462     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00463 >
00464 auto
00465 scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00466
00467 template
00468 <
00469     template<typename, const index_t, const index_t, typename> class Multivector,
00470     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00471 >
00472 auto
00473 real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00474
00475 template
00476 <
00477     template<typename, const index_t, const index_t, typename> class Multivector,
00478     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00479 >
00480 auto
00481 imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00482
00483 template
00484 <
00485     template<typename, const index_t, const index_t, typename> class Multivector,
00486     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00487 >
00488 auto
00489 pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00490
00491 template
00492 <
00493     template<typename, const index_t, const index_t, typename> class Multivector,
00494     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00495 >
00496 auto
00497 odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00498
00499 template
00500 <
00501     template<typename, const index_t, const index_t, typename> class Multivector,
00502     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00503 >
00504 auto
00505 even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00506
00507 template
00508 <
00509     template<typename, const index_t, const index_t, typename> class Multivector,
00510     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00511 >
00512 auto
00513 vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>;
00514
00515 template
00516 <
00517     template<typename, const index_t, const index_t, typename> class Multivector,
00518     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00519 >
00520 auto
00521 involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00522
00523 template
00524 <
00525     template<typename, const index_t, const index_t, typename> class Multivector,
00526     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00527 >
00528 auto
00529 reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00530
00531 template
00532 <
00533     template<typename, const index_t, const index_t, typename> class Multivector,
00534     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00535 >
00536 auto
00537 conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00538
00539 template
00540 <
00541     template<typename, const index_t, const index_t, typename> class Multivector,
00542     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00543 >
00544 auto
00545 quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00546
00547 template
00548 <
00549     template<typename, const index_t, const index_t, typename> class Multivector,
00550     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00551 >
00552 auto
00553 quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00554
00555 template
00556 <

```

```

00560     template<typename, const index_t, const index_t, typename> class Multivector,
00561     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00562 >
00563 auto
00564 norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00565
00566 template
00567 <
00568     template<typename, const index_t, const index_t, typename> class Multivector,
00569     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00570 >
00571 auto
00572 abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00573
00574 template
00575 <
00576     template<typename, const index_t, const index_t, typename> class Multivector,
00577     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00578 >
00579 auto
00580 max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T;
00581
00582 template
00583 <
00584     template<typename, const index_t, const index_t, typename> class Multivector,
00585     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00586 >
00587 auto
00588 complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00589 Multivector<Scalar_T,LO,HI,Tune_P>;
00590
00591 template
00592 <
00593     template<typename, const index_t, const index_t, typename> class Multivector,
00594     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00595 >
00596 auto
00597 elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00598
00599 template
00600 <
00601     template<typename, const index_t, const index_t, typename> class Multivector,
00602     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00603 >
00604 auto
00605 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00606       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00607       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00608
00609 template
00610 <
00611     template<typename, const index_t, const index_t, typename> class Multivector,
00612     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00613 >
00614 auto
00615 sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00616
00617 // Transcendental functions
00618
00619 template
00620 <
00621     template<typename, const index_t, const index_t, typename> class Multivector,
00622     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00623 >
00624 auto
00625 clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00626 Multivector<Scalar_T,LO,HI,Tune_P>;
00627
00628 template
00629 <
00630     template<typename, const index_t, const index_t, typename> class Multivector,
00631     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00632 >
00633 auto
00634 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00635     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00636     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00637
00638 template
00639 <
00640     template<typename, const index_t, const index_t, typename> class Multivector,
00641     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00642 >
00643 auto
00644 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00645
00646 template
00647 <
00648     template<typename, const index_t, const index_t, typename> class Multivector,
00649     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00650 >
00651 auto
00652 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00653
00654 template
00655 <

```

```

00656     template<typename, const index_t, const index_t, typename> class Multivector,
00657     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00658 >
00659 auto
00660 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00661     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00662     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00663
00665 template
00666 <
00667     template<typename, const index_t, const index_t, typename> class Multivector,
00668     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00669 >
00670 auto
00671 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00672
00674 template
00675 <
00676     template<typename, const index_t, const index_t, typename> class Multivector,
00677     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00678 >
00679 auto
00680 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00681     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00682     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00683
00685 template
00686 <
00687     template<typename, const index_t, const index_t, typename> class Multivector,
00688     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00689 >
00690 auto
00691 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00692
00694 template
00695 <
00696     template<typename, const index_t, const index_t, typename> class Multivector,
00697     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00698 >
00699 auto
00700 cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00701
00703 template
00704 <
00705     template<typename, const index_t, const index_t, typename> class Multivector,
00706     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00707 >
00708 auto
00709 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00710     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00711     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00712
00714 template
00715 <
00716     template<typename, const index_t, const index_t, typename> class Multivector,
00717     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00718 >
00719 auto
00720 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00721
00723 template
00724 <
00725     template<typename, const index_t, const index_t, typename> class Multivector,
00726     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00727 >
00728 auto
00729 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00730     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00731     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00732
00734 template
00735 <
00736     template<typename, const index_t, const index_t, typename> class Multivector,
00737     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00738 >
00739 auto
00740 sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00741
00743 template
00744 <
00745     template<typename, const index_t, const index_t, typename> class Multivector,
00746     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00747 >
00748 auto
00749 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00750     const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00751     const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;

```

```

00752
00753 template
00754 <
00755     template<typename, const index_t, const index_t, typename> class Multivector,
00756     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00757 >
00758 auto
00759 asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00760
00761 template
00762 <
00763     template<typename, const index_t, const index_t, typename> class Multivector,
00764     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00765 >
00766 auto
00767 sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00768
00769 template
00770 <
00771     template<typename, const index_t, const index_t, typename> class Multivector,
00772     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00773 >
00774 auto
00775 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00776        const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00777        const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00778
00779 template
00780 <
00781     template<typename, const index_t, const index_t, typename> class Multivector,
00782     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00783 >
00784 auto
00785 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00786
00787 template
00788 <
00789     template<typename, const index_t, const index_t, typename> class Multivector,
00790     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00791 >
00792 auto
00793 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00794      const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00795      const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00796
00797 template
00798 <
00799     template<typename, const index_t, const index_t, typename> class Multivector,
00800     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00801 >
00802 auto
00803 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00804
00805 template
00806 <
00807     template<typename, const index_t, const index_t, typename> class Multivector,
00808     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00809 >
00810 auto
00811 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00812       const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00813       const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00814
00815 template
00816 <
00817     template<typename, const index_t, const index_t, typename> class Multivector,
00818     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00819 >
00820 auto
00821 atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00822
00823 template
00824 <
00825     template<typename, const index_t, const index_t, typename> class Multivector,
00826     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00827 >
00828 auto
00829 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00830
00831 template
00832 <
00833     template<typename, const index_t, const index_t, typename> class Multivector,
00834     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00835 >
00836 auto
00837 tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00838
00839 template
00840 <
00841     template<typename, const index_t, const index_t, typename> class Multivector,
00842     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00843 >
00844 auto
00845 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00846        const Multivector<Scalar_T,LO,HI,Tune_P>& i,

```

```

00849         const bool prechecked = false) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00850
00852     template
00853     <
00854         template<typename, const index_t, const index_t, typename> class Multivector,
00855         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00856     >
00857     auto
00858     atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>;
00859 }
00860 #endif // _GLUCAT_CLIFFORD_ALGEBRA_H

```

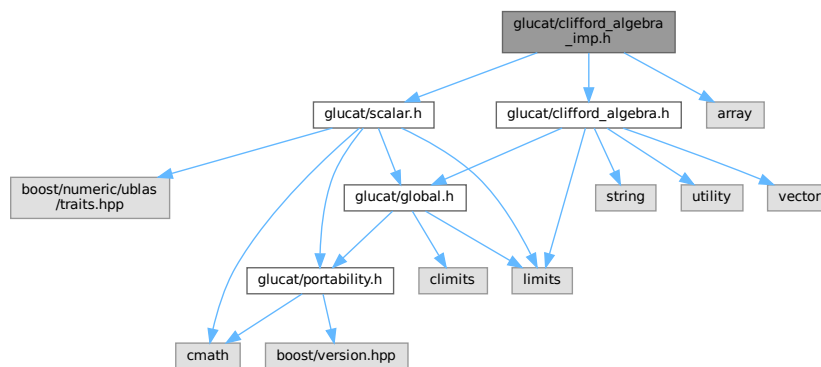
## 7.3 glucat/clifford\_algebra\_imp.h File Reference

```

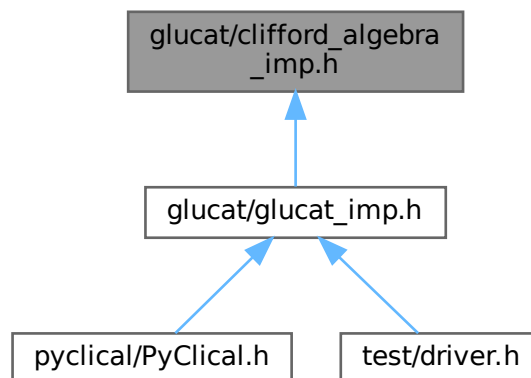
#include "glucat/clifford_algebra.h"
#include "glucat/scalar.h"
#include <array>

```

Include dependency graph for clifford\_algebra\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)

## Functions

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator!= (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> bool`  
*Test for inequality of multivector and scalar.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator!= (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for inequality of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::error\_squared\_tol (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Quadratic norm error tolerance relative to a specific multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::error\_squared (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold) -> Scalar_T`  
*Relative or absolute error using the quadratic norm.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs, const Scalar_T threshold, const Scalar_T tolerance) -> bool`  
*Test for approximate equality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::approx\_equal (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> bool`  
*Test for approximate equality of multivectors.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const Scalar_T &scr) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum of multivector and scalar.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator+ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator+ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric sum.*





- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator/ (const Scalar_T &scr, const Multivector< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Quotient of scalar and multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator/ (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::operator| (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::inv (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Geometric multiplicative inverse.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Integer power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, template< typename, const index\_t, const index\_t, typename > class RHS, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, const RHS< Scalar_T, LO, HI, Tune_P > &rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Multivector power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::outer\_pow (const Multivector< Scalar_T, LO, HI, Tune_P > &lhs, int rhs) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Outer product power of multivector.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::scalar (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::real (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Real part: synonym for scalar part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::imag (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`  
*Imaginary part: deprecated (always 0)*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::pure (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`  
*Pure part.*
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::even (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Even part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::odd (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Odd part.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::vector\_part (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const std::vector< Scalar_←_T >`

*Vector part of multivector, as a `vector_t` with respect to frame()*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::involute (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Main involution, each  $\{i\}$  is replaced by  $-\{i\}$  in each term, eg.  $\{1\}*\{2\} \rightarrow (-\{2\})*(-\{1\})$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::reverse (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Reversion, eg.  $\{1\}*\{2\} \rightarrow \{2\}*\{1\}$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::conj (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Conjugation,  $rev \circ inv \implies inv \circ rev$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::quad (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T quadratic form  $== (rev(x)*x)(0)$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::norm (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Scalar\_T norm  $==$  sum of norm of coordinates.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Absolute value  $== \sqrt{norm}$*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::max\_abs (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> Scalar_T`

*Maximum of absolute values of components of multivector: multivector infinity norm.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::complexifier (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of -1 which commutes with all members of the frame of the given multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::elliptic (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`
- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`static void glucat::check\_complex (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false)`

*Check that  $i$  is a valid complexifier for  $val$ .*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::sqrt (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::clifford\_exp (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::log (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::cosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::acosh (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse hyperbolic cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Cosine of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::cos (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Cosine of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::acos (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`



*Inverse hyperbolic tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::tan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Tangent of multivector.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val, const Multivector< Scalar_T, LO, HI, Tune_P > &i, const bool prechecked=false) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector with specified complexifier.*

- `template<template< typename, const index\_t, const index\_t, typename > class Multivector, typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::atan (const Multivector< Scalar_T, LO, HI, Tune_P > &val) -> const Multivector< Scalar_T, LO, HI, Tune_P >`

*Inverse tangent of multivector.*

## 7.4 clifford\_algebra\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00002 #define _GLUCAT_CLIFFORD_ALGEBRA_IMP_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   clifford_algebra_imp.h : Implement common Clifford algebra functions
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2021 by Paul C. Leopardi
00009   *****/
00010
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00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   See also Arvind Raja's original header comments in glucat.h
00032   *****/
00033
00034 // References for algorithms:
00035 // [AS]:
00036 // Milton Abramowicz and Irene A. Stegun, "Handbook of mathematical functions",
00037 // Dover 1972, first published 1965.
00038 // [CHKL]:
00039 // Sheung Hun Cheng, Nicholas J. Higham, Charles S. Kenney and Alan J. Laub,
00040 // "Approximating the Logarithm of a Matrix to Specified Accuracy", 1999.
00041 // ftp://ftp.ma.man.ac.uk/pub/narep/narep353.ps.gz
00042 // [GL]:
00043 // Gene H. Golub and Charles F. van Loan,
00044 // "Matrix Computations", 3rd ed., Johns Hopkins UP, 1996.
```

```

00045 // [GW]:
00046 // C.F. Gerald and P.O. Wheatley, "Applied Numerical Analysis",
00047 // 6th Edition, Addison-Wesley, 1999.
00048 // [H]:
00049 // Nicholas J. Higham
00050 // "The Scaling and Squaring Method for the Matrix Exponential Revisited",
00051 // SIAM Journal on Matrix Analysis and Applications,
00052 // Vol. 26, Issue 4 (2005), pp. 1179-1193.
00053 // [Z]:
00054 // Doron Zeilberger, "PADE" (Maple code), 2002.
00055 // http://www.math.rutgers.edu/~zeilberg/tokhniot/PADE
00056
00057 #include "glucat/clifford_algebra.h"
00058 #include "glucat/scalar.h"
00059
00060 #include <array>
00061
00062 namespace glucat
00063 {
00064     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00065     auto
00066     clifford_algebra<Scalar_T,Index_Set_T,Multivector_T>::
00067     classname() -> const std::string
00068     { return "clifford_algebra"; }
00069
00070     template< typename Scalar_T, typename Index_Set_T, typename Multivector_T>
00071     const
00072     Scalar_T
00073     clifford_algebra<Scalar_T,Index_Set_T,Multivector_T>::
00074     default_truncation = std::numeric_limits<Scalar_T>::epsilon();
00075
00076     template
00077     <
00078         template<typename, const index_t, const index_t, typename> class Multivector,
00079         template<typename, const index_t, const index_t, typename> class RHS,
00080         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00081     >
00082     inline
00083     auto
00084     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00085     bool
00086     { return !(lhs == rhs); }
00087
00088     template< template<typename, const index_t, const index_t, typename> class Multivector,
00089               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00090     inline
00091     auto
00092     operator!= (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> bool
00093     { return !(lhs == scr); }
00094
00095     template< template<typename, const index_t, const index_t, typename> class Multivector,
00096               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097     inline
00098     auto
00099     operator!= (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00100     { return !(rhs == scr); }
00101
00102     template
00103     <
00104         template<typename, const index_t, const index_t, typename> class Multivector,
00105         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00106     >
00107     auto
00108     error_squared_tol(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00109     {
00110         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00111         static const auto scalar_eps = std::numeric_limits<Scalar_T>::epsilon();
00112         static const auto nbr_different_bits =
00113             std::numeric_limits<Scalar_T>::digits / Tune_P::denom_different_bits +
00114             Tune_P::extra_different_bits;
00115         static const auto abs_tol = scalar_eps *
00116             numeric_traits<Scalar_T>::pow(Scalar_T(2), nbr_different_bits);
00117         using framed_multi_t = typename multivector_t::framed_multi_t;
00118         const auto nbr_terms = double(framed_multi_t(val).truncated(scalar_eps).nbr_terms());
00119         return abs_tol * abs_tol * std::max(Scalar_T(nbr_terms), Scalar_T(1));
00120     }
00121
00122     template
00123     <
00124         template<typename, const index_t, const index_t, typename> class Multivector,
00125         template<typename, const index_t, const index_t, typename> class RHS,
00126         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00127     >
00128     inline
00129     auto
00130     error_squared(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00131                  const RHS<Scalar_T,LO,HI,Tune_P>& rhs,

```

```

00136         const Scalar_T threshold) -> Scalar_T
00137     {
00138         const auto relative = norm(rhs) > threshold;
00139         const auto abs_norm_diff = norm(rhs-lhs);
00140         return (relative)
00141             ? abs_norm_diff/norm(rhs)
00142             : abs_norm_diff;
00143     }
00144
00146     template
00147     <
00148         template<typename, const index_t, const index_t, typename> class Multivector,
00149         template<typename, const index_t, const index_t, typename> class RHS,
00150         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00151     >
00152     inline
00153     auto
00154     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00155                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs,
00156                 const Scalar_T threshold,
00157                 const Scalar_T tolerance) -> bool
00158     { return error_squared(lhs, rhs, threshold) < tolerance; }
00159
00161     template
00162     <
00163         template<typename, const index_t, const index_t, typename> class Multivector,
00164         template<typename, const index_t, const index_t, typename> class RHS,
00165         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00166     >
00167     inline
00168     auto
00169     approx_equal(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs,
00170                 const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> bool
00171     {
00172         const Scalar_T rhs_tol = error_squared_tol(rhs);
00173         return approx_equal(lhs, rhs, rhs_tol, rhs_tol);
00174     }
00175
00177     template< template<typename, const index_t, const index_t, typename> class Multivector,
00178               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00179     inline
00180     auto
00181     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
00182     Multivector<Scalar_T,LO,HI,Tune_P>
00183     {
00184         auto result = lhs;
00185         return result += scr;
00186     }
00187
00188     template< template<typename, const index_t, const index_t, typename> class Multivector,
00189               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00190     inline
00191     auto
00192     operator+ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00193     Multivector<Scalar_T,LO,HI,Tune_P>
00194     {
00195         return rhs + scr;
00196     }
00197
00198     template
00199     <
00200         template<typename, const index_t, const index_t, typename> class Multivector,
00201         template<typename, const index_t, const index_t, typename> class RHS,
00202         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00203     >
00204     inline
00205     auto
00206     operator+ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00207     const Multivector<Scalar_T,LO,HI,Tune_P>
00208     {
00209         auto result = lhs;
00210         return result += rhs;
00211     }
00212
00213     template< template<typename, const index_t, const index_t, typename> class Multivector,
00214               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215     inline
00216     auto
00217     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
00218     Multivector<Scalar_T,LO,HI,Tune_P>
00219     {
00220         auto result = lhs;
00221         return result -= scr;
00222     }
00223
00224     template< template<typename, const index_t, const index_t, typename> class Multivector,
00225               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >

```



```

00226     inline
00227     auto
00228     operator- (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00229     { return -rhs + scr; }
00230
00232     template
00233     <
00234         template<typename, const index_t, const index_t, typename> class Multivector,
00235         template<typename, const index_t, const index_t, typename> class RHS,
00236         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00237     >
00238     inline
00239     auto
00240     operator- (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00241     {
00242         auto result = lhs;
00243         return result -= rhs;
00244     }
00245
00247     template< template<typename, const index_t, const index_t, typename> class Multivector,
00248               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00249     inline
00250     auto
00251     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00252     {
00253         auto result = lhs;
00254         return result *= scr;
00255     }
00256
00258     template< template<typename, const index_t, const index_t, typename> class Multivector,
00259               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00260     inline
00261     auto
00262     operator* (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00263     { // Note: this assumes that scalar commutes with multivector.
00264       // This excludes Clifford algebras over non-commuting rings.
00265         return rhs * scr;
00266     }
00267
00269     template
00270     <
00271         template<typename, const index_t, const index_t, typename> class Multivector,
00272         template<typename, const index_t, const index_t, typename> class RHS,
00273         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00274     >
00275     inline
00276     auto
00277     operator* (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00278     {
00279         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00280         return lhs * multivector_t(rhs);
00281     }
00282
00284     template
00285     <
00286         template<typename, const index_t, const index_t, typename> class Multivector,
00287         template<typename, const index_t, const index_t, typename> class RHS,
00288         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00289     >
00290     inline
00291     auto
00292     operator^ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00293     {
00294         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00295         return lhs ^ multivector_t(rhs);
00296     }
00297
00299     template
00300     <
00301         template<typename, const index_t, const index_t, typename> class Multivector,
00302         template<typename, const index_t, const index_t, typename> class RHS,
00303         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00304     >
00305     inline
00306     auto
00307     operator& (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
const Multivector<Scalar_T,LO,HI,Tune_P>
00308     {
00309         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00310         return lhs & multivector_t(rhs);
00311     }

```



```

00312
00313 template
00314 <
00315     template<typename, const index_t, const index_t, typename> class Multivector,
00316     template<typename, const index_t, const index_t, typename> class RHS,
00317     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00318 >
00319 inline
00320 auto
00321 operator% (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00322 const Multivector<Scalar_T,LO,HI,Tune_P>
00323 {
00324     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00325     return lhs % multivector_t(rhs);
00326 }
00327
00328 template
00329 <
00330     template<typename, const index_t, const index_t, typename> class Multivector,
00331     template<typename, const index_t, const index_t, typename> class RHS,
00332     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00333 >
00334 inline
00335 auto
00336 star (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00337 Scalar_T
00338 {
00339     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00340     return star(lhs, multivector_t(rhs));
00341 }
00342
00343 template< template<typename, const index_t, const index_t, typename> class Multivector,
00344           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00345 inline
00346 auto
00347 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const Scalar_T& scr) -> const
00348 Multivector<Scalar_T,LO,HI,Tune_P>
00349 {
00350     auto result = lhs;
00351     return result /= scr;
00352 }
00353
00354 template< template<typename, const index_t, const index_t, typename> class Multivector,
00355           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00356 inline
00357 auto
00358 operator/ (const Scalar_T& scr, const Multivector<Scalar_T,LO,HI,Tune_P>& rhs) -> const
00359 Multivector<Scalar_T,LO,HI,Tune_P>
00360 {
00361     Multivector<Scalar_T,LO,HI,Tune_P> result = scr;
00362     return result /= rhs;
00363 }
00364
00365 template
00366 <
00367     template<typename, const index_t, const index_t, typename> class Multivector,
00368     template<typename, const index_t, const index_t, typename> class RHS,
00369     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00370 >
00371 inline
00372 auto
00373 operator/ (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00374 const Multivector<Scalar_T,LO,HI,Tune_P>
00375 {
00376     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00377     return lhs / multivector_t(rhs);
00378 }
00379
00380 template
00381 <
00382     template<typename, const index_t, const index_t, typename> class Multivector,
00383     template<typename, const index_t, const index_t, typename> class RHS,
00384     typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00385 >
00386 inline
00387 auto
00388 operator| (const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) ->
00389 const Multivector<Scalar_T,LO,HI,Tune_P>
00390 {
00391     using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00392     return lhs | multivector_t(rhs);
00393 }
00394
00395 template< template<typename, const index_t, const index_t, typename> class Multivector,
00396           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00397 inline
00398 auto

```

```

00400     inv(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00401     { return val.inv(); }
00402
00404     template< template<typename, const index_t, const index_t, typename> class Multivector,
00405               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00406     auto
00407     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00408     {
00409         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00410         if (lhs == Scalar_T(0))
00411         {
00412             using traits_t = numeric_traits<Scalar_T>;
00413             return
00414                 (rhs < 0)
00415                 ? traits_t::NaN()
00416                 : (rhs == 0)
00417                 ? Scalar_T(1)
00418                 : Scalar_T(0);
00419         }
00420         auto result = multivector_t(Scalar_T(1));
00421         auto power =
00422             (rhs < 0)
00423             ? lhs.inv()
00424             : lhs;
00425         for (auto
00426             k = std::abs(rhs);
00427             k != 0;
00428             k /= 2)
00429         {
00430             if (k % 2)
00431                 result *= power;
00432             power *= power;
00433         }
00434         return result;
00435     }
00436
00438     template
00439     <
00440         template<typename, const index_t, const index_t, typename> class Multivector,
00441         template<typename, const index_t, const index_t, typename> class RHS,
00442         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00443     >
00444     inline
00445     auto
00446     pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, const RHS<Scalar_T,LO,HI,Tune_P>& rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00447     {
00448         using traits_t = numeric_traits<Scalar_T>;
00449
00450         if (lhs == Scalar_T(0))
00451         {
00452             const Scalar_T m = rhs.scalar();
00453             if (rhs == m)
00454                 return
00455                     (m < 0)
00456                     ? traits_t::NaN()
00457                     : (m == 0)
00458                     ? Scalar_T(1)
00459                     : Scalar_T(0);
00460             else
00461                 return Scalar_T(0);
00462         }
00463         return exp(log(lhs) * rhs);
00464     }
00465
00467     template< template<typename, const index_t, const index_t, typename> class Multivector,
00468               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00469     auto
00470     outer_pow(const Multivector<Scalar_T,LO,HI,Tune_P>& lhs, int rhs) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00471     { return lhs.outer_pow(rhs); }
00472
00474     template< template<typename, const index_t, const index_t, typename> class Multivector,
00475               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00476     inline
00477     auto
00478     scalar(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00479     { return val.scalar(); }
00480
00482     template< template<typename, const index_t, const index_t, typename> class Multivector,
00483               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00484     inline
00485     auto
00486     real(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00487     { return val.scalar(); }
00488

```

```

00490     template
00491     <
00492         template<typename, const index_t, const index_t, typename> class Multivector,
00493         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P
00494     >
00495     inline
00496     auto
00497     imag(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00498     { return Scalar_T(0); }
00499
00500     template< template<typename, const index_t, const index_t, typename> class Multivector,
00501               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00502     inline
00503     auto
00504     pure(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00505     { return val - val.scalar(); }
00506
00507     template< template<typename, const index_t, const index_t, typename> class Multivector,
00508               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00509     inline
00510     auto
00511     even(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00512     { return val.even(); }
00513
00514     template< template<typename, const index_t, const index_t, typename> class Multivector,
00515               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00516     inline
00517     auto
00518     odd(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00519     { return val.odd(); }
00520
00521     template< template<typename, const index_t, const index_t, typename> class Multivector,
00522               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00523     inline
00524     auto
00525     vector_part(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const std::vector<Scalar_T>
00526     { return val.vector_part(); }
00527
00528     template< template<typename, const index_t, const index_t, typename> class Multivector,
00529               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00530     inline
00531     auto
00532     involute(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00533     { return val.involute(); }
00534
00535     template< template<typename, const index_t, const index_t, typename> class Multivector,
00536               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00537     inline
00538     auto
00539     reverse(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00540     { return val.reverse(); }
00541
00542     template< template<typename, const index_t, const index_t, typename> class Multivector,
00543               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00544     inline
00545     auto
00546     conj(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00547     { return val.conj(); }
00548
00549     template< template<typename, const index_t, const index_t, typename> class Multivector,
00550               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00551     inline
00552     auto
00553     quad(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00554     { return val.quad(); }
00555
00556     template< template<typename, const index_t, const index_t, typename> class Multivector,
00557               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00558     inline
00559     auto
00560     norm(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00561     { return val.norm(); }
00562
00563     template< template<typename, const index_t, const index_t, typename> class Multivector,
00564               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00565     inline
00566     auto
00567     abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00568     { return numeric_traits<Scalar_T>::sqrt(val.norm()); }
00569
00570     template< template<typename, const index_t, const index_t, typename> class Multivector,
00571               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00572     inline
00573     auto
00574     max_abs(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> Scalar_T
00575     { return val.max_abs(); }
00576
00577
00578
00579
00580
00581
00582
00583
00584
00585
00586
00587

```

```

00589     template< template<typename, const index_t, const index_t, typename> class Multivector,
00590               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00591     auto
00592     complexifier(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
Multivector<Scalar_T,LO,HI,Tune_P>
00593     {
00594         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00595         using traits_t = numeric_traits<Scalar_T>;
00596
00597         auto frm = val.frame();
00598         using array_t = std::array<index_t, 4>;
00599         auto incp = array_t{0, 2, 1, 0};
00600         auto incq = array_t{1, 0, 0, 0};
00601         auto bott = pos_mod((frm.count_pos() - frm.count_neg()), 4);
00602         for (auto
00603             k = index_t(0);
00604             k != incp[bott];
00605             k++)
00606             for (auto
00607                 idx = index_t(1);
00608                 idx != HI+1;
00609                 ++idx)
00610                 if (!frm[idx])
00611                 {
00612                     frm.set(idx);
00613                     break;
00614                 }
00615         for (auto
00616             k = index_t(0);
00617             k != incq[bott];
00618             k++)
00619             for (auto
00620                 idx = index_t(-1);
00621                 idx != LO-1;
00622                 --idx)
00623                 if (!frm[idx])
00624                 {
00625                     frm.set(idx);
00626                     break;
00627                 }
00628         auto new_bott = pos_mod(frm.count_pos() - frm.count_neg(), 4);
00629
00630         if ((incp[new_bott] == 0) && (incq[new_bott] == 0))
00631             return multivector_t(frm, Scalar_T(1));
00632         else
00633             // Return IEEE NaN or -Inf
00634             return traits_t::NaN();
00635     }
00636
00637     template< template<typename, const index_t, const index_t, typename> class Multivector,
00638               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00639     inline
00640     auto
00641     elliptic(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00642     { return complexifier(val); }
00643
00644     template< template<typename, const index_t, const index_t, typename> class Multivector,
00645               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00646     inline
00647     static
00648     void
00649     check_complex(const Multivector<Scalar_T,LO,HI,Tune_P>& val,
00650                  const Multivector<Scalar_T,LO,HI,Tune_P>& i, const bool prechecked = false)
00651     {
00652         if (!prechecked)
00653         {
00654             using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00655             using index_set_t = typename multivector_t::index_set_t;
00656             using error_t = typename multivector_t::error_t;
00657
00658             const auto i_frame = i.frame();
00659             // We need i to be a complexifier whose frame is large enough to represent val
00660             if (complexifier(i) != i ||
00661                 (val.frame() | i_frame) != i_frame ||
00662                 complexifier(val).frame().count() > i_frame.count())
00663                 throw error_t("check_complex(val, i): i is not a valid complexifier for val");
00664         }
00665     }
00666
00667     template< template<typename, const index_t, const index_t, typename> class Multivector,
00668               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00669     inline
00670     auto
00671     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00672          bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00673     { return sqrt(val, i, prechecked); }
00674
00675
00676
00677

```

```

00679     template< template<typename, const index_t, const index_t, typename> class Multivector,
00680               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00681     inline
00682     auto
00683     sqrt(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00684     { return sqrt(val, complexifier(val), true); }
00685
00686     template< template<typename, const index_t, const index_t, typename> class Multivector,
00687               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00688     auto
00689     clifford_exp(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const
00690     Multivector<Scalar_T,LO,HI,Tune_P>
00691     {
00692         // Scaling and squaring Pade' approximation of matrix exponential
00693         // Reference: [GL], Section 11.3, p572-576
00694         // Reference: [H]
00695
00696         using traits_t = numeric_traits<Scalar_T>;
00697
00698         const auto scalar_val = val.scalar();
00699         const auto scalar_exp = traits_t::exp(scalar_val);
00700         if (traits_t::isNaN_or_isInf(scalar_exp))
00701             return traits_t::NaN();
00702         if (val == scalar_val)
00703             return scalar_exp;
00704
00705         using multivector_t = Multivector<Scalar_T,LO,HI,Tune_P>;
00706         auto A = val - scalar_val;
00707         const auto pure_scale2 = A.norm();
00708
00709         if (traits_t::isNaN_or_isInf(pure_scale2))
00710             return traits_t::NaN();
00711         if (pure_scale2 == Scalar_T(0))
00712             return scalar_exp;
00713
00714         const auto ilog2_scale =
00715             std::max(0, traits_t::to_int(ceil((log2(pure_scale2) +
00716             Scalar_T(A.frame().count())/Scalar_T(2))) - 3));
00717         const auto i_scale = traits_t::pow(Scalar_T(2), ilog2_scale);
00718         if (traits_t::isNaN_or_isInf(i_scale))
00719             return traits_t::NaN();
00720
00721         A /= i_scale;
00722         multivector_t pure_exp;
00723         {
00724             using limits_t = std::numeric_limits<Scalar_T>;
00725             const auto nbr_even_powers = 2*(limits_t::digits / 32) + 4;
00726             using nbr_t = decltype(nbr_even_powers);
00727
00728             // Create an array of coefficients
00729             const auto max_power = 2*nbr_even_powers + 1;
00730             static std::array<Scalar_T, max_power+1> c;
00731             if (c[0] != Scalar_T(1))
00732             {
00733                 c[0] = Scalar_T(1);
00734                 for (auto
00735                     k = decltype(max_power)(0);
00736                     k != max_power;
00737                     ++k)
00738                     c[k+1] = c[k]*(max_power-k) / ((2*max_power-k)*(k+1));
00739             }
00740
00741             // Create an array of even powers
00742             std::array<multivector_t, nbr_even_powers> AA;
00743             AA[0] = A * A;
00744             AA[1] = AA[0] * AA[0];
00745             for (auto
00746                 k = nbr_t(2);
00747                 k != nbr_even_powers;
00748                 ++k)
00749                 AA[k] = AA[k-2] * AA[1];
00750
00751             // Use compensated summation to calculate U and AV
00752             auto residual = multivector_t();
00753             auto U = multivector_t(c[0]);
00754             for (auto
00755                 k = nbr_t(0);
00756                 k != nbr_even_powers;
00757                 ++k)
00758             {
00759                 const auto& term = AA[k]*c[2*k + 2] - residual;
00760                 const auto& sum = U + term;
00761                 residual = (sum - U) - term;
00762                 U = sum;
00763             }
00764             residual = multivector_t();
00765             auto AV = multivector_t(c[1]);

```

```

00765     for (auto
00766         k = nbr_t(0);
00767         k != nbr_even_powers;
00768         ++k)
00769     {
00770         const auto& term = AA[k]*c[2*k + 3] - residual;
00771         const auto& sum = AV + term;
00772         residual = (sum - AV) - term;
00773         AV = sum;
00774     }
00775     AV *= A;
00776     pure_exp = (U+AV) / (U-AV);
00777 }
00778 for (auto
00779     k = decltype(ilog2_scale)(0);
00780     k != ilog2_scale;
00781     ++k)
00782     pure_exp *= pure_exp;
00783 return pure_exp * scalar_exp;
00784 }
00785
00786 template< template<typename, const index_t, const index_t, typename> class Multivector,
00787           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788 inline
00789 auto
00790 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00791 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00792 { return log(val, i, prechecked); }
00793
00794 template< template<typename, const index_t, const index_t, typename> class Multivector,
00795           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796 inline
00797 auto
00798 log(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00799 { return log(val, complexifier(val), true); }
00800
00801 template< template<typename, const index_t, const index_t, typename> class Multivector,
00802           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00803 inline
00804 auto
00805 cosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00806 {
00807     using traits_t = numeric_traits<Scalar_T>;
00808     if (val.isnan())
00809         return traits_t::NaN();
00810     const auto& s = val.scalar();
00811     if (val == s)
00812         return traits_t::cosh(s);
00813     return (exp(val)+exp(-val)) / Scalar_T(2);
00814 }
00815
00816 // Reference: [AS], Section 4.6, p86-89
00817 template< template<typename, const index_t, const index_t, typename> class Multivector,
00818           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00819 inline
00820 auto
00821 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00822 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00823 {
00824     using traits_t = numeric_traits<Scalar_T>;
00825     check_complex(val, i, prechecked);
00826     if (val.isnan())
00827         return traits_t::NaN();
00828     const auto radical = sqrt(val*val - Scalar_T(1), i, true);
00829     return (norm(val + radical) >= norm(val))
00830         ? log(val + radical, i, true)
00831         : -log(val - radical, i, true);
00832 }
00833
00834 // Reference: [AS], Section 4.6, p86-89
00835 template< template<typename, const index_t, const index_t, typename> class Multivector,
00836           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00837 inline
00838 auto
00839 acosh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00840 { return acosh(val, complexifier(val), true); }
00841
00842 template< template<typename, const index_t, const index_t, typename> class Multivector,
00843           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00844 auto
00845 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00846 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00847 {
00848     using traits_t = numeric_traits<Scalar_T>;
00849     if (val.isnan())

```

```

00855         return traits_t::NaN();
00856
00857     const auto& s = val.scalar();
00858     if (val == s)
00859         return traits_t::cos(s);
00860
00861     check_complex(val, i, prechecked);
00862
00863     static const auto& twopi = Scalar_T(2) * traits_t::pi();
00864     const auto& z = i *
00865         (val - s + traits_t::fmod(s, twopi));
00866     return (exp(z)+exp(-z)) / Scalar_T(2);
00867 }
00868
00870 template< template<typename, const index_t, const index_t, typename> class Multivector,
00871           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00872 inline
00873 auto
00874 cos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00875 { return cos(val, complexifier(val), true); }
00876
00878 // Reference: [AS], Section 4.4, p79-83
00879 template< template<typename, const index_t, const index_t, typename> class Multivector,
00880           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00881 inline
00882 auto
00883 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00884 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00885 {
00886     using traits_t = numeric_traits<Scalar_T>;
00887     if (val.isnan())
00888         return traits_t::NaN();
00889
00890     const auto& s = val.scalar();
00891     if (val == s && traits_t::abs(s) <= Scalar_T(1))
00892         return traits_t::acos(s);
00893
00894     check_complex(val, i, prechecked);
00895     return i * acosh(val, i, true);
00896 }
00897
00898 // Reference: [AS], Section 4.4, p79-83
00899 template< template<typename, const index_t, const index_t, typename> class Multivector,
00900           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00901 inline
00902 auto
00903 acos(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00904 { return acos(val, complexifier(val), true); }
00905
00907 template< template<typename, const index_t, const index_t, typename> class Multivector,
00908           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00909 inline
00910 auto
00911 sinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00912 {
00913     using traits_t = numeric_traits<Scalar_T>;
00914     if (val.isnan())
00915         return traits_t::NaN();
00916
00917     const auto& s = val.scalar();
00918     if (val == s)
00919         return traits_t::sinh(s);
00920
00921     return (exp(val)-exp(-val)) / Scalar_T(2);
00922 }
00923
00925 // Reference: [AS], Section 4.6, p86-89
00926 template< template<typename, const index_t, const index_t, typename> class Multivector,
00927           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00928 inline
00929 auto
00930 asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00931 bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00932 {
00933     using traits_t = numeric_traits<Scalar_T>;
00934     check_complex(val, i, prechecked);
00935     if (val.isnan())
00936         return traits_t::NaN();
00937
00938     const auto radical = sqrt(val*val + Scalar_T(1), i, true);
00939     return (norm(val + radical) >= norm(val))
00940         ? log( val + radical, i, true)
00941         : -log(-val + radical, i, true);
00942 }
00943
00944 // Reference: [AS], Section 4.6, p86-89
00945 template< template<typename, const index_t, const index_t, typename> class Multivector,

```

```

00946         typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00947     inline
00948     auto
00949     asinh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00950     { return asinh(val, complexifier(val), true); }
00951
00952     template< template<typename, const index_t, const index_t, typename> class Multivector,
00953             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00954     auto
00955     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
00956     prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00957     {
00958         using traits_t = numeric_traits<Scalar_T>;
00959         if (val.isnan())
00960             return traits_t::NaN();
00961
00962         const auto& s = val.scalar();
00963         if (val == s)
00964             return traits_t::sin(s);
00965
00966         check_complex(val, i, prechecked);
00967
00968         static const auto& twopi = Scalar_T(2) * traits_t::pi();
00969         const auto& z = i *
00970             (val - s + traits_t::fmod(s, twopi));
00971         return i * (exp(-z)-exp(z)) / Scalar_T(2);
00972     }
00973
00974     template< template<typename, const index_t, const index_t, typename> class Multivector,
00975             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00976     inline
00977     auto
00978     sin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00979     { return sin(val, complexifier(val), true); }
00980
00981     // Reference: [AS], Section 4.4, p79-83
00982     template< template<typename, const index_t, const index_t, typename> class Multivector,
00983             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00984     inline
00985     auto
00986     asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
00987     bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
00988     {
00989         using traits_t = numeric_traits<Scalar_T>;
00990         if (val.isnan())
00991             return traits_t::NaN();
00992
00993         const auto& s = val.scalar();
00994         if (val == s && traits_t::abs(s) <= Scalar_T(1))
00995             return traits_t::asin(s);
00996
00997         check_complex(val, i, prechecked);
00998         return -i * asinh(i * val, i, true);
00999     }
01000
01001     // Reference: [AS], Section 4.4, p79-83
01002     template< template<typename, const index_t, const index_t, typename> class Multivector,
01003             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01004     inline
01005     auto
01006     asin(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01007     { return asin(val, complexifier(val), true); }
01008
01009     template< template<typename, const index_t, const index_t, typename> class Multivector,
01010             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01011     inline
01012     auto
01013     tanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01014     {
01015         using traits_t = numeric_traits<Scalar_T>;
01016         if (val.isnan())
01017             return traits_t::NaN();
01018
01019         const auto& s = val.scalar();
01020         if (val == s)
01021             return traits_t::tanh(s);
01022
01023         return sinh(val) / cosh(val);
01024     }
01025
01026     // Reference: [AS], Section 4.6, p86-89
01027     template< template<typename, const index_t, const index_t, typename> class Multivector,
01028             typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01029     inline
01030     auto
01031     atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01032     bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>

```



```

01036 {
01037     using traits_t = numeric_traits<Scalar_T>;
01038     check_complex(val, i, prechecked);
01039     return val.isnan()
01040         ? traits_t::NaN()
01041         : (norm(val + Scalar_T(1)) > norm(val - Scalar_T(1)))
01042           ? (log(val + Scalar_T(1), i, true) - log(-val + Scalar_T(1), i, true)) / Scalar_T(2)
01043           : log((val + Scalar_T(1)) / (-val + Scalar_T(1)), i, true) / Scalar_T(2);
01044 }
01045
01046 // Reference: [AS], Section 4.6, p86-89
01047 template< template<typename, const index_t, const index_t, typename> class Multivector,
01048           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01049 inline
01050 auto
01051 atanh(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01052 { return atanh(val, complexifier(val), true); }
01053
01054 template< template<typename, const index_t, const index_t, typename> class Multivector,
01055           typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01056 inline
01057 auto
01058 tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i, bool
01059 prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01060 {
01061     {
01062         using traits_t = numeric_traits<Scalar_T>;
01063         if (val.isnan())
01064             return traits_t::NaN();
01065
01066         const auto& s = val.scalar();
01067         if (val == s)
01068             return traits_t::tan(s);
01069
01070         check_complex(val, i, prechecked);
01071         return sin(val, i, true) / cos(val, i, true);
01072     }
01073
01074     template< template<typename, const index_t, const index_t, typename> class Multivector,
01075               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01076     inline
01077     auto
01078     tan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01079     { return tan(val, complexifier(val), true); }
01080
01081     // Reference: [AS], Section 4.4, p79-83
01082     template< template<typename, const index_t, const index_t, typename> class Multivector,
01083               typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01084     inline
01085     auto
01086     atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val, const Multivector<Scalar_T,LO,HI,Tune_P>& i,
01087           bool prechecked) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01088     {
01089         {
01090             using traits_t = numeric_traits<Scalar_T>;
01091             if (val.isnan())
01092                 return traits_t::NaN();
01093
01094             const auto& s = val.scalar();
01095             if (val == s)
01096                 return traits_t::atan(s);
01097
01098             check_complex(val, i, prechecked);
01099             return -i * atanh(i * val, i, true);
01100         }
01101
01102         // Reference: [AS], Section 4.4, p79-83
01103         template< template<typename, const index_t, const index_t, typename> class Multivector,
01104                   typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01105         inline
01106         auto
01107         atan(const Multivector<Scalar_T,LO,HI,Tune_P>& val) -> const Multivector<Scalar_T,LO,HI,Tune_P>
01108         { return atan(val, complexifier(val), true); }
01109     }
01110 }
01111 #endif // _GLUCAT_CLIFFORD_ALGEBRA_IMP_H

```

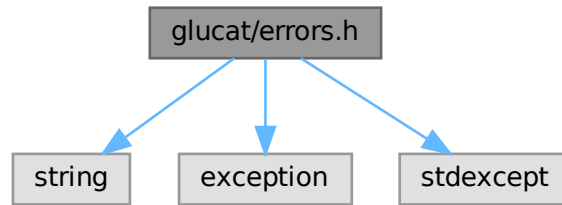
## 7.5 glucat/errors.h File Reference

```

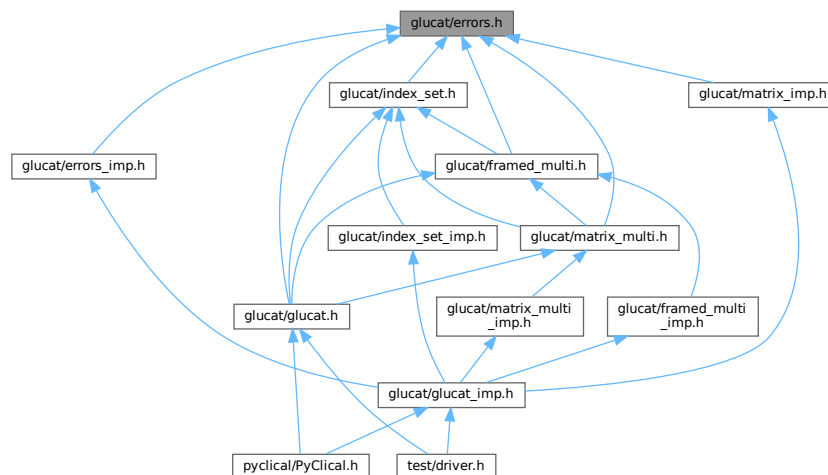
#include <string>
#include <exception>

```

```
#include <stdexcept>
Include dependency graph for errors.h:
```



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::glucat\\_error](#)  
*Abstract exception class.*
- class [glucat::error< Class\\_T >](#)  
*Specific exception class.*

## Namespaces

- namespace [glucat](#)

## 7.6 errors.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_H
00002 #define _GLUCAT_ERRORS_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors.h : Declare error classes and functions
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include <string>
00035 #include <exception>
00036 #include <stdexcept>
00037
00038 namespace glucat
00039 {
00040     class glucat_error : public std::logic_error
00041     {
00042     public:
00043         glucat_error(const std::string& context, const std::string& msg)
00044             : logic_error(msg), name(context)
00045         { }
00046         ~glucat_error() noexcept override = default;
00047         virtual auto heading() const noexcept -> const std::string =0;
00048         virtual auto classname() const noexcept -> const std::string =0;
00049         virtual void print_error_msg() const =0;
00050         std::string name;
00051     };
00052
00053     template< class Class_T >
00054     class error : public glucat_error
00055     {
00056     public:
00057         error(const std::string& msg);
00058         error(const std::string& context, const std::string& msg);
00059         auto heading() const noexcept -> const std::string override;
00060         auto classname() const noexcept -> const std::string override;
00061         void print_error_msg() const override;
00062     };
00063
00064 #endif // _GLUCAT_ERRORS_H

```

## 7.7 glucat/errors\_imp.h File Reference

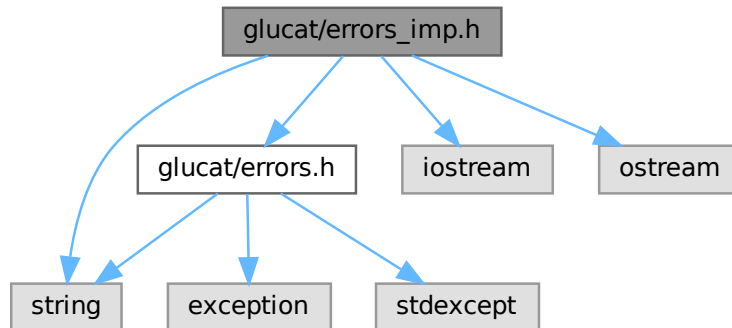
```

#include "glucat/errors.h"
#include <string>
#include <iostream>

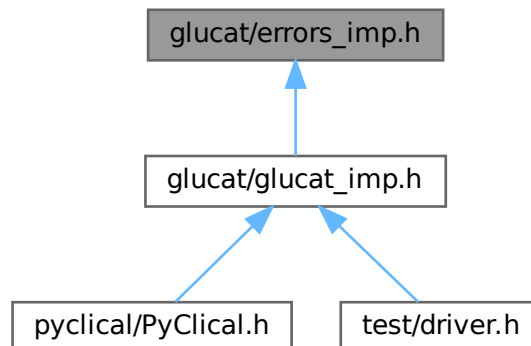
```

```
#include <ostream>
```

Include dependency graph for errors\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## 7.8 errors\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_ERRORS_IMP_H
00002 #define _GLUCAT_ERRORS_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     errors_imp.h : Define error functions
00006     -----
  
```

```

00007      begin                : Sun 2001-12-20
00008      copyright            : (C) 2001-2007 by Paul C. Leopardi
00009      *****
00010
00011      This library is free software: you can redistribute it and/or modify
00012      it under the terms of the GNU Lesser General Public License as published
00013      by the Free Software Foundation, either version 3 of the License, or
00014      (at your option) any later version.
00015
00016      This library is distributed in the hope that it will be useful,
00017      but WITHOUT ANY WARRANTY; without even the implied warranty of
00018      MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019      GNU Lesser General Public License for more details.
00020
00021      You should have received a copy of the GNU Lesser General Public License
00022      along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024      *****
00025      This library is based on a prototype written by Arvind Raja and was
00026      licensed under the LGPL with permission of the author. See Arvind Raja,
00027      "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028      in Ablamowicz, Lounesto and Parra (eds.)
00029      "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030      *****
00031      See also Arvind Raja's original header comments in glucat.h
00032      *****/
00033
00034 #include "glucat/errors.h"
00035
00036 #include <string>
00037 #include <iostream>
00038 #include <ostream>
00039
00040 namespace glucat
00041 {
00042     template< class Class_T >
00043     error<Class_T>::
00044     error(const std::string& msg)
00045     : glucat_error(Class_T::classname(), msg)
00046     { }
00047
00048     template< class Class_T >
00049     error<Class_T>::
00050     error(const std::string& context, const std::string& msg)
00051     : glucat_error(context, msg)
00052     { }
00053
00054     template< class Class_T >
00055     auto
00056     error<Class_T>::
00057     heading() const noexcept -> const std::string
00058     { return "Error in glucat: "; }
00059
00060     template< class Class_T >
00061     auto
00062     error<Class_T>::
00063     classname() const noexcept -> const std::string
00064     { return name; }
00065
00066     template< class Class_T >
00067     void
00068     error<Class_T>::
00069     print_error_msg() const
00070     { std::cerr << heading() << classname() << std::endl << what() << std::endl; }
00071 }
00072 #endif // _GLUCAT_ERRORS_IMP_H
00073

```

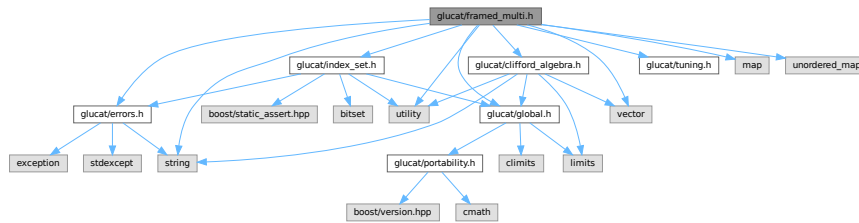
## 7.9 glucat/framed\_multi.h File Reference

```

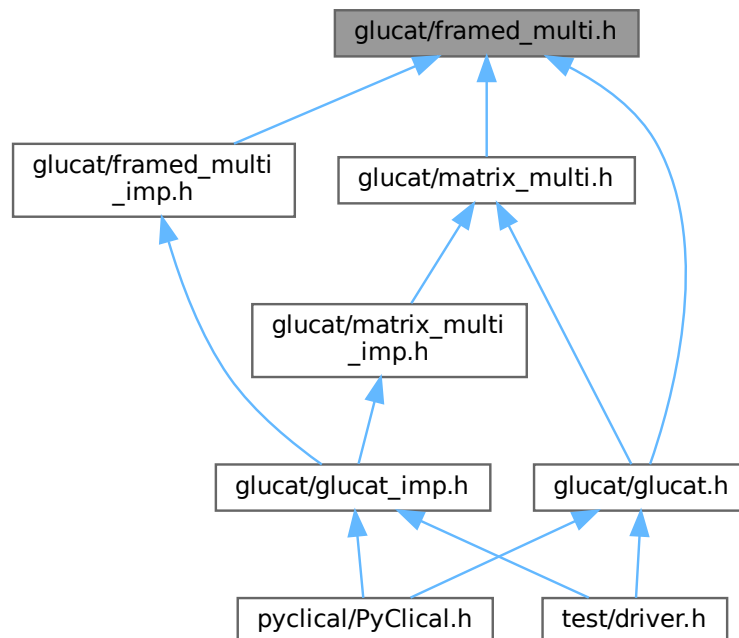
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include <string>
#include <utility>
#include <map>

```

```
#include <unordered_map>
#include <vector>
Include dependency graph for framed_multi.h:
```



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::index_set_hash< LO, HI >`
- class `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >`  
*A `framed_multi<Scalar_T,LO,HI,Tune_P>` is a framed approximation to a multivector.*
- class `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::hash_size_t`
- class `glucat::framed_multi< Scalar_T, LO, HI, Tune_P >::var_term`  
*Variable term.*
- struct `std::numeric_limits< glucat::framed_multi< Scalar_T, LO, HI, Tune_P > >`  
*Numeric limits for `framed_multi` inherit limits for the corresponding scalar type.*

## Namespaces

- namespace [glucat](#)
- namespace [std](#)

## Functions

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator\\*](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator^](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator&](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator%](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::star](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator/](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator|](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &lhs, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator>>](#) (std::istream &s, [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &  
*Read multivector from input.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator<<](#) (std::ostream &os, const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI>  
auto [glucat::operator<<](#) (std::ostream &os, const std::pair< const [index\\_set](#)< LO, HI >, Scalar\_T > &term) -> std::ostream &  
*Write term to output.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::exp](#) (const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [framed\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >  
*Exponential of multivector.*
- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI>  
static auto [glucat::crd\\_of\\_mult](#) (const std::pair< const [index\\_set](#)< LO, HI >, Scalar\_T > &lhs, const std::pair< const [index\\_set](#)< LO, HI >, Scalar\_T > &rhs) -> Scalar\_T

*Coordinate of product of terms.*

- `template<typename Scalar_T, const index_t LO, const index_t HI>`  
`auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`

*Product of terms.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const framed_multi< Scalar_T, LO, HI, Tune_P > &val, const framed_multi< Scalar_T, LO, HI, Tune_P > &i, bool prechecked) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

## 7.10 framed\_multi.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_FRAMED_MULTI_H
00002 #define _GLUCAT_FRAMED_MULTI_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     framed_multi.h : Declare a class for the framed representation of a multivector
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039
00040 #if defined(_GLUCAT_USE_BOOST_POOL_ALLOC)
00041 // Use the Boost pool allocator
00042 #include <boost/pool/poolfwd.hpp>
00043 #endif
00044
00045 #include <string>
00046 #include <utility>
00047 #include <map>
00048 #include <unordered_map>
00049 #include <vector>
00050
00051 namespace glucat
00052 {
00053     // Forward declarations for friends
00054
00055     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00056     class framed_multi; // forward
00057 }
```



```

00058     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00059     class matrix_multi; // forward
00060
00062     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063     auto
00064     operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00065
00067     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00068     auto
00069     operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00070
00072     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00073     auto
00074     operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00075
00077     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00078     auto
00079     operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00080
00082     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00083     auto
00084     star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T;
00085
00087     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00088     auto
00089     operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00090
00092     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00093     auto
00094     operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00095
00097     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00098     auto
00099     operator> (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00100
00102     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00103     auto
00104     operator< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00105
00107     template< typename Scalar_T, const index_t LO, const index_t HI >
00108     auto
00109     operator<< (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
std::ostream&;
00110
00112     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113     auto
00114     exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00115
00116     template< const index_t LO, const index_t HI>
00117     class index_set_hash
00118     {
00119     public:
00120         using index_set_t = index_set<LO, HI>;
00121         inline auto operator()(index_set_t val) const -> size_t { return val.hash_fn(); }
00122     };
00123
00125     template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<> >
00126     class framed_multi :
00127     public clifford_algebra< Scalar_T, index_set<LO,HI>, framed_multi<Scalar_T,LO,HI,Tune_P> >,
00128     private std::unordered_map< index_set<LO,HI>, Scalar_T, index_set_hash<LO,HI> >
00129     {
00130     public:
00131         using multivector_t = framed_multi;
00132         using framed_multi_t = multivector_t;
00133         using scalar_t = Scalar_T;
00134         using tune_p = Tune_P;
00135         using index_set_t = index_set<LO, HI>;
00136         using term_t = std::pair<const index_set_t, Scalar_T>;
00137         using vector_t = std::vector<Scalar_T>;
00138         using error_t = error<multivector_t>;
00139         using matrix_multi_t = matrix_multi<Scalar_T,LO,HI,Tune_P >;
00140         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00141         friend class matrix_multi;
00142         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00143         friend class framed_multi;
00144     private:

```

```

00146     class var_term; // forward
00147     using var_term_t = class var_term;
00148     using matrix_t = typename matrix_multi_t::matrix_t;
00149     using sorted_map_t = std::map< index_set_t, Scalar_T, std::less<const index_set_t> >;
00150     using map_t = std::unordered_map<index_set_t, Scalar_T, index_set_hash<LO, HI>;
00151
00152     class hash_size_t
00153     {
00154     public:
00155         hash_size_t(size_t hash_size)
00156             : n(hash_size)
00157             { };
00158         auto operator()() const -> size_t
00159         { return n; }
00160     private:
00161         size_t n;
00162     };
00163
00164     using framed_pair_t = std::pair<const multivector_t, const multivector_t>;
00165     using size_type = typename map_t::size_type;
00166     using iterator = typename map_t::iterator;
00167     using const_iterator = typename map_t::const_iterator;
00168
00169 public:
00170     static auto classname() -> const std::string;
00171     ~framed_multi() override = default;
00172     framed_multi();
00173
00174 private:
00175     framed_multi(const hash_size_t& hash_size);
00176
00177 public:
00178     template< typename Other_Scalar_T >
00179     framed_multi(const framed_multi<Other_Scalar_T, LO, HI, Tune_P>& val);
00180     template< typename Other_Scalar_T >
00181     framed_multi(const framed_multi<Other_Scalar_T, LO, HI, Tune_P>& val,
00182                 const index_set_t frm, const bool prechecked = false);
00183     framed_multi(const framed_multi_t& val,
00184                 const index_set_t frm, const bool prechecked = false);
00185     framed_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00186     framed_multi(const index_set_t ist, const Scalar_T& crd,
00187                 const index_set_t frm, const bool prechecked = false);
00188     framed_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00189     framed_multi(const int scr, const index_set_t frm = index_set_t());
00190     framed_multi(const vector_t& vec,
00191                 const index_set_t frm, const bool prechecked = false);
00192     framed_multi(const std::string& str);
00193     framed_multi(const std::string& str,
00194                 const index_set_t frm, const bool prechecked = false);
00195     framed_multi(const char* str)
00196     { *this = framed_multi(std::string(str)); };
00197     framed_multi(const char* str,
00198                 const index_set_t frm, const bool prechecked = false)
00199     { *this = framed_multi(std::string(str), frm, prechecked); };
00200     template< typename Other_Scalar_T >
00201     framed_multi(const matrix_multi<Other_Scalar_T, LO, HI, Tune_P >& val);
00202     template< typename Other_Scalar_T >
00203     auto fast_matrix_multi(const index_set_t frm) const -> const
00204     matrix_multi<Other_Scalar_T, LO, HI, Tune_P >;
00205     auto fast_framed_multi() const -> const framed_multi_t;
00206
00207     _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00208
00209     auto nbr_terms() const -> unsigned long;
00210
00211     static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const multivector_t;
00212
00213     // Friend declarations
00214
00215     friend auto
00216     operator* <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00217     friend auto
00218     operator^ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00219     friend auto
00220     operator& <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00221     friend auto
00222     operator% <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00223     friend auto
00224     star <>(const multivector_t& lhs, const multivector_t& rhs) -> Scalar_T;
00225     friend auto
00226     operator/ <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00227     friend auto
00228     operator| <>(const multivector_t& lhs, const multivector_t& rhs) -> const multivector_t;
00229
00230     friend auto
00231     operator<< <>(std::istream& s, multivector_t& val) -> std::istream&;
00232     friend auto
00233     operator<< <>(std::ostream& os, const multivector_t& val) -> std::ostream&;

```

```

00253     friend auto
00254         operator<< (<std::ostream& os, const term_t& term) -> std::ostream&;
00255
00256     friend auto
00257         exp<>(const multivector_t& val) -> const multivector_t;
00258
00260     auto operator+= (const term_t& term) -> multivector_t&;
00261
00262 private:
00264     auto fold(const index_set_t frm) const -> multivector_t;
00266     auto unfold(const index_set_t frm) const -> multivector_t;
00268     auto centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&;
00270     auto centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&;
00272     auto centre_qp1_pm1(index_t& p, index_t& q) -> multivector_t&;
00274     auto divide(const index_set_t ist) const -> const framed_pair_t;
00276     auto fast(const index_t level, const bool odd) const -> const matrix_t;
00277
00279     class var_term :
00280     public std::pair<index_set<LO,HI>, Scalar_T>
00281     {
00282     public:
00283         using var_pair_t = std::pair<index_set<LO, HI>, Scalar_T>;
00284
00286         static auto classname() -> const std::string
00287         { return "var_term"; };
00289         ~var_term() = default;
00291         var_term()
00292         : var_pair_t(index_set_t(), Scalar_T(1))
00293         { };
00295         var_term(const index_set_t ist, const Scalar_T& crd = Scalar_T(1))
00296         : var_pair_t(ist, crd)
00297         { };
00299         auto operator*= (const term_t& rhs) -> var_term_t&
00300         {
00301             this->second *= rhs.second * this->first.sign_of_mult(rhs.first);
00302             this->first ^= rhs.first;
00303             return *this;
00304         }
00305     };
00306 };
00307
00308 // Non-members
00309
00311 template< typename Scalar_T, const index_t LO, const index_t HI >
00312 inline
00313 static
00314 auto
00315 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00316             const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T;
00317
00319 template< typename Scalar_T, const index_t LO, const index_t HI >
00320 auto
00321 operator*
00322 (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
00323  const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const index_set<LO,HI>,
00324 Scalar_T>;
00326 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327 auto
00328 sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
00329 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00331
00333 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00334 auto
00335 exp(const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00336
00338 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00339 auto
00340 log(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
00341 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>;
00342
00344 namespace std
00345 {
00346     template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00347     struct numeric_limits< glucat::framed_multi<Scalar_T,LO,HI,Tune_P> > :
00348     public numeric_limits<Scalar_T>
00349     { };
00350 }
00351 #endif // _GLUCAT_FRAMED_MULTI_H

```



## Macros

- `#define _GLUCAT_HASH_N(x) (x)`
- `#define _GLUCAT_HASH_SIZE_T(x) (typename multivector_t::hash_size_t)(x)`

## Functions

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator* (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator^ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator& (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator% (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi<`  
`Scalar_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::star (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_T, LO,`  
`HI, Tune_P > &rhs) -> Scalar_T`  
*Hestenes scalar product.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator/ (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator| (const framed_multi< Scalar_T, LO, HI, Tune_P > &lhs, const framed_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &rhs) -> const framed_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator<< (std::ostream &os, const framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std_↵`  
`::ostream &`  
*Write multivector to output.*
- `template<typename Scalar_T , const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const std::pair< const index_set< LO, HI >, Scalar_T > &term)`  
`-> std::ostream &`  
*Write term to output.*
- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::operator>> (std::istream &s, framed_multi< Scalar_T, LO, HI, Tune_P > &val) -> std::istream`  
`&`  
*Read multivector from input.*
- `template<typename Scalar_T , const index_t LO, const index_t HI>`  
`static auto glucat::crd_of_mult (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std_↵`  
`::pair< const index_set< LO, HI >, Scalar_T > &rhs) -> Scalar_T`  
*Coordinate of product of terms.*
- `template<typename Scalar_T , const index_t LO, const index_t HI>`  
`auto glucat::operator* (const std::pair< const index_set< LO, HI >, Scalar_T > &lhs, const std::pair< const`  
`index_set< LO, HI >, Scalar_T > &rhs) -> const std::pair< const index_set< LO, HI >, Scalar_T >`

*Product of terms.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::sqrt (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::exp (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val) -> const framed\_multi< Scalar_T,`  
`LO, HI, Tune_P >`

*Exponential of multivector.*

- `template<typename Scalar_T , const index\_t LO, const index\_t HI, typename Tune_P >`  
`auto glucat::log (const framed\_multi< Scalar_T, LO, HI, Tune_P > &val, const framed\_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const framed\_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

## 7.11.1 Macro Definition Documentation

### 7.11.1.1 `_GLUCAT_HASH_N`

```
#define _GLUCAT_HASH_N(  
    x ) (x)
```

Definition at line 54 of file [framed\\_multi\\_imp.h](#).

### 7.11.1.2 `_GLUCAT_HASH_SIZE_T`

```
#define _GLUCAT_HASH_SIZE_T(  
    x ) (typename multivector_t::hash_size_t) (x)
```

Definition at line 55 of file [framed\\_multi\\_imp.h](#).

Referenced by [glucat::framed\\_multi< Scalar\\_T, LO, HI, Tune\\_P >::framed\\_multi\(\)](#), [glucat::operator%\(\)](#), [glucat::operator&\(\)](#), [glucat::operator\\*\(\)](#), and [glucat::operator^\(\)](#).

## 7.12 `framed_multi_imp.h`

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_FRAMED_MULTI_IMP_H
00002 #define _GLUCAT_FRAMED_MULTI_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     framed_multi_imp.h : Implement the coordinate map representation of a
00006     Clifford algebra element
00007     -----
00008     begin                : Sun 2001-12-09
00009     copyright            : (C) 2001-2021 by Paul C. Leopardi
00010     *****/
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
```

```

00023     along with this library.  If not, see <http://www.gnu.org/licenses/>.
00024
00025     *****
00026 This library is based on a prototype written by Arvind Raja and was
00027 licensed under the LGPL with permission of the author. See Arvind Raja,
00028 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029 in Ablamowicz, Lounesto and Parra (eds.)
00030 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031     *****
00032     See also Arvind Raja's original header comments in glucat.h
00033     *****/
00034
00035 #include "glucat/framed_multi.h"
00036
00037 #include "glucat/scalar.h"
00038 #include "glucat/random.h"
00039 #include "glucat/generation.h"
00040 #include "glucat/matrix.h"
00041
00042 #include <sstream>
00043 #include <fstream>
00044
00045 namespace glucat
00046 {
00047     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00048     auto
00049     framed_multi<Scalar_T,LO,HI,Tune_P>::
00050     classname() -> const std::string
00051     { return "framed_multi"; }
00052
00053 #define _GLUCAT_HASH_N(x) (x)
00054 #define _GLUCAT_HASH_SIZE_T(x) (typename multivector_t::hash_size_t)(x)
00055
00056     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00057     framed_multi<Scalar_T,LO,HI,Tune_P>::
00058     framed_multi()
00059     : map_t(_GLUCAT_HASH_N(0))
00060     { }
00061
00062     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00063     framed_multi<Scalar_T,LO,HI,Tune_P>::
00064     framed_multi(const hash_size_t& hash_size)
00065     : map_t(_GLUCAT_HASH_N(hash_size()))
00066     { }
00067
00068     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00069     template< typename Other_Scalar_T >
00070     framed_multi<Scalar_T,LO,HI,Tune_P>::
00071     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00072     : map_t(_GLUCAT_HASH_N(val.size()))
00073     {
00074         for (auto& val_term : val)
00075             this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00076     }
00077
00078     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00079     template< typename Other_Scalar_T >
00080     framed_multi<Scalar_T,LO,HI,Tune_P>::
00081     framed_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00082                 const index_set_t frm, const bool prechecked)
00083     : map_t(_GLUCAT_HASH_N(val.size()))
00084     {
00085         if (!prechecked && (val.frame() | frm) != frm)
00086             throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00087         for (auto& val_term : val)
00088             this->insert(term_t(val_term.first, numeric_traits<Scalar_T>::to_scalar_t(val_term.second)));
00089     }
00090
00091     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00092     framed_multi<Scalar_T,LO,HI,Tune_P>::
00093     framed_multi(const multivector_t& val,
00094                 const index_set_t frm, const bool prechecked)
00095     : map_t(_GLUCAT_HASH_N(val.size()))
00096     {
00097         if (!prechecked && (val.frame() | frm) != frm)
00098             throw error_t("multivector_t(val,frm): cannot initialize with value outside of frame");
00099         for (auto& val_term : val)
00100             this->insert(val_term);
00101     }
00102
00103     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00104     framed_multi<Scalar_T,LO,HI,Tune_P>::
00105     framed_multi(const index_set_t ist, const Scalar_T& crd)
00106     : map_t(_GLUCAT_HASH_N(1))
00107     {
00108         if (crd != Scalar_T(0))
00109             this->insert(term_t(ist, crd));
00110     }

```

```

00117     }
00118
00120     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00121     framed_multi<Scalar_T,LO,HI,Tune_P>::
00122     framed_multi(const index_set_t ist, const Scalar_T& crd,
00123                 const index_set_t frm, const bool prechecked)
00124     : map_t(_GLUCAT_HASH_N(1))
00125     {
00126         if (!prechecked && (ist | frm) != frm)
00127             throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00128         if (crd != Scalar_T(0))
00129             this->insert(term_t(ist, crd));
00130     }
00131
00133     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00134     framed_multi<Scalar_T,LO,HI,Tune_P>::
00135     framed_multi(const Scalar_T& scr, const index_set_t frm)
00136     : map_t(_GLUCAT_HASH_N(1))
00137     {
00138         if (scr != Scalar_T(0))
00139             this->insert(term_t(index_set_t(), scr));
00140     }
00141
00143     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00144     framed_multi<Scalar_T,LO,HI,Tune_P>::
00145     framed_multi(const int scr, const index_set_t frm)
00146     : map_t(_GLUCAT_HASH_N(1))
00147     {
00148         if (scr != Scalar_T(0))
00149             this->insert(term_t(index_set_t(), Scalar_T(scr)));
00150     }
00151
00153     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00154     framed_multi<Scalar_T,LO,HI,Tune_P>::
00155     framed_multi(const vector_t& vec,
00156                 const index_set_t frm, const bool prechecked)
00157     : map_t(_GLUCAT_HASH_N(vec.size()))
00158     {
00159         if (!prechecked && index_t(vec.size()) != frm.count())
00160             throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00161         auto idx = frm.min();
00162         const auto frm_end = frm.max()+1;
00163         for (auto& crd : vec)
00164         {
00165             *this += term_t(index_set_t(idx), crd);
00166             for (
00167                 ++idx;
00168                 idx != frm_end && !frm[idx];
00169                 ++idx)
00170                 ;
00171         }
00172     }
00173
00175     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00176     framed_multi<Scalar_T,LO,HI,Tune_P>::
00177     framed_multi(const std::string& str)
00178     : map_t(_GLUCAT_HASH_N(0))
00179     {
00180         std::istringstream ss(str);
00181         ss >> *this;
00182         if (!ss)
00183             throw error_t("multivector_t(str): could not parse string");
00184         // Peek to see if the end of the string has been reached.
00185         ss.peek();
00186         if (!ss.eof())
00187             throw error_t("multivector_t(str): could not parse entire string");
00188     }
00189
00191     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00192     framed_multi<Scalar_T,LO,HI,Tune_P>::
00193     framed_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00194     : map_t(_GLUCAT_HASH_N(0))
00195     {
00196         if (prechecked)
00197             *this = multivector_t(str);
00198         else
00199             *this = multivector_t(multivector_t(str), frm, false);
00200     }
00201
00203     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00204     template< typename Other_Scalar_T >
00205     framed_multi<Scalar_T,LO,HI,Tune_P>::
00206     framed_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00207     : map_t(_GLUCAT_HASH_N(1))
00208     {
00209         if (val == Other_Scalar_T(0))
00210             return;

```



```

00211
00212     const auto dim = val.m_matrix.size1();
00213     using traits_t = numeric_traits<Scalar_T>;
00214     if (dim == 1)
00215     {
00216         this->insert(term_t(index_set_t(), traits_t::to_scalar_t(val.m_matrix(0, 0))));
00217         return;
00218     }
00219     if (dim >= Tune_P::inv_fast_dim_threshold)
00220     try
00221     {
00222         *this = (val.template fast_framed_multi<Scalar_T>()).truncated();
00223         return;
00224     }
00225     catch (const glucat_error& e)
00226     { }
00227
00228     const auto val_norm = traits_t::to_scalar_t(val.norm());
00229     if (traits_t::isNaN_or_isInf(val_norm))
00230     {
00231         *this = traits_t::NaN();
00232         return;
00233     }
00234     const auto frm = val.frame();
00235     const auto algebra_dim = set_value_t(1) << frm.count();
00236     auto result = multivector_t(
00237         _GLUCAT_HASH_SIZE_T(std::min<size_t>(algebra_dim, matrix::nnz(val.m_matrix))));
00238     for (auto
00239         stv = set_value_t(0);
00240         stv != algebra_dim;
00241         stv++)
00242     {
00243         const auto ist = index_set_t(stv, frm, true);
00244         const auto crd =
00245             traits_t::to_scalar_t(matrix::inner<Other_Scalar_T>(val.basis_element(ist), val.m_matrix));
00246         if (crd != Scalar_T(0))
00247             result.insert(term_t(ist, crd));
00248     }
00249     *this = result.truncated();
00250 }
00251
00252 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00253 auto
00254 framed_multi<Scalar_T,LO,HI,Tune_P>::
00255 operator== (const multivector_t& rhs) const -> bool
00256 {
00257     {
00258         if (this->size() != rhs.size())
00259             return false;
00260         const auto rhs_end = rhs.end();
00261         for (auto& this_term : *this)
00262         {
00263             const const_iterator& rhs_it = rhs.find(this_term.first);
00264             if (rhs_it == rhs_end || rhs_it->second != this_term.second)
00265                 return false;
00266         }
00267         return true;
00268     }
00269
00270 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00271 inline
00272 auto
00273 framed_multi<Scalar_T,LO,HI,Tune_P>::
00274 operator== (const Scalar_T& scr) const -> bool
00275 {
00276     {
00277         switch (this->size())
00278         {
00279             case 0:
00280                 return scr == Scalar_T(0);
00281             case 1:
00282             {
00283                 const auto& this_it = this->begin();
00284                 return this_it->first == index_set_t() && this_it->second == scr;
00285             }
00286             default:
00287                 return false;
00288         }
00289     }
00290
00291 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00292 inline
00293 auto
00294 framed_multi<Scalar_T,LO,HI,Tune_P>::
00295 operator+= (const Scalar_T& scr) -> multivector_t&
00296 {
00297     {
00298         *this += term_t(index_set_t(), scr);
00299         return *this;
00300     }

```

```

00301
00303 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00304 inline
00305 auto
00306 framed_multi<Scalar_T,LO,HI,Tune_P>::
00307 operator+= (const multivector_t& rhs) -> multivector_t&
00308 { // simply add terms
00309     for (auto& rhs_term : rhs)
00310         *this += rhs_term;
00311     return *this;
00312 }
00313
00315 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00316 inline
00317 auto
00318 framed_multi<Scalar_T,LO,HI,Tune_P>::
00319 operator-= (const Scalar_T& scr) -> multivector_t&
00320 {
00321     *this += term_t(index_set_t(), -scr);
00322     return *this;
00323 }
00324
00326 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00327 inline
00328 auto
00329 framed_multi<Scalar_T,LO,HI,Tune_P>::
00330 operator-= (const multivector_t& rhs) -> multivector_t&
00331 {
00332     for (auto& rhs_term : rhs)
00333         *this += term_t(rhs_term.first, -(rhs_term.second));
00334     return *this;
00335 }
00336
00338 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00339 inline
00340 auto
00341 framed_multi<Scalar_T,LO,HI,Tune_P>::
00342 operator- () const -> const multivector_t
00343 { // multiply coordinates of all terms by -1
00344     auto result = *this;
00345     for (auto& result_term : result)
00346         result_term.second *= Scalar_T(-1);
00347     return result;
00348 }
00349
00351 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00352 auto
00353 framed_multi<Scalar_T,LO,HI,Tune_P>::
00354 operator*= (const Scalar_T& scr) -> multivector_t&
00355 { // multiply coordinates of all terms by scalar
00356     using traits_t = numeric_traits<Scalar_T>;
00357
00358     if (traits_t::isNaN_or_isInf(scr))
00359         return *this = traits_t::NaN();
00360     if (scr == Scalar_T(0))
00361         if (this->isnan())
00362             *this = traits_t::NaN();
00363     else
00364         this->clear();
00365     else
00366         for (auto& this_term : *this)
00367             this_term.second *= scr;
00368     return *this;
00369 }
00370
00372 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00373 auto
00374 operator* (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00375 {
00376     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00377     using traits_t = numeric_traits<Scalar_T>;
00378
00379     if (lhs.isnan() || rhs.isnan())
00380         return traits_t::NaN();
00381
00382     const double lhs_size = lhs.size();
00383     const double rhs_size = rhs.size();
00384     const auto our_frame = lhs.frame() | rhs.frame();
00385     const auto frm_count = our_frame.count();
00386     const auto algebra_dim = set_value_t(1) << frm_count;
00387     const auto direct_mult = lhs_size * rhs_size <= double(algebra_dim);
00388     if (direct_mult)
00389     { // If we have a sparse multiply, store the result directly
00390         auto result = multivector_t(
00391             _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00392         for (auto& lhs_term : lhs)

```

```

00393         for (auto& rhs_term : rhs)
00394             result += lhs_term * rhs_term;
00395     return result;
00396 }
00397 else
00398 { // Past a certain threshold, the matrix algorithm is fastest
00399     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00400     return matrix_multi_t(lhs, our_frame, true) *
00401         matrix_multi_t(rhs, our_frame, true);
00402 }
00403 }
00404
00406 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00407 inline
00408 auto
00409 framed_multi<Scalar_T,LO,HI,Tune_P>::
00410 operator*= (const multivector_t& rhs) -> multivector_t&
00411 { return *this = *this * rhs; }
00412
00414 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00415 auto
00416 operator^ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00417 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00418 { // Arvind Raja's original reference:
00419     // "old clical, outerproduct(p,q:pterm):pterm in file compmod.pas"
00420
00421     if (lhs.empty() || rhs.empty())
00422         return Scalar_T(0);
00423
00424     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00425     using index_set_t = typename multivector_t::index_set_t;
00426     using term_t = typename multivector_t::term_t;
00427
00428     const auto empty_set = index_set_t();
00429
00430     const double lhs_size = lhs.size();
00431     const double rhs_size = rhs.size();
00432     const auto lhs_frame = lhs.frame();
00433     const auto rhs_frame = rhs.frame();
00434     const auto our_frame = lhs_frame | rhs_frame;
00435     const auto algebra_dim = set_value_t(1) << our_frame.count();
00436     auto result = multivector_t(
00437         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00438     const auto lhs_end = lhs.end();
00439     const auto rhs_end = rhs.end();
00440
00441     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00442     {
00443         for (auto
00444             result_stv = set_value_t(0);
00445             result_stv != algebra_dim;
00446             ++result_stv)
00447         {
00448             const auto result_ist = index_set_t(result_stv, our_frame, true);
00449             const auto lhs_result_frame = lhs_frame & result_ist;
00450             const auto lhs_result_dim = set_value_t(1) << lhs_result_frame.count();
00451             auto result_crd = Scalar_T(0);
00452             for (auto
00453                 lhs_stv = set_value_t(0);
00454                 lhs_stv != lhs_result_dim;
00455                 ++lhs_stv)
00456             {
00457                 const auto lhs_ist = index_set_t(lhs_stv, lhs_result_frame, true);
00458                 const auto rhs_ist = result_ist ^ lhs_ist;
00459                 if ((rhs_ist | rhs_frame) == rhs_frame)
00460                 {
00461                     const auto lhs_it = lhs.find(lhs_ist);
00462                     if (lhs_it != lhs_end)
00463                     {
00464                         const auto rhs_it = rhs.find(rhs_ist);
00465                         if (rhs_it != rhs_end)
00466                             result_crd += crd_of_mult(*lhs_it, *rhs_it);
00467                     }
00468                 }
00469             }
00470             if (result_crd != Scalar_T(0))
00471                 result.insert(term_t(result_ist, result_crd));
00472         }
00473     }
00474     return result;
00475 }
00476 else
00477 {
00478     for (auto& lhs_term : lhs)
00479     for (auto& rhs_term : rhs)
00480     if ((lhs_term.first & rhs_term.first) == empty_set)
00481         result += lhs_term * rhs_term;
00482     return result;

```

```

00481     }
00482 }
00483
00484 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00485 inline
00486 auto
00487 framed_multi<Scalar_T,LO,HI,Tune_P>::
00488 operator^= (const multivector_t& rhs) -> multivector_t&
00489 { return *this = *this ^ rhs; }
00490
00491 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00492 auto
00493 operator& (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00494 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00495 { // Arvind Raja's original reference:
00496   // "old clical, innerproduct(p,q:pterm):pterm in file compmod.pas"
00497
00498   if (lhs.empty() || rhs.empty())
00499     return Scalar_T(0);
00500
00501   using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00502   using index_set_t = typename multivector_t::index_set_t;
00503   using term_t = typename multivector_t::term_t;
00504
00505   const auto lhs_end = lhs.end();
00506   const auto rhs_end = rhs.end();
00507   const double lhs_size = lhs.size();
00508   const double rhs_size = rhs.size();
00509
00510   const auto lhs_frame = lhs.frame();
00511   const auto rhs_frame = rhs.frame();
00512
00513   const auto our_frame = lhs_frame | rhs_frame;
00514   const auto algebra_dim = set_value_t(1) << our_frame.count();
00515   auto result = multivector_t(
00516     _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00517   if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00518   {
00519     for (auto
00520       result_stv = set_value_t(0);
00521       result_stv != algebra_dim;
00522       ++result_stv)
00523     {
00524       const auto result_ist = index_set_t(result_stv, our_frame, true);
00525       const auto comp_frame = our_frame & ~result_ist;
00526       const auto comp_dim = set_value_t(1) << comp_frame.count();
00527       auto result_crd = Scalar_T(0);
00528       for (auto
00529         comp_stv = set_value_t(1);
00530         comp_stv != comp_dim;
00531         ++comp_stv)
00532       {
00533         const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00534         const auto our_ist = result_ist ^ comp_ist;
00535         if ((our_ist | lhs_frame) == lhs_frame)
00536         {
00537           const auto lhs_it = lhs.find(our_ist);
00538           if (lhs_it != lhs_end)
00539           {
00540             const auto rhs_it = rhs.find(comp_ist);
00541             if (rhs_it != rhs_end)
00542               result_crd += crd_of_mult(*lhs_it, *rhs_it);
00543           }
00544         }
00545       }
00546       if (result_stv != 0)
00547       {
00548         if ((our_ist | rhs_frame) == rhs_frame)
00549         {
00550           const auto rhs_it = rhs.find(our_ist);
00551           if (rhs_it != rhs_end)
00552           {
00553             const auto lhs_it = lhs.find(comp_ist);
00554             if (lhs_it != lhs_end)
00555               result_crd += crd_of_mult(*lhs_it, *rhs_it);
00556           }
00557         }
00558       }
00559     }
00560     if (result_crd != Scalar_T(0))
00561       result.insert(term_t(result_ist, result_crd));
00562   }
00563 }
00564 else
00565 {
00566   const auto empty_set = index_set_t();
00567   for (auto& lhs_term : lhs)
00568   {

```

```

00569         const auto lhs_ist = lhs_term.first;
00570         if (lhs_ist != empty_set)
00571             for (auto& rhs_term : rhs)
00572             {
00573                 const auto rhs_ist = rhs_term.first;
00574                 if (rhs_ist != empty_set)
00575                 {
00576                     const auto our_ist = lhs_ist | rhs_ist;
00577                     if ((lhs_ist == our_ist) || (rhs_ist == our_ist))
00578                         result += lhs_term * rhs_term;
00579                 }
00580             }
00581     }
00582 }
00583 return result;
00584 }
00585
00586 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00587 inline
00588 auto
00589 framed_multi<Scalar_T,LO,HI,Tune_P>::
00590 operator+=( const multivector_t& rhs) -> multivector_t&
00591 { return *this = *this & rhs; }
00592
00593 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00594 auto
00595 operator% (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00596 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00597 {
00598     // Reference: Leo Dorst, "Honing geometric algebra for its use in the computer sciences",
00599     // in Geometric Computing with Clifford Algebras, ed. G. Sommer,
00600     // Springer 2001, Chapter 6, pp. 127-152.
00601     // http://staff.science.uva.nl/~leo/clifford/index.html
00602
00603     if (lhs.empty() || rhs.empty())
00604         return Scalar_T(0);
00605
00606     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00607     using index_set_t = typename multivector_t::index_set_t;
00608     using term_t = typename multivector_t::term_t;
00609     using map_t = typename multivector_t::map_t;
00610
00611     const auto lhs_end = lhs.end();
00612     const auto rhs_end = rhs.end();
00613     const double lhs_size = lhs.size();
00614     const double rhs_size = rhs.size();
00615     const auto lhs_frame = lhs.frame();
00616     const auto rhs_frame = rhs.frame();
00617
00618     const auto our_frame = lhs_frame | rhs_frame;
00619     const auto algebra_dim = set_value_t(1) << our_frame.count();
00620     auto result = multivector_t(
00621         _GLUCAT_HASH_SIZE_T(size_t(std::min(lhs_size * rhs_size, double(algebra_dim)))));
00622
00623     if (lhs_size * rhs_size > double(Tune_P::products_size_threshold))
00624     {
00625         for (auto
00626             result_stv = set_value_t(0);
00627             result_stv != algebra_dim;
00628             ++result_stv)
00629         {
00630             const auto result_ist = index_set_t(result_stv, our_frame, true);
00631             const auto comp_frame = lhs_frame & ~result_ist;
00632             const auto comp_dim = set_value_t(1) << comp_frame.count();
00633             auto result_crd = Scalar_T(0);
00634             for (auto
00635                 comp_stv = set_value_t(0);
00636                 comp_stv != comp_dim;
00637                 ++comp_stv)
00638             {
00639                 const auto comp_ist = index_set_t(comp_stv, comp_frame, true);
00640                 const auto rhs_ist = result_ist ^ comp_ist;
00641                 if ((rhs_ist | rhs_frame) == rhs_frame)
00642                 {
00643                     const auto rhs_it = rhs.find(rhs_ist);
00644                     if (rhs_it != rhs_end)
00645                     {
00646                         const auto lhs_it = lhs.find(comp_ist);
00647                         if (lhs_it != lhs_end)
00648                             result_crd += crd_of_mult(*lhs_it, *rhs_it);
00649                     }
00650                 }
00651             }
00652             if (result_crd != Scalar_T(0))
00653                 result.insert(term_t(result_ist, result_crd));
00654         }
00655     }
00656 }

```

```

00657     else
00658     {
00659         for (auto& rhs_term : rhs)
00660         {
00661             const auto rhs_list = rhs_term.first;
00662             for (auto& lhs_term : lhs)
00663             {
00664                 const index_set_t lhs_list = lhs_term.first;
00665                 if ((lhs_list | rhs_list) == rhs_list)
00666                     result += lhs_term * rhs_term;
00667             }
00668         }
00669     }
00670     return result;
00671 }
00672
00673 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00674 inline
00675 auto
00676 framed_multi<Scalar_T,LO,HI,Tune_P>::
00677 operator%=(const multivector_t& rhs) -> multivector_t&
00678 { return *this = *this % rhs; }
00679
00680 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00681 auto
00682 star(const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const framed_multi<Scalar_T,LO,HI,Tune_P>& rhs)
00683 -> Scalar_T
00684 {
00685     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00686
00687     auto result = Scalar_T(0);
00688     const auto small_star_large = lhs.size() < rhs.size();
00689     const auto* smallp =
00690         small_star_large
00691         ? &lhs
00692         : &rhs;
00693     const auto* largep =
00694         small_star_large
00695         ? &rhs
00696         : &lhs;
00697
00698     for (auto& small_term : *smallp)
00699     {
00700         const auto small_list = small_term.first;
00701         const auto large_crd = (*largep)[small_list];
00702         if (large_crd != Scalar_T(0))
00703             result += small_list.sign_of_square() * small_term.second * large_crd;
00704     }
00705     return result;
00706 }
00707
00708 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00709 auto
00710 framed_multi<Scalar_T,LO,HI,Tune_P>::
00711 operator/=(const Scalar_T& scr) -> multivector_t&
00712 { // Divide coordinates of all terms by scr
00713     using traits_t = numeric_traits<Scalar_T>;
00714
00715     if (traits_t::isNaN(scr))
00716         return *this = traits_t::NaN();
00717     if (traits_t::isInf(scr))
00718         if (this->isnan())
00719             *this = traits_t::NaN();
00720         else
00721             this->clear();
00722     else
00723         for (auto& this_term : *this)
00724             this_term.second /= scr;
00725     return *this;
00726 }
00727
00728 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00729 inline
00730 auto
00731 operator/ (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00732 framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00733 {
00734     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00735     using traits_t = numeric_traits<Scalar_T>;
00736     using index_set_t = typename multivector_t::index_set_t;
00737     using matrix_multi_t = typename multivector_t::matrix_multi_t;
00738
00739     if (rhs == Scalar_T(0))
00740         return traits_t::NaN();
00741
00742     const auto our_frame = lhs.frame() | rhs.frame();
00743     return matrix_multi_t(lhs, our_frame, true) / matrix_multi_t(rhs, our_frame, true);
00744 }

```

```

00746     }
00747
00749     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00750     inline
00751     auto
00752     framed_multi<Scalar_T,LO,HI,Tune_P>::
00753     operator/= (const multivector_t& rhs) -> multivector_t&
00754     { return *this = *this / rhs; }
00755
00757     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00758     inline
00759     auto
00760     operator| (const framed_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
00761     framed_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
00762     {
00763         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00764         using matrix_multi_t = typename multivector_t::matrix_multi_t;
00765         return matrix_multi_t(rhs) * matrix_multi_t(lhs) / matrix_multi_t(rhs.involute());
00766     }
00767
00769     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00770     inline
00771     auto
00772     framed_multi<Scalar_T,LO,HI,Tune_P>::
00773     operator|= (const multivector_t& rhs) -> multivector_t&
00774     { return *this = *this | rhs; }
00775
00777     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00778     inline
00779     auto
00780     framed_multi<Scalar_T,LO,HI,Tune_P>::
00781     inv() const -> const multivector_t
00782     {
00783         auto result = matrix_multi_t(Scalar_T(1), this->frame());
00784         return result /= matrix_multi_t(*this);
00785     }
00786
00788     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00789     auto
00790     framed_multi<Scalar_T,LO,HI,Tune_P>::
00791     pow(int m) const -> const multivector_t
00792     { return glucat::pow(*this, m); }
00793
00795     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00796     auto
00797     framed_multi<Scalar_T,LO,HI,Tune_P>::
00798     outer_pow(int m) const -> const multivector_t
00799     {
00800         if (m < 0)
00801             throw error_t("outer_pow(int): negative exponent");
00802         auto result = multivector_t(Scalar_T(1));
00803         auto a = *this;
00804         for (;
00805             m != 0;
00806             m >= 1, a = a ^ a)
00807             if (m & 1)
00808                 result ^= a;
00809         return result;
00810     }
00811
00813     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00814     inline
00815     auto
00816     framed_multi<Scalar_T,LO,HI,Tune_P>::
00817     frame() const -> const index_set_t
00818     {
00819         auto result = index_set_t();
00820         for (auto& this_term : *this)
00821             result |= this_term.first;
00822         return result;
00823     }
00824
00826     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00827     inline
00828     auto
00829     framed_multi<Scalar_T,LO,HI,Tune_P>::
00830     grade() const -> index_t
00831     {
00832         auto result = index_t(0);
00833         for (auto& this_term : *this)
00834             result = std::max(result, this_term.first.count());
00835         return result;
00836     }
00837
00839     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00840     inline

```

```

00841     auto
00842     framed_multi<Scalar_T,LO,HI,Tune_P>::
00843     operator[] (const index_set_t ist) const -> Scalar_T
00844     {
00845         const auto& this_it = this->find(ist);
00846         if (this_it == this->end())
00847             return Scalar_T(0);
00848         else
00849             return this_it->second;
00850     }
00851
00852 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00853 auto
00854 framed_multi<Scalar_T,LO,HI,Tune_P>::
00855 operator() (index_t grade) const -> const multivector_t
00856 {
00857     if ((grade < 0) || (grade > HI-LO))
00858         return Scalar_T(0);
00859     else
00860     {
00861         auto result = multivector_t();
00862         for (auto& this_term : *this)
00863             if (this_term.first.count() == grade)
00864                 result += this_term;
00865         return result;
00866     }
00867 }
00868
00869 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00870 inline
00871 auto
00872 framed_multi<Scalar_T,LO,HI,Tune_P>::
00873 scalar() const -> Scalar_T
00874 { return (*this)[index_set_t()]; }
00875
00876 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00877 inline
00878 auto
00879 framed_multi<Scalar_T,LO,HI,Tune_P>::
00880 pure() const -> const multivector_t
00881 { return *this - this->scalar(); }
00882
00883 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00884 auto
00885 framed_multi<Scalar_T,LO,HI,Tune_P>::
00886 even() const -> const multivector_t
00887 { // even part of x, sum of the pure(count) with even count
00888     auto result = multivector_t();
00889     for (auto& this_term : *this)
00890         if ((this_term.first.count() % 2) == 0)
00891             result.insert(this_term);
00892     return result;
00893 }
00894
00895 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00896 auto
00897 framed_multi<Scalar_T,LO,HI,Tune_P>::
00898 odd() const -> const multivector_t
00899 { // even part of x, sum of the pure(count) with even count
00900     auto result = multivector_t();
00901     for (auto& this_term : *this)
00902         if ((this_term.first.count() % 2) == 1)
00903             result.insert(this_term);
00904     return result;
00905 }
00906
00907 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00908 auto
00909 framed_multi<Scalar_T,LO,HI,Tune_P>::
00910 vector_part() const -> const vector_t
00911 { return this->vector_part(this->frame(), true); }
00912
00913 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00914 auto
00915 framed_multi<Scalar_T,LO,HI,Tune_P>::
00916 vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00917 {
00918     if (!prechecked && (this->frame() | frm) != frm)
00919         throw error_t("vector_part(frm): value is outside of requested frame");
00920     auto result = vector_t();
00921     result.reserve(frm.count());
00922     const auto frm_end = frm.max()+1;
00923     for (auto
00924         idx = frm.min();
00925         idx != frm_end;
00926         ++idx)
00927         // Frame may contain indices which do not correspond to a grade 1 term but

```



```

00935         // frame cannot omit any index corresponding to a grade 1 term
00936         if (frm[idx])
00937             result.push_back((*this)[index_set_t(idx)]);
00938     return result;
00939 }
00940
00942 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00943 auto
00944 framed_multi<Scalar_T,LO,HI,Tune_P>::
00945 involute() const -> const multivector_t
00946 {
00947     auto result = *this;
00948     for (auto& result_term : result)
00949     { // for a k-vector u, involute(u) == (-1)^k * u
00950         if ((result_term.first.count() % 2) == 1)
00951             result_term.second *= Scalar_T(-1);
00952     }
00953     return result;
00954 }
00955
00957 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00958 auto
00959 framed_multi<Scalar_T,LO,HI,Tune_P>::
00960 reverse() const -> const multivector_t
00961 {
00962     auto result = *this;
00963     for (auto& result_term : result)
00964         // For a k-vector u, reverse(u) = { -u, k == 2,3 (mod 4)
00965         // { u, k == 0,1 (mod 4)
00966         switch (result_term.first.count() % 4)
00967         {
00968             case 2:
00969             case 3:
00970                 result_term.second *= Scalar_T(-1);
00971                 break;
00972             default:
00973                 break;
00974         }
00975     return result;
00976 }
00977
00979 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00980 auto
00981 framed_multi<Scalar_T,LO,HI,Tune_P>::
00982 conj() const -> const multivector_t
00983 {
00984     auto result = *this;
00985     for (auto& result_term : result)
00986         // For a k-vector u, conj(u) = { -u, k == 1,2 (mod 4)
00987         // { u, k == 0,3 (mod 4)
00988         switch (result_term.first.count() % 4)
00989         {
00990             case 1:
00991             case 2:
00992                 result_term.second *= Scalar_T(-1);
00993                 break;
00994             default:
00995                 break;
00996         }
00997     return result;
00998 }
00999
01001 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01002 auto
01003 framed_multi<Scalar_T,LO,HI,Tune_P>::
01004 quad() const -> Scalar_T
01005 {
01006     // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
01007     // ref: old clical: quadfunction(p:pter):pterm in file compmod.pas
01008     auto result = Scalar_T(0);
01009     for (auto& this_term : *this)
01010     {
01011         const auto sign =
01012             (this_term.first.count_neg() % 2)
01013             ? -Scalar_T(1)
01014             : Scalar_T(1);
01015         result += sign * (this_term.second) * (this_term.second);
01016     }
01017     return result;
01018 }
01019
01021 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01022 auto
01023 framed_multi<Scalar_T,LO,HI,Tune_P>::
01024 norm() const -> Scalar_T
01025 {
01026     using traits_t = numeric_traits<Scalar_T>;

```

```

01027
01028     auto result = Scalar_T(0);
01029     for (auto& this_term : *this)
01030     {
01031         const auto abs_crd = traits_t::abs(this_term.second);
01032         result += abs_crd * abs_crd;
01033     }
01034     return result;
01035 }
01036
01037 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01038 auto
01039 framed_multi<Scalar_T,LO,HI,Tune_P>::
01040 max_abs() const -> Scalar_T
01041 {
01042     using traits_t = numeric_traits<Scalar_T>;
01043
01044     auto result = Scalar_T(0);
01045     for (auto& this_term : *this)
01046     {
01047         const auto abs_crd = traits_t::abs(this_term.second);
01048         if (abs_crd > result)
01049             result = abs_crd;
01050     }
01051     return result;
01052 }
01053
01054 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01055 auto
01056 framed_multi<Scalar_T,LO,HI,Tune_P>::
01057 random(const index_set_t frm, Scalar_T fill) -> const multivector_t
01058 {
01059     using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01060     using index_set_t = typename multivector_t::index_set_t;
01061     using term_t = typename multivector_t::term_t;
01062
01063     using random_generator_t = random_generator<Scalar_T>;
01064     auto& generator = random_generator_t::generator();
01065
01066     fill =
01067         (fill < Scalar_T(0))
01068         ? Scalar_T(0)
01069         : (fill > Scalar_T(1))
01070         ? Scalar_T(1)
01071         : fill;
01072     const auto algebra_dim = set_value_t(1) << frm.count();
01073     using traits_t = numeric_traits<Scalar_T>;
01074     const auto mean_abs = traits_t::sqrt(Scalar_T(double(algebra_dim)));
01075     auto result = multivector_t();
01076     for (auto
01077         stv = set_value_t(0);
01078         stv != algebra_dim;
01079         ++stv)
01080     {
01081         if (generator.uniform() < fill)
01082         {
01083             const auto& result_crd = generator.normal() / mean_abs;
01084             result.insert(term_t(index_set_t(stv, frm, true), result_crd));
01085         }
01086     }
01087     return result;
01088 }
01089
01090 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01091 inline
01092 void
01093 framed_multi<Scalar_T,LO,HI,Tune_P>::
01094 write(const std::string& msg) const
01095 { std::cout << msg << std::endl << " " << (*this) << std::endl; }
01096
01097 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01098 inline
01099 void
01100 framed_multi<Scalar_T,LO,HI,Tune_P>::
01101 write(std::ofstream& ofile, const std::string& msg) const
01102 {
01103     if (!ofile)
01104         throw error_t("write(ofile,msg): cannot write to output file");
01105     ofile << msg << std::endl << " " << (*this) << std::endl;
01106 }
01107
01108 template< typename Map_T,typename Sorted_Map_T >
01109 class sorted_range
01110 {
01111 public:
01112     using map_t = Map_T;
01113     using sorted_map_t = Sorted_Map_T;
01114     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01115 }
01116

```

```

01119     sorted_range (Sorted_Map_T &sorted_val, const Map_T& val)
01120     {
01121         for (auto& val_term : val)
01122             sorted_val.insert(val_term);
01123         sorted_begin = sorted_val.begin();
01124         sorted_end   = sorted_val.end();
01125     }
01126     sorted_iterator sorted_begin;
01127     sorted_iterator sorted_end;
01128 };
01129
01130 template< typename Sorted_Map_T >
01131 class sorted_range< Sorted_Map_T, Sorted_Map_T >
01132 {
01133 public:
01134     using map_t = Sorted_Map_T;
01135     using sorted_map_t = Sorted_Map_T;
01136     using sorted_iterator = typename Sorted_Map_T::const_iterator;
01137
01138     sorted_range (Sorted_Map_T &sorted_val, const Sorted_Map_T& val)
01139     : sorted_begin( val.begin() ),
01140       sorted_end( val.end() )
01141     { }
01142     sorted_iterator sorted_begin;
01143     sorted_iterator sorted_end;
01144 };
01145
01146 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01147 auto
01148 operator<< (std::ostream& os, const framed_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
01149 {
01150     using limits_t = std::numeric_limits<Scalar_T>;
01151     if (val.empty())
01152         os << 0;
01153     else if (val.isnan())
01154         os << limits_t::quiet_NaN();
01155     else if (val.isinf())
01156     {
01157         const Scalar_T& inf = limits_t::infinity();
01158         os << (scalar(val) < 0.0 ? -inf : inf);
01159     }
01160     else
01161     {
01162         using traits_t = numeric_traits<Scalar_T>;
01163         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01164         Scalar_T truncation;
01165         switch (os.flags() & std::ios::floatfield)
01166         {
01167             case std::ios_base::scientific:
01168                 truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()) + 1);
01169                 break;
01170             case std::ios_base::fixed:
01171                 truncation = Scalar_T(1) / (traits_t::pow(Scalar_T(10), int(os.precision())) *
01172 val.max_abs());
01173                 break;
01174             case std::ios_base::fixed | std::ios_base::scientific:
01175                 truncation = multivector_t::default_truncation;
01176                 break;
01177             default:
01178                 truncation = Scalar_T(1) / traits_t::pow(Scalar_T(10), int(os.precision()));
01179                 break;
01180         }
01181         auto truncated_val = val.truncated(truncation);
01182         if (truncated_val.empty())
01183             os << 0;
01184         else
01185         {
01186             using map_t = typename multivector_t::map_t;
01187             using sorted_map_t = typename multivector_t::sorted_map_t;
01188             using sorted_iterator = typename sorted_map_t::const_iterator;
01189             auto sorted_val = sorted_map_t();
01190             const auto sorted_val_range = sorted_range< map_t, sorted_map_t >(sorted_val, truncated_val);
01191             auto sorted_it = sorted_val_range.sorted_begin;
01192             os << *sorted_it;
01193             for (++sorted_it;
01194                 sorted_it != sorted_val_range.sorted_end;
01195                 ++sorted_it)
01196             {
01197                 const Scalar_T& scr = sorted_it->second;
01198                 if (scr >= 0.0)
01199                     os << '+';
01200                 os << *sorted_it;
01201             }
01202         }
01203     }
01204     return os;
01205 }

```

```

01206
01208     template< typename Scalar_T, const index_t LO, const index_t HI >
01209     auto
01210     operator« (std::ostream& os, const std::pair< const index_set<LO,HI>, Scalar_T >& term) ->
std::ostream&
01211     {
01212         const auto second_as_double = numeric_traits<Scalar_T>::to_double(term.second);
01213         const auto use_double =
01214             (os.precision() <= std::numeric_limits<double>::digits10) ||
01215             (term.second == Scalar_T(second_as_double));
01216         if (term.first.count() == 0)
01217             if (use_double)
01218                 os « second_as_double;
01219             else
01220                 os « term.second;
01221         else if (term.second == Scalar_T(-1))
01222             {
01223                 os « '-';
01224                 os « term.first;
01225             }
01226         else if (term.second != Scalar_T(1))
01227             {
01228                 if (use_double)
01229                     {
01230                         auto tol = std::pow(10.0, -os.precision());
01231                         if ( std::fabs(second_as_double + 1.0) < tol )
01232                             os « '-';
01233                         else if ( std::fabs(second_as_double - 1.0) >= tol )
01234                             os « second_as_double;
01235                     }
01236                 else
01237                     os « term.second;
01238                 os « term.first;
01239             }
01240         else
01241             os « term.first;
01242         return os;
01243     }
01244
01246     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01247     auto
01248     operator» (std::istream& s, framed_multi<Scalar_T,LO,HI,Tune_P> & val) -> std::istream&
01249     { // Input looks like 1.0-2.0{1,2}+3.2{3,4}.
01250         using multivector_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
01251         // Parsing variables.
01252         auto local_val = multivector_t();
01253         auto c = 0;
01254         // Parsing control variables.
01255         auto negative = false;
01256         auto expect_term = true;
01257         // The multivector may begin with '+' or '-'. Check for this.
01258         c = s.peek();
01259         if (s.good() && (c == int('+') || c == int('-')))
01260             { // A '-' here negates the following term.
01261                 negative = (c == int('-'));
01262                 // Consume the '+' or '-'.
01263                 s.get();
01264             }
01265         while (s.good())
01266             { // Parse a term.
01267                 // A term consists of an optional scalar, followed by an optional index set.
01268                 // At least one of the two must be present.
01269                 // Default coordinate is Scalar_T(1).
01270                 auto coordinate = Scalar_T(1);
01271                 // Default index set is empty.
01272                 auto ist = index_set<LO,HI>();
01273                 // First, check for an opening brace.
01274                 c = s.peek();
01275                 if (s.good())
01276                     { // If the character is not an opening brace,
01277                         // a coordinate value is expected here.
01278                         if (c != int('{'))
01279                             { // Try to read a coordinate value.
01280                                 double coordinate_as_double;
01281                                 s » coordinate_as_double;
01282                                 // Reading the coordinate may have resulted in an end of file condition.
01283                                 // This is not a failure.
01284                                 if (s)
01285                                     coordinate = Scalar_T(coordinate_as_double);
01286                             }
01287                         }
01288                 else
01289                     { // End of file here ends parsing while a term may still be expected.
01290                         break;
01291                     }
01292                 // Coordinate is now Scalar_T(1) or a Scalar_T value.
01293                 // Parse an optional index set.

```

```

01294     if (s.good())
01295     {
01296         c = s.peek();
01297         if (s.good() && c == int('{'))
01298         { // Try to read index set.
01299             s » ist;
01300         }
01301     }
01302     // Reading the term may have resulted in an end of file condition.
01303     // This is not a failure.
01304     if (s)
01305     {
01306         // Immediately after parsing a term, another term is not expected.
01307         expect_term = false;
01308         if (coordinate != Scalar_T(0))
01309         {
01310             // Add the term to the local multivector.
01311             coordinate =
01312                 negative
01313                 ? -coordinate
01314                 : coordinate;
01315             using term_t = typename multivector_t::term_t;
01316             local_val += term_t(ist, coordinate);
01317         }
01318     }
01319     // Check if anything follows the current term.
01320     if (s.good())
01321     {
01322         c = s.peek();
01323         if (s.good())
01324         { // Only '+' and '-' are valid here.
01325             if (c == int('+') || c == int('-'))
01326             { // A '-' here negates the following term.
01327                 negative = (c == int('-'));
01328                 // Consume the '+' or '-'.
01329                 s.get();
01330                 // Immediately after '+' or '-',
01331                 // expect another term.
01332                 expect_term = true;
01333             }
01334             else
01335             { // Any other character here is a not failure,
01336                 // but still ends the parsing of the multivector.
01337                 break;
01338             }
01339         }
01340     }
01341 }
01342 // If a term is still expected, this is a failure.
01343 if (expect_term)
01344     s.clear(std::istream::failbit);
01345 // End of file is not a failure.
01346 if (s)
01347 { // The multivector has been successfully parsed.
01348     val = local_val;
01349 }
01350 return s;
01351 }
01352
01353 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01354 auto
01355 framed_multi<Scalar_T,LO,HI,Tune_P>::
01356 nbr_terms () const -> unsigned long
01357 { return this->size(); }
01358
01359 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01360 inline
01361 auto
01362 framed_multi<Scalar_T,LO,HI,Tune_P>::
01363 operator+= (const term_t& term) -> multivector_t&
01364 { // Do not insert terms with 0 coordinate
01365     if (term.second != Scalar_T(0))
01366     {
01367         const auto& this_it = this->find(term.first);
01368         if (this_it == this->end())
01369             this->insert(term);
01370         else if (this_it->second + term.second == Scalar_T(0))
01371             // Erase term if resulting coordinate is 0
01372             this->erase(this_it);
01373         else
01374             this_it->second += term.second;
01375     }
01376     return *this;
01377 }
01378
01379 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01380 auto

```

```

01384   framed_multi<Scalar_T,LO,HI,Tune_P>::
01385   isinf() const -> bool
01386   {
01387       using traits_t = numeric_traits<Scalar_T>;
01388
01389       if (std::numeric_limits<Scalar_T>::has_infinity)
01390           for (auto& this_term : *this)
01391               if (traits_t::isInf(this_term.second))
01392                   return true;
01393       return false;
01394   }
01395
01397   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01398   auto
01399   framed_multi<Scalar_T,LO,HI,Tune_P>::
01400   isnan() const -> bool
01401   {
01402       using traits_t = numeric_traits<Scalar_T>;
01403
01404       if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
01405           for (auto& this_term : *this)
01406               if (traits_t::isNaN(this_term.second))
01407                   return true;
01408       return false;
01409   }
01410
01412   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01413   auto
01414   framed_multi<Scalar_T,LO,HI,Tune_P>::
01415   truncated(const Scalar_T& limit) const -> const multivector_t
01416   {
01417       using traits_t = numeric_traits<Scalar_T>;
01418
01419       if (this->isnan() || this->isinf())
01420           return *this;
01421       const auto truncation = traits_t::abs(limit);
01422       const auto top = max_abs();
01423       auto result = multivector_t();
01424       if (top != Scalar_T(0))
01425           for (auto& this_term : *this)
01426               if (traits_t::abs(this_term.second) > top * truncation)
01427                   result.insert(this_term);
01428       return result;
01429   }
01430
01432   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01433   auto
01434   framed_multi<Scalar_T,LO,HI,Tune_P>::
01435   fold(const index_set_t frm) const -> multivector_t
01436   {
01437       if (frm.is_contiguous())
01438           return *this;
01439       else
01440       {
01441           auto result = multivector_t();
01442           for (auto& this_term : *this)
01443               result.insert(term_t(this_term.first.fold(frm), this_term.second));
01444           return result;
01445       }
01446   }
01447
01449   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01450   auto
01451   framed_multi<Scalar_T,LO,HI,Tune_P>::
01452   unfold(const index_set_t frm) const -> multivector_t
01453   {
01454       if (frm.is_contiguous())
01455           return *this;
01456       else
01457       {
01458           auto result = multivector_t();
01459           for (auto& this_term : *this)
01460               result.insert(term_t(this_term.first.unfold(frm), this_term.second));
01461           return result;
01462       }
01463   }
01464
01466   // Reference: [L] 16.4 Periodicity of 8, p216
01467   template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01468   auto
01469   framed_multi<Scalar_T,LO,HI,Tune_P>::
01470   centre_pm4_qp4(index_t& p, index_t& q) -> multivector_t&
01471   {
01472       // We add 4 to q by subtracting 4 from p
01473       if (q+4 > -LO)
01474           throw error_t("centre_pm4_qp4(p,q): LO is too high to represent this value");
01475       if (this->frame().max() > p-4)

```

```

01476     {
01477         using index_pair_t = typename index_set_t::index_pair_t;
01478         const auto pm3210 = index_set_t(index_pair_t(p-3,p), true);
01479         const auto qm4321 = index_set_t(index_pair_t(-q-4,-q-1), true);
01480         const auto& tqm4321 = term_t(qm4321, Scalar_T(1));
01481         auto result = multivector_t();
01482         for (auto& this_term : *this)
01483         {
01484             const auto ist = this_term.first;
01485             if (ist.max() > p-4)
01486             {
01487                 auto var_term = var_term_t();
01488                 for (auto
01489                     n = index_t(0);
01490                     n != index_t(4);
01491                     ++n)
01492                     if (ist[n+p-3])
01493                         var_term *= term_t(index_set_t(n-q-4), Scalar_T(1)) * tqm4321;
01494                 // Mask out {p-3}..{p}
01495                 result.insert(term_t(ist & ~pm3210, this_term.second) *
01496                               term_t(var_term.first, var_term.second));
01497             }
01498             else
01499                 result.insert(this_term);
01500         }
01501         *this = result;
01502     }
01503     p -=4; q += 4;
01504     return *this;
01505 }
01506
01507 // Reference: [L] 16.4 Periodicity of 8, p216
01508 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01509 auto
01510 framed_multi<Scalar_T,LO,HI,Tune_P>::
01511 centre_pp4_qm4(index_t& p, index_t& q) -> multivector_t&
01512 {
01513     // We add 4 to p by subtracting 4 from q
01514     if (p+4 > HI)
01515         throw error_t("centre_pp4_qm4(p,q): HI is too low to represent this value");
01516     if (this->frame().min() < -q+4)
01517     {
01518         using index_pair_t = typename index_set_t::index_pair_t;
01519         const auto qp0123 = index_set_t(index_pair_t(-q,-q+3), true);
01520         const auto pp1234 = index_set_t(index_pair_t(p+1,p+4), true);
01521         const auto& tpp1234 = term_t(pp1234, Scalar_T(1));
01522         auto result = multivector_t();
01523         for (auto& this_term : *this)
01524         {
01525             index_set_t ist = this_term.first;
01526             if (ist.min() < -q+4)
01527             {
01528                 auto var_term = var_term_t();
01529                 for (auto
01530                     n = index_t(0);
01531                     n != index_t(4);
01532                     ++n)
01533                     if (ist[n-q])
01534                         var_term *= term_t(index_set_t(n+p+1), Scalar_T(1)) * tpp1234;
01535                 // Mask out {-q}..{-q+3}
01536                 result.insert(term_t(var_term.first, var_term.second) *
01537                               term_t(ist & ~qp0123, this_term.second));
01538             }
01539             else
01540                 result.insert(this_term);
01541         }
01542         *this = result;
01543     }
01544     p +=4; q -= 4;
01545     return *this;
01546 }
01547
01548 // Reference: [P] Proposition 15.20, p 131
01549 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01550 auto
01551 framed_multi<Scalar_T,LO,HI,Tune_P>::
01552 centre_qpl_pml(index_t& p, index_t& q) -> multivector_t&
01553 {
01554     if (q+1 > HI)
01555         throw error_t("centre_qpl_pml(p,q): HI is too low to represent this value");
01556     if (p-1 > -LO)
01557         throw error_t("centre_qpl_pml(p,q): LO is too high to represent this value");
01558     const auto qpl = index_set_t(q+1);
01559     const auto& tqpl = term_t(qpl, Scalar_T(1));
01560     auto result = multivector_t();
01561     for (auto& this_term : *this)
01562     {
01563         // ...
01564     }

```

```

01565     const auto ist = this_term.first;
01566     auto var_term = var_term_t(index_set_t(), this_term.second);
01567     for (auto
01568         n = -q;
01569         n != p;
01570         ++n)
01571         if (n != 0 && ist[n])
01572             var_term *= term_t(index_set_t(-n) | qp1, Scalar_T(1));
01573     if (p != 0 && ist[p])
01574         var_term *= tqpl;
01575     result.insert(term_t(var_term.first, var_term.second));
01576 }
01577 index_t orig_p = p;
01578 p = q+1;
01579 q = orig_p-1;
01580 return *this = result;
01581 }
01582
01583 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01584 auto
01585 framed_multi<Scalar_T,LO,HI,Tune_P>::
01586 divide(const index_set_t ist) const -> const framed_pair_t
01587 {
01588     auto quo = multivector_t();
01589     auto rem = multivector_t();
01590     for (auto& this_term : *this)
01591         if ((this_term.first | ist) == this_term.first)
01592             quo.insert(term_t(this_term.first ^ ist, this_term.second));
01593         else
01594             rem.insert(this_term);
01595     return framed_pair_t(quo, rem);
01596 }
01597
01598 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01599 auto
01600 framed_multi<Scalar_T,LO,HI,Tune_P>::
01601 fast(const index_t level, const bool odd) const -> const matrix_t
01602 {
01603     // Assume val is already folded and centred
01604     if (this->empty())
01605     {
01606         using matrix_index_t = typename matrix_multi_t::matrix_index_t;
01607         const auto dim = matrix_index_t(1) << level;
01608         auto result = matrix_t(dim, dim);
01609         result.clear();
01610         return result;
01611     }
01612     if (level == 0)
01613         return matrix::unit<matrix_t>(1) * this->scalar();
01614
01615     using basis_matrix_t = typename matrix_multi_t::basis_matrix_t;
01616     using basis_scalar_t = typename basis_matrix_t::value_type;
01617
01618     const auto& I = matrix::unit<basis_matrix_t>(2);
01619     auto J = basis_matrix_t(2,2,2);
01620     J.clear();
01621     J(0,1) = basis_scalar_t(-1);
01622     J(1,0) = basis_scalar_t(1);
01623     auto K = J;
01624     K(0,1) = basis_scalar_t(1);
01625     auto JK = I;
01626     JK(0,0) = basis_scalar_t(-1);
01627
01628     const auto ist_mn = index_set_t(-level);
01629     const auto ist_pn = index_set_t(level);
01630     if (level == 1)
01631     {
01632         if (odd)
01633             return matrix_t(J) * (*this)[ist_mn] + matrix_t(K) * (*this)[ist_pn];
01634         else
01635             return matrix_t(I) * this->scalar() + matrix_t(JK) * (*this)[ist_mn ^ ist_pn];
01636     }
01637     else
01638     {
01639         const auto& pair_mn = this->divide(ist_mn);
01640         const auto& quo_mn = pair_mn.first;
01641         const auto& rem_mn = pair_mn.second;
01642         const auto& pair_quo_mnpn = quo_mn.divide(ist_pn);
01643         const auto& val_mnpn = pair_quo_mnpn.first;
01644         const auto& val_mn = pair_quo_mnpn.second;
01645         const auto& pair_rem_mnpn = rem_mn.divide(ist_pn);
01646         const auto& val_pn = pair_rem_mnpn.first;
01647         const auto& val_l = pair_rem_mnpn.second;
01648         using matrix::kron;
01649         if (odd)
01650             return - kron(JK, val_l.fast(level-1, 1))
01651                 + kron(I, val_mnpn.fast(level-1, 1))

```



```

01654         + kron(J, val_mn.fast (level-1, 0))
01655         + kron(K, val_pn.fast (level-1, 0));
01656     else
01657         return kron(I, val_l.fast (level-1, 0))
01658         + kron(JK, val_mnpn.fast (level-1, 0))
01659         + kron(K, val_mn.fast (level-1, 1))
01660         - kron(J, val_pn.fast (level-1, 1));
01661     }
01662 }
01663
01664 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01665 template< typename Other_Scalar_T >
01666 auto
01667 framed_multi<Scalar_T,LO,HI,Tune_P>::
01668 fast_matrix_multi(const index_set_t frm) const -> const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>
01669 {
01670     // Fold val
01671     auto val = this->fold(frm);
01672     auto p = frm.count_pos();
01673     auto q = frm.count_neg();
01674     const auto bott_offset = gen::offset_to_super[pos_mod(p - q, 8)];
01675     p += std::max(bott_offset, index_t(0));
01676     q -= std::min(bott_offset, index_t(0));
01677     if (p > HI)
01678         throw error_t("fast_matrix_multi(frm): HI is too low to represent this value");
01679     if (q > -LO)
01680         throw error_t("fast_matrix_multi(frm): LO is too high to represent this value");
01681     // Centre val
01682     while (p - q > 4)
01683         val.centre_pm4_qp4(p, q);
01684     while (p - q < -3)
01685         val.centre_pp4_qm4(p, q);
01686     if (p - q > 1)
01687         val.centre_qpl_pml(p, q);
01688     const index_t level = (p + q)/2;
01689
01690     // Do the fast transform
01691     const auto& ev_val = val.even();
01692     const auto& od_val = val.odd();
01693     return matrix_multi<Other_Scalar_T,LO,HI,Tune_P>(ev_val.fast(level, 0) + od_val.fast(level, 1),
01694     frm);
01695 }
01696
01697 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01698 inline
01699 auto
01700 framed_multi<Scalar_T,LO,HI,Tune_P>::
01701 fast_framed_multi() const -> const multivector_t
01702 { return *this; }
01703
01704 template< typename Scalar_T, const index_t LO, const index_t HI >
01705 inline
01706 static
01707 auto
01708 crd_of_mult(const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01709             const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> Scalar_T
01710 { return lhs.first.sign_of_mult(rhs.first) * lhs.second * rhs.second; }
01711
01712 template< typename Scalar_T, const index_t LO, const index_t HI >
01713 inline
01714 auto
01715 operator* (const std::pair<const index_set<LO,HI>, Scalar_T>& lhs,
01716            const std::pair<const index_set<LO,HI>, Scalar_T>& rhs) -> const std::pair<const
01717 index_set<LO,HI>, Scalar_T>
01718 {
01719     using term_t = std::pair<const index_set<LO,HI>, Scalar_T>;
01720     return term_t(lhs.first ^ rhs.first, crd_of_mult(lhs, rhs));
01721 }
01722
01723 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01724 auto
01725 sqrt(const framed_multi<Scalar_T,LO,HI,Tune_P>& val, const framed_multi<Scalar_T,LO,HI,Tune_P>& i,
01726 bool prechecked) -> const framed_multi<Scalar_T,LO,HI,Tune_P>
01727 {
01728     using traits_t = numeric_traits<Scalar_T>;
01729     if (val.isnan())
01730         return traits_t::NaN();
01731     check_complex(val, i, prechecked);
01732
01733     const auto realval = val.scalar();
01734     if (val == realval)
01735     {
01736         if (realval < Scalar_T(0))
01737             return i * traits_t::sqrt(-realval);
01738         else
01739             return traits_t::sqrt(realval);
01740     }
01741 }

```

```

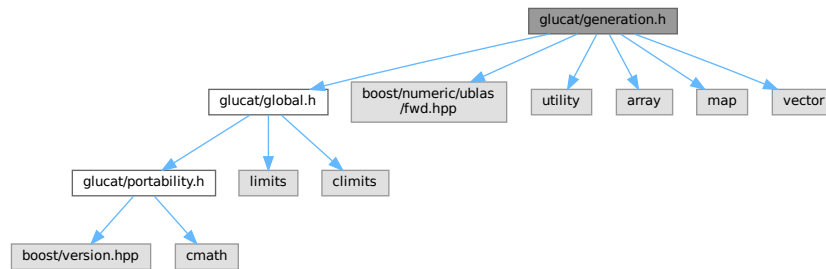
01742     }
01743     using matrix_multi_t = typename framed_multi<Scalar_T, LO, HI, Tune_P>::matrix_multi_t;
01744     return sqrt(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01745 }
01746
01747 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01748 auto
01749 exp(const framed_multi<Scalar_T, LO, HI, Tune_P>& val) -> const framed_multi<Scalar_T, LO, HI, Tune_P>
01750 {
01751     using traits_t = numeric_traits<Scalar_T>;
01752     if (val.isnan())
01753         return traits_t::NaN();
01754
01755     const auto s = scalar(val);
01756     if (val == s)
01757         return traits_t::exp(s);
01758
01759     const double size = val.size();
01760     const auto frm_count = val.frame().count();
01761     const auto algebra_dim = set_value_t(1) << frm_count;
01762
01763     if( (size * size <= double(algebra_dim)) || (frm_count < Tune_P::mult_matrix_threshold))
01764     {
01765         switch (Tune_P::function_precision)
01766         {
01767             case precision_demoted:
01768             {
01769                 using demoted_scalar_t = typename traits_t::demoted::type;
01770                 using demoted_multivector_t = framed_multi<demoted_scalar_t, LO, HI, Tune_P>;
01771
01772                 const auto& demoted_val = demoted_multivector_t(val);
01773                 return clifford_exp(demoted_val);
01774             }
01775             break;
01776             case precision_promoted:
01777             {
01778                 using promoted_scalar_t = typename traits_t::promoted::type;
01779                 using promoted_multivector_t = framed_multi<promoted_scalar_t, LO, HI, Tune_P>;
01780
01781                 const auto& promoted_val = promoted_multivector_t(val);
01782                 return clifford_exp(promoted_val);
01783             }
01784             break;
01785             default:
01786                 return clifford_exp(val);
01787         }
01788     }
01789     else
01790     {
01791         using matrix_multi_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
01792         return exp(matrix_multi_t(val));
01793     }
01794 }
01795
01796 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01797 auto
01800 log(const framed_multi<Scalar_T, LO, HI, Tune_P>& val, const framed_multi<Scalar_T, LO, HI, Tune_P>& i,
bool prechecked) -> const framed_multi<Scalar_T, LO, HI, Tune_P>
01801 {
01802     using traits_t = numeric_traits<Scalar_T>;
01803     if (val == Scalar_T(0) || val.isnan())
01804         return traits_t::NaN();
01805
01806     check_complex(val, i, prechecked);
01807
01808     const auto realval = val.scalar();
01809     if (val == realval)
01810     {
01811         if (realval < Scalar_T(0))
01812             return i * traits_t::pi() + traits_t::log(-realval);
01813         else
01814             return traits_t::log(realval);
01815     }
01816     using matrix_multi_t = typename framed_multi<Scalar_T, LO, HI, Tune_P>::matrix_multi_t;
01817     return log(matrix_multi_t(val), matrix_multi_t(i), prechecked);
01818 }
01819 }
01820 #endif // _GLUCAT_FRAMED_MULTI_IMP_H

```

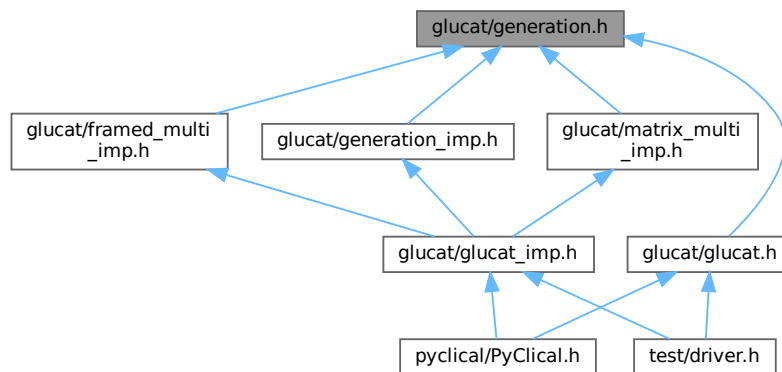
## 7.13 glucat/generation.h File Reference

```
#include "glucat/global.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <utility>
#include <array>
#include <map>
#include <vector>
```

Include dependency graph for generation.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class `glucat::gen::generator_table< Matrix_T >`  
Table of generators for specific signatures.

### Namespaces

- namespace `glucat`
- namespace `glucat::gen`

## Typedefs

- using `glucat::gen::signature_t = std::pair<index_t, index_t>`  
*A signature is a pair of indices, p, q, with  $p == \text{frame.max}()$ ,  $q == -\text{frame.min}()$*

## Variables

- static const `std::array< index_t, 8 > glucat::gen::offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1}`  
*Offsets between the current signature and that of the real superalgebra.*

## 7.14 generation.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GENERATION_H
00002 #define _GLUCAT_GENERATION_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   generation.h : Declare functions for generation of the matrix representation
00006   -----
00007   begin                : Wed Jan 23 2002
00008   copyright            : (C) 2002-2012 by Paul C. Leopardi
00009 *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024 *****/
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030 *****/
00031 See also Arvind Raja's original header comments in glucat.h
00032 *****/
00033
00034 #include "glucat/global.h"
00035
00036 #include <boost/numeric/ublas/fwd.hpp>
00037
00038 #include <utility>
00039 #include <array>
00040 #include <map>
00041 #include <vector>
00042
00043 namespace glucat { namespace gen
00044 {
00045     namespace ublas = boost::numeric::ublas;
00046
00047     using signature_t = std::pair<index_t, index_t>;
00048
00049     template< class Matrix_T >
00050     class generator_table :
00051     private std::map< signature_t, std::vector<Matrix_T> >
00052     {
00053     public:
00054         auto operator() (const index_t p, const index_t q) -> const Matrix_T*;
00055         static auto generator() -> generator_table<Matrix_T>&;
00056     private:
00057         auto gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&;
00058         void gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig);
00059         void gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig);
00060         void gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig);
00061         void gen_from_qp1_pml(const std::vector<Matrix_T>& old, const signature_t sig);
00062     }
00063 }
00064 }
```

```

00075     friend class friend_for_private_destructor;
00076     // Enforce singleton
00077     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00078     generator_table() = default;
00079     ~generator_table() = default;
00080 public:
00081     generator_table(const generator_table&) = delete;
00082     auto operator= (const generator_table&) -> generator_table& = delete;
00083 };
00084
00086 static const std::array<index_t, 8> offset_to_super = {0,-1, 0,-1,-2, 3, 2, 1};
00087
00088 } }
00089 #endif // _GLUCAT_GENERATION_H

```

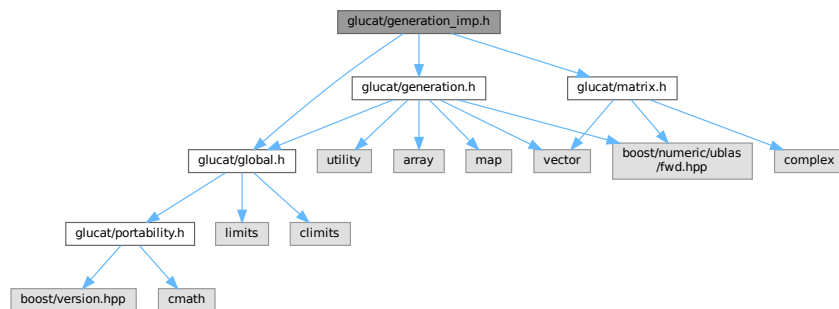
## 7.15 glucat/generation\_imp.h File Reference

```

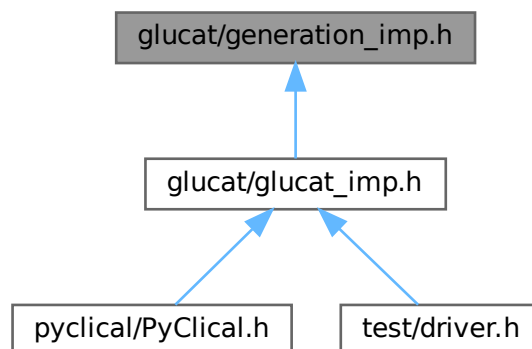
#include "glucat/global.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"

```

Include dependency graph for generation\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)
- namespace [glucat::gen](#)

## 7.16 generation\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GENERATION_IMP_H
00002 #define _GLUCAT_GENERATION_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     generation_imp.h : Implement functions for generation of the matrix representation
00006     -----
00007     begin                : Wed Jan 23 2002
00008     copyright             : (C) 2002-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/generation.h"
00036 #include "glucat/matrix.h"
00037
00038 namespace glucat { namespace gen
00039 {
00040     // References for algorithms:
00041     // [M]: Scott Meyers, "Effective C++" Second Edition, Addison-Wesley, 1998.
00042     // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00043     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00044
00045     // Reference: [M] Item 47
00046     template< class Matrix_T >
00047     auto
00048     generator_table<Matrix_T>::
00049     generator() -> generator_table<Matrix_T>&
00050     { static generator_table<Matrix_T> g; return g;}
00051
00052     // Reference: [P] Table 15.27, p 133
00053     template< class Matrix_T >
00054     inline
00055     auto
00056     generator_table<Matrix_T>::
00057     operator() (const index_t p, const index_t q) -> const Matrix_T*
00058     {
00059         const auto bott = pos_mod(p-q, 8);
00060         switch(bott)
00061         {
00062             case 0:
00063             case 2:
00064                 // Construct generators
00065                 return &(gen_vector(p, q)[q]);
00066             default:
00067                 // Select generators from the vector for a larger frame
00068                 const auto super_p = p + std::max(offset_to_super[bott], index_t(0));
00069                 const auto super_q = q - std::min(offset_to_super[bott], index_t(0));
00070                 return &(gen_vector(super_p, super_q)[super_q]);
00071         }
00072     }
00073 }
00074 }

```

```

00075
00076 template< class Matrix_T >
00077 auto
00078 generator_table<Matrix_T>::
00079 gen_vector(const index_t p, const index_t q) -> const std::vector<Matrix_T>&
00080 {
00081     using result_t = std::vector<Matrix_T>;
00082     const auto card = p + q;
00083     const auto bias = p - q;
00084     const auto bott = pos_mod(bias, 8);
00085     const auto sig = signature_t(p, q);
00086     if (this->find(sig) == this->end())
00087     {
00088         switch(bott)
00089         {
00090             case 0:
00091                 if (bias < 0)
00092                     // Construct generators for p,q given generators for p+4,q-4
00093                     gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00094                 else if (bias > 0)
00095                     // Construct generators for p,q given generators for p-4,q+4
00096                     gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00097                 else if (card == 0)
00098                 { // Base case. Save a generator vector containing one matrix, size 1.
00099                     auto result = result_t(1, matrix::unit<Matrix_T>(1));
00100                     this->insert(make_pair(sig, result));
00101                 }
00102                 else
00103                     // Construct generators for p,q given generators for p-1,q-1
00104                     gen_from_pml_qml(gen_vector(p-1, q-1), sig);
00105                 break;
00106             case 2:
00107                 if (bias < 2)
00108                     // Construct generators for p,q given generators for p+4,q-4
00109                     gen_from_pp4_qm4(gen_vector(p+4, q-4), sig);
00110                 else if (bias > 2)
00111                     // Construct generators for p,q given generators for p-4,q+4
00112                     gen_from_pm4_qp4(gen_vector(p-4, q+4), sig);
00113                 else
00114                     // Construct generators for p,q given generators for q+1,p-1
00115                     gen_from_qp1_pml(gen_vector(q+1, p-1), sig);
00116                 break;
00117             default:
00118                 break;
00119         }
00120     }
00121     return (*this)[sig];
00122 }
00123
00124 // Reference: [P] Proposition 15.17, p 131
00125 template< class Matrix_T >
00126 void
00127 generator_table<Matrix_T>::
00128 gen_from_pml_qml(const std::vector<Matrix_T>& old, const signature_t sig)
00129 {
00130     const auto new_size = old.size() + 2;
00131     using size_t = decltype(new_size);
00132     using result_t = std::vector<Matrix_T>;
00133     auto result = result_t(new_size);
00134
00135     const auto old_dim = old[0].size1();
00136     const auto& eye = matrix::unit<Matrix_T>(old_dim);
00137
00138     auto neg = Matrix_T(2,2,2);
00139     neg(0,1) = -1;
00140     neg(1,0) = 1;
00141
00142     auto pos = neg;
00143     pos(0,1) = 1;
00144
00145     auto dup = Matrix_T(2,2,2);
00146     dup(0,0) = 1;
00147     dup(1,1) = -1;
00148
00149     result[0] = matrix::mono_kron(neg, eye);
00150     for (auto
00151         k = size_t(1);
00152         k != new_size-1;
00153         ++k)
00154         result[k] = matrix::mono_kron(dup, old[k-1]);
00155     result[new_size-1] = matrix::mono_kron(pos, eye);
00156
00157     // Save the resulting generator array.
00158     this->insert(make_pair(sig, result));
00159 }
00160
00161 // Reference: [L] 16.4 Periodicity of 8, p216
00162 template< class Matrix_T >
00163 void

```

```

00165 generator_table<Matrix_T>::
00166 gen_from_pm4_qp4(const std::vector<Matrix_T>& old, const signature_t sig)
00167 {
00168     const auto old_size = old.size();
00169     using size_t = decltype(old_size);
00170     using result_t = std::vector<Matrix_T>;
00171     auto result = result_t(old_size);
00172
00173     auto h = old[0];
00174     for (auto
00175         k = size_t(1);
00176         k != size_t(4);
00177         ++k)
00178         h = matrix::mono_prod(old[k], h);
00179
00180     for (auto
00181         k = size_t(0);
00182         k != old_size-4;
00183         ++k)
00184         result[k] = old[k+4];
00185     for (auto
00186         k = old_size-4;
00187         k != old_size;
00188         ++k)
00189         result[k] = matrix::mono_prod(old[k+4-old_size], h);
00190     // Save the resulting generator array.
00191     this->insert(make_pair(sig, result));
00192 }
00193
00194 // Reference: [L] 16.4 Periodicity of 8, p216
00195 template< class Matrix_T >
00196 void
00197 generator_table<Matrix_T>::
00198 gen_from_pp4_qm4(const std::vector<Matrix_T>& old, const signature_t sig)
00199 {
00200     const auto old_size = old.size();
00201     using size_t = decltype(old_size);
00202     using result_t = std::vector<Matrix_T>;
00203     auto result = result_t(old_size);
00204
00205     auto h = old[old_size-1];
00206     for (auto
00207         k = size_t(1);
00208         k != size_t(4);
00209         ++k)
00210         h = matrix::mono_prod(old[old_size-1-k], h);
00211
00212     for (auto
00213         k = size_t(0);
00214         k != size_t(4);
00215         ++k)
00216         result[k] = matrix::mono_prod(old[k+old_size-4], h);
00217     for (auto
00218         k = size_t(4);
00219         k != old_size;
00220         ++k)
00221         result[k] = old[k-4];
00222     // Save the resulting generator array.
00223     this->insert(make_pair(sig, result));
00224 }
00225
00226 // Reference: [P] Proposition 15.20, p 131
00227 template< class Matrix_T >
00228 void
00229 generator_table<Matrix_T>::
00230 gen_from_qp1_pml(const std::vector<Matrix_T>& old, const signature_t sig)
00231 {
00232     const auto old_size = old.size();
00233     using size_t = decltype(old_size);
00234     using result_t = std::vector<Matrix_T>;
00235     auto result = result_t(old_size);
00236
00237     const auto& h = old[old_size-1];
00238     for (auto
00239         k = size_t(0);
00240         k != old_size-1;
00241         ++k)
00242         result[k] = matrix::mono_prod(old[old_size-2-k], h);
00243     result[old_size-1] = h;
00244
00245     // Save the resulting generator array.
00246     this->insert(make_pair(sig, result));
00247 }
00248 } }
00249 } }
00250 } }
00251 } }
00252 #endif // _GLUCAT_GENERATION_IMP_H

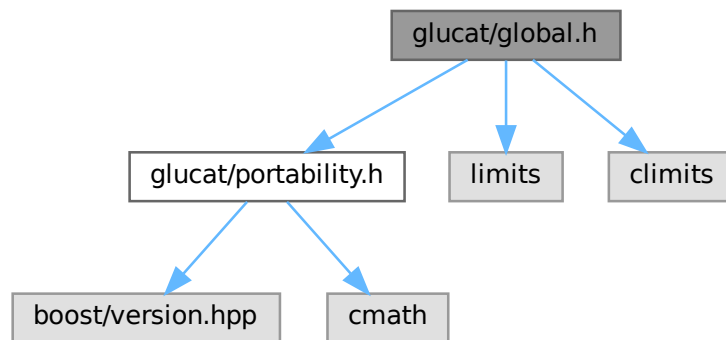
```



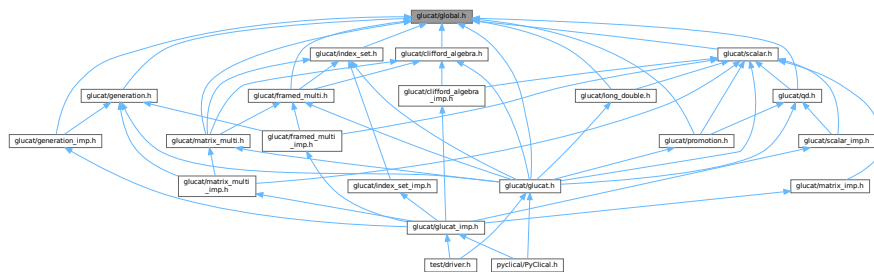
## 7.17 glucat/global.h File Reference

```
#include "glucat/portability.h"
#include <limits>
#include <climits>
```

Include dependency graph for global.h:



This graph shows which files directly or indirectly include this file:



### Classes

- struct `glucat::CTAssertion< true >`
- class `glucat::compare_types< LHS_T, RHS_T >`  
*Type comparison.*
- class `glucat::compare_types< T, T >`
- class `glucat::bool_to_type< truth_value >`  
*Bool to type.*

### Namespaces

- namespace `glucat`

## Macros

- `#define _GLUCAT_CTAssert(expr, msg) namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }`

## Typedefs

- using [glucat::index\\_t](#) = int  
*Size of index\_t should be enough to represent LO, HI.*
- using [glucat::set\\_value\\_t](#) = unsigned long  
*Size of set\_value\_t should be enough to contain index\_set<LO,HI>*

## Functions

- [glucat::\\_GLUCAT\\_CTAssert](#) (std::numeric\_limits< unsigned char >::radix==2, CannotDetermineBitsPerChar) const [index\\_t](#) BITS\_PER\_CHAR  
*If radix of unsigned char is not 2, we can't easily determine number of bits from sizeof.*
- [glucat::\\_GLUCAT\\_CTAssert](#) (\_GLUCAT\_BITS\_PER\_ULONG==BITS\_PER\_SET\_VALUE, BitsPerULongDoesNotMatchSetValueT) const [index\\_t](#) DEFAULT\_LO  
*Default lowest index in an index set.*
- template<typename LHS\_T, typename RHS\_T >  
auto [glucat::pos\\_mod](#) (LHS\_T lhs, RHS\_T rhs) -> LHS\_T  
*Modulo function which works reliably for lhs < 0.*

## Variables

- const double [glucat::MS\\_PER\\_S](#) = 1000.0  
*Timing constant: deprecated here - moved to [test/timing.h](#).*
- const [index\\_t](#) [glucat::BITS\\_PER\\_SET\\_VALUE](#) = std::numeric\_limits<[set\\_value\\_t](#)>::digits  
*Number of bits in set\_value\_t.*
- const [index\\_t](#) [glucat::DEFAULT\\_HI](#) = [index\\_t](#)(BITS\_PER\_SET\_VALUE / 2)  
*Default highest index in an index set.*

## 7.17.1 Macro Definition Documentation

### 7.17.1.1 \_GLUCAT\_CTAssert

```
#define _GLUCAT_CTAssert(  
    expr,  
    msg ) namespace { struct msg { glucat::CTAssertion<(expr)> ERROR_##msg; }; }
```

Definition at line 48 of file [global.h](#).

## 7.18 global.h

[Go to the documentation of this file.](#)

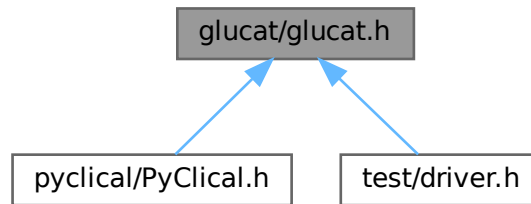
```

00001 #ifndef _GLUCAT_GLOBAL_H
00002 #define _GLUCAT_GLOBAL_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     global.h : Global declarations
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/portability.h"
00035
00036 #include <limits>
00037 #include <climits>
00038
00039 namespace glucat
00040 {
00041     // References:
00042     // [AA]: A. Alexandrescu, "Modern C++ Design", Addison-Wesley, 2001.
00043
00044     // Reference: [AA], p. 25
00045     template<bool> struct CTAAssertion;
00046     template<> struct CTAAssertion<true> { };
00047     #define _GLUCAT_CTAssert(expr, msg) \
00048         namespace { struct msg { glucat::CTAAssertion<(expr)> ERROR_##msg; }; }
00049
00050
00051     // Reference: [AA], pp. 34--37
00052     template < typename LHS_T, typename RHS_T >
00053     class compare_types
00054     {
00055     public:
00056         enum { are_same = false };
00057     };
00058     template < typename T >
00059     class compare_types<T, T>
00060     {
00061     public:
00062         enum { are_same = true };
00063     };
00064
00065
00066     // Reference: [AA], 2.4, p. 29
00067     template< bool truth_value >
00068     class bool_to_type
00069     {
00070     private:
00071         enum { value = truth_value };
00072     };
00073
00074
00075     // Global types which determine sizes
00076     using index_t = int;
00077     using set_value_t = unsigned long;
00078
00079
00080     // Global constants
00081     const double MS_PER_S = 1000.0;
00082
00083     // Constants which determine sizes
00084
00085     // Bits per unsigned long
00086     #if (ULONG_MAX == (4294967295UL))

```



This graph shows which files directly or indirectly include this file:



## 7.20 glucat.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GLUCAT_H
00002 #define _GLUCAT_GLUCAT_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   glucat.h : Organize GluCat header files for applications
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2021 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   Arvind Raja's original header comments and references follow.
00032   *****/
00033   // clifford algebra package, Arvind.Raja@hut.fi
00034   // ref: Press et.al. "Numerical Recipes in C", 2nd ed., C.U.P., 1992.
00035   // ref: LEDA, v 3.0, Stefan N\aher, Max-Planck-Institut f\ur Informatik
00036   // ref: Stroustrup B., "The C++ Programming Language", 2nd ed.,
00037   // Addison-Wesley, 1991.
00038   // ref: R. Sedgewick, "Algorithms in C++", Addison-Wesley, 1992.
00039   // ref: S. Meyers, "Effective C++ ", Addison-Wesley, 1992.
00040   *****/
00041
00042 #include "glucat/portability.h"
00043
00044 #include "glucat/global.h"
00045
00046 #include "glucat/errors.h"
00047
00048 #include "glucat/index_set.h"
00049
00050 #include "glucat/scalar.h"
00051
00052 #include "glucat/long_double.h"
00053
00054 #include "glucat/qd.h"

```

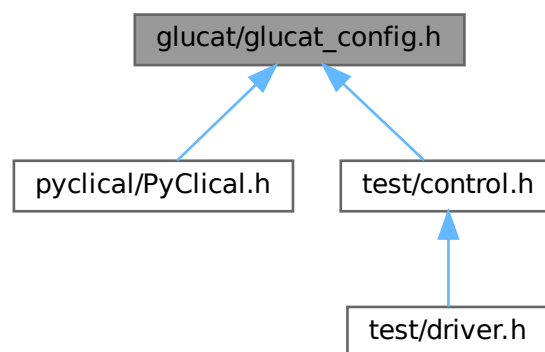
```

00055
00056 #include "glucat/promotion.h"
00057
00058 #include "glucat/random.h"
00059
00060 #include "glucat/clifford_algebra.h"
00061
00062 #include "glucat/tuning.h"
00063
00064 #include "glucat/framed_multi.h"
00065
00066 #include "glucat/generation.h"
00067
00068 #include "glucat/matrix.h"
00069
00070 #include "glucat/matrix_multi.h"
00071
00072 #endif // _GLUCAT_GLUCAT_H

```

## 7.21 glucat/glucat\_config.h File Reference

This graph shows which files directly or indirectly include this file:



### Macros

- #define GLUCAT\_HAVE\_CXX11 1
- #define GLUCAT\_HAVE\_INTTYPES\_H 1
- #define GLUCAT\_HAVE\_STDINT\_H 1
- #define GLUCAT\_HAVE\_STDIO\_H 1
- #define GLUCAT\_HAVE\_STDLIB\_H 1
- #define GLUCAT\_HAVE\_STRINGS\_H 1
- #define GLUCAT\_HAVE\_STRING\_H 1
- #define GLUCAT\_HAVE\_SYS\_STAT\_H 1
- #define GLUCAT\_HAVE\_SYS\_TYPES\_H 1
- #define GLUCAT\_HAVE\_UNISTD\_H 1
- #define GLUCAT\_PACKAGE "glucat"
- #define GLUCAT\_PACKAGE\_BUGREPORT ""
- #define GLUCAT\_PACKAGE\_NAME "glucat"
- #define GLUCAT\_PACKAGE\_STRING "glucat 0.12.0"
- #define GLUCAT\_PACKAGE\_TARNAME "glucat"

- `#define GLUCAT_PACKAGE_URL ""`
- `#define GLUCAT_PACKAGE_VERSION "0.12.0"`
- `#define GLUCAT_STDC_HEADERS 1`
- `#define GLUCAT_VERSION "0.12.0"`

## 7.21.1 Macro Definition Documentation

### 7.21.1.1 GLUCAT\_HAVE\_CXX11

```
#define GLUCAT_HAVE_CXX11 1
```

Definition at line 20 of file [glucat\\_config.h](#).

### 7.21.1.2 GLUCAT\_HAVE\_INTTYPES\_H

```
#define GLUCAT_HAVE_INTTYPES_H 1
```

Definition at line 28 of file [glucat\\_config.h](#).

### 7.21.1.3 GLUCAT\_HAVE\_STDINT\_H

```
#define GLUCAT_HAVE_STDINT_H 1
```

Definition at line 39 of file [glucat\\_config.h](#).

### 7.21.1.4 GLUCAT\_HAVE\_STDIO\_H

```
#define GLUCAT_HAVE_STDIO_H 1
```

Definition at line 44 of file [glucat\\_config.h](#).

### 7.21.1.5 GLUCAT\_HAVE\_STDLIB\_H

```
#define GLUCAT_HAVE_STDLIB_H 1
```

Definition at line 49 of file [glucat\\_config.h](#).

### 7.21.1.6 GLUCAT\_HAVE\_STRING\_H

```
#define GLUCAT_HAVE_STRING_H 1
```

Definition at line 59 of file [glucat\\_config.h](#).

#### 7.21.1.7 GLUCAT\_HAVE\_STRINGS\_H

```
#define GLUCAT_HAVE_STRINGS_H 1
```

Definition at line 54 of file [glucat\\_config.h](#).

#### 7.21.1.8 GLUCAT\_HAVE\_SYS\_STAT\_H

```
#define GLUCAT_HAVE_SYS_STAT_H 1
```

Definition at line 64 of file [glucat\\_config.h](#).

#### 7.21.1.9 GLUCAT\_HAVE\_SYS\_TYPES\_H

```
#define GLUCAT_HAVE_SYS_TYPES_H 1
```

Definition at line 69 of file [glucat\\_config.h](#).

#### 7.21.1.10 GLUCAT\_HAVE\_UNISTD\_H

```
#define GLUCAT_HAVE_UNISTD_H 1
```

Definition at line 74 of file [glucat\\_config.h](#).

#### 7.21.1.11 GLUCAT\_PACKAGE

```
#define GLUCAT_PACKAGE "glucat"
```

Definition at line 79 of file [glucat\\_config.h](#).

#### 7.21.1.12 GLUCAT\_PACKAGE\_BUGREPORT

```
#define GLUCAT_PACKAGE_BUGREPORT ""
```

Definition at line 84 of file [glucat\\_config.h](#).

#### 7.21.1.13 GLUCAT\_PACKAGE\_NAME

```
#define GLUCAT_PACKAGE_NAME "glucat"
```

Definition at line 89 of file [glucat\\_config.h](#).

Referenced by [glucat::control\\_t::control\\_t\(\)](#).



#### 7.21.1.14 GLUCAT\_PACKAGE\_STRING

```
#define GLUCAT_PACKAGE_STRING "glucat 0.12.0"
```

Definition at line 94 of file [glucat\\_config.h](#).

#### 7.21.1.15 GLUCAT\_PACKAGE\_TARNAME

```
#define GLUCAT_PACKAGE_TARNAME "glucat"
```

Definition at line 99 of file [glucat\\_config.h](#).

#### 7.21.1.16 GLUCAT\_PACKAGE\_URL

```
#define GLUCAT_PACKAGE_URL ""
```

Definition at line 104 of file [glucat\\_config.h](#).

#### 7.21.1.17 GLUCAT\_PACKAGE\_VERSION

```
#define GLUCAT_PACKAGE_VERSION "0.12.0"
```

Definition at line 109 of file [glucat\\_config.h](#).

#### 7.21.1.18 GLUCAT\_STDC\_HEADERS

```
#define GLUCAT_STDC_HEADERS 1
```

Definition at line 116 of file [glucat\\_config.h](#).

#### 7.21.1.19 GLUCAT\_VERSION

```
#define GLUCAT_VERSION "0.12.0"
```

Definition at line 121 of file [glucat\\_config.h](#).

Referenced by [glucat::control\\_t::control\\_t\(\)](#).

## 7.22 glucat\_config.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_GLUCAT_CONFIG_H
00002 #define _GLUCAT_GLUCAT_CONFIG_H 1
00003
00004 /* glucat/glucat_config.h. Generated automatically at end of configure. */
00005 /* config.h. Generated from config.h.in by configure. */
00006 /* config.h.in. Generated from configure.ac by autoheader. */
00007
00008 /* Define to dummy `main' function (if any) required to link to the Fortran
00009    libraries. */
00010 /* #undef F77_DUMMY_MAIN */
00011
00012 /* Define if F77 and FC dummy `main' functions are identical. */
00013 /* #undef FC_DUMMY_MAIN_EQ_F77 */
00014
00015 /* Define if you have a BLAS library. */
00016 /* #undef HAVE_BLAS */
00017
00018 /* define if the compiler supports basic C++11 syntax */
00019 #ifndef GLUCAT_HAVE_CXX11
00020 #define GLUCAT_HAVE_CXX11 1
00021 #endif
00022
00023 /* define if the compiler supports basic C++14 syntax */
00024 /* #undef HAVE_CXX14 */
00025
00026 /* Define to 1 if you have the <inttypes.h> header file. */
00027 #ifndef GLUCAT_HAVE_INTTYPES_H
00028 #define GLUCAT_HAVE_INTTYPES_H 1
00029 #endif
00030
00031 /* Define if you have LAPACK library. */
00032 /* #undef HAVE_LAPACK */
00033
00034 /* Define to 1 if you have the `lmf' library (-lmf). */
00035 /* #undef HAVE_LIBIMF */
00036
00037 /* Define to 1 if you have the <stdint.h> header file. */
00038 #ifndef GLUCAT_HAVE_STDINT_H
00039 #define GLUCAT_HAVE_STDINT_H 1
00040 #endif
00041
00042 /* Define to 1 if you have the <stdio.h> header file. */
00043 #ifndef GLUCAT_HAVE_STDIO_H
00044 #define GLUCAT_HAVE_STDIO_H 1
00045 #endif
00046
00047 /* Define to 1 if you have the <stdlib.h> header file. */
00048 #ifndef GLUCAT_HAVE_STDLIB_H
00049 #define GLUCAT_HAVE_STDLIB_H 1
00050 #endif
00051
00052 /* Define to 1 if you have the <strings.h> header file. */
00053 #ifndef GLUCAT_HAVE_STRINGS_H
00054 #define GLUCAT_HAVE_STRINGS_H 1
00055 #endif
00056
00057 /* Define to 1 if you have the <string.h> header file. */
00058 #ifndef GLUCAT_HAVE_STRING_H
00059 #define GLUCAT_HAVE_STRING_H 1
00060 #endif
00061
00062 /* Define to 1 if you have the <sys/stat.h> header file. */
00063 #ifndef GLUCAT_HAVE_SYS_STAT_H
00064 #define GLUCAT_HAVE_SYS_STAT_H 1
00065 #endif
00066
00067 /* Define to 1 if you have the <sys/types.h> header file. */
00068 #ifndef GLUCAT_HAVE_SYS_TYPES_H
00069 #define GLUCAT_HAVE_SYS_TYPES_H 1
00070 #endif
00071
00072 /* Define to 1 if you have the <unistd.h> header file. */
00073 #ifndef GLUCAT_HAVE_UNISTD_H
00074 #define GLUCAT_HAVE_UNISTD_H 1
00075 #endif
00076
00077 /* Name of package */
00078 #ifndef GLUCAT_PACKAGE
00079 #define GLUCAT_PACKAGE "glucat"
00080 #endif
00081
00082 /* Define to the address where bug reports for this package should be sent. */

```

```

00083 #ifndef GLUCAT_PACKAGE_BUGREPORT
00084 #define GLUCAT_PACKAGE_BUGREPORT ""
00085 #endif
00086
00087 /* Define to the full name of this package. */
00088 #ifndef GLUCAT_PACKAGE_NAME
00089 #define GLUCAT_PACKAGE_NAME "glucat"
00090 #endif
00091
00092 /* Define to the full name and version of this package. */
00093 #ifndef GLUCAT_PACKAGE_STRING
00094 #define GLUCAT_PACKAGE_STRING "glucat 0.12.0"
00095 #endif
00096
00097 /* Define to the one symbol short name of this package. */
00098 #ifndef GLUCAT_PACKAGE_TARNAME
00099 #define GLUCAT_PACKAGE_TARNAME "glucat"
00100 #endif
00101
00102 /* Define to the home page for this package. */
00103 #ifndef GLUCAT_PACKAGE_URL
00104 #define GLUCAT_PACKAGE_URL ""
00105 #endif
00106
00107 /* Define to the version of this package. */
00108 #ifndef GLUCAT_PACKAGE_VERSION
00109 #define GLUCAT_PACKAGE_VERSION "0.12.0"
00110 #endif
00111
00112 /* Define to 1 if all of the C90 standard headers exist (not just the ones
00113    required in a freestanding environment). This macro is provided for
00114    backward compatibility; new code need not use it. */
00115 #ifndef GLUCAT_STDC_HEADERS
00116 #define GLUCAT_STDC_HEADERS 1
00117 #endif
00118
00119 /* Version number of package */
00120 #ifndef GLUCAT_VERSION
00121 #define GLUCAT_VERSION "0.12.0"
00122 #endif
00123
00124 /* once: _GLUCAT_GLUCAT_CONFIG_H */
00125 #endif

```

## 7.23 glucat/glucat\_imp.h File Reference

```

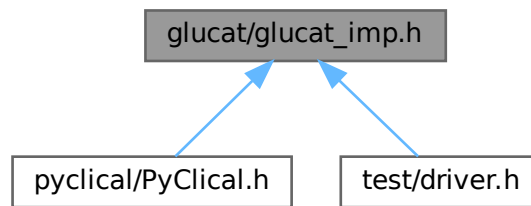
#include "glucat/errors_imp.h"
#include "glucat/index_set_imp.h"
#include "glucat/scalar_imp.h"
#include "glucat/clifford_algebra_imp.h"
#include "glucat/random.h"
#include "glucat/framed_multi_imp.h"
#include "glucat/matrix_imp.h"
#include "glucat/generation_imp.h"
#include "glucat/matrix_multi_imp.h"

```

Include dependency graph for glucat\_imp.h:



This graph shows which files directly or indirectly include this file:



## 7.24 glucat\_imp.h

[Go to the documentation of this file.](#)

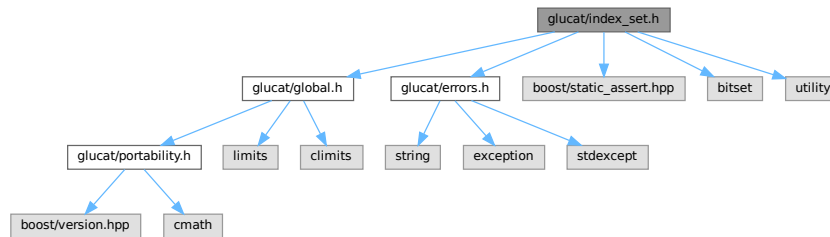
```

00001 #ifndef _GLUCAT_GLUCAT_IMP_H
00002 #define _GLUCAT_GLUCAT_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     glucat_imp.h : Organize GluCat template definitions which cannot be compiled separately
00006     -----
00007     begin                : Sun 2001-12-25
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     For Arvind Raja's original header comments, see glucat.h
00032     *****/
00033
00034 // Template definitions which cannot be compiled separately
00035
00036 #include "glucat/errors_imp.h"
00037
00038 #include "glucat/index_set_imp.h"
00039
00040 #include "glucat/scalar_imp.h"
00041
00042 #include "glucat/clifford_algebra_imp.h"
00043
00044 #include "glucat/random.h"
00045
00046 #include "glucat/framed_multi_imp.h"
00047
00048 #include "glucat/matrix_imp.h"
00049
00050 #include "glucat/generation_imp.h"
00051
00052 #include "glucat/matrix_multi_imp.h"
00053
00054 #endif // _GLUCAT_GLUCAT_IMP_H
  
```

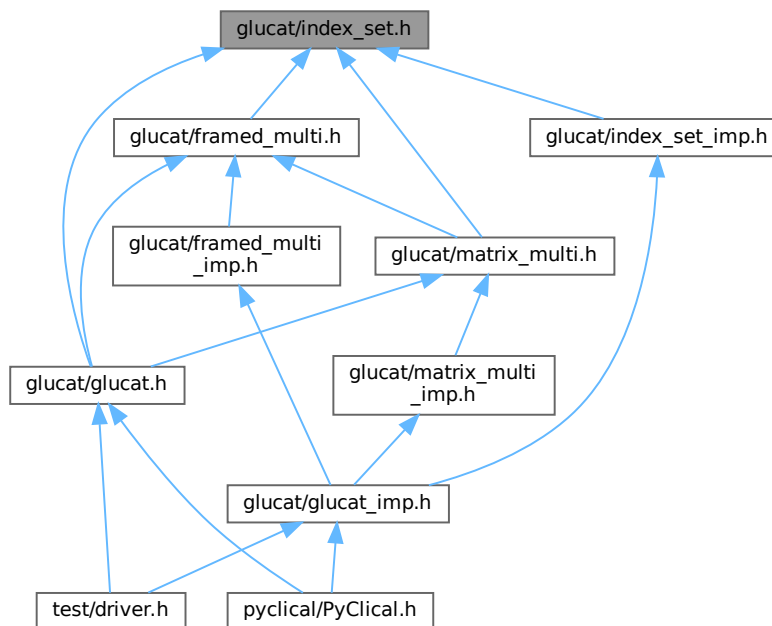
## 7.25 glucat/index\_set.h File Reference

```
#include "glucat/global.h"
#include "glucat/errors.h"
#include <boost/static_assert.hpp>
#include <bitset>
#include <utility>
```

Include dependency graph for index\_set.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class [glucat::index\\_set< LO, HI >](#)  
*Index set class based on std::bitset<> in Gnu standard C++ library.*
- class [glucat::index\\_set< LO, HI >::reference](#)  
*Index set member reference.*

## Namespaces

- namespace [glucat](#)

## Functions

- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator^ (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Symmetric set difference: exclusive or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator& (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Set intersection: and.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator| (const index\_set< LO, HI > &lhs, const index\_set< LO, HI > &rhs) -> const index\_set< LO, HI >`  
*Set union: or.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::compare (const index\_set< LO, HI > &a, const index\_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `glucat::GLUCAT\_CTAssert (sizeof(set\_value\_t) >=sizeof(std::bitset< DEFAULT\_HI-DEFAULT\_LO >),`  
`Default_index_set_too_big_for_value) template< const index\_t LO`  
*Size of [set\\_value\\_t](#) should be enough to contain [bitset](#)<[DEFAULT\\_HI](#)-[DEFAULT\\_LO](#)>*
- `const index\_t HI auto glucat::operator<< (std::ostream &os, const index\_set< LO, HI > &ist) -> std::ostream &`
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::operator>> (std::istream &s, index\_set< LO, HI > &ist) -> std::istream &`  
*Read in index set.*
- `auto glucat::sign\_of\_square (index\_t j) -> int`  
*Square of generator {j}.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::min\_neg (const index\_set< LO, HI > &ist) -> index\_t`  
*Minimum negative index, or 0 if none.*
- `template<const index\_t LO, const index\_t HI>`  
`auto glucat::max\_pos (const index\_set< LO, HI > &ist) -> index\_t`  
*Maximum positive index, or 0 if none.*

## 7.26 [index\\_set.h](#)

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_INDEX_SET_H
00002 #define _GLUCAT_INDEX_SET_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set.h : Declare a class for a set of non-zero integer indices
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright             : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
```

```

00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036
00037 #include <boost/static_assert.hpp>
00038
00039 #include <bitset>
00040 #include <utility>
00041
00042 namespace glucat
00043 {
00044     template<const index_t LO, const index_t HI>
00045     class index_set; // forward
00046
00047     template<const index_t LO, const index_t HI>
00048     auto
00049     operator^ (const index_set<LO,HI>& lhs,
00050               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00051
00052     template<const index_t LO, const index_t HI>
00053     auto
00054     operator& (const index_set<LO,HI>& lhs,
00055               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00056
00057     template<const index_t LO, const index_t HI>
00058     auto
00059     operator| (const index_set<LO,HI>& lhs,
00060               const index_set<LO,HI>& rhs) -> const index_set<LO,HI>;
00061
00062     // -1 if a<b, +1 if a>b, 0 if a==b
00063     template<const index_t LO, const index_t HI>
00064     auto
00065     compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int;
00066
00067     template<const index_t LO, const index_t HI>
00068     class index_set :
00069     private std::bitset<HI-LO>
00070     {
00071     private:
00072         BOOST_STATIC_ASSERT((LO <= 0) && (0 <= HI) && (LO < HI) && \
00073                             (~LO < _GLUCAT_BITS_PER_ULONG) && \
00074                             (HI < _GLUCAT_BITS_PER_ULONG) && \
00075                             (HI-LO <= _GLUCAT_BITS_PER_ULONG));
00076     public:
00077         using bitset_t = std::bitset<HI - LO>;
00078         using error_t = error<index_set>;
00079
00080         using index_set_t = index_set;
00081         using index_pair_t = std::pair<index_t, index_t>;
00082
00083         static const index_t v_lo = LO;
00084         static const index_t v_hi = HI;
00085
00086         static auto classname() -> const std::string;
00087         index_set() = default;
00088         index_set(const bitset_t bst);
00089         index_set(const index_t idx);
00090         index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked = false);
00091         index_set(const index_pair_t& range, const bool prechecked = false);
00092         index_set(const std::string& str);
00093
00094         auto operator== (const index_set_t rhs) const -> bool;
00095         auto operator!= (const index_set_t rhs) const -> bool;
00096         auto operator~ () const -> index_set_t;
00097         auto operator^= (const index_set_t rhs) -> index_set_t&;
00098         auto operator&= (const index_set_t rhs) -> index_set_t&;
00099         auto operator|= (const index_set_t rhs) -> index_set_t&;
00100         auto operator[] (const index_t idx) const -> bool;
00101         auto test(const index_t idx) const -> bool;
00102         auto set() -> index_set_t&;
00103         auto set(const index_t idx) -> index_set_t&;
00104         auto set(const index_t idx, const int val) -> index_set_t&;

```

```

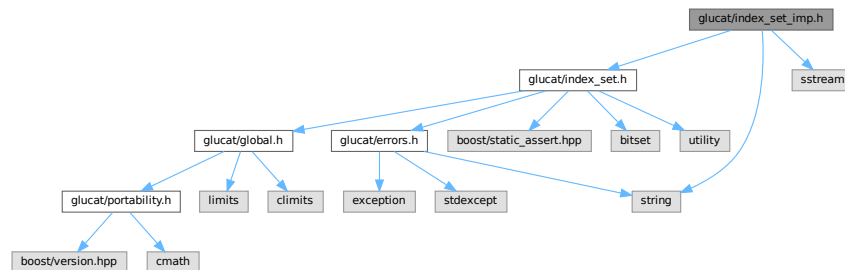
00127     auto reset() -> index_set_t&;
00129     auto reset(const index_t idx) -> index_set_t&;
00131     auto flip() -> index_set_t&;
00133     auto flip(const index_t idx) -> index_set_t&;
00135     auto count() const -> index_t;
00137     auto count_neg() const -> index_t;
00139     auto count_pos() const -> index_t;
00141     auto min() const -> index_t;
00143     auto max() const -> index_t;
00144
00145     // Functions which support Clifford algebra operations
00147     auto operator< (const index_set_t rhs) const -> bool;
00149     auto is_contiguous () const -> bool;
00151     auto fold () const -> const index_set_t;
00153     auto fold (const index_set_t frm, const bool prechecked = false) const -> const
index_set_t;
00155     auto unfold (const index_set_t frm, const bool prechecked = false) const -> const
index_set_t;
00157     auto value_of_fold (const index_set_t frm) const -> set_value_t;
00159     auto sign_of_mult (const index_set_t ist) const -> int;
00161     auto sign_of_square () const -> int;
00162
00164     auto hash_fn () const -> size_t;
00165
00166     // Friends
00167     friend auto operator^<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00168     friend auto operator&<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00169     friend auto operator|<> (const index_set_t& lhs, const index_set_t& rhs) -> const index_set_t;
00170     friend auto compare<> (const index_set_t& lhs, const index_set_t& rhs) -> int;
00171
00172     // Member reference:
00173     class reference;
00174     friend class reference;
00175
00177     class reference {
00178     friend class index_set;
00179
00180     public:
00182     reference() = delete;
00183     reference (index_set_t& ist, index_t idx);
00184     ~reference () = default;
00186     auto operator== (const reference& c_j) const -> bool;
00188     auto operator= (const bool x) -> reference&;
00190     auto operator= (const reference& c_j) -> reference&;
00192     auto operator~ () const -> bool;
00194     operator bool () const;
00196     auto flip() -> reference&;
00197
00198     private:
00199     index_set_t* m_pst;
00200     index_t m_idx;
00201     };
00203     auto operator[](index_t idx) -> reference;
00204 private:
00206     auto lex_less_than (const index_set_t rhs) const -> bool;
00207     };
00208
00210     _GLUCAT_CTAssert(sizeof(set_value_t) >= sizeof(std::bitset<DEFAULT_HI-DEFAULT_LO>),
00211         Default_index_set_too_big_for_value)
00212
00213     // non-members
00214
00216     template<const index_t LO, const index_t HI>
00217     auto
00218     operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&;
00219
00221     template<const index_t LO, const index_t HI>
00222     auto
00223     operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&;
00224
00225     // Functions which support Clifford algebra operations
00227     auto sign_of_square(index_t j) -> int;
00228
00230     template<const index_t LO, const index_t HI>
00231     auto
00232     min_neg(const index_set<LO,HI>& ist) -> index_t;
00233
00235     template<const index_t LO, const index_t HI>
00236     auto
00237     max_pos(const index_set<LO,HI>& ist) -> index_t;
00238 }
00239 #endif // _GLUCAT_INDEX_SET_H

```

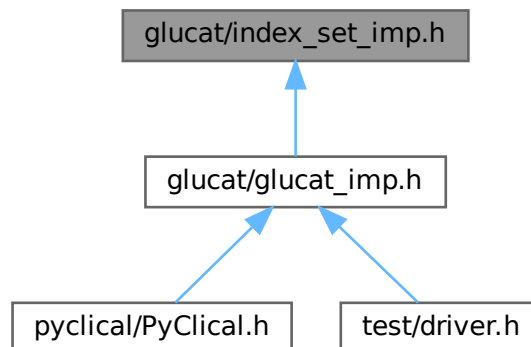


## 7.27 glucat/index\_set\_imp.h File Reference

```
#include "glucat/index_set.h"
#include <string>
#include <sstream>
Include dependency graph for index_set_imp.h:
```



This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Functions

- template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
 auto [glucat::operator^](#) (const [index\\_set](#)< LO, HI > &lhs, const [index\\_set](#)< LO, HI > &rhs) -> const [index\\_set](#)< LO, HI >  
*Symmetric set difference: exclusive or.*
- template<const [index\\_t](#) LO, const [index\\_t](#) HI>  
 auto [glucat::operator&](#) (const [index\\_set](#)< LO, HI > &lhs, const [index\\_set](#)< LO, HI > &rhs) -> const [index\\_set](#)< LO, HI >

*Set intersection: and.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator| (const index_set< LO, HI > &lhs, const index_set< LO, HI > &rhs) -> const index_set< LO, HI >`

*Set union: or.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::compare (const index_set< LO, HI > &a, const index_set< LO, HI > &b) -> int`  
*"lexicographic compare" eg. {3,4,5} is less than {3,7,8}*
- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator<< (std::ostream &os, const index_set< LO, HI > &ist) -> std::ostream &`

*Write out index set.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::operator>> (std::istream &s, index_set< LO, HI > &ist) -> std::istream &`

*Read in index set.*

- `static auto glucat::inverse_reversed_gray (unsigned long x) -> unsigned long`

*Inverse reversed Gray code.*

- `static auto glucat::inverse_gray (unsigned long x) -> unsigned long`

*Inverse Gray code.*

- `auto glucat::sign_of_square (index_t j) -> int`

*Square of generator {j}.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::min_neg (const index_set< LO, HI > &ist) -> index_t`

*Minimum negative index, or 0 if none.*

- `template<const index_t LO, const index_t HI>`  
`auto glucat::max_pos (const index_set< LO, HI > &ist) -> index_t`

*Maximum positive index, or 0 if none.*

## 7.28 index\_set\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_INDEX_SET_IMP_H
00002 #define _GLUCAT_INDEX_SET_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     index_set_imp.h : Implement a class for a set of non-zero integer indices
00006                      -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/index_set.h"
```

```

00035
00036 #include <string>
00037 #include <sstream>
00038
00039 namespace glucat
00040 {
00041     // References for algorithms:
00042     // [JA]: Joerg Arndt, "Algorithms for programmers", http://www.jjj.de/fxt/fxtbook.pdf
00043     //       Chapter 1, Bit wizardry, http://www.jjj.de/bitwizardry/bitwizardrypage.html
00044     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00045
00046     template<const index_t LO, const index_t HI>
00047     inline
00048     auto
00049     index_set<LO,HI>::
00050     classname() -> const std::string
00051     { return "index_set"; }
00052
00053     template<const index_t LO, const index_t HI>
00054     index_set<LO,HI>::
00055     index_set(const index_t idx)
00056     { this->set(idx); }
00057
00058     template<const index_t LO, const index_t HI>
00059     index_set<LO,HI>::
00060     index_set(const bitset_t bst):
00061     bitset_t(bst)
00062     { }
00063
00064     template<const index_t LO, const index_t HI>
00065     index_set<LO,HI>::
00066     index_set(const set_value_t folded_val, const index_set_t frm, const bool prechecked)
00067     {
00068         if (!prechecked && folded_val >= (set_value_t(1) << frm.count()))
00069             throw error_t("index_set(val,frm): cannot create: value gives an index set outside of frame");
00070         const index_set_t folded_frame = frm.fold();
00071         const index_t min_index = folded_frame.min();
00072         const index_t skip = min_index > 0 ? 1 : 0;
00073         const index_set_t folded_set = index_set_t(bitset_t(folded_val) << (min_index - skip - LO));
00074         *this = folded_set.unfold(frm);
00075     }
00076
00077     template<const index_t LO, const index_t HI>
00078     index_set<LO,HI>::
00079     index_set(const index_pair_t& range, const bool prechecked)
00080     {
00081         if (!prechecked && (range.first < LO || range.second > HI))
00082             throw error_t("index_set(range): cannot create: range is too large");
00083         const index_t begin_bit = (range.first < 0)
00084             ? range.first-LO
00085             : range.first-LO-1;
00086         const index_t end_bit = (range.second < 0)
00087             ? range.second-LO+1
00088             : range.second-LO;
00089         unsigned long mask = ( (end_bit == _GLUCAT_BITS_PER_ULONGLONG)
00090             ? -1UL
00091             : (1UL << end_bit)-1UL)
00092             & ~((1UL << begin_bit)-1UL);
00093         *this = bitset_t(mask);
00094     }
00095
00096     template<const index_t LO, const index_t HI>
00097     index_set<LO,HI>::
00098     index_set(const std::string& str)
00099     {
00100         std::istringstream ss(str);
00101         ss >> *this;
00102         if (!ss)
00103             throw error_t("index_set_t(str): could not parse string");
00104         // Peek to see if the end of the string has been reached.
00105         ss.peek();
00106         if (!ss.eof())
00107             throw error_t("index_set_t(str): could not parse entire string");
00108     }
00109
00110     template<const index_t LO, const index_t HI>
00111     inline
00112     auto
00113     index_set<LO,HI>::
00114     operator== (const index_set_t rhs) const -> bool
00115     {
00116         const auto* pthis = static_cast<const bitset_t*>(this);
00117         return *pthis == static_cast<bitset_t>(rhs);
00118     }
00119
00120     template<const index_t LO, const index_t HI>
00121     inline

```

```

00129     auto
00130     index_set<LO,HI>::
00131     operator!= (const index_set_t rhs) const -> bool
00132     {
00133         const auto* pthis = static_cast<const bitset_t*>(this);
00134         return *pthis != static_cast<bitset_t>(rhs);
00135     }
00136
00137     template<const index_t LO, const index_t HI>
00138     inline
00139     auto
00140     index_set<LO,HI>::
00141     operator~ () const -> index_set_t
00142     { return bitset_t::operator~(); }
00143
00144     template<const index_t LO, const index_t HI>
00145     inline
00146     auto
00147     index_set<LO,HI>::
00148     operator^= (const index_set_t rhs) -> index_set_t&
00149     {
00150         bitset_t* pthis = this;
00151         *pthis ^= static_cast<bitset_t>(rhs);
00152         return *this;
00153     }
00154
00155     template<const index_t LO, const index_t HI>
00156     inline
00157     auto
00158     index_set<LO,HI>::
00159     operator^ (const index_set<LO,HI>& lhs,
00160                const index_set<LO,HI>& rhs) -> const
00161     index_set<LO,HI>
00162     {
00163         using index_set_t = index_set<LO, HI>;
00164         using bitset_t = typename index_set_t::bitset_t;
00165         return static_cast<bitset_t>(lhs) ^ static_cast<bitset_t>(rhs);
00166     }
00167
00168     template<const index_t LO, const index_t HI>
00169     inline
00170     auto
00171     index_set<LO,HI>::
00172     operator&= (const index_set_t rhs) -> index_set_t&
00173     {
00174         bitset_t* pthis = this;
00175         *pthis &= static_cast<bitset_t>(rhs);
00176         return *this;
00177     }
00178
00179     template<const index_t LO, const index_t HI>
00180     inline
00181     auto
00182     index_set<LO,HI>::
00183     operator& (const index_set<LO,HI>& lhs,
00184                const index_set<LO,HI>& rhs) -> const
00185     index_set<LO,HI>
00186     {
00187         using index_set_t = index_set<LO, HI>;
00188         using bitset_t = typename index_set_t::bitset_t;
00189         return static_cast<bitset_t>(lhs) & static_cast<bitset_t>(rhs);
00190     }
00191
00192     template<const index_t LO, const index_t HI>
00193     inline
00194     auto
00195     index_set<LO,HI>::
00196     operator|= (const index_set_t rhs) -> index_set_t&
00197     {
00198         bitset_t* pthis = this;
00199         *pthis |= static_cast<bitset_t>(rhs);
00200         return *this;
00201     }
00202
00203     template<const index_t LO, const index_t HI>
00204     inline
00205     auto
00206     index_set<LO,HI>::
00207     operator| (const index_set<LO,HI>& lhs,
00208                const index_set<LO,HI>& rhs) -> const
00209     index_set<LO,HI>
00210     {
00211         using index_set_t = index_set<LO, HI>;
00212         using bitset_t = typename index_set_t::bitset_t;
00213         return static_cast<bitset_t>(lhs) | static_cast<bitset_t>(rhs);
00214     }
00215
00216     template<const index_t LO, const index_t HI>
00217     inline
00218     auto

```

```

00224 index_set<LO,HI>::
00225 operator[] (const index_t idx) -> reference
00226 { return reference(*this, idx); }
00227
00229 template<const index_t LO, const index_t HI>
00230 inline
00231 auto
00232 index_set<LO,HI>::
00233 operator[] (const index_t idx) const -> bool
00234 { return this->test(idx); }
00235
00237 template<const index_t LO, const index_t HI>
00238 inline
00239 auto
00240 index_set<LO,HI>::
00241 test(const index_t idx) const -> bool
00242 {
00243     // Reference: [JA], 1.2.1
00244     return (idx < 0)
00245         ? bool(bitset_t::to_ulong() & (1UL << (idx - LO)))
00246         : (idx > 0)
00247         ? bool(bitset_t::to_ulong() & (1UL << (idx - LO - 1)))
00248         : false;
00249 }
00250
00252 template<const index_t LO, const index_t HI>
00253 inline
00254 auto
00255 index_set<LO,HI>::
00256 set() -> index_set_t&
00257 {
00258     bitset_t::set();
00259     return *this;
00260 }
00261
00263 template<const index_t LO, const index_t HI>
00264 inline
00265 auto
00266 index_set<LO,HI>::
00267 set(index_t idx) -> index_set_t&
00268 {
00269     if (idx > 0)
00270         bitset_t::set(idx-LO-1);
00271     else if (idx < 0)
00272         bitset_t::set(idx-LO);
00273     return *this;
00274 }
00275
00277 template<const index_t LO, const index_t HI>
00278 inline
00279 auto
00280 index_set<LO,HI>::
00281 set(const index_t idx, const int val) -> index_set_t&
00282 {
00283     if (idx > 0)
00284         bitset_t::set(idx-LO-1, val);
00285     else if (idx < 0)
00286         bitset_t::set(idx-LO, val);
00287     return *this;
00288 }
00289
00291 template<const index_t LO, const index_t HI>
00292 inline
00293 auto
00294 index_set<LO,HI>::
00295 reset() -> index_set_t&
00296 {
00297     bitset_t::reset();
00298     return *this;
00299 }
00300
00302 template<const index_t LO, const index_t HI>
00303 inline
00304 auto
00305 index_set<LO,HI>::
00306 reset(const index_t idx) -> index_set_t&
00307 {
00308     if (idx > 0)
00309         bitset_t::reset(idx-LO-1);
00310     else if (idx < 0)
00311         bitset_t::reset(idx-LO);
00312     return *this;
00313 }
00314
00316 template<const index_t LO, const index_t HI>
00317 inline
00318 auto

```

```

00319 index_set<LO,HI>::
00320 flip() -> index_set<LO,HI>&
00321 {
00322     bitset_t::flip();
00323     return *this;
00324 }
00325
00326 template<const index_t LO, const index_t HI>
00327 inline
00328 auto
00329 index_set<LO,HI>::
00330 flip(const index_t idx) -> index_set_t&
00331 {
00332     {
00333         if (idx > 0)
00334             bitset_t::flip(idx-LO-1);
00335         else if (idx < 0)
00336             bitset_t::flip(idx-LO);
00337         return *this;
00338     }
00339
00340 template<const index_t LO, const index_t HI>
00341 inline
00342 auto
00343 index_set<LO,HI>::
00344 count() const -> index_t
00345 {
00346     unsigned long val = bitset_t::to_ulong();
00347     // Reference: [JA], 1.3
00348     if (val == 0)
00349         return 0;
00350     else
00351     {
00352         index_t result = 1;
00353         while (val &= val-1)
00354             ++result;
00355         return result;
00356     }
00357 }
00358
00359 template<const index_t LO, const index_t HI>
00360 inline
00361 auto
00362 index_set<LO,HI>::
00363 count_neg() const -> index_t
00364 {
00365     static const index_set_t lo_mask = bitset_t((1UL << -LO) - 1UL);
00366     const index_set_t neg_part = *this & lo_mask;
00367     return neg_part.count();
00368 }
00369
00370 template<const index_t LO, const index_t HI>
00371 inline
00372 auto
00373 index_set<LO,HI>::
00374 count_pos() const -> index_t
00375 {
00376     const auto* pthis = static_cast<const bitset_t*>(this);
00377     const index_set_t pos_part = *pthis >> -LO;
00378     return pos_part.count();
00379 }
00380
00381 #if (_GLUCAT_BITS_PER_ULONG == 64)
00382 template<const index_t LO, const index_t HI>
00383 inline
00384 auto
00385 index_set<LO,HI>::
00386 min() const -> index_t
00387 {
00388     // Reference: [JA], 1.3
00389     unsigned long val = bitset_t::to_ulong();
00390     if (val == 0)
00391         return 0;
00392     else
00393     {
00394         val -= val & (val-1); // isolate lowest bit
00395
00396         index_t idx = 0;
00397         const index_t nbits = HI - LO;
00398
00399         if (nbits > 8)
00400         {
00401             if (val & 0xffffffff00000000ul)
00402                 idx += 32;
00403             if (val & 0xffff0000ffff0000ul)
00404                 idx += 16;
00405             if (val & 0xff00ff00ff00ff00ul)
00406                 idx += 8;
00407         }
00408     }
00409 }

```

```

00411     }
00412     if (val & 0xf0f0f0f0f0f0f0ul)
00413         idx += 4;
00414     if (val & 0xccccccccccccccul)
00415         idx += 2;
00416     if (val & 0aaaaaaaaaaaaaaul)
00417         idx += 1;
00418
00419     return idx + ((idx < -LO) ? LO : LO+1);
00420 }
00421 }
00422 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00423 template<const index_t LO, const index_t HI>
00424 inline
00425 index_t
00426 index_set<LO,HI>::
00427 min() const
00428 {
00429     // Reference: [JA], 1.3
00430     unsigned long val = bitset_t::to_ulong();
00431     if (val == 0)
00432         return 0;
00433     else
00434     {
00435         val -= val & (val-1); // isolate lowest bit
00436
00437         index_t idx = 0;
00438         const index_t nbits = HI - LO;
00439         if (nbits > 8)
00440         {
00441             if (val & 0xffff0000ul)
00442                 idx += 16;
00443             if (val & 0xff00ff00ul)
00444                 idx += 8;
00445         }
00446         if (val & 0xf0f0f0f0ul)
00447             idx += 4;
00448         if (val & 0xccccccccul)
00449             idx += 2;
00450         if (val & 0aaaaaaaaul)
00451             idx += 1;
00452
00453         return idx + ((idx < -LO) ? LO : LO+1);
00454     }
00455 }
00456 }
00457 #else
00458 template<const index_t LO, const index_t HI>
00459 auto
00460 index_set<LO,HI>::
00461 min() const -> index_t
00462 {
00463     for (auto
00464         idx = LO;
00465         idx != 0;
00466         ++idx)
00467         if (this->test(idx))
00468             return idx;
00469     for (auto
00470         idx = index_t(1);
00471         idx <= HI;
00472         ++idx)
00473         if (this->test(idx))
00474             return idx;
00475     return 0;
00476 }
00477 }
00478 #endif
00479
00480 #if (_GLUCAT_BITS_PER_ULONG == 64)
00481 template<const index_t LO, const index_t HI>
00482 inline
00483 auto
00484 index_set<LO,HI>::
00485 max() const -> index_t
00486 {
00487     // Reference: [JA], 1.6
00488     auto val = bitset_t::to_ulong();
00489     if (val == 0)
00490         return 0;
00491     else
00492     {
00493         auto idx = index_t(0);
00494         const auto nbits = HI - LO;
00495         if (nbits > 8)
00496         {
00497             if (val & 0xffffffff00000000ul)
00498                 { val >>= 32; idx += 32; }
00499             if (val & 0x00000000ffff0000ul)

```

```

00501         { val >>= 16; idx += 16; }
00502         if (val & 0x000000000000ff00ul)
00503         { val >>= 8; idx += 8; }
00504     }
00505     if (val & 0x00000000000000f0ul)
00506     { val >>= 4; idx += 4; }
00507     if (val & 0x000000000000000c0ul)
00508     { val >>= 2; idx += 2; }
00509     if (val & 0x0000000000000002ul)
00510     { idx += 1; }
00511     return idx + ((idx < -LO) ? LO : LO+1);
00512 }
00513 }
00514 #elif (_GLUCAT_BITS_PER_ULONG == 32)
00515 template<const index_t LO, const index_t HI>
00516 inline
00517 auto
00518 index_set<LO,HI>::
00519 max() const -> index_t
00520 {
00521     // Reference: [JA], 1.6
00522     auto val = bitset_t::to_ulong();
00523     if (val == 0)
00524         return 0;
00525     else
00526     {
00527         auto idx = index_t(0);
00528         const auto nbits = HI - LO;
00529         if (nbits > 8)
00530         {
00531             if (val & 0xffff0000ul)
00532             { val >>= 16; idx += 16; }
00533             if (val & 0x0000ff00ul)
00534             { val >>= 8; idx += 8; }
00535         }
00536         if (val & 0x000000f0ul)
00537         { val >>= 4; idx += 4; }
00538         if (val & 0x0000000c0ul)
00539         { val >>= 2; idx += 2; }
00540         if (val & 0x00000002ul)
00541         { idx += 1; }
00542         return idx + ((idx < -LO) ? LO : LO+1);
00543     }
00544 }
00545 }
00546 #else
00547 template<const index_t LO, const index_t HI>
00548 auto
00549 index_set<LO,HI>::
00550 max() const -> index_t
00551 {
00552     for (auto
00553         idx = HI;
00554         idx != 0;
00555         --idx)
00556         if (this->test(idx))
00557             return idx;
00558     for (auto
00559         idx = index_t(-1);
00560         idx >= LO;
00561         --idx)
00562         if (this->test(idx))
00563             return idx;
00564     return 0;
00565 }
00566 #endif
00567 // eg. {3,4,5} is less than {3,7,8}
00568 template<const index_t LO, const index_t HI>
00569 inline
00570 auto
00571 compare(const index_set<LO,HI>& a, const index_set<LO,HI>& b) -> int
00572 {
00573     return (a == b)
00574         ? 0
00575         : a.lex_less_than(b)
00576         ? -1
00577         : 1;
00578 }
00579 // eg. {3,4,5} is less than {3,7,8}
00580 template<const index_t LO, const index_t HI>
00581 inline
00582 auto
00583 index_set<LO,HI>::
00584 lex_less_than(const index_set_t rhs) const -> bool
00585 { return bitset_t::to_ulong() < rhs.bitset_t::to_ulong(); }
00586
00587
00588
00589
00590
00591

```



```

00593 // Order by count, then order lexicographically within the equivalence class of count.
00594 template<const index_t LO, const index_t HI>
00595 inline
00596 auto
00597 index_set<LO,HI>::
00598 operator< (const index_set_t rhs) const -> bool
00599 {
00600     const auto this_grade = this->count();
00601     const auto rhs_grade = rhs.count();
00602     return (this_grade < rhs_grade)
00603         ? true
00604         : (this_grade > rhs_grade)
00605         ? false
00606         : this->lex_less_than(rhs);
00607 }
00608
00609 template<const index_t LO, const index_t HI>
00610 auto
00611 operator<< (std::ostream& os, const index_set<LO,HI>& ist) -> std::ostream&
00612 {
00613     {
00614         index_t i;
00615         os << '{';
00616         for (i = LO;
00617              (i <= HI) && !(ist[i]);
00618              ++i)
00619             { }
00620         if (i <= HI)
00621             os << i;
00622         for (++i;
00623              i <= HI;
00624              ++i)
00625             if (ist[i])
00626                 os << ',' << i;
00627         os << '}';
00628         return os;
00629     }
00630
00631 template<const index_t LO, const index_t HI>
00632 auto
00633 operator>> (std::istream& s, index_set<LO,HI>& ist) -> std::istream&
00634 {
00635     // Parsing variables.
00636     auto i = index_t(0);
00637     using index_set_t = index_set<LO,HI>;
00638     auto local_ist = index_set_t();
00639     // Parsing control variables.
00640     auto parse_index_list = true;
00641     auto expect_closing_brace = false;
00642     auto expect_index = false;
00643     // Parse an optional opening brace.
00644     auto c = s.peek();
00645     // If there is a failure or end of file, this ends parsing.
00646     if (!s.good())
00647         parse_index_list = false;
00648     else
00649     { // Check for an opening brace.
00650         expect_closing_brace = (c == int('{'));
00651         if (expect_closing_brace)
00652         { // Consume the opening brace.
00653             s.get();
00654             // The next character may be a closing brace,
00655             // indicating the empty index set.
00656             c = s.peek();
00657             if (s.good() && (c == int('}')))
00658             { // A closing brace has been parsed and is no longer expected.
00659                 expect_closing_brace = false;
00660                 // Consume the closing brace.
00661                 s.get();
00662                 // This ends parsing.
00663                 parse_index_list = false;
00664             }
00665         }
00666     }
00667     if (s.good() && parse_index_list)
00668     { // Parse an optional index list.
00669         // The index list starts with a first index.
00670         for (s >> i;
00671              !s.fail();
00672              s >> i)
00673         { // An index has been parsed. Check to see if it is in range.
00674             if ((i < LO) || (i > HI))
00675             { // An index out of range is a failure.
00676                 s.clear(std::istream::failbit);
00677                 break;
00678             }
00679             // Add the index to the index set local_ist.
00680             local_ist.set(i);
00681         }
00682     }

```

```

00682         // Immediately after parsing an index, an index is no longer expected.
00683         expect_index = false;
00684         // Reading the index may have resulted in an end of file condition.
00685         // If so, this ends the index list.
00686         if (s.eof())
00687             break;
00688         // The index list continues with a comma, and
00689         // may be ended by a closing brace, if it was begun with an opening brace.
00690         // Parse a possible comma or closing brace.
00691         c = s.peek();
00692         if (!s.good())
00693             break;
00694         // First, test for a closing brace, if expected.
00695         if (expect_closing_brace && (c == int('}')))
00696         { // Consume the closing brace.
00697             s.get();
00698             // Immediately after parsing the closing brace, it is no longer expected.
00699             expect_closing_brace = false;
00700             // A closing brace ends the index list.
00701             break;
00702         }
00703         // Now test for a comma.
00704         if (c == int(','))
00705         { // Consume the comma.
00706             s.get();
00707             // A index is expected after the comma.
00708             expect_index = true;
00709         }
00710         else
00711         { // Any other character here is a failure.
00712             s.clear(std::istream::failbit);
00713             break;
00714         }
00715     }
00716 }
00717 // If an index or a closing brace is still expected, this is a failure.
00718 if (expect_index || expect_closing_brace)
00719     s.clear(std::istream::failbit);
00720 // End of file is not a failure.
00721 if (s)
00722 { // The index set has been successfully parsed.
00723     ist = local_ist;
00724 }
00725 return s;
00726 }
00727
00728 template<const index_t LO, const index_t HI>
00729 inline
00730 auto
00731 index_set<LO,HI>::
00732 is_contiguous () const -> bool
00733 {
00734     {
00735         const auto min_index = this->min();
00736         const auto max_index = this->max();
00737         return (min_index < 0 && max_index > 0)
00738             ? max_index - min_index == this->count()
00739             : (min_index == 1 || max_index == -1) &&
00740               (max_index - min_index == this->count() - 1);
00741     }
00742 }
00743
00744 template<const index_t LO, const index_t HI>
00745 inline
00746 auto
00747 index_set<LO,HI>::
00748 fold() const -> const
00749 index_set<LO,HI>
00750 { return this->fold(*this, true); }
00751
00752 template<const index_t LO, const index_t HI>
00753 auto
00754 index_set<LO,HI>::
00755 fold(const index_set_t frm, const bool prechecked) const -> const
00756 index_set<LO,HI>
00757 {
00758     {
00759         if (!prechecked && ((*this | frm) != frm))
00760             throw error_t("fold(frm): cannot fold from outside of frame");
00761         const auto frm_min = frm.min();
00762         const auto frm_max = frm.max();
00763         auto result = index_set_t();
00764         auto fold_idx = index_t(-1);
00765         for (auto
00766             unfold_idx = fold_idx;
00767             unfold_idx >= frm_min;
00768             --unfold_idx)
00769             if (frm.test(unfold_idx))
00770                 // result.set(fold_idx--, this->test(unfold_idx));
00771         {

```

```

00772         if (this->test(unfold_idx))
00773             result.set(fold_idx);
00774         --fold_idx;
00775     }
00776     fold_idx = index_t(1);
00777     for (auto
00778         unfold_idx = fold_idx;
00779         unfold_idx <= frm_max;
00780         ++unfold_idx)
00781         if (frm.test(unfold_idx))
00782             // result.set(fold_idx++, this->test(unfold_idx));
00783         {
00784             if (this->test(unfold_idx))
00785                 result.set(fold_idx);
00786             ++fold_idx;
00787         }
00788     return result;
00789 }
00790
00791 template<const index_t LO, const index_t HI>
00792 auto
00793 index_set<LO,HI>::
00794 unfold(const index_set_t frm, const bool prechecked) const -> const index_set_t
00795 {
00796     {
00797         const char* msg =
00798             "unfold(frm): cannot unfold into a smaller frame";
00799         const auto frm_min = frm.min();
00800         const auto frm_max = frm.max();
00801         auto result = index_set_t();
00802         auto fold_idx = index_t(-1);
00803         for (auto
00804             unfold_idx = fold_idx;
00805             unfold_idx >= frm_min;
00806             --unfold_idx)
00807             if (frm.test(unfold_idx))
00808                 if (this->test(fold_idx--))
00809                     result.set(unfold_idx);
00810         if (!prechecked && ((fold_idx+1) > this->min()))
00811             throw error_t(msg);
00812         fold_idx = index_t(1);
00813         for (auto
00814             unfold_idx = fold_idx;
00815             unfold_idx <= frm_max;
00816             ++unfold_idx)
00817             if (frm.test(unfold_idx))
00818                 if (this->test(fold_idx++))
00819                     result.set(unfold_idx);
00820         if (!prechecked && ((fold_idx-1) < this->max()))
00821             throw error_t(msg);
00822         return result;
00823     }
00824
00825 template<const index_t LO, const index_t HI>
00826 inline
00827 auto
00828 index_set<LO,HI>::
00829 value_of_fold(const index_set_t frm) const -> set_value_t
00830 {
00831     {
00832         const auto min_index = frm.fold().min();
00833         if (min_index == 0)
00834             return 0;
00835         else
00836         {
00837             const auto folded_set = this->fold(frm);
00838             const auto skip = min_index > 0 ? index_t(1) : index_t(0);
00839             return folded_set.bitset_t::to_ulong() > (min_index-LO-skip);
00840         }
00841     }
00842
00843 inline
00844 static
00845 auto inverse_reversed_gray(unsigned long x) -> unsigned long
00846 {
00847     {
00848         // Reference: [JA]
00849 #if (_GLUCAT_BITS_PER_ULONG >= 64)
00850         x ^= x << 32; // for 64-bit words
00851 #endif
00852         x ^= x << 16; // reversed_gray ** 16
00853         x ^= x << 8;  // reversed_gray ** 8
00854         x ^= x << 4;  // reversed_gray ** 4
00855         x ^= x << 2;  // reversed_gray ** 2
00856         x ^= x << 1;  // reversed_gray ** 1
00857         return x;
00858     }
00859
00860 inline
00861 static

```

```

00863 auto inverse_gray(unsigned long x) -> unsigned long
00864 {
00865     // Reference: [JA]
00866     #if (_GLUCAT_BITS_PER_ULONG >= 64)
00867         x ^= x » 32; // for 64-bit words
00868     #endif
00869     x ^= x » 16; // gray ** 16
00870     x ^= x » 8; // gray ** 8
00871     x ^= x » 4; // gray ** 4
00872     x ^= x » 2; // gray ** 2
00873     x ^= x » 1; // gray ** 1
00874     return x;
00875 }
00876
00877 template<const index_t LO, const index_t HI>
00878 auto
00880 index_set<LO,HI>::
00881 sign_of_mult(const index_set_t rhs) const -> int
00882 {
00883     // Implemented using Walsh functions and Gray codes.
00884     // Reference: [L] Chapter 21, 21.3
00885     // Reference: [JA]
00886     const auto uthis = this->bitset_t::to_ulong();
00887     const auto urhs = rhs.bitset_t::to_ulong();
00888     const auto nbits = HI - LO;
00889     auto negative = 0UL;
00890     if (nbits > 8)
00891     {
00892         // Set h to be the inverse reversed Gray code of rhs.
00893         // This sets each bit of h to be the cumulative ^ of
00894         // the same and lower bits of rhs.
00895         const auto h = inverse_reversed_gray(urhs);
00896         // Set k to be the inverse Gray code of *this & h.
00897         // This sets the low bit of k to be parity(*this & h).
00898         const auto k = inverse_gray(uthis & h);
00899         // Set q to be the inverse Gray code of the positive part of *this & rhs.
00900         const auto q = inverse_gray((uthis & urhs) » -LO);
00901         negative = k ^ q;
00902     }
00903     else
00904     {
00905         auto h = 0UL;
00906         for (auto
00907             j = index_t(0);
00908             j < -LO;
00909             ++j)
00910         {
00911             h ^= urhs » j;
00912             negative ^= h & (uthis » j);
00913         }
00914         for (auto
00915             j = index_t(-LO);
00916             j < nbits;
00917             ++j)
00918         {
00919             negative ^= h & (uthis » j);
00920             h ^= urhs » j;
00921         }
00922     }
00923     return 1 - int((negative & 1) « 1);
00924 }
00925
00927 template<const index_t LO, const index_t HI>
00928 inline
00929 auto
00930 index_set<LO,HI>::
00931 sign_of_square() const -> int
00932 {
00933     auto result = 1 - int((this->count_neg() % 2) « 1);
00934     switch (this->count() % 4)
00935     {
00936     case 2:
00937     case 3:
00938         result *= -1;
00939         break;
00940     default:
00941         break;
00942     }
00943     return result;
00944 }
00945
00947 template<const index_t LO, const index_t HI>
00948 inline
00949 auto
00950 index_set<LO,HI>::
00951 hash_fn() const -> size_t
00952 {

```

```

00953     static const auto lo_mask = (1UL « -LO) - 1UL;
00954     const auto uthis = bitset_t::to_ulong();
00955     const auto neg_part = uthis & lo_mask;
00956     const auto pos_part = uthis » -LO;
00957     return size_t(neg_part ^ pos_part);
00958 }
00959
00961     inline
00962     auto
00963     sign_of_square(index_t j) -> int
00964     { return (j < 0) ? -1 : 1; }
00965
00967     template<const index_t LO, const index_t HI>
00968     inline
00969     auto
00970     min_neg(const index_set<LO,HI>& ist) -> index_t
00971     { return std::min(ist.min(), 0); }
00972
00974     template<const index_t LO, const index_t HI>
00975     inline
00976     auto
00977     max_pos(const index_set<LO,HI>& ist) -> index_t
00978     { return std::max(ist.max(), 0); }
00979
00980 // index_set reference
00981
00983     template<const index_t LO, const index_t HI>
00984     inline
00985     index_set<LO,HI>::reference::
00986     reference( index_set_t& ist, index_t idx ) :
00987         m_pst(&ist),
00988         m_idx(idx)
00989     { }
00990
00992     template<const index_t LO, const index_t HI>
00993     inline
00994     auto
00995     index_set<LO,HI>::reference::
00996     operator== (const reference& c_j) const -> bool
00997     { return m_pst == c_j.m_pst && m_idx == c_j.m_idx; }
00998
01000     template<const index_t LO, const index_t HI>
01001     inline
01002     auto
01003     index_set<LO,HI>::reference::
01004     operator= (bool x) -> reference&
01005     {
01006         if ( x )
01007             m_pst->set(m_idx);
01008         else
01009             m_pst->reset(m_idx);
01010         return *this;
01011     }
01012
01014     template<const index_t LO, const index_t HI>
01015     inline
01016     auto
01017     index_set<LO,HI>::reference::
01018     operator= (const reference& c_j) -> reference&
01019     {
01020         if (&c_j != this && c_j != *this)
01021         {
01022             if ( (c_j.m_pst)[c_j.m_idx] )
01023                 m_pst->set(m_idx);
01024             else
01025                 m_pst->reset(m_idx);
01026         }
01027         return *this;
01028     }
01029
01031     template<const index_t LO, const index_t HI>
01032     inline
01033     auto
01034     index_set<LO,HI>::reference::
01035     operator~ () const -> bool
01036     { return !(m_pst->test(m_idx)); }
01037
01039     template<const index_t LO, const index_t HI>
01040     inline
01041     index_set<LO,HI>::reference::
01042     operator bool () const
01043     { return m_pst->test(m_idx); }
01044
01046     template<const index_t LO, const index_t HI>
01047     inline
01048     auto
01049     index_set<LO,HI>::reference::

```

```

01050     flip() -> reference&
01051     {
01052         m_pst->flip(m_idx);
01053         return *this;
01054     }
01055 }
01056 #endif // _GLUCAT_INDEX_SET_IMP_H

```

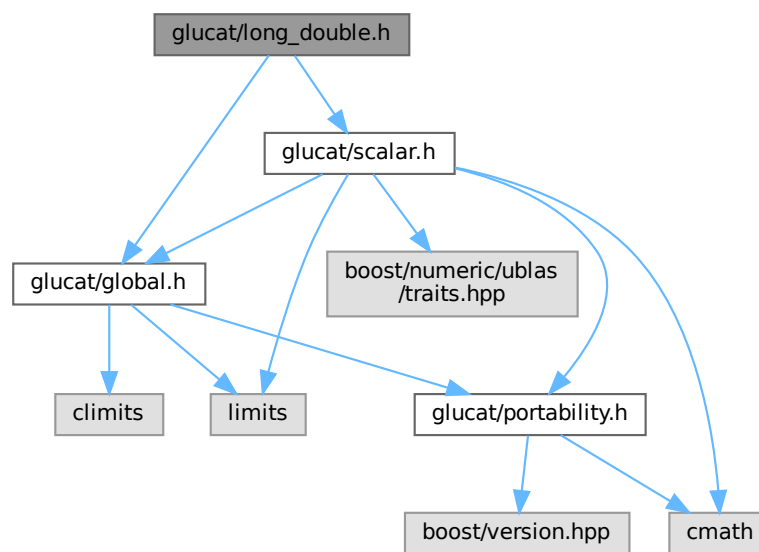
## 7.29 glucat/long\_double.h File Reference

```

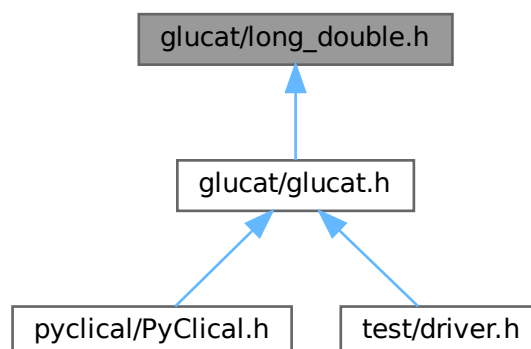
#include "glucat/global.h"
#include "glucat/scalar.h"

```

Include dependency graph for long\_double.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## Variables

- static const long double `glucat::l_pi` = 3.1415926535897932384626433832795029L
- static const long double `glucat::l_ln2` = 0.6931471805599453094172321214581766L

## 7.30 long\_double.h

Go to the documentation of this file.

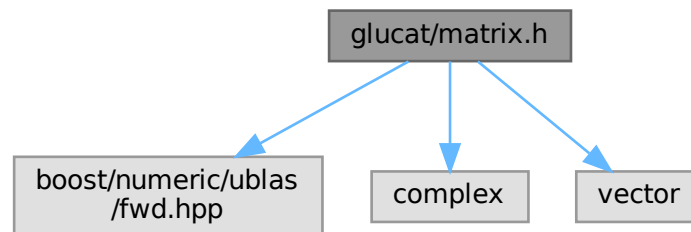
```

00001 #ifndef _GLUCAT_LONG_DOUBLE_H
00002 #define _GLUCAT_LONG_DOUBLE_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     long_double.h : Define std functions for long double
00006     -----
00007     begin                : 2001-12-18
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010     This library is free software: you can redistribute it and/or modify
00011     it under the terms of the GNU Lesser General Public License as published
00012     by the Free Software Foundation, either version 3 of the License, or
00013     (at your option) any later version.
00014
00015     This library is distributed in the hope that it will be useful,
00016     but WITHOUT ANY WARRANTY; without even the implied warranty of
00017     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00018     GNU Lesser General Public License for more details.
00019
00020     You should have received a copy of the GNU Lesser General Public License
00021     along with this library. If not, see <http://www.gnu.org/licenses/>.
00022
00023     *****/
00024     This library is based on a prototype written by Arvind Raja and was
00025     licensed under the LGPL with permission of the author. See Arvind Raja,
00026     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00027     in Ablamowicz, Lounesto and Parra (eds.)
00028     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00029     *****/
00030     See also Arvind Raja's original header comments and references in glucat.h
00031     *****/
00032
00033 #include "glucat/global.h"
00034 #include "glucat/scalar.h"
00035
00036 namespace glucat
00037 {
00038     #if defined(__USE_GNU)
00039         static const long double l_pi = M_PI;
00040         static const long double l_ln2 = M_LN2;
00041     #else
00042         static const long double l_pi = 3.1415926535897932384626433832795029L;
00043         static const long double l_ln2 = 0.6931471805599453094172321214581766L;
00044     #endif
00045
00046     template<>
00047     inline
00048     auto
00049     numeric_traits<long double>::
00050     pi() -> long double
00051     { return l_pi; }
00052
00053     template<>
00054     inline
00055     auto
00056     numeric_traits<long double>::
00057     ln_2() -> long double
00058     { return l_ln2; }
00059 }
00060 #endif // _GLUCAT_LONG_DOUBLE_H

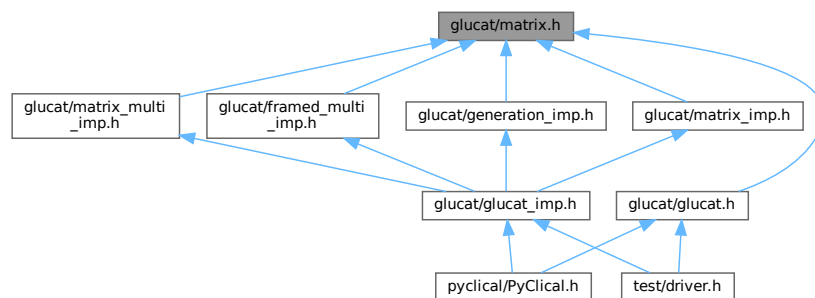
```

## 7.31 glucat/matrix.h File Reference

```
#include <boost/numeric/ublas/fwd.hpp>
#include <complex>
#include <vector>
Include dependency graph for matrix.h:
```



This graph shows which files directly or indirectly include this file:



### Classes

- struct [glucat::matrix::eig\\_genus< Matrix\\_T >](#)  
*Structure containing classification of eigenvalues.*

### Namespaces

- namespace [glucat](#)
- namespace [glucat::matrix](#)

### Typedefs

- using [glucat::matrix::eig\\_case\\_t](#)  
*Classification of eigenvalues of a matrix.*



## Functions

- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Kronecker tensor product of matrices - as per Matlab kron.*
- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::mono\\_kron](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Sparse Kronecker tensor product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::nork](#) (const LHS\_T &lhs, const RHS\_T &rhs, const bool mono=true) -> const RHS\_T  
*Left inverse of Kronecker product.*
- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::signed\\_perm\\_nork](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> const RHS\_T  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::nnz](#) (const Matrix\_T &m) -> typename Matrix\_T::size\_type  
*Number of non-zeros.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::isinf](#) (const Matrix\_T &m) -> bool  
*Infinite.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::isnan](#) (const Matrix\_T &m) -> bool  
*Not a Number.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::unit](#) (const typename Matrix\_T::size\_type n) -> const Matrix\_T  
*Unit matrix - as per Matlab eye.*
- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::mono\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
 expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of monomial matrices.*
- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::sparse\\_prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_↵  
 expression< RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of sparse matrices.*
- template<typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::prod](#) (const ublas::matrix\_expression< LHS\_T > &lhs, const ublas::matrix\_expression<  
 RHS\_T > &rhs) -> const typename RHS\_T::expression\_type  
*Product of matrices.*
- template<typename Scalar\_T , typename LHS\_T , typename RHS\_T >  
 auto [glucat::matrix::inner](#) (const LHS\_T &lhs, const RHS\_T &rhs) -> Scalar\_T  
*Inner product:  $\text{sum}(x(i,j)*y(i,j))/x.\text{nrows}()$*
- template<typename Matrix\_T >  
 auto [glucat::matrix::norm\\_frob2](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Square of Frobenius norm.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::trace](#) (const Matrix\_T &val) -> typename Matrix\_T::value\_type  
*Matrix trace.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::eigenvalues](#) (const Matrix\_T &val) -> std::vector< std::complex< double > >  
*Eigenvalues of a matrix.*
- template<typename Matrix\_T >  
 auto [glucat::matrix::classify\\_eigenvalues](#) (const Matrix\_T &val) -> [eig\\_genus](#)< Matrix\_T >  
*Classify the eigenvalues of a matrix.*

## 7.32 matrix.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_H
00002 #define _GLUCAT_MATRIX_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix.h : Declare common matrix functions
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009                        : uBLAS interface contributed by Joerg Walter
00010     *****/
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025     *****/
00026     This library is based on a prototype written by Arvind Raja and was
00027     licensed under the LGPL with permission of the author. See Arvind Raja,
00028     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029     in Ablamowicz, Lounesto and Parra (eds.)
00030     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031     *****/
00032     See also Arvind Raja's original header comments in glucat.h
00033     *****/
00034
00035 #include <boost/numeric/ublas/fwd.hpp>
00036
00037 #include <complex>
00038 #include <vector>
00039
00040 namespace glucat
00041 {
00042     namespace ublas = boost::numeric::ublas;
00043
00044     namespace matrix
00045     {
00046         template< typename LHS_T, typename RHS_T >
00047         auto
00048         kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00049         RHS_T;
00050
00051         template< typename LHS_T, typename RHS_T >
00052         auto
00053         mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00054         RHS_T;
00055
00056         template< typename LHS_T, typename RHS_T >
00057         auto
00058         nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono = true) -> const
00059         RHS_T;
00060
00061         template< typename LHS_T, typename RHS_T >
00062         auto
00063         signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00064         RHS_T;
00065
00066         template< typename Matrix_T >
00067         auto
00068         nnz(const Matrix_T& m) -> typename Matrix_T::size_type;
00069
00070         template< typename Matrix_T >
00071         auto
00072         isinf(const Matrix_T& m) -> bool;
00073
00074         template< typename Matrix_T >
00075         auto
00076         isnan(const Matrix_T& m) -> bool;
00077
00078         template< typename Matrix_T >
00079         auto
00080         unit(const typename Matrix_T::size_type n) -> const
00081         Matrix_T;
00082
00083     }
00084
00085 }

```

```

00092     template< typename LHS_T, typename RHS_T >
00093     auto
00094     mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00095              const ublas::matrix_expression<RHS_T>& rhs) -> const
00096     typename RHS_T::expression_type;
00097
00099     template< typename LHS_T, typename RHS_T >
00100     auto
00101     sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00102               const ublas::matrix_expression<RHS_T>& rhs) -> const
00103     typename RHS_T::expression_type;
00104
00106     template< typename LHS_T, typename RHS_T >
00107     auto
00108     prod(const ublas::matrix_expression<LHS_T>& lhs,
00109          const ublas::matrix_expression<RHS_T>& rhs) -> const
00110     typename RHS_T::expression_type;
00111
00113     template< typename Scalar_T, typename LHS_T, typename RHS_T >
00114     auto
00115     inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T;
00116
00118     template< typename Matrix_T >
00119     auto
00120     norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type;
00121
00123     template< typename Matrix_T >
00124     auto
00125     trace(const Matrix_T& val) -> typename Matrix_T::value_type;
00126
00128     template< typename Matrix_T >
00129     auto
00130     eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >;
00131
00133     using eig_case_t = enum {
00134         safe_eigs,
00135         neg_real_eigs,
00136         both_eigs};
00137
00139     template< typename Matrix_T >
00140     struct eig_genus
00141     {
00142         using Scalar_T = typename Matrix_T::value_type;
00144         bool m_is_singular = false;
00146         eig_case_t m_eig_case = safe_eigs;
00148         Scalar_T m_safe_arg = Scalar_T(0);
00149     };
00150
00152     template< typename Matrix_T >
00153     auto
00154     classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>;
00155 }
00156 }
00157
00158 #endif // _GLUCAT_MATRIX_H

```

## 7.33 glucat/matrix\_imp.h File Reference

```

#include "glucat/errors.h"
#include "glucat/scalar.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/vector.hpp>
#include <boost/numeric/ublas/vector_proxy.hpp>
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/bindings/lapack/driver/gees.hpp>
#include <boost/numeric/bindings/ublas.hpp>
#include <set>

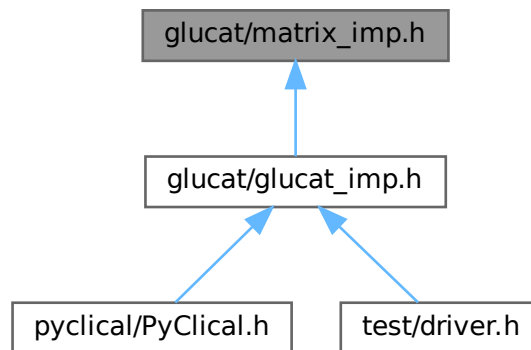
```

```
#include <vector>
```

Include dependency graph for `matrix_imp.h`:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`
- namespace `glucat::matrix`

## Functions

- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Kronecker tensor product of matrices - as per Matlab kron.*
- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::mono_kron (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Sparse Kronecker tensor product of monomial matrices.*
- `template<typename LHS_T, typename RHS_T >`  
`void glucat::matrix::nork_range (RHS_T &result, const typename LHS_T::const_iterator2 lhs_it2, const RHS_T &rhs, const typename RHS_T::size_type res_s1, const typename RHS_T::size_type res_s2)`  
*Utility routine for nork: calculate result for a range of indices.*
- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::nork (const LHS_T &lhs, const RHS_T &rhs, const bool mono=true) -> const RHS_T`  
*Left inverse of Kronecker product.*
- `template<typename LHS_T, typename RHS_T >`  
`auto glucat::matrix::signed_perm_nork (const LHS_T &lhs, const RHS_T &rhs) -> const RHS_T`  
*Left inverse of Kronecker product where lhs is a signed permutation matrix.*

- `template<typename Matrix_T >`  
`auto glucat::matrix::nnz (const Matrix_T &m) -> typename Matrix_T::size_type`  
*Number of non-zeros.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::isinf (const Matrix_T &m) -> bool`  
*Infinite.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::isnan (const Matrix_T &m) -> bool`  
*Not a Number.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::unit (const typename Matrix_T::size_type n) -> const Matrix_T`  
*Unit matrix - as per Matlab eye.*
- `template<typename LHS_T , typename RHS_T >`  
`auto glucat::matrix::mono_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_↵`  
`expression< RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of monomial matrices.*
- `template<typename LHS_T , typename RHS_T >`  
`auto glucat::matrix::sparse_prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_↵`  
`expression< RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of sparse matrices.*
- `template<typename LHS_T , typename RHS_T >`  
`auto glucat::matrix::prod (const ublas::matrix_expression< LHS_T > &lhs, const ublas::matrix_expression<`  
`RHS_T > &rhs) -> const typename RHS_T::expression_type`  
*Product of matrices.*
- `template<typename Scalar_T , typename LHS_T , typename RHS_T >`  
`auto glucat::matrix::inner (const LHS_T &lhs, const RHS_T &rhs) -> Scalar_T`  
*Inner product:  $\sum(x(i,j)*y(i,j))/x.nrows()$*
- `template<typename Matrix_T >`  
`auto glucat::matrix::norm_frob2 (const Matrix_T &val) -> typename Matrix_T::value_type`  
*Square of Frobenius norm.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::trace (const Matrix_T &val) -> typename Matrix_T::value_type`  
*Matrix trace.*
- `template<typename Matrix_T >`  
`static auto glucat::matrix::to_lapack (const Matrix_T &val) -> ublas::matrix< double, ublas::column_major >`  
*Convert matrix to LAPACK format.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::eigenvalues (const Matrix_T &val) -> std::vector< std::complex< double > >`  
*Eigenvalues of a matrix.*
- `template<typename Matrix_T >`  
`auto glucat::matrix::classify_eigenvalues (const Matrix_T &val) -> eig_genus< Matrix_T >`  
*Classify the eigenvalues of a matrix.*

## 7.34 matrix\_imp.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_MATRIX_IMP_H
00002 #define _GLUCAT_MATRIX_IMP_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   matrix_imp.h : Implement common matrix functions
00006   -----
00007   begin                : Sun 2001-12-09
00008   copyright            : (C) 2001-2012 by Paul C. Leopardi
00009                       : uBLAS interface contributed by Joerg Walter
```

```

00010 *****
00011
00012     This library is free software: you can redistribute it and/or modify
00013     it under the terms of the GNU Lesser General Public License as published
00014     by the Free Software Foundation, either version 3 of the License, or
00015     (at your option) any later version.
00016
00017     This library is distributed in the hope that it will be useful,
00018     but WITHOUT ANY WARRANTY; without even the implied warranty of
00019     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020     GNU Lesser General Public License for more details.
00021
00022     You should have received a copy of the GNU Lesser General Public License
00023     along with this library. If not, see <http://www.gnu.org/licenses/>.
00024
00025 *****
00026 This library is based on a prototype written by Arvind Raja and was
00027 licensed under the LGPL with permission of the author. See Arvind Raja,
00028 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00029 in Ablamowicz, Lounesto and Parra (eds.)
00030 "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00031 *****
00032     See also Arvind Raja's original header comments in glucat.h
00033 *****/
00034
00035 #include "glucat/errors.h"
00036 #include "glucat/scalar.h"
00037 #include "glucat/matrix.h"
00038
00039 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00040 #   pragma GCC diagnostic push
00041 #   pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00042 # endif
00043 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00044 #   include <boost/serialization/array_wrapper.hpp>
00045 # endif
00046 #include <boost/numeric/ublas/vector.hpp>
00047 #include <boost/numeric/ublas/vector_proxy.hpp>
00048 #include <boost/numeric/ublas/matrix.hpp>
00049 #include <boost/numeric/ublas/matrix_expression.hpp>
00050 #include <boost/numeric/ublas/matrix_proxy.hpp>
00051 #include <boost/numeric/ublas/matrix_sparse.hpp>
00052 #include <boost/numeric/ublas/operation.hpp>
00053 #include <boost/numeric/ublas/operation_sparse.hpp>
00054
00055 #if defined(_GLUCAT_USE_BINDINGS)
00056 #   include <boost/numeric/bindings/lapack/driver/gees.hpp>
00057 #   include <boost/numeric/bindings/ublas.hpp>
00058 #endif
00059
00060 #if defined(_GLUCAT_USE_BLAZE)
00061 #include <blaze/Math.h>
00062 #include <blaze/math/DynamicMatrix.h>
00063 #include <blaze/math/DynamicVector.h>
00064 #endif
00065
00066 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00067 #   pragma GCC diagnostic pop
00068 # endif
00069
00070 #include <set>
00071 #include <vector>
00072
00073 namespace glucat { namespace matrix
00074 {
00075     // References for algorithms:
00076     // [v]: C. F. van Loan and N. Pitsianis, "Approximation with Kronecker products",
00077     // in Linear Algebra for Large Scale and Real-Time Applications, Marc S. Moonen,
00078     // Gene H. Golub, and Bart L. R. Moor (eds.), 1993, pp. 293--314.
00079
00080     template< typename LHS_T, typename RHS_T >
00081     auto
00082     kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00083     RHS_T
00084     {
00085         {
00086             const auto rhs_s1 = rhs.size1();
00087             const auto rhs_s2 = rhs.size2();
00088             auto result = RHS_T(lhs.size1()*rhs_s1, lhs.size2()*rhs_s2);
00089             result.clear();
00090
00091             for (auto
00092                 lhs_it1 = lhs.begin1();
00093                 lhs_it1 != lhs.end1();
00094                 ++lhs_it1)
00095                 for (auto
00096                     lhs_it2 = lhs_it1.begin();
00097                     lhs_it2 != lhs_it1.end();

```

```

00098         ++lhs_it2)
00099     {
00100         const auto start1 = rhs_s1 * lhs_it2.index1();
00101         const auto start2 = rhs_s2 * lhs_it2.index2();
00102         const auto& lhs_val = *lhs_it2;
00103         for (auto
00104             rhs_it1 = rhs.begin1();
00105             rhs_it1 != rhs.end1();
00106             ++rhs_it1)
00107             for (auto
00108                 rhs_it2 = rhs_it1.begin();
00109                 rhs_it2 != rhs_it1.end();
00110                 ++rhs_it2)
00111                 result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00112     }
00113     return result;
00114 }
00115
00117 template< typename LHS_T, typename RHS_T >
00118 auto
00119 mono_kron(const LHS_T& lhs, const RHS_T& rhs) -> const
00120 RHS_T
00121 {
00122     const auto rhs_s1 = rhs.size1();
00123     const auto rhs_s2 = rhs.size2();
00124     const auto dim = lhs.size1()*rhs_s1;
00125     auto result = RHS_T(dim, dim, dim);
00126     result.clear();
00127
00128     for (auto
00129         lhs_it1 = lhs.begin1();
00130         lhs_it1 != lhs.end1();
00131         ++lhs_it1)
00132     {
00133         const auto lhs_it2 = lhs_it1.begin();
00134         const auto start1 = rhs_s1 * lhs_it2.index1();
00135         const auto start2 = rhs_s2 * lhs_it2.index2();
00136         const auto& lhs_val = *lhs_it2;
00137         for (auto
00138             rhs_it1 = rhs.begin1();
00139             rhs_it1 != rhs.end1();
00140             ++rhs_it1)
00141         {
00142             const auto rhs_it2 = rhs_it1.begin();
00143             result(start1 + rhs_it2.index1(), start2 + rhs_it2.index2()) = lhs_val * *rhs_it2;
00144         }
00145     }
00146     return result;
00147 }
00148
00150 template< typename LHS_T, typename RHS_T >
00151 void
00152 nork_range(RHS_T& result,
00153            const typename LHS_T::const_iterator2 lhs_it2,
00154            const RHS_T& rhs,
00155            const typename RHS_T::size_type res_s1,
00156            const typename RHS_T::size_type res_s2)
00157 {
00158     // Definition matches [v] Section 4, Theorem 4.1.
00159     const auto start1 = res_s1 * lhs_it2.index1();
00160     const auto start2 = res_s2 * lhs_it2.index2();
00161     using ublas::range;
00162     const auto& rang1 = range(start1, start1 + res_s1);
00163     const auto& rang2 = range(start2, start2 + res_s2);
00164     using matrix_range_t = ublas::matrix_range<const RHS_T>;
00165     const auto& rhs_range = matrix_range_t(rhs, rang1, rang2);
00166     using Scalar_T = typename RHS_T::value_type;
00167     const auto lhs_val = numeric_traits<Scalar_T>::to_scalar_t(*lhs_it2);
00168     for (auto
00169         rhs_it1 = rhs_range.begin1();
00170         rhs_it1 != rhs_range.end1();
00171         ++rhs_it1)
00172         for (auto
00173             rhs_it2 = rhs_it1.begin();
00174             rhs_it2 != rhs_it1.end();
00175             ++rhs_it2)
00176             result(rhs_it2.index1(), rhs_it2.index2()) += lhs_val * *rhs_it2;
00177 }
00178
00180 template< typename LHS_T, typename RHS_T >
00181 auto
00182 nork(const LHS_T& lhs, const RHS_T& rhs, const bool mono) -> const
00183 RHS_T
00184 {
00185     // nork(A, kron(A, B)) is close to B
00186     // Definition matches [v] Section 4, Theorem 4.1.
00187     const auto lhs_s1 = lhs.size1();

```

```

00188     const auto lhs_s2 = lhs.size2();
00189     const auto rhs_s1 = rhs.size1();
00190     const auto rhs_s2 = rhs.size2();
00191     const auto res_s1 = rhs_s1 / lhs_s1;
00192     const auto res_s2 = rhs_s2 / lhs_s2;
00193     using Scalar_T = typename RHS_T::value_type;
00194     const auto norm_frob2_lhs = norm_frob2(lhs);
00195     if (!mono)
00196     {
00197         using error_t = error<RHS_T>;
00198         if (rhs_s1 == 0)
00199             throw error_t("matrix", "nork: number of rows must not be 0");
00200         if (rhs_s2 == 0)
00201             throw error_t("matrix", "nork: number of cols must not be 0");
00202         if (res_s1 * lhs_s1 != rhs_s1)
00203             throw error_t("matrix", "nork: incompatible numbers of rows");
00204         if (res_s2 * lhs_s2 != rhs_s2)
00205             throw error_t("matrix", "nork: incompatible numbers of cols");
00206         if (norm_frob2_lhs == Scalar_T(0))
00207             throw error_t("matrix", "nork: LHS must not be 0");
00208     }
00209     auto result = RHS_T(res_s1, res_s2);
00210     result.clear();
00211     for (auto
00212         lhs_it1 = lhs.begin1();
00213         lhs_it1 != lhs.end1();
00214         ++lhs_it1)
00215     for (auto
00216         lhs_it2 = lhs_it1.begin();
00217         lhs_it2 != lhs_it1.end();
00218         ++lhs_it2)
00219         if (*lhs_it2 != Scalar_T(0))
00220             nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00221     result /= norm_frob2_lhs;
00222     return result;
00223 }
00224
00226 template< typename LHS_T, typename RHS_T >
00227 auto
00228 signed_perm_nork(const LHS_T& lhs, const RHS_T& rhs) -> const
00229 RHS_T
00230 {
00231     // signed_perm_nork(A, kron(A, B)) is close to B
00232     // Definition matches [v] Section 4, Theorem 4.1.
00233     const auto lhs_s1 = lhs.size1();
00234     const auto lhs_s2 = lhs.size2();
00235     const auto rhs_s1 = rhs.size1();
00236     const auto rhs_s2 = rhs.size2();
00237     const auto res_s1 = rhs_s1 / lhs_s1;
00238     const auto res_s2 = rhs_s2 / lhs_s2;
00239     using Scalar_T = typename RHS_T::value_type;
00240     const auto norm_frob2_lhs = Scalar_T( double(lhs_s1) );
00241     auto result = RHS_T(res_s1, res_s2);
00242     result.clear();
00243     for (auto
00244         lhs_it1 = lhs.begin1();
00245         lhs_it1 != lhs.end1();
00246         ++lhs_it1)
00247     {
00248         const auto lhs_it2 = lhs_it1.begin();
00249         nork_range<LHS_T, RHS_T>(result, lhs_it2, rhs, res_s1, res_s2);
00250     }
00251     result /= norm_frob2_lhs;
00252     return result;
00253 }
00254
00256 template< typename Matrix_T >
00257 auto
00258 nnz(const Matrix_T& m) -> typename Matrix_T::size_type
00259 {
00260     using size_t = typename Matrix_T::size_type;
00261     auto result = size_t(0);
00262     for (auto
00263         it1 = m.begin1();
00264         it1 != m.end1();
00265         ++it1)
00266     for (auto& entry : it1)
00267         if (entry != 0)
00268             ++result;
00269     return result;
00270 }
00271
00273 template< typename Matrix_T >
00274 auto
00275 isinf(const Matrix_T& m) -> bool
00276 {
00277     using Scalar_T = typename Matrix_T::value_type;

```



```

00278     for (auto
00279         it1 = m.begin1();
00280         it1 != m.end1();
00281         ++it1)
00282         for (auto& entry : it1)
00283             if (numeric_traits<Scalar_T>::isInf(entry))
00284                 return true;
00285     return false;
00286 }
00287
00288
00290 template< typename Matrix_T >
00291 auto
00292 isnan(const Matrix_T& m) -> bool
00293 {
00294     using Scalar_T = typename Matrix_T::value_type;
00295     for (auto
00296         it1 = m.begin1();
00297         it1 != m.end1();
00298         ++it1)
00299         for (auto& entry : it1)
00300             if (numeric_traits<Scalar_T>::isNaN(entry))
00301                 return true;
00302     return false;
00303 }
00304
00305
00307 template< typename Matrix_T >
00308 inline
00309 auto
00310 unit(const typename Matrix_T::size_type dim) -> const
00311 Matrix_T
00312 {
00313     using Scalar_T = typename Matrix_T::value_type;
00314     return ublas::identity_matrix<Scalar_T>(dim);
00315 }
00316
00318 template< typename LHS_T, typename RHS_T >
00319 auto
00320 mono_prod(const ublas::matrix_expression<LHS_T>& lhs,
00321           const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00322 {
00323     using rhs_expression_t = const RHS_T;
00324     using matrix_row_t = typename ublas::matrix_row<rhs_expression_t>;
00325
00326     const auto dim = lhs().size1();
00327     // The following assumes that RHS_T is a sparse matrix type.
00328     auto result = RHS_T(dim, dim, dim);
00329     for (auto
00330         lhs_row = lhs().begin1();
00331         lhs_row != lhs().end1();
00332         ++lhs_row)
00333     {
00334         const auto& lhs_it = lhs_row.begin();
00335         if (lhs_it != lhs_row.end())
00336         {
00337             const auto& rhs_row = matrix_row_t(rhs(), lhs_it.index2());
00338             const auto& rhs_it = rhs_row.begin();
00339             if (rhs_it != rhs_row.end())
00340                 result(lhs_it.index1(), rhs_it.index()) = (*lhs_it) * (*rhs_it);
00341         }
00342     }
00343     return result;
00344 }
00345
00347 template< typename LHS_T, typename RHS_T >
00348 inline
00349 auto
00350 sparse_prod(const ublas::matrix_expression<LHS_T>& lhs,
00351            const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00352 {
00353     using expression_t = typename RHS_T::expression_type;
00354     return ublas::sparse_prod<expression_t>(lhs(), rhs());
00355 }
00356
00358 template< typename LHS_T, typename RHS_T >
00359 inline
00360 auto
00361 prod(const ublas::matrix_expression<LHS_T>& lhs,
00362      const ublas::matrix_expression<RHS_T>& rhs) -> const typename RHS_T::expression_type
00363 {
00364     const auto dim = lhs().size1();
00365     RHS_T result(dim, dim);
00366     ublas::axpy_prod(lhs, rhs, result, true);
00367     return result;
00368 }
00369

```

```

00371 template< typename Scalar_T, typename LHS_T, typename RHS_T >
00372 auto
00373 inner(const LHS_T& lhs, const RHS_T& rhs) -> Scalar_T
00374 {
00375     auto result = Scalar_T(0);
00376     for (auto
00377         lhs_it1 = lhs.begin1();
00378         lhs_it1 != lhs.end1();
00379         ++lhs_it1)
00380     for (auto
00381         lhs_it2 = lhs_it1.begin();
00382         lhs_it2 != lhs_it1.end();
00383         ++lhs_it2)
00384     {
00385         const auto& rhs_val = rhs(lhs_it2.index1(), lhs_it2.index2());
00386         if (rhs_val != Scalar_T(0))
00387             result += (*lhs_it2) * rhs_val;
00388     }
00389     return result / lhs.size1();
00390 }
00391
00393 template< typename Matrix_T >
00394 auto
00395 norm_frob2(const Matrix_T& val) -> typename Matrix_T::value_type
00396 {
00397     using Scalar_T = typename Matrix_T::value_type;
00398
00399     auto result = Scalar_T(0);
00400     for (auto
00401         val_it1 = val.begin1();
00402         val_it1 != val.end1();
00403         ++val_it1)
00404     for (auto& val_entry : val_it1)
00405     {
00406         if (numeric_traits<Scalar_T>::isNaN(val_entry))
00407             return numeric_traits<Scalar_T>::NaN();
00408         result += val_entry * val_entry;
00409     }
00410     return result;
00411 }
00412
00414 template< typename Matrix_T >
00415 auto
00416 trace(const Matrix_T& val) -> typename Matrix_T::value_type
00417 {
00418     using Scalar_T = typename Matrix_T::value_type;
00419
00420     auto result = Scalar_T(0);
00421     auto dim = val.size1();
00422     for (auto
00423         ndx = decltype(dim)(0);
00424         ndx != dim;
00425         ++ndx)
00426     {
00427         const Scalar_T crd = val(ndx, ndx);
00428         if (numeric_traits<Scalar_T>::isNaN(crd))
00429             return numeric_traits<Scalar_T>::NaN();
00430         result += crd;
00431     }
00432     return result;
00433 }
00434
00435 #if defined(_GLUCAT_USE_BINDINGS)
00437 template< typename Matrix_T >
00438 static
00439 auto
00440 to_lapack(const Matrix_T& val) -> ublas::matrix<double, ublas::column_major>
00441 {
00442     const auto s1 = val.size1();
00443     const auto s2 = val.size2();
00444
00445     using lapack_matrix_t = typename ublas::matrix<double, ublas::column_major>;
00446     auto result = lapack_matrix_t(s1, s2);
00447     result.clear();
00448
00449     using Scalar_T = typename Matrix_T::value_type;
00450     using traits_t = numeric_traits<Scalar_T>;
00451
00452     for (auto
00453         val_it1 = val.begin1();
00454         val_it1 != val.end1();
00455         ++val_it1)
00456     for (auto
00457         val_it2 = val_it1.begin();
00458         val_it2 != val_it1.end();
00459         ++val_it2)
00460         result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);

```

```

00461
00462     return result;
00463 }
00464 #endif
00465
00466 #if defined(_GLUCAT_USE_BLAZE)
00467     template< typename Matrix_T >
00468     static
00469     auto
00470     to_blaze(const Matrix_T& val) -> blaze::DynamicMatrix<double, blaze::rowMajor>
00471     {
00472     {
00473         const auto s1 = val.size1();
00474         const auto s2 = val.size2();
00475
00476         using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00477         auto result = blaze_matrix_t(s1, s2);
00478
00479         using Scalar_T = typename Matrix_T::value_type;
00480         using traits_t = numeric_traits<Scalar_T>;
00481
00482         for (auto
00483             val_it1 = val.begin1();
00484             val_it1 != val.end1();
00485             ++val_it1)
00486             for (auto
00487                 val_it2 = val_it1.begin();
00488                 val_it2 != val_it1.end();
00489                 ++val_it2)
00490                 result(val_it2.index1(), val_it2.index2()) = traits_t::to_double(*val_it2);
00491
00492         return result;
00493     }
00494 }
00495 #endif
00496
00497     template< typename Matrix_T >
00498     auto
00499     eigenvalues(const Matrix_T& val) -> std::vector< std::complex<double> >
00500     {
00501     {
00502         using complex_t = std::complex<double>;
00503         using complex_vector_t = typename std::vector<complex_t>;
00504
00505         const auto dim = val.size1();
00506         auto lambda = complex_vector_t(dim);
00507
00508         #if defined(_GLUCAT_USE_BINDINGS)
00509         namespace lapack = boost::numeric::bindings::lapack;
00510         using lapack_matrix_t = typename ublas::matrix<double, ublas::column_major>;
00511
00512         auto T = to_lapack(val);
00513         auto V = T;
00514         using vector_t = typename ublas::vector<double>;
00515         auto real_lambda = vector_t(dim);
00516         auto imag_lambda = vector_t(dim);
00517         fortran_int_t sdim = 0;
00518
00519         lapack::gees('N', 'N', nullptr, T, sdim, real_lambda, imag_lambda, V);
00520
00521         for (auto
00522             k = decltype(dim)(0);
00523             k != dim;
00524             ++k)
00525             lambda[k] = complex_t(real_lambda[k], imag_lambda[k]);
00526         #endif
00527         #if defined(_GLUCAT_USE_BLAZE)
00528         using blaze_matrix_t = typename blaze::DynamicMatrix<double, blaze::rowMajor>;
00529         using complex_t = std::complex<double>;
00530         using blaze_complex_vector_t = blaze::DynamicVector<complex_t, blaze::columnVector>;
00531
00532         auto blaze_val = to_blaze(val);
00533         auto blaze_lambda = blaze_complex_vector_t(dim);
00534         blaze::geev(blaze_val, blaze_lambda);
00535
00536         for (auto
00537             k = decltype(dim)(0);
00538             k != dim;
00539             ++k)
00540             lambda[k] = blaze_lambda[k];
00541         #endif
00542         return lambda;
00543     }
00544 }
00545
00546     template< typename Matrix_T >
00547     auto
00548     classify_eigenvalues(const Matrix_T& val) -> eig_genus<Matrix_T>
00549     {
00550     {
00551         using Scalar_T = typename Matrix_T::value_type;

```

```

00551     eig_genus<Matrix_T> result;
00552
00553     using complex_t = std::complex<double>;
00554     using complex_vector_t = typename std::vector<complex_t>;
00555     auto lambda = eigenvalues(val);
00556
00557     std::set<double> arg_set;
00558
00559     using vector_index_t = typename complex_vector_t::size_type;
00560     const auto dim = lambda.size();
00561     static const auto epsilon =
00562         std::max(std::numeric_limits<double>::epsilon(),
00563             numeric_traits<Scalar_T>::to_double(std::numeric_limits<Scalar_T>::epsilon()));
00564     static const auto zero_eig_tol = 4096.0*epsilon;
00565
00566     bool neg_real_eig_found = false;
00567     bool imag_eig_found = false;
00568     bool zero_eig_found = false;
00569
00570     for (auto
00571         k = decltype(dim)(0);
00572         k != dim;
00573         ++k)
00574     {
00575         const auto lambda_k = lambda[k];
00576         arg_set.insert(std::arg(lambda_k));
00577
00578         const auto real_lambda_k = std::real(lambda_k);
00579         const auto imag_lambda_k = std::imag(lambda_k);
00580         const auto norm_tol = 4096.0*epsilon*std::norm(lambda_k);
00581
00582         if (!neg_real_eig_found &&
00583             real_lambda_k < -epsilon &&
00584             (imag_lambda_k == 0.0 ||
00585              imag_lambda_k * imag_lambda_k < norm_tol))
00586             neg_real_eig_found = true;
00587         if (!imag_eig_found &&
00588             imag_lambda_k > epsilon &&
00589             (real_lambda_k == 0.0 ||
00590              real_lambda_k * real_lambda_k < norm_tol))
00591             imag_eig_found = true;
00592         if (!zero_eig_found &&
00593             std::norm(lambda_k) < zero_eig_tol)
00594             zero_eig_found = true;
00595     }
00596
00597     if (zero_eig_found)
00598         result.m_is_singular = true;
00599
00600     static const auto pi = numeric_traits<double>::pi();
00601     if (neg_real_eig_found)
00602     {
00603         if (imag_eig_found)
00604             result.m_eig_case = both_eigs;
00605         else
00606         {
00607             result.m_eig_case = neg_real_eigs;
00608             result.m_safe_arg = Scalar_T(-pi / 2.0);
00609         }
00610     }
00611
00612     if (result.m_eig_case == both_eigs)
00613     {
00614         auto arg_it = arg_set.begin();
00615         auto first_arg = *arg_it;
00616         auto best_arg = first_arg;
00617         auto best_diff = 0.0;
00618         auto previous_arg = first_arg;
00619         for (++arg_it;
00620             arg_it != arg_set.end();
00621             ++arg_it)
00622         {
00623             const auto arg_diff = *arg_it - previous_arg;
00624             if (arg_diff > best_diff)
00625             {
00626                 best_diff = arg_diff;
00627                 best_arg = previous_arg;
00628             }
00629             previous_arg = *arg_it;
00630         }
00631         const auto arg_diff = first_arg + 2.0*pi - previous_arg;
00632         if (arg_diff > best_diff)
00633         {
00634             best_diff = arg_diff;
00635             best_arg = previous_arg;
00636         }
00637         result.m_safe_arg = Scalar_T(pi - (best_arg + best_diff / 2.0));

```

```

00638     }
00639     return result;
00640 }
00641 } }
00642
00643 #endif // _GLUCAT_MATRIX_IMP_H

```

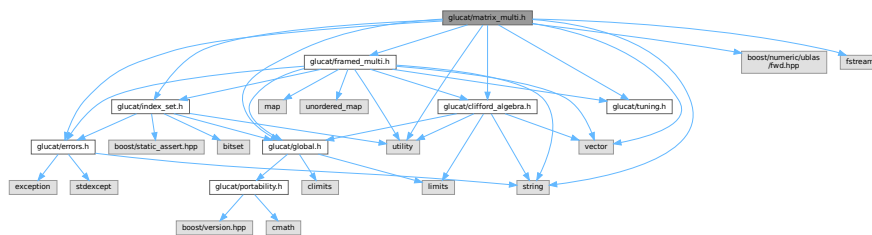
## 7.35 glucat/matrix\_multi.h File Reference

```

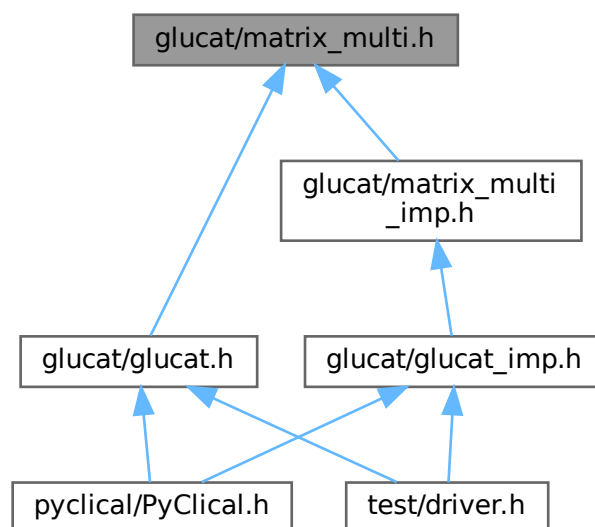
#include "glucat/global.h"
#include "glucat/errors.h"
#include "glucat/index_set.h"
#include "glucat/clifford_algebra.h"
#include "glucat/tuning.h"
#include "glucat/framed_multi.h"
#include <boost/numeric/ublas/fwd.hpp>
#include <fstream>
#include <string>
#include <utility>
#include <vector>

```

Include dependency graph for matrix\_multi.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class `glucat::matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*A `matrix_multi<Scalar_T,LO,HI,Tune_P>` is a matrix approximation to a multivector.*
- struct `std::numeric_limits< glucat::matrix_multi< Scalar_T, LO, HI, Tune_P > >`  
*Numeric limits for `matrix_multi` inherit limits for the corresponding scalar type.*

## Namespaces

- namespace `glucat`
- namespace `std`

## Functions

- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator*` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric product.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator^` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Outer product.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator&` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Inner product.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator%` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Left contraction.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::star` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> `Scalar_T`  
*Hestenes scalar product.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator/` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Geometric quotient.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator|` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs) -> const `matrix_multi< Scalar_T, LO, HI, Tune_P >`  
*Transformation via twisted adjoint action.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator>>` (std::istream &s, `matrix_multi< Scalar_T, LO, HI, Tune_P >` &val) -> std::istream &  
*Read multivector from input.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator<<` (std::ostream &os, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &val) -> std::ostream &  
*Write multivector to output.*
- template<typename Scalar\_T , const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::reframe` (const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs, const `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs, `matrix_multi< Scalar_T, LO, HI, Tune_P >` &lhs\_reframed, `matrix_multi< Scalar_T, LO, HI, Tune_P >` &rhs\_reframed) -> const `index_set< LO, HI >`

*Find a common frame for operands of a binary operator.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::matrix_sqrt (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Square root of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_T, LO,`  
`HI, Tune_P > &i, bool prechecked) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::matrix_log (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val, const matrix_multi< Scalar_↵`  
`_T, LO, HI, Tune_P > &i, const index_t level) -> const matrix_multi< Scalar_T, LO, HI, Tune_P >`

*Natural logarithm of multivector with specified complexifier.*

- `template<typename Scalar_T , const index_t LO, const index_t HI, typename Tune_P >`  
`auto glucat::exp (const matrix_multi< Scalar_T, LO, HI, Tune_P > &val) -> const matrix_multi< Scalar_T,`  
`LO, HI, Tune_P >`

*Exponential of multivector.*

## 7.36 matrix\_multi.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_MATRIX_MULTI_H
00002 #define _GLUCAT_MATRIX_MULTI_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_multi.h : Declare a class for the matrix representation of a multivector
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/errors.h"
00036 #include "glucat/index_set.h"
00037 #include "glucat/clifford_algebra.h"
00038 #include "glucat/tuning.h"
00039 #include "glucat/framed_multi.h"
00040
00041 #include <boost/numeric/ublas/fwd.hpp>
00042
00043 #include <fstream>
00044 #include <string>
```

```

00045 #include <utility>
00046 #include <vector>
00047
00048 namespace glucat
00049 {
00050     namespace ublas = boost::numeric::ublas;
00051
00052     // Forward declarations for friends
00053
00054     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00055     class framed_multi; // forward
00056
00057     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00058     class matrix_multi; // forward
00059
00060     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00061     auto
00062     operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00063
00064     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00065     auto
00066     operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00067
00068     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00069     auto
00070     operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00071
00072     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00073     auto
00074     operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00075
00076     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00077     auto
00078     operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00079
00080     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00081     auto
00082     star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T;
00083
00084     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00085     auto
00086     operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00087
00088     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00089     auto
00090     operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00091
00092     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00093     auto
00094     operator» (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&;
00095
00096     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00097     auto
00098     operator« (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&;
00099
00100     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00101     auto
00102     reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>;
00103
00104     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00105     auto
00106     sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00107
00108     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00109     auto
00110     matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00111
00112     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00113     auto
00114     log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00115
00116     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00117     auto
00118     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00119
00120     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00121     auto
00122     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00123
00124     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00125     auto
00126     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00127
00128     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00129     auto
00130     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00131
00132     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00133     auto
00134     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00135

```



```

00136     template< typename Scalar_T = double, const index_t LO = DEFAULT_LO, const index_t HI = DEFAULT_HI,
typename Tune_P = tuning<> >
00137     class matrix_multi :
00138     public clifford_algebra< Scalar_T, index_set<LO,HI>, matrix_multi<Scalar_T,LO,HI,Tune_P> >
00139     {
00140     public:
00141         using multivector_t = matrix_multi;
00142         using matrix_multi_t = multivector_t;
00143         using scalar_t = Scalar_T;
00144         using tune_p = Tune_P;
00145         using index_set_t = index_set<LO, HI>;
00146         using term_t = std::pair<const index_set_t, Scalar_T>;
00147         using vector_t = std::vector<Scalar_T>;
00148         using error_t = error<multivector_t>;
00149         using framed_multi_t = framed_multi<Scalar_T,LO,HI,Tune_P>;
00150         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00151         friend class framed_multi;
00152         template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00153         friend class matrix_multi;
00154
00155     private:
00156         using orientation_t = ublas::row_major;
00157         using basis_matrix_t = ublas::compressed_matrix<int, orientation_t>;
00158         using matrix_t = ublas::matrix<Scalar_T, orientation_t>;
00159         using matrix_index_t = typename matrix_t::size_type;
00160
00161     public:
00162         static auto classname() -> const std::string;
00163         ~matrix_multi() override = default;
00164         matrix_multi();
00165         template< typename Other_Scalar_T >
00166         matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00167         template< typename Other_Scalar_T >
00168         matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00169             const index_set_t frm, const bool prechecked = false);
00170         matrix_multi(const multivector_t& val,
00171             const index_set_t frm, const bool prechecked = false);
00172         matrix_multi(const index_set_t ist, const Scalar_T& crd = Scalar_T(1));
00173         matrix_multi(const index_set_t ist, const Scalar_T& crd,
00174             const index_set_t frm, const bool prechecked = false);
00175         matrix_multi(const Scalar_T& scr, const index_set_t frm = index_set_t());
00176         matrix_multi(const int scr, const index_set_t frm = index_set_t());
00177         matrix_multi(const vector_t& vec,
00178             const index_set_t frm, const bool prechecked = false);
00179         matrix_multi(const std::string& str);
00180         matrix_multi(const std::string& str,
00181             const index_set_t frm, const bool prechecked = false);
00182         matrix_multi(const char* str)
00183         { *this = matrix_multi(std::string(str)); };
00184         matrix_multi(const char* str,
00185             const index_set_t frm, const bool prechecked = false)
00186         { *this = matrix_multi(std::string(str), frm, prechecked); };
00187         template< typename Other_Scalar_T >
00188         matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val);
00189         template< typename Other_Scalar_T >
00190         matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& val,
00191             const index_set_t frm, const bool prechecked = false);
00192         auto fast_matrix_multi(const index_set_t frm) const -> const matrix_multi_t;
00193         template< typename Other_Scalar_T >
00194         auto fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00195
00196     private:
00197         template< typename Matrix_T >
00198         matrix_multi(const Matrix_T& mtx, const index_set_t frm);
00199         matrix_multi(const matrix_t& mtx, const index_set_t frm);
00200         auto basis_element(const index_set<LO,HI>& ist) const -> const basis_matrix_t;
00201
00202     public:
00203         _GLUCAT_CLIFFORD_ALGEBRA_OPERATIONS
00204
00205         auto operator= (const multivector_t& rhs) -> multivector_t&;
00206
00207         static auto random(const index_set_t frm, Scalar_T fill = Scalar_T(1)) -> const matrix_multi_t;
00208
00209         // Friend declarations
00210
00211         friend auto
00212         operator* <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00213         friend auto
00214         operator^ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00215         friend auto
00216         operator& <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00217         friend auto
00218         operator% <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00219         friend auto

```

```

00244     star      <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> Scalar_T;
00245     friend auto
00246     operator/ <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00247     friend auto
00248     operator| <>(const matrix_multi_t& lhs, const matrix_multi_t& rhs) -> const matrix_multi_t;
00249
00250     friend auto
00251     operator« <>(std::istream& s, multivector_t& val) -> std::istream&;
00252     friend auto
00253     operator« <>(std::ostream& os, const multivector_t& val) -> std::ostream&;
00254     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00255     friend auto
00256     reframe (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs,      const
matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs,
00257             matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& lhs_reframed,
matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& rhs_reframed) -> const
index_set<Other_LO,Other_HI>;
00258     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00259     friend auto
00260     matrix_sqrt (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00261                 const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00262                 const index_t level)
00263     -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00264     template< typename Other_Scalar_T, const index_t Other_LO, const index_t Other_HI, typename
Other_Tune_P >
00265     friend auto
00266     matrix_log (const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& val,
00267                const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>& i,
00268                const index_t level)
00269     -> const matrix_multi<Other_Scalar_T,Other_LO,Other_HI,Other_Tune_P>;
00270
00272     auto      operator+= (const term_t& rhs) -> multivector_t&;
00273
00274     private:
00275         // Data members
00276
00278         index_set_t      m_frame;
00280         matrix_t          m_matrix;
00281     };
00282
00283     // Non-members
00284
00286     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00287     auto
00288     exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>;
00289
00290 }
00291
00292 namespace std
00293 {
00295     template < typename Scalar_T, const glucat::index_t LO, const glucat::index_t HI, typename Tune_P >
00296     struct numeric_limits< glucat::matrix_multi<Scalar_T,LO,HI,Tune_P> > :
00297     public numeric_limits<Scalar_T>
00298     { };
00299 }
00300 #endif // _GLUCAT_MATRIX_MULTI_H

```

## 7.37 glucat/matrix\_multi\_imp.h File Reference

```

#include "glucat/matrix_multi.h"
#include "glucat/scalar.h"
#include "glucat/generation.h"
#include "glucat/matrix.h"
#include <boost/numeric/ublas/matrix.hpp>
#include <boost/numeric/ublas/matrix_expression.hpp>
#include <boost/numeric/ublas/matrix_proxy.hpp>
#include <boost/numeric/ublas/matrix_sparse.hpp>
#include <boost/numeric/ublas/operation.hpp>
#include <boost/numeric/ublas/operation_sparse.hpp>
#include <boost/numeric/ublas/triangular.hpp>
#include <boost/numeric/ublas/lu.hpp>
#include <boost/numeric/ublas/io.hpp>
#include <fstream>

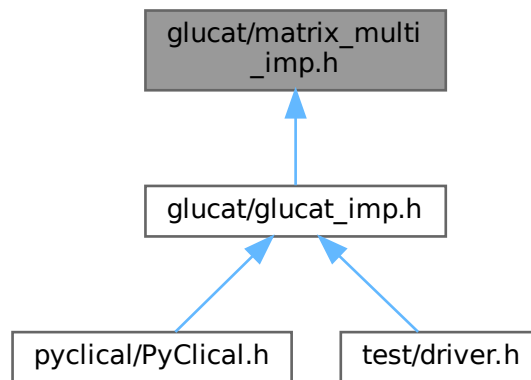
```

```
#include <iomanip>
#include <array>
#include <iostream>
```

Include dependency graph for matrix\_multi\_imp.h:



This graph shows which files directly or indirectly include this file:



## Classes

- class [glucat::basis\\_table< Scalar\\_T, LO, HI, Matrix\\_T >](#)  
*Table of basis elements used as a cache by basis\_element()*
- struct [pade::pade\\_sqrt\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade::pade\\_sqrt\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(sqrt(1+x),x,n,n)*
- struct [pade::pade\\_sqrt\\_numer< float >](#)
- struct [pade::pade\\_sqrt\\_denom< float >](#)
- struct [pade::pade\\_sqrt\\_numer< long double >](#)
- struct [pade::pade\\_sqrt\\_denom< long double >](#)
- struct [pade::pade\\_sqrt\\_numer< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< dd\\_real >](#)
- struct [pade::pade\\_sqrt\\_numer< qd\\_real >](#)
- struct [pade::pade\\_sqrt\\_denom< qd\\_real >](#)
- struct [pade::pade\\_log\\_numer< Scalar\\_T >](#)  
*Coefficients of numerator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*
- struct [pade::pade\\_log\\_denom< Scalar\\_T >](#)  
*Coefficients of denominator polynomials of Pade approximations produced by Pade1(log(1+x),x,n,n)*

- struct `pade::pade_log_numer< float >`
- struct `pade::pade_log_denom< float >`
- struct `pade::pade_log_numer< long double >`
- struct `pade::pade_log_denom< long double >`
- struct `pade::pade_log_numer< dd_real >`
- struct `pade::pade_log_denom< dd_real >`
- struct `pade::pade_log_numer< qd_real >`
- struct `pade::pade_log_denom< qd_real >`

## Namespaces

- namespace `glucat`
- namespace `pade`

## Functions

- auto `glucat::offset_level` (const `index_t` p, const `index_t` q) -> `index_t`  
*Determine the log2 dim corresponding to signature p, q.*
- template<typename Matrix\_Index\_T, const `index_t` LO, const `index_t` HI>  
static auto `glucat::folded_dim` (const `index_set`< LO, HI > &sub) -> Matrix\_Index\_T  
*Determine the matrix dimension of the fold of a subalgebra.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::reframe` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs, `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs\_reframed, `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs\_reframed) -> const `index_set`< LO, HI >  
*Find a common frame for operands of a binary operator.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator*` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >  
*Geometric product.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator^` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >  
*Outer product.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator&` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >  
*Inner product.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator%` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >  
*Left contraction.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::star` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> Scalar\_T  
*Hestenes scalar product.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator/` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >  
*Geometric quotient.*
- template<typename Scalar\_T, const `index_t` LO, const `index_t` HI, typename Tune\_P >  
auto `glucat::operator|` (const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &lhs, const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P > &rhs) -> const `matrix_multi`< Scalar\_T, LO, HI, Tune\_P >

*Transformation via twisted adjoint action.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator<<](#) (std::ostream &os, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::ostream &

*Write multivector to output.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::operator>>](#) (std::istream &s, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> std::istream &

*Read multivector from input.*

- template<typename Multivector\_T , typename Matrix\_T , typename Basis\_Matrix\_T >  
static auto [glucat::fast](#) (const Matrix\_T &X, [index\\_t](#) level) -> Multivector\_T

*Inverse generalized Fast Fourier Transform.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P , const size\_t Size>  
static auto [glucat::pade\\_approx](#) (const std::array< Scalar\_T, Size > &numer, const std::array< Scalar\_T, Size > &denom, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &X) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static void [glucat::db\\_step](#) ([matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &M, [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &Y)

*Single step of product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [glucat::db\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 4)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Product form of Denman-Beavers square root iteration.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [glucat::cr\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, Scalar\_T norm\_Y\_tol=std::pow(std::numeric\_limits< Scalar\_T >::epsilon(), 1)) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Cyclic reduction square root iteration.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::matrix\\_sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::sqrt](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Square root of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [glucat::pade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Pade' approximation of log.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
static auto [glucat::cascade\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Incomplete square root cascade and Pade' approximation of log.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::matrix\\_log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, const [index\\_t](#) level) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::log](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val, const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &i, bool prechecked) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Natural logarithm of multivector with specified complexifier.*

- template<typename Scalar\_T , const [index\\_t](#) LO, const [index\\_t](#) HI, typename Tune\_P >  
auto [glucat::exp](#) (const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P > &val) -> const [matrix\\_multi](#)< Scalar\_T, LO, HI, Tune\_P >

*Exponential of multivector.*

## 7.38 matrix\_multi\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_MATRIX_MULTI_IMP_H
00002 #define _GLUCAT_MATRIX_MULTI_IMP_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     matrix_multi_imp.h : Implement the matrix representation of a multivector
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/matrix_multi.h"
00035
00036 #include "glucat/scalar.h"
00037 #include "glucat/generation.h"
00038 #include "glucat/matrix.h"
00039
00040 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00041 # pragma GCC diagnostic push
00042 # pragma GCC diagnostic ignored "-Wunused-local-typedefs"
00043 # endif
00044 # if defined(_GLUCAT_HAVE_BOOST_SERIALIZATION_ARRAY_WRAPPER_H)
00045 # include <boost/serialization/array_wrapper.hpp>
00046 # endif
00047 #include <boost/numeric/ublas/matrix.hpp>
00048 #include <boost/numeric/ublas/matrix_expression.hpp>
00049 #include <boost/numeric/ublas/matrix_proxy.hpp>
00050 #include <boost/numeric/ublas/matrix_sparse.hpp>
00051 #include <boost/numeric/ublas/operation.hpp>
00052 #include <boost/numeric/ublas/operation_sparse.hpp>
00053 #include <boost/numeric/ublas/triangular.hpp>
00054 #include <boost/numeric/ublas/lu.hpp>
00055 #include <boost/numeric/ublas/io.hpp>
00056 # if defined(_GLUCAT_GCC_IGNORE_UNUSED_LOCAL_TYPEDEFS)
00057 # pragma GCC diagnostic pop
00058 # endif
00059
00060 #include <fstream>
00061 #include <iomanip>
00062 #include <array>
00063 #include <iostream>
00064
00065 namespace glucat
00066 {
00067     // References for algorithms:
00068     // [CHKL]:
00069     // [L]: Pertti Lounesto, "Clifford algebras and spinors", Cambridge UP, 1997.
00070     // [MB]: Beatrice Meini, "The Matrix Square Root From a New Functional Perspective:
00071     // Theoretical Results and Computational Issues", SIAM Journal on
00072     // Matrix Analysis and Applications 26(2):362-376, 2004.
00073     // [P]: Ian R. Porteous, "Clifford algebras and the classical groups", Cambridge UP, 1995.
00074
00075     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00076     auto
00077     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00078     classname() -> const std::string
00079     { return "matrix_multi"; }
00080
00081     // Reference: [P] Table 15.27, p 133
00082     inline

```

```

00085     auto
00086     offset_level(const index_t p, const index_t q) -> index_t
00087     {
00088         // Offsets between the log2 of the matrix dimension for the current signature
00089         // and that of the real superalgebra
00090         static const std::array<int, 8> offset_log2_dim = {0, 1, 0, 1, 1, 2, 1, 1};
00091         const index_t bott = pos_mod(p-q, 8);
00092         return (p+q)/2 + offset_log2_dim[bott];
00093     }
00094
00095     // Reference: [P] Table 15.27, p 133
00096     template< typename Matrix_Index_T, const index_t LO, const index_t HI >
00097     inline
00098     static
00099     auto
00100     folded_dim( const index_set<LO,HI>& sub ) -> Matrix_Index_T
00101     { return 1 « offset_level(sub.count_pos(), sub.count_neg()); }
00102
00103     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00104     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00105     matrix_multi()
00106     : m_frame( index_set_t() ),
00107       m_matrix( matrix_t( 1, 1 ) )
00108     { this->m_matrix.clear(); }
00109
00110     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00111     template< typename Other_Scalar_T >
00112     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00113     matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val)
00114     : m_frame( val.m_frame ), m_matrix( val.m_matrix.size(), val.m_matrix.size() )
00115     {
00116         this->m_matrix.clear();
00117         for (auto
00118             val_it1 = val.m_matrix.begin();
00119             val_it1 != val.m_matrix.end();
00120             ++val_it1)
00121             for (auto
00122                 val_it2 = val_it1.begin();
00123                 val_it2 != val_it1.end();
00124                 ++val_it2)
00125                 this->m_matrix(val_it2.index1(), val_it2.index2()) =
00126                 numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00127     }
00128
00129     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00130     template< typename Other_Scalar_T >
00131     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00132     matrix_multi(const matrix_multi<Other_Scalar_T,LO,HI,Tune_P>& val, const index_set_t frm, const bool
00133     prechecked)
00134     : m_frame( frm )
00135     {
00136         if (frm != val.m_frame)
00137             *this = multivector_t(framed_multi_t(val), frm);
00138         else
00139         {
00140             const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00141             this->m_matrix.resize(dim, dim, false);
00142             this->m_matrix.clear();
00143             for (auto
00144                 val_it1 = val.m_matrix.begin();
00145                 val_it1 != val.m_matrix.end();
00146                 ++val_it1)
00147                 for (auto
00148                     val_it2 = val_it1.begin();
00149                     val_it2 != val_it1.end();
00150                     ++val_it2)
00151                     this->m_matrix(val_it2.index1(), val_it2.index2()) =
00152                     numeric_traits<Scalar_T>::to_scalar_t(*val_it2);
00153         }
00154     }
00155
00156     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00157     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00158     matrix_multi(const multivector_t& val, const index_set_t frm, const bool prechecked)
00159     : m_frame( frm )
00160     {
00161         if (frm != val.m_frame)
00162             *this = multivector_t(framed_multi_t(val), frm);
00163         else
00164             this->m_matrix = val.m_matrix;
00165     }
00166
00167     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00168     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00169     matrix_multi(const index_set_t ist, const Scalar_T& crd)
00170     : m_frame( ist )
00171     {

```

```

00175     const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00176     this->m_matrix.resize(dim, dim, false);
00177     this->m_matrix.clear();
00178     *this += term_t(ist, crd);
00179 }
00180
00182 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00183 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00184 matrix_multi(const index_set_t ist, const Scalar_T& crd, const index_set_t frm, const bool
prechecked)
00185 : m_frame( frm )
00186 {
00187     if (!prechecked && (ist | frm) != frm)
00188         throw error_t("multivector_t(ist,crd,frm): cannot initialize with value outside of frame");
00189     const matrix_index_t dim = folded_dim<matrix_index_t>(frm);
00190     this->m_matrix.resize(dim, dim, false);
00191     this->m_matrix.clear();
00192     *this += term_t(ist, crd);
00193 }
00194
00196 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00197 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00198 matrix_multi(const Scalar_T& scr, const index_set_t frm)
00199 : m_frame( frm )
00200 {
00201     const auto dim = folded_dim<matrix_index_t>(frm);
00202     this->m_matrix.resize(dim, dim, false);
00203     this->m_matrix.clear();
00204     *this += term_t(index_set_t(), scr);
00205 }
00206
00208 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00209 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00210 matrix_multi(const int scr, const index_set_t frm)
00211 { *this = multivector_t(Scalar_T(scr), frm); }
00212
00214 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00215 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00216 matrix_multi(const vector_t& vec,
00217             const index_set_t frm, const bool prechecked)
00218 : m_frame( frm )
00219 {
00220     if (!prechecked && index_t(vec.size()) != frm.count())
00221         throw error_t("multivector_t(vec,frm): cannot initialize with vector not matching frame");
00222     const auto dim = folded_dim<matrix_index_t>(frm);
00223     this->m_matrix.resize(dim, dim, false);
00224     this->m_matrix.clear();
00225     auto idx = frm.min();
00226     const auto frm_end = frm.max()+1;
00227     for (auto& crd : vec)
00228     {
00229         *this += term_t(index_set_t(idx), crd);
00230         for (
00231             ++idx;
00232             idx != frm_end && !frm[idx];
00233             ++idx)
00234             ;
00235     }
00236 }
00237
00239 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00240 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00241 matrix_multi(const std::string& str)
00242 { *this = framed_multi_t(str); }
00243
00245 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00246 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00247 matrix_multi(const std::string& str, const index_set_t frm, const bool prechecked)
00248 { *this = multivector_t(framed_multi_t(str), frm, prechecked); }
00249
00251 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00252 template< typename Other_Scalar_T >
00253 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00254 matrix_multi(const framed_multi_t<Other_Scalar_T,LO,HI,Tune_P>& val)
00255 : m_frame( val.frame() )
00256 {
00257     if (val.size() >= Tune_P::fast_size_threshold)
00258     try
00259     {
00260         *this = val.template fast_matrix_multi<Scalar_T>(this->m_frame);
00261         return;
00262     }
00263     catch (const glucat_error& e)
00264     { }
00265     const auto dim = folded_dim<matrix_index_t>(this->m_frame);
00266     this->m_matrix.resize(dim, dim, false);
00267     this->m_matrix.clear();

```



```

00268
00269     using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00270     for (auto& val_term : val)
00271         *this += val_term;
00272 }
00273
00275 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00276 template< typename Other_Scalar_T >
00277 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00278 matrix_multi(const framed_multi<Other_Scalar_T,LO,HI,Tune_P>& framed_val, const index_set_t frm,
const bool prechecked)
00279 {
00280     const auto val = framed_val.truncated();
00281     const auto our_frame = val.frame() | frm;
00282     if (val.size() >= Tune_P::fast_size_threshold)
00283     {
00284         try
00285         {
00286             *this = val.template fast_matrix_multi<Scalar_T>(our_frame);
00287             return;
00288         }
00289         catch (const glucat_error& e)
00290         { }
00291         this->m_frame = our_frame;
00292         const auto dim = folded_dim<matrix_index_t>(our_frame);
00293         this->m_matrix.resize(dim, dim, false);
00294         this->m_matrix.clear();
00295
00296         using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
00297         for (auto& val_term : val)
00298             *this += val_term;
00299     }
00300
00301 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00302 template< typename Matrix_T >
00303 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00304 matrix_multi(const Matrix_T& mtx, const index_set_t frm)
00305 : m_frame( frm ), m_matrix( mtx.size1(), mtx.size2() )
00306 {
00307     this->m_matrix.clear();
00308
00309     for (auto
00310         mtx_it1 = mtx.begin1();
00311         mtx_it1 != mtx.end1();
00312         ++mtx_it1)
00313     {
00314         for (auto
00315             mtx_it2 = mtx_it1.begin();
00316             mtx_it2 != mtx_it1.end();
00317             ++mtx_it2)
00318             this->m_matrix(mtx_it2.index1(), mtx_it2.index2()) =
00319                 numeric_traits<Scalar_T>::to_scalar_t(*mtx_it2);
00320     }
00321
00322 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00323 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00324 matrix_multi(const matrix_t& mtx, const index_set_t frm)
00325 : m_frame( frm ), m_matrix( mtx )
00326 { }
00327
00328 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00329 auto
00330 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00331 operator= (const multivector_t& rhs) -> multivector_t&
00332 {
00333     // Check for assignment to self
00334     if (this == &rhs)
00335         return *this;
00336     this->m_frame = rhs.m_frame;
00337     this->m_matrix = rhs.m_matrix;
00338     return *this;
00339 }
00340
00342 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00343 inline
00344 auto
00345 reframe (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs,
00346         matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs_reframed,
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs_reframed) -> const index_set<LO,HI>
00347 {
00348     using index_set_t = index_set<LO, HI>;
00349     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00350     using framed_multi_t = typename multivector_t::framed_multi_t;
00351     // Determine the initial common frame
00352     index_set_t our_frame = lhs.m_frame | rhs.m_frame;
00353     framed_multi_t framed_lhs;
00354     framed_multi_t framed_rhs;
00355     if ((lhs.m_frame != our_frame) || (rhs.m_frame != our_frame))

```

```

00356     {
00357         // The common frame may expand as a result of the transform to framed_multi_t
00358         framed_lhs = framed_multi_t(lhs);
00359         framed_rhs = framed_multi_t(rhs);
00360         our_frame |= framed_lhs.frame() | framed_rhs.frame();
00361     }
00362     // Do the reframing only where necessary
00363     if (lhs.m_frame != our_frame)
00364         lhs_reframed = multivector_t(framed_lhs, our_frame, true);
00365     if (rhs.m_frame != our_frame)
00366         rhs_reframed = multivector_t(framed_rhs, our_frame, true);
00367     return our_frame;
00368 }
00369
00370 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00371 auto
00372 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00373 operator== (const multivector_t& rhs) const -> bool
00374 {
00375     // Ensure that there is no aliasing
00376     if (this == &rhs)
00377         return true;
00378
00379     // Operate only within a common frame
00380     multivector_t lhs_reframed;
00381     multivector_t rhs_reframed;
00382     const index_set_t our_frame = reframe(*this, rhs, lhs_reframed, rhs_reframed);
00383     const multivector_t& lhs_ref = (this->m_frame == our_frame)
00384         ? *this
00385         : lhs_reframed;
00386     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00387         ? rhs
00388         : rhs_reframed;
00389
00390     return ublas::norm_inf(lhs_ref.m_matrix - rhs_ref.m_matrix) == 0;
00391 }
00392
00393 // Test for equality of multivector and scalar
00394 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00395 inline
00396 auto
00397 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00398 operator== (const Scalar_T& scr) const -> bool
00399 {
00400     if (scr != Scalar_T(0))
00401         return *this == multivector_t(framed_multi_t(scr), this->m_frame, true);
00402     else if (ublas::norm_inf(this->m_matrix) != 0)
00403         return false;
00404     else
00405     {
00406         const matrix_index_t dim = this->m_matrix.size1();
00407         return !(dim == 1 && this->isnan());
00408     }
00409 }
00410
00411 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00412 inline
00413 auto
00414 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00415 operator+= (const Scalar_T& scr) -> multivector_t&
00416 { return *this += term_t(index_set_t(), scr); }
00417
00418 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00419 inline
00420 auto
00421 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00422 operator+= (const multivector_t& rhs) -> multivector_t&
00423 {
00424     // Ensure that there is no aliasing
00425     if (this == &rhs)
00426         return *this *= Scalar_T(2);
00427
00428     // Operate only within a common frame
00429     multivector_t rhs_reframed;
00430     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00431     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00432         ? rhs
00433         : rhs_reframed;
00434
00435     noalias(this->m_matrix) += rhs_ref.m_matrix;
00436     return *this;
00437 }
00438
00439 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00440 inline
00441 auto
00442 matrix_multi<Scalar_T,LO,HI,Tune_P>::

```

```

00447 operator-= (const Scalar_T& scr) -> multivector_t&
00448 { return *this += term_t(index_set_t(), -scr); }
00449
00451 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00452 inline
00453 auto
00454 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00455 operator-= (const multivector_t& rhs) -> multivector_t&
00456 {
00457     // Ensure that there is no aliasing
00458     if (this == &rhs)
00459         return *this = Scalar_T(0);
00460
00461     // Operate only within a common frame
00462     multivector_t rhs_reframed;
00463     const index_set_t our_frame = reframe(*this, rhs, *this, rhs_reframed);
00464     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00465         ? rhs
00466         : rhs_reframed;
00467
00468     noalias(this->m_matrix) -= rhs_ref.m_matrix;
00469     return *this;
00470 }
00471
00473 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00474 inline
00475 auto
00476 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00477 operator- () const -> const multivector_t
00478 { return multivector_t(-(this->m_matrix), this->m_frame); }
00479
00481 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00482 inline
00483 auto
00484 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00485 operator*= (const Scalar_T& scr) -> multivector_t&
00486 { // multiply coordinates of all terms by scalar
00487
00488     using traits_t = numeric_traits<Scalar_T>;
00489     if (traits_t::isNaN_or_isInf(scr) || this->isnan())
00490         return *this = traits_t::NaN();
00491     if (scr == Scalar_T(0))
00492         *this = Scalar_T(0);
00493     else
00494         this->m_matrix *= scr;
00495     return *this;
00496 }
00497
00499 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00500 inline
00501 auto
00502 operator* (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00503 {
00504     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00505     using index_set_t = typename multivector_t::index_set_t;
00506
00507     if (lhs.isnan() || rhs.isnan())
00508         return numeric_traits<Scalar_T>::NaN();
00509
00510     // Operate only within a common frame
00511     multivector_t lhs_reframed;
00512     multivector_t rhs_reframed;
00513     const index_set_t our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00514     const multivector_t& lhs_ref = (lhs.m_frame == our_frame)
00515         ? lhs
00516         : lhs_reframed;
00517     const multivector_t& rhs_ref = (rhs.m_frame == our_frame)
00518         ? rhs
00519         : rhs_reframed;
00520
00521     using matrix_t = typename multivector_t::matrix_t;
00522     using matrix_index_t = typename matrix_t::size_type;
00523
00524     const matrix_index_t dim = lhs_ref.m_matrix.size();
00525     multivector_t result = multivector_t(matrix_t(dim, dim), our_frame);
00526     result.m_matrix.clear();
00527     ublas::axpy_prod(lhs_ref.m_matrix, rhs_ref.m_matrix, result.m_matrix, true);
00528     return result;
00529 }
00530
00532 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00533 inline
00534 auto
00535 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00536 operator*= (const multivector_t& rhs) -> multivector_t&
00537 { return *this = *this * rhs; }

```

```

00538
00540     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00541     inline
00542     auto
00543     operator^ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00544     {
00545         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00546         using framed_multi_t = typename multivector_t::framed_multi_t;
00547         return framed_multi_t(lhs) ^ framed_multi_t(rhs);
00548     }
00549
00551     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00552     inline
00553     auto
00554     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00555     operator^= (const multivector_t& rhs) -> multivector_t&
00556     { return *this = *this ^ rhs; }
00557
00559     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00560     inline
00561     auto
00562     operator& (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00563     {
00564         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00565         using framed_multi_t = typename multivector_t::framed_multi_t;
00566         return framed_multi_t(lhs) & framed_multi_t(rhs);
00567     }
00568
00570     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00571     inline
00572     auto
00573     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00574     operator&= (const multivector_t& rhs) -> multivector_t&
00575     { return *this = *this & rhs; }
00576
00578     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00579     inline
00580     auto
00581     operator% (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00582     {
00583         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00584         using framed_multi_t = typename multivector_t::framed_multi_t;
00585         return framed_multi_t(lhs) % framed_multi_t(rhs);
00586     }
00587
00589     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00590     inline
00591     auto
00592     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00593     operator%= (const multivector_t& rhs) -> multivector_t&
00594     { return *this = *this % rhs; }
00595
00597     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00598     inline
00599     auto
00600     star(const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs)
-> Scalar_T
00601     { return (lhs * rhs).scalar(); }
00602
00604     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00605     inline
00606     auto
00607     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00608     operator/= (const Scalar_T& scr) -> multivector_t&
00609     { return *this *= Scalar_T(1)/scr; }
00610
00612     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00613     auto
00614     operator/ (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const
matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00615     {
00616         using traits_t = numeric_traits<Scalar_T>;
00617
00618         if (lhs.isnan() || rhs.isnan())
00619             return traits_t::NaN();
00620
00621         if (rhs == Scalar_T(0))
00622             return traits_t::NaN();
00623
00624         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
00625         using index_set_t = typename multivector_t::index_set_t;
00626
00627         // Operate only within a common frame
00628         multivector_t lhs_reframed;

```

```

00629     multivector_t rhs_reframed;
00630     const auto our_frame = reframe(lhs, rhs, lhs_reframed, rhs_reframed);
00631     const auto& lhs_ref = (lhs.m_frame == our_frame)
00632         ? lhs
00633         : lhs_reframed;
00634     const auto& rhs_ref = (rhs.m_frame == our_frame)
00635         ? rhs
00636         : rhs_reframed;
00637
00638     // Solve result == lhs_ref/rhs_ref <=> result*rhs_ref == lhs_ref
00639     // We now solve X == B/A
00640     // (where X == result, B == lhs_ref.m_matrix and A == rhs_ref.m_matrix)
00641     // X == B/A <=> X*A == B <=> AT*XT == BT
00642     // So, we solve AT*XT == BT
00643
00644     using matrix_t = typename multivector_t::matrix_t;
00645     using matrix_index_t = typename matrix_t::size_type;
00646
00647     const auto& AT = matrix_t(ublas::trans(rhs_ref.m_matrix));
00648     auto LU = AT;
00649
00650     using permutation_t = ublas::permutation_matrix<matrix_index_t>;
00651
00652     auto pvector = permutation_t(AT.size1());
00653     if (! ublas::lu_factorize(LU, pvector))
00654     {
00655         const auto& BT = matrix_t(ublas::trans(lhs_ref.m_matrix));
00656         auto XT = BT;
00657         ublas::lu_substitute(LU, pvector, XT);
00658         if (matrix::isnan(XT))
00659             return traits_t::NaN();
00660
00661         // Iterative refinement.
00662         // Reference: Nicholas J. Higham, "Accuracy and Stability of Numerical Algorithms",
00663         // SIAM, 1996, ISBN 0-89871-355-2, Chapter 11
00664         if (Tune_P::div_max_steps > 0)
00665         {
00666             // matrix_t R = ublas::prod(AT, XT) - BT;
00667             auto R = matrix_t(-BT);
00668             ublas::axpy_prod(AT, XT, R, false);
00669             if (matrix::isnan(R))
00670                 return traits_t::NaN();
00671
00672             auto nr = Scalar_T(ublas::norm_inf(R));
00673             if ( nr != Scalar_T(0) && !traits_t::isNaN_or_isInf(nr) )
00674             {
00675                 auto XTnew = XT;
00676                 auto nrold = nr + Scalar_T(1);
00677                 for (auto
00678                     step = 0;
00679                     step != Tune_P::div_max_steps &&
00680                     nr < nrold &&
00681                     nr != Scalar_T(0) &&
00682                     nr == nr;
00683                     ++step)
00684                 {
00685                     nrold = nr;
00686                     if (step != 0)
00687                         XT = XTnew;
00688                     auto& D = R;
00689                     ublas::lu_substitute(LU, pvector, D);
00690                     XTnew -= D;
00691                     // noalias(R) = ublas::prod(AT, XTnew) - BT;
00692                     R = -BT;
00693                     ublas::axpy_prod(AT, XTnew, R, false);
00694                     nr = ublas::norm_inf(R);
00695                 }
00696             }
00697         }
00698         return multivector_t(ublas::trans(XT), our_frame);
00699     }
00700     else
00701         // AT is singular. Return NaN
00702         return traits_t::NaN();
00703 }
00704
00706 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00707 inline
00708 auto
00709 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00710 operator/=(const multivector_t& rhs) -> multivector_t&
00711 { return *this = *this / rhs; }
00712
00714 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00715 inline
00716 auto
00717 operator| (const matrix_multi<Scalar_T,LO,HI,Tune_P>& lhs, const

```

```

matrix_multi<Scalar_T,LO,HI,Tune_P>& rhs) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
00718 { return rhs * lhs / rhs.involute(); }
00719
00721 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00722 inline
00723 auto
00724 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00725 operator|= (const multivector_t& rhs) -> multivector_t&
00726 { return *this = rhs * *this / rhs.involute(); }
00727
00729 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00730 inline
00731 auto
00732 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00733 inv() const -> const multivector_t
00734 { return multivector_t(Scalar_T(1), this->m_frame) / *this; }
00735
00737 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00738 inline
00739 auto
00740 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00741 pow(int m) const -> const multivector_t
00742 { return glucat::pow(*this, m); }
00743
00745 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00746 auto
00747 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00748 outer_pow(int m) const -> const multivector_t
00749 {
00750     if (m < 0)
00751         throw error_t("outer_pow(m): negative exponent");
00752     framed_multi_t a = *this;
00753     return a.outer_pow(m);
00754 }
00755
00757 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00758 inline
00759 auto
00760 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00761 grade() const -> index_t
00762 { return framed_multi_t(*this).grade(); }
00763
00765 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00766 inline
00767 auto
00768 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00769 frame() const -> const index_set_t
00770 { return this->m_frame; }
00771
00773 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00774 inline
00775 auto
00776 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00777 operator[] (const index_set_t ist) const -> Scalar_T
00778 {
00779     // Use matrix inner product only if ist is in frame
00780     if ( (ist | this->m_frame) == this->m_frame)
00781         return matrix::inner<Scalar_T>(this->basis_element(ist), this->m_matrix);
00782     else
00783         return Scalar_T(0);
00784 }
00785
00787 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00788 inline
00789 auto
00790 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00791 operator() (index_t grade) const -> const multivector_t
00792 {
00793     if ((grade < 0) || (grade > HI-LO))
00794         return 0;
00795     else
00796         return (framed_multi_t(*this))(grade);
00797 }
00798
00800 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00801 inline
00802 auto
00803 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00804 scalar() const -> Scalar_T
00805 {
00806     const matrix_index_t dim = this->m_matrix.size1();
00807     return matrix::trace(this->m_matrix) / Scalar_T( double(dim) );
00808 }
00809
00811 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00812 inline
00813 auto

```

```

00814     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00815     pure() const -> const multivector_t
00816     { return *this - this->scalar(); }
00817
00819     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00820     inline
00821     auto
00822     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00823     even() const -> const multivector_t
00824     { return framed_multi_t(*this).even(); }
00825
00827     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00828     inline
00829     auto
00830     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00831     odd() const -> const multivector_t
00832     { return framed_multi_t(*this).odd(); }
00833
00835     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00836     auto
00837     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00838     vector_part() const -> const vector_t
00839     { return this->vector_part(this->frame(), true); }
00840
00842     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00843     auto
00844     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00845     vector_part(const index_set_t frm, const bool prechecked) const -> const vector_t
00846     {
00847         if (!prechecked && (this->frame() | frm) != frm)
00848             throw error_t("vector_part(frm): value is outside of requested frame");
00849         vector_t result;
00850         // If we need to enlarge the frame we may as well use a framed_multi_t
00851         if (this->frame() != frm)
00852             return framed_multi_t(*this).vector_part(frm, true);
00853
00854         const auto begin_index = frm.min();
00855         const auto end_index = frm.max()+1;
00856         for (auto
00857             idx = begin_index;
00858             idx != end_index;
00859             ++idx)
00860             if (frm[idx])
00861                 // Frame may contain indices which do not correspond to a grade 1 term but
00862                 // frame cannot omit any index corresponding to a grade 1 term
00863                 result.push_back(
00864                     matrix::inner<Scalar_T>(this->basis_element(index_set_t(idx)),
00865                     this->m_matrix));
00866         return result;
00867     }
00868
00870     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00871     inline
00872     auto
00873     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00874     involute() const -> const multivector_t
00875     { return framed_multi_t(*this).involute(); }
00876
00878     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00879     inline
00880     auto
00881     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00882     reverse() const -> const multivector_t
00883     { return framed_multi_t(*this).reverse(); }
00884
00886     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00887     inline
00888     auto
00889     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00890     conj() const -> const multivector_t
00891     { return framed_multi_t(*this).conj(); }
00892
00894     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00895     inline
00896     auto
00897     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00898     quad() const -> Scalar_T
00899     { // scalar(conj(x)*x) = 2*quad(even(x)) - quad(x)
00900         // Arvind Raja ref: "old clical: quadfunction(p:pterm):pterm in file compmod.pas"
00901         return framed_multi_t(*this).quad();
00902     }
00903
00905     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00906     inline
00907     auto
00908     matrix_multi<Scalar_T,LO,HI,Tune_P>::
00909     norm() const -> Scalar_T

```

```

00910 {
00911     const matrix_index_t dim = this->m_matrix.size1();
00912     return matrix::norm_frob2(this->m_matrix) / Scalar_T( double(dim) );
00913 }
00914
00916 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00917 inline
00918 auto
00919 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00920 max_abs() const -> Scalar_T
00921 { return framed_multi_t(*this).max_abs(); }
00922
00924 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00925 auto
00926 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00927 random(const index_set<LO,HI> frm, Scalar_T fill) -> const multivector_t
00928 {
00929     return framed_multi<Scalar_T,LO,HI,Tune_P>::random(frm, fill);
00930 }
00931
00933 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00934 inline
00935 void
00936 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00937 write(const std::string& msg) const
00938 { framed_multi_t(*this).write(msg); }
00939
00941 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00942 inline
00943 void
00944 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00945 write(std::ofstream& ofile, const std::string& msg) const
00946 {
00947     if (!ofile)
00948         throw error_t("write(ofile,msg): cannot write to output file");
00949     framed_multi_t(*this).write(ofile, msg);
00950 }
00951
00953 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00954 inline
00955 auto
00956 operator<< (std::ostream& os, const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::ostream&
00957 {
00958     os << typename matrix_multi<Scalar_T,LO,HI,Tune_P>::framed_multi_t(val);
00959     return os;
00960 }
00961
00963 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00964 inline
00965 auto
00966 operator>> (std::istream& s, matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> std::istream&
00967 { // Input looks like 1.0-2.0{1,2}+3.2{3,4}
00968     framed_multi<Scalar_T,LO,HI,Tune_P> local;
00969     s >> local;
00970     // If s.bad() then we have a corrupt input
00971     // otherwise we are fine and can copy the resulting matrix_multi
00972     if (!s.bad())
00973         val = local;
00974     return s;
00975 }
00976
00978 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00979 inline
00980 auto
00981 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00982 isinf() const -> bool
00983 {
00984     if (std::numeric_limits<Scalar_T>::has_infinity)
00985         return matrix::isinf(this->m_matrix);
00986     else
00987         return false;
00988 }
00989
00991 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
00992 inline
00993 auto
00994 matrix_multi<Scalar_T,LO,HI,Tune_P>::
00995 isnan() const -> bool
00996 {
00997     if (std::numeric_limits<Scalar_T>::has_quiet_NaN)
00998         return matrix::isnan(this->m_matrix);
00999     else
01000         return false;
01001 }
01002
01004 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01005 inline

```



```

01006     auto
01007     matrix_multi<Scalar_T,LO,HI,Tune_P>::
01008     truncated(const Scalar_T& limit) const -> const multivector_t
01009     { return framed_multi_t(*this).truncated(limit); }
01010
01011     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01012     inline
01013     auto
01014     matrix_multi<Scalar_T,LO,HI,Tune_P>::
01015     operator+= (const term_t& term) -> multivector_t&
01016     {
01017         if (term.second != Scalar_T(0))
01018             this->m_matrix.plus_assign(matrix_t(this->basis_element(term.first)) * term.second);
01019         return *this;
01020     }
01021
01022     template< typename Multivector_T, typename Matrix_T, typename Basis_Matrix_T >
01023     static
01024     auto
01025     fast(const Matrix_T& X, index_t level) -> Multivector_T
01026     {
01027         using framed_multi_t = Multivector_T;
01028
01029         using index_set_t = typename framed_multi_t::index_set_t;
01030         using Scalar_T = typename framed_multi_t::scalar_t;
01031         using matrix_t = Matrix_T;
01032         using basis_matrix_t = Basis_Matrix_T;
01033         using basis_scalar_t = typename basis_matrix_t::value_type;
01034         using traits_t = numeric_traits<Scalar_T>;
01035
01036         if (level == 0)
01037             return framed_multi_t(traits_t::to_scalar_t(X(0,0)));
01038
01039         if (ublas::norm_inf(X) == 0)
01040             return Scalar_T(0);
01041
01042         const basis_matrix_t& I = matrix::unit<basis_matrix_t>(2);
01043         basis_matrix_t J(2,2,2);
01044         J.clear();
01045         J(0,1) = basis_scalar_t(-1);
01046         J(1,0) = basis_scalar_t(1);
01047         basis_matrix_t K = J;
01048         K(0,1) = basis_scalar_t(1);
01049         basis_matrix_t JK = I;
01050         JK(0,0) = basis_scalar_t(-1);
01051
01052         using matrix::signed_perm_nork;
01053         const index_set_t ist_mn = index_set_t(-level);
01054         const index_set_t ist_pn = index_set_t(level);
01055         const index_set_t ist_mnpn = ist_mn | ist_pn;
01056         if (level == 1)
01057         {
01058             using term_t = typename framed_multi_t::term_t;
01059             const Scalar_T i_x = traits_t::to_scalar_t(signed_perm_nork(I, X)(0, 0));
01060             const Scalar_T j_x = traits_t::to_scalar_t(signed_perm_nork(J, X)(0, 0));
01061             const Scalar_T k_x = traits_t::to_scalar_t(signed_perm_nork(K, X)(0, 0));
01062             const Scalar_T jk_x = traits_t::to_scalar_t(signed_perm_nork(JK, X)(0, 0));
01063             framed_multi_t
01064             result = i_x;
01065             result += term_t(ist_mn, j_x); // j_x * mn;
01066             result += term_t(ist_pn, k_x); // k_x * pn;
01067             return result += term_t(ist_mnpn, jk_x); // jk_x * mnpn;
01068         }
01069         else
01070         {
01071             const framed_multi_t& mn = framed_multi_t(ist_mn);
01072             const framed_multi_t& pn = framed_multi_t(ist_pn);
01073             const framed_multi_t& mnpn = framed_multi_t(ist_mnpn);
01074             const framed_multi_t& i_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01075             (signed_perm_nork(I, X), level-1);
01076             const framed_multi_t& j_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01077             (signed_perm_nork(J, X), level-1);
01078             const framed_multi_t& k_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01079             (signed_perm_nork(K, X), level-1);
01080             const framed_multi_t& jk_x = fast<framed_multi_t, matrix_t, basis_matrix_t>
01081             (signed_perm_nork(JK, X), level-1);
01082             framed_multi_t
01083             result = i_x.even() - jk_x.odd();
01084             result += (j_x.even() - k_x.odd()) * mn;
01085             result += (k_x.even() - j_x.odd()) * pn;
01086             return result += (jk_x.even() - i_x.odd()) * mnpn;
01087         }
01088     }
01089 }
01090
01091     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01092     inline
01093     auto

```

```

01096 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01097 fast_matrix_multi(const index_set_t frm) const -> const multivector_t
01098 {
01099     if (this->m_frame == frm)
01100         return *this;
01101     else
01102         return (this->template fast_framed_multi<Scalar_T>()).template fast_matrix_multi<Scalar_T>(frm);
01103 }
01104
01106 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01107 template <typename Other_Scalar_T>
01108 auto
01109 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01110 fast_framed_multi() const -> const framed_multi<Other_Scalar_T,LO,HI,Tune_P>
01111 {
01112     // Determine the amount of off-centering needed
01113     index_t p = this->m_frame.count_pos();
01114     index_t q = this->m_frame.count_neg();
01115
01116     const index_t bott = pos_mod(p-q, 8);
01117     p += std::max(gen::offset_to_super[bott],index_t(0));
01118     q -= std::min(gen::offset_to_super[bott],index_t(0));
01119
01120     const index_t orig_p = p;
01121     const index_t orig_q = q;
01122     while (p-q > 4)
01123         { p -= 4; q += 4; }
01124     while (p-q < -3)
01125         { p += 4; q -= 4; }
01126     if (p-q > 1)
01127     {
01128         index_t old_p = p;
01129         p = q+1;
01130         q = old_p-1;
01131     }
01132     const index_t level = (p+q)/2;
01133
01134     // Do the inverse fast transform
01135     using framed_multi_t = framed_multi<Other_Scalar_T,LO,HI,Tune_P>;
01136     framed_multi_t val = fast<framed_multi_t, matrix_t, basis_matrix_t>(this->m_matrix, level);
01137
01138     // Off-centre val
01139     switch (pos_mod(orig_p-orig_q, 8))
01140     {
01141     case 2:
01142     case 3:
01143     case 4:
01144         val.centre_qp1_pm1(p, q);
01145         break;
01146     default:
01147         break;
01148     }
01149     if (orig_p-orig_q > 4)
01150         while (p != orig_p)
01151             val.centre_pp4_qm4(p, q);
01152     if (orig_p-orig_q < -3)
01153         while (p != orig_p)
01154             val.centre_pm4_qp4(p, q);
01155
01156     // Return unfolded val
01157     return val.unfold(this->m_frame);
01158 }
01159
01161 template< typename Scalar_T, const index_t LO, const index_t HI, typename Matrix_T >
01162 class basis_table :
01163 public std::map< std::pair< const index_set<LO,HI>, const index_set<LO,HI> >,
01164               Matrix_T* >
01165 {
01166 public:
01167     static auto basis() -> basis_table& { static basis_table b; return b;}
01168 private:
01169     friend class friend_for_private_destructor;
01170     // Enforce singleton
01171     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
01172     basis_table() = default;
01173     ~basis_table() = default;
01174 public:
01175     basis_table(const basis_table&) = delete;
01176     auto operator= (const basis_table&) -> basis_table& = delete;
01177 };
01178
01180 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01181 auto
01182 matrix_multi<Scalar_T,LO,HI,Tune_P>::
01183 basis_element(const index_set_t& ist) const -> const basis_matrix_t
01184 {
01185     using index_set_pair_t = std::pair<const index_set_t, const index_set_t>;

```

```

01190     const auto& unfolded_pair = index_set_pair_t(ist, this->m_frame);
01191
01192     using basis_table_t = basis_table<Scalar_T, LO, HI, basis_matrix_t>;
01193     auto& basis_cache = basis_table_t::basis();
01194
01195     const auto frame_count = this->m_frame.count();
01196     const auto use_cache = frame_count <= index_t(Tune_P::basis_max_count);
01197
01198     if (use_cache)
01199     {
01200         const auto basis_it = basis_cache.find(unfolded_pair);
01201         if (basis_it != basis_cache.end())
01202             return *(basis_it->second);
01203     }
01204     const auto folded_set = ist.fold(this->m_frame);
01205     const auto folded_frame = this->m_frame.fold();
01206     const auto& folded_pair = index_set_pair_t(folded_set, folded_frame);
01207     using basis_pair_t = std::pair<const index_set_pair_t, basis_matrix_t*>;
01208     if (use_cache)
01209     {
01210         const auto basis_it = basis_cache.find(folded_pair);
01211         if (basis_it != basis_cache.end())
01212         {
01213             auto* result_ptr = basis_it->second;
01214             basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01215             return *result_ptr;
01216         }
01217     }
01218     const auto folded_max = folded_frame.max();
01219     const auto folded_min = folded_frame.min();
01220     const auto p = std::max(folded_max, index_t(0));
01221     const auto q = std::max(index_t(-folded_min), index_t(0));
01222     const auto* e = (gen::generator_table<basis_matrix_t>::generator())(p, q);
01223     const auto dim = matrix_index_t(1) << offset_level(p, q);
01224     auto result = matrix::unit<basis_matrix_t>(dim);
01225     for (auto
01226         k = folded_min;
01227         k <= folded_max;
01228         ++k)
01229         if (folded_set[k])
01230             result = matrix::mono_prod(result, e[k]);
01231     if (use_cache)
01232     {
01233         auto* result_ptr = new basis_matrix_t(result);
01234         basis_cache.insert(basis_pair_t(folded_pair, result_ptr));
01235         basis_cache.insert(basis_pair_t(unfolded_pair, result_ptr));
01236     }
01237     return result;
01238 }
01239
01240 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P, const size_t Size
01241 >
01242 inline
01243 static
01244 auto
01245 pade_approx(
01246     const std::array<Scalar_T, Size>& numer,
01247     const std::array<Scalar_T, Size>& denom,
01248     const matrix_multi<Scalar_T, LO, HI, Tune_P>& X) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
01249 {
01250     // Pade' approximation
01251     // Reference: [GW], Section 4.3, pp318-322
01252     // Reference: [GL], Section 11.3, p572-576.
01253
01254     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
01255     using traits_t = numeric_traits<Scalar_T>;
01256
01257     if (X.isnan())
01258         return traits_t::NaN();
01259
01260     // Array size is assumed to be even
01261     const auto nbr_even_powers = Size/2 - 1;
01262
01263     // Create an array of even powers
01264     auto XX = std::vector<multivector_t>(nbr_even_powers);
01265     XX[0] = X * X;
01266     XX[1] = XX[0] * XX[0];
01267     for (auto
01268         k = size_t(2);
01269         k != nbr_even_powers;
01270         ++k)
01271         XX[k] = XX[k-2] * XX[1];
01272
01273     // Calculate numerator N and denominator D
01274     auto N = multivector_t(numer[1]);
01275     for (auto
01276         k = size_t(0);

```

```

01277         k != nbr_even_powers;
01278         ++k)
01279     N += XX[k] * numer[2*k + 3];
01280 N *= X;
01281 N += numer[0];
01282 for (auto
01283     k = size_t(0);
01284     k != nbr_even_powers;
01285     ++k)
01286     N += XX[k] * numer[2*k + 2];
01287 auto D = multivector_t(denom[1]);
01288 for (auto
01289     k = size_t(0);
01290     k != nbr_even_powers;
01291     ++k)
01292     D += XX[k] * denom[2*k + 3];
01293 D *= X;
01294 D += denom[0];
01295 for (auto
01296     k = size_t(0);
01297     k != nbr_even_powers;
01298     ++k)
01299     D += XX[k] * denom[2*k + 2];
01300 return N / D;
01301 }
01302
01303 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01304 inline
01305 static
01306 void
01307 db_step(matrix_multi<Scalar_T,LO,HI,Tune_P>& M, matrix_multi<Scalar_T,LO,HI,Tune_P>& Y)
01308 {
01309     // Reference: [CHKL]
01310     const auto& invM = inv(M);
01311     M = ((M + invM)/Scalar_T(2) + Scalar_T(1)) / Scalar_T(2);
01312     Y *= (invM + Scalar_T(1)) / Scalar_T(2);
01313 }
01314
01315 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01316 static
01317 auto
01318 db_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01319         Scalar_T norm_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 4)) -> const
01320 matrix_multi<Scalar_T,LO,HI,Tune_P>
01321 {
01322     // Reference: [CHKL]
01323     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01324     if (val == Scalar_T(0))
01325         return val;
01326     static const auto sqrt_max_steps = Tune_P::db_sqrt_max_steps;
01327     auto M = val;
01328     auto Y = val;
01329     for (auto
01330         step = 0;
01331         step != sqrt_max_steps && norm(M - Scalar_T(1)) > norm_tol;
01332         ++step)
01333     {
01334         if (Y.isnan())
01335             return numeric_traits<Scalar_T>::NaN();
01336         db_step(M, Y);
01337     }
01338     return Y;
01339 }
01340
01341 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01342 static
01343 auto
01344 cr_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01345         Scalar_T norm_Y_tol=std::pow(std::numeric_limits<Scalar_T>::epsilon(), 1)) -> const
01346 matrix_multi<Scalar_T,LO,HI,Tune_P>
01347 {
01348     // Reference: [MB]
01349     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01350     if (val == Scalar_T(0))
01351         return val;
01352     static const auto sqrt_max_steps = Tune_P::cr_sqrt_max_steps;
01353     auto Z = Scalar_T(2) * (Scalar_T(1) + val);
01354     auto Y = Scalar_T(1) - val;
01355     using traits_t = numeric_traits<Scalar_T>;
01356     auto norm_Y = norm(Y);
01357     for (auto
01358         step = 0;

```

```

01365         step != sqrt_max_steps && norm_Y > norm_Y_tol;
01366         ++step)
01367     {
01368         const auto old_norm_Y = norm_Y;
01369         Y = (-Y / Z) * Y;
01370         norm_Y = norm(Y);
01371         if (Y.isnan() || (norm_Y > old_norm_Y * Scalar_T(2)))
01372             return numeric_traits<Scalar_T>::NaN();
01373
01374         Z += Y * Scalar_T(2);
01375     }
01376     return Z / Scalar_T(4);
01377 }
01378 }
01379
01380 namespace pade {
01381     // Reference: [Z], Padel
01382     template< typename Scalar_T >
01383     struct pade_sqrt_numer
01384     {
01385     {
01386         using array = std::array<Scalar_T, 14>;
01387         static const array number;
01388     };
01389     template< typename Scalar_T >
01390     const typename pade_sqrt_numer<Scalar_T>::array pade_sqrt_numer<Scalar_T>::number =
01391     {
01392         1.0,                27.0/4.0,            81.0/4.0,            2277.0/64.0,
01393         10395.0/256.0,       32319.0/1024.0,       8721.0/512.0,       26163.0/4096.0,
01394         53703.0/32768.0,     36465.0/131072.0,    3861.0/131072.0,    7371.0/4194304.0,
01395         819.0/16777216.0,    27.0/67108864.0
01396     };
01397
01398     // Reference: [Z], Padel
01399     template< typename Scalar_T >
01400     struct pade_sqrt_denom
01401     {
01402     {
01403         using array = std::array<Scalar_T, 14>;
01404         static const array denom;
01405     };
01406     template< typename Scalar_T >
01407     const typename pade_sqrt_denom<Scalar_T>::array pade_sqrt_denom<Scalar_T>::denom =
01408     {
01409         1.0,                25.0/4.0,            69.0/4.0,            1771.0/64.0,
01410         7315.0/256.0,        20349.0/1024.0,       4845.0/512.0,       12597.0/4096.0,
01411         21879.0/32768.0,    12155.0/131072.0,    1001.0/131072.0,    1365.0/4194304.0,
01412         91.0/16777216.0,    1.0/67108864.0
01413     };
01414
01415     template< >
01416     struct pade_sqrt_numer<float>
01417     {
01418         using array = std::array<float, 10>;
01419         static const array number;
01420     };
01421     const typename pade_sqrt_numer<float>::array pade_sqrt_numer<float>::number =
01422     {
01423         1.0,                19.0/4.0,            19.0/2.0,            665.0/64.0,
01424         1729.0/256.0,        2717.0/1024.0,       627.0/1024.0,       627.0/8192.0,
01425         285.0/65536.0,       19.0/262144.0
01426     };
01427     template< >
01428     struct pade_sqrt_denom<float>
01429     {
01430         using array = std::array<float, 10>;
01431         static const array denom;
01432     };
01433     const typename pade_sqrt_denom<float>::array pade_sqrt_denom<float>::denom =
01434     {
01435         1.0,                17.0/4.0,            15.0/2.0,            455.0/64.0,
01436         1001.0/256.0,        1287.0/1024.0,       231.0/1024.0,       165.0/8192.0,
01437         45.0/65536.0,        1.0/262144.0
01438     };
01439
01440     template< >
01441     struct pade_sqrt_numer<long double>
01442     {
01443         using array = std::array<long double, 18>;
01444         static const array number;
01445     };
01446     const typename pade_sqrt_numer<long double>::array pade_sqrt_numer<long double>::number =
01447     {
01448         1.0L,                35.0L/4.0L,            35.0L,                5425.0L/64.0L,
01449         35525.0L/256.0L,     166257.0L/1024.0L,     143325.0L/1024.0L,     740025.0L/8192.0L,
01450         2877875.0L/65536.0L,  4206125.0L/262144.0L,  572033.0L/131072.0L,  1820105.0L/2097152.0L,
01451         1028755.0L/8388608.0L, 395675.0L/33554432.0L, 24225.0L/33554432.0L,  6783.0L/268435456.0L,
01452         1785.0L/4294967296.0L, 35.0L/17179869184.0L
01453     };

```

```

01454     template< >
01455     struct pade_sqrt_denom<long double>
01456     {
01457         using array = std::array<long double, 18>;
01458         static const array denom;
01459     };
01460     const typename pade_sqrt_denom<long double>::array pade_sqrt_denom<long double>::denom =
01461     {
01462         1.0L,                33.0L/4.0L,                31.0L,                4495.0L/64.0L,
01463         27405.0L/256.0L,      118755.0L/1024.0L,      94185.0L/1024.0L,      444015.0L/8192.0L,
01464         1562275.0L/65536.0L,  2042975.0L/262144.0L,  245157.0L/131072.0L,  676039.0L/2097152.0L,
01465         323323.0L/8388608.0L, 101745.0L/33554432.0L, 4845.0L/33554432.0L, 969.0L/268435456.0L,
01466         153.0L/4294967296.0L, 1.0L/17179869184.0L
01467     };
01468
01469 #if defined(_GLUCAT_USE_QD)
01470     template< >
01471     struct pade_sqrt_number<dd_real>
01472     {
01473         using array = std::array<dd_real, 22>;
01474         static const array numer;
01475     };
01476     const typename pade_sqrt_number<dd_real>::array pade_sqrt_number<dd_real>::numer =
01477     {
01478         dd_real("1"),                dd_real("43")/dd_real("4"),
01479         dd_real("215")/dd_real("4"),  dd_real("10621")/dd_real("64"),
01480         dd_real("90687")/dd_real("256"), dd_real("567987")/dd_real("1024"),
01481         dd_real("168861")/dd_real("256"), dd_real("1246355")/dd_real("2048"),
01482         dd_real("7228859")/dd_real("16384"), dd_real("16583853")/dd_real("65536"),
01483         dd_real("7538115")/dd_real("65536"), dd_real("173376645")/dd_real("4194304"),
01484         dd_real("195747825")/dd_real("16777216"), dd_real("171655785")/dd_real("67108864"),
01485         dd_real("14375115")/dd_real("33554432"), dd_real("14375115")/dd_real("268435456"),
01486         dd_real("20764055")/dd_real("4294967296"), dd_real("5167525")/dd_real("17179869184"),
01487         dd_real("206701")/dd_real("17179869184"), dd_real("76153")/dd_real("274877906944"),
01488         dd_real("3311")/dd_real("1099511627776"), dd_real("43")/dd_real("4398046511104")
01489     };
01490     template< >
01491     struct pade_sqrt_denom<dd_real>
01492     {
01493         using array = std::array<dd_real, 22>;
01494         static const array denom;
01495     };
01496     const typename pade_sqrt_denom<dd_real>::array pade_sqrt_denom<dd_real>::denom =
01497     {
01498         dd_real("1"),                dd_real("41")/dd_real("4"),
01499         dd_real("195")/dd_real("4"),  dd_real("9139")/dd_real("64"),
01500         dd_real("73815")/dd_real("256"), dd_real("435897")/dd_real("1024"),
01501         dd_real("121737")/dd_real("256"), dd_real("840565")/dd_real("2048"),
01502         dd_real("4539051")/dd_real("16384"), dd_real("9641775")/dd_real("65536"),
01503         dd_real("4032015")/dd_real("65536"), dd_real("84672315")/dd_real("4194304"),
01504         dd_real("86493225")/dd_real("16777216"), dd_real("67863915")/dd_real("67108864"),
01505         dd_real("5014575")/dd_real("33554432"), dd_real("4345965")/dd_real("268435456"),
01506         dd_real("5311735")/dd_real("4294967296"), dd_real("1081575")/dd_real("17179869184"),
01507         dd_real("33649")/dd_real("17179869184"), dd_real("8855")/dd_real("274877906944"),
01508         dd_real("231")/dd_real("1099511627776"), dd_real("1")/dd_real("4398046511104")
01509     };
01510
01511     template< >
01512     struct pade_sqrt_number<qd_real>
01513     {
01514         using array = std::array<qd_real, 34>;
01515         static const array numer;
01516     };
01517     const typename pade_sqrt_number<qd_real>::array pade_sqrt_number<qd_real>::numer =
01518     {
01519         qd_real("1"),                qd_real("67")/qd_real("4"),
01520         qd_real("134"),              qd_real("43617")/qd_real("64"),
01521         qd_real("633485")/qd_real("256"), qd_real("6992857")/qd_real("1024"),
01522         qd_real("15246721")/qd_real("1024"), qd_real("215632197")/qd_real("8192"),
01523         qd_real("2518145487")/qd_real("65536"),
01524         qd_real("12301285425")/qd_real("262144"),
01525         qd_real("6344873535")/qd_real("131072"),
01526         qd_real("89075432355")/qd_real("2097152"),
01527         qd_real("267226297065")/qd_real("8388608"),
01528         qd_real("687479618945")/qd_real("33554432"),
01529         qd_real("379874182975")/qd_real("33554432"),
01530         qd_real("1443521895305")/qd_real("268435456"),
01531         qd_real("9425348845815")/qd_real("4294967296"),
01532         qd_real("13195488384141")/qd_real("17179869184"),
01533         qd_real("987417498133")/qd_real("4294967296"),
01534         qd_real("8055248011085")/qd_real("137438953472"),
01535         qd_real("6958363175533")/qd_real("549755813888"),
01536         qd_real("5056698705201")/qd_real("219902325552"),
01537         qd_real("766166470485")/qd_real("219902325552"),
01538         qd_real("766166470485")/qd_real("17592186044416"),
01539         qd_real("623623871325")/qd_real("140737488355328"),
01540         qd_real("203123203803")/qd_real("562949953421312"),

```

```

01532     qd_real("6478601247")/qd_real("281474976710656"),
qd_real("5038912081")/qd_real("4503599627370496"),
01533     qd_real("719844583")/qd_real("18014398509481984"),
qd_real("71853815")/qd_real("72057594037927936"),
01534     qd_real("1165197")/qd_real("72057594037927936"),
qd_real("87703")/qd_real("576460752303423488"),
01535     qd_real("12529")/qd_real("18446744073709551616"),
qd_real("67")/qd_real("73786976294838206464")
01536 };
01537 template< >
01538 struct pade_sqrt_denom<qd_real>
01539 {
01540     using array = std::array<qd_real, 34>;
01541     static const array denom;
01542 };
01543 const typename pade_sqrt_denom<qd_real>::array pade_sqrt_denom<qd_real>::denom =
01544 {
01545     qd_real("1"),
01546     qd_real("126"),
01547     qd_real("557845")/qd_real("256"),
01548     qd_real("12515965")/qd_real("1024"),
01549     qd_real("1916797311")/qd_real("65536"),
qd_real("8996462475")/qd_real("262144"),
01550     qd_real("4450881435")/qd_real("131072"),
qd_real("59826782925")/qd_real("2097152"),
01551     qd_real("171503444385")/qd_real("8388608"),
qd_real("420696483235")/qd_real("33554432"),
01552     qd_real("221120793075")/qd_real("33554432"),
qd_real("797168807855")/qd_real("268435456"),
01553     qd_real("4923689695575")/qd_real("4294967296"),
qd_real("6499270398159")/qd_real("17179869184"),
01554     qd_real("456864812569")/qd_real("4294967296"),
qd_real("3486599885395")/qd_real("137438953472"),
01555     qd_real("2804116503573")/qd_real("549755813888"),
qd_real("1886827875075")/qd_real("2199023255552"),
01556     qd_real("263012370465")/qd_real("2199023255552"),
qd_real("240141729555")/qd_real("17592186044416"),
01557     qd_real("176848560525")/qd_real("140737488355328"),
qd_real("51538723353")/qd_real("562949953421312"),
01558     qd_real("1450433115")/qd_real("281474976710656"),
qd_real("977699359")/qd_real("4503599627370496"),
01559     qd_real("118183439")/qd_real("18014398509481984"),
qd_real("9652005")/qd_real("72057594037927936"),
01560     qd_real("121737")/qd_real("72057594037927936"),
qd_real("6545")/qd_real("576460752303423488"),
01561     qd_real("561")/qd_real("18446744073709551616"),
qd_real("1")/qd_real("73786976294838206464")
01562 };
01563 #endif
01564 }
01565
01566 namespace glucat
01567 {
01568     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01569     auto
01570     matrix_sqrt(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01571                 const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01572                 const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01573     {
01574         // Reference: [GW], Section 4.3, pp318-322
01575         // Reference: [GL], Section 11.3, p572-576
01576         // Reference: [Z], Padel
01577
01578         using traits_t = numeric_traits<Scalar_T>;
01579
01580         if (val.isnan())
01581             return traits_t::NaN();
01582
01583         const auto scr_val = val.scalar();
01584         if (val == scr_val)
01585         {
01586             if (scr_val < Scalar_T(0))
01587                 return i * traits_t::sqrt(-scr_val);
01588             else
01589                 return traits_t::sqrt(scr_val);
01590         }
01591     }
01592
01593     // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01594     const auto scale =
01595         (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < Scalar_T(1))
01596         ? scr_val
01597         : (scr_val < Scalar_T(0))
01598         ? -abs(val)
01599         : abs(val);
01600     const auto sqrt_scale = traits_t::sqrt(traits_t::abs(scale));
01601     if (traits_t::isNaN_or_isInf(sqrt_scale))
01602         return traits_t::NaN();

```

```

01603
01604     using multivector_t = matrix_multi<Scalar_T, LO, HI, Tune_P>;
01605     auto rescale = multivector_t(sqrt_scale);
01606     if (scale < Scalar_T(0))
01607         rescale = i * sqrt_scale;
01608
01609     const auto& unitval = val / scale;
01610     static const auto max_norm = Scalar_T(1.0/4.0);
01611     auto use_approx_sqrt = true;
01612     auto use_cr_sqrt = false;
01613     auto scaled_result = multivector_t();
01614 #if defined(_GLUCAT_USE_EIGENVALUES)
01615     static const auto sqrt_2 = traits_t::sqrt(Scalar_T(2));
01616     if (level == 0)
01617     {
01618         using matrix_t = typename multivector_t::matrix_t;
01619
01620         // What kind of eigenvalues does the matrix contain?
01621         const auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
01622         const index_t next_level =
01623             (genus.m_is_singular)
01624             ? level
01625             : level + 1;
01626         switch (genus.m_eig_case)
01627         {
01628             case matrix::neg_real_eigs:
01629                 scaled_result = matrix_sqrt(-i * unitval, i, next_level) * (i + Scalar_T(1)) / sqrt_2;
01630                 use_approx_sqrt = false;
01631                 break;
01632             case matrix::both_eigs:
01633                 {
01634                     const auto safe_arg = genus.m_safe_arg;
01635                     scaled_result = matrix_sqrt(exp(i*safe_arg) * unitval, i, next_level) * exp(-i*safe_arg) /
01636                     Scalar_T(2));
01637                     use_approx_sqrt = false;
01638                     break;
01639                 default:
01640                     break;
01641                 }
01642             use_cr_sqrt = genus.m_is_singular;
01643         }
01644 #endif
01645         if (use_approx_sqrt)
01646         {
01647             scaled_result =
01648                 (norm(unitval - Scalar_T(1)) < max_norm)
01649                 // Pade' approximation of square root
01650                 ? pade_approx(pade::pade_sqrt_numer<Scalar_T>::numer,
01651                             pade::pade_sqrt_denom<Scalar_T>::denom,
01652                             unitval - Scalar_T(1))
01653                 // Product form of Denman-Beavers square root iteration
01654                 : (use_cr_sqrt)
01655                   ? cr_sqrt(unitval)
01656                   : db_sqrt(unitval);
01657         }
01658         return (scaled_result.isnan() ||
01659             !approx_equal(pow(scaled_result, 2), unitval))
01660             ? traits_t::NaN()
01661             : scaled_result * rescale;
01662     }
01663
01664     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01665     auto
01666     sqrt(const matrix_multi<Scalar_T, LO, HI, Tune_P>& val, const matrix_multi<Scalar_T, LO, HI, Tune_P>& i,
01667         bool prechecked) -> const matrix_multi<Scalar_T, LO, HI, Tune_P>
01668     {
01669         // Reference: [GW], Section 4.3, pp318-322
01670         // Reference: [GL], Section 11.3, p572-576
01671         // Reference: [Z], Pade1
01672
01673         using traits_t = numeric_traits<Scalar_T>;
01674
01675         if (val.isnan())
01676             return traits_t::NaN();
01677
01678         check_complex(val, i, prechecked);
01679
01680         switch (Tune_P::function_precision)
01681         {
01682             case precision_demoted:
01683                 {
01684                     using demoted_scalar_t = typename traits_t::demoted::type;
01685                     using demoted_multivector_t = matrix_multi<demoted_scalar_t, LO, HI, Tune_P>;
01686
01687                     const auto& demoted_val = demoted_multivector_t(val);
01688                     const auto& demoted_i = demoted_multivector_t(i);

```



```

01689         return matrix_sqrt(demoted_val, demoted_i, 0);
01690     }
01691     break;
01692     case precision_promoted:
01693     {
01694         using promoted_scalar_t = typename traits_t::promoted::type;
01695         using promoted_multivector_t = matrix_multi<promoted_scalar_t, LO, HI, Tune_P>;
01696
01697         const auto& promoted_val = promoted_multivector_t(val);
01698         const auto& promoted_i = promoted_multivector_t(i);
01699
01700         return matrix_sqrt(promoted_val, promoted_i, 0);
01701     }
01702     break;
01703     default:
01704         return matrix_sqrt(val, i, 0);
01705     }
01706 }
01707 }
01708 }
01709
01710 namespace pade {
01711     // Reference: [Z], Padel
01712     template< typename Scalar_T >
01713     struct pade_log_number
01714     {
01715         using array = std::array<Scalar_T, 14>;
01716         static const array number;
01717     };
01718     template< typename Scalar_T >
01719     const typename pade_log_number<Scalar_T>::array pade_log_number<Scalar_T>::number =
01720     {
01721         {
01722             0.0,          1.0,          6.0,          4741.0/300.0,
01723             1441.0/60.0,  107091.0/4600.0,  8638.0/575.0,  263111.0/40250.0,
01724             153081.0/80500.0,  395243.0/1101240.0,  28549.0/688275.0,  605453.0/228813200.0,
01725             785633.0/10296594000.0,  1145993.0/1873980108000.0
01726         };
01727     };
01728     // Reference: [Z], Padel
01729     template< typename Scalar_T >
01730     struct pade_log_denom
01731     {
01732         using array = std::array<Scalar_T, 14>;
01733         static const array denom;
01734     };
01735     template< typename Scalar_T >
01736     const typename pade_log_denom<Scalar_T>::array pade_log_denom<Scalar_T>::denom =
01737     {
01738         {
01739             1.0,          13.0/2.0,          468.0/25.0,          1573.0/50.0,
01740             1573.0/46.0,  11583.0/460.0,          10296.0/805.0,  2574.0/575.0,
01741             11583.0/10925.0,  143.0/874.0,          572.0/37145.0,  117.0/148580.0,
01742             13.0/742900.0,  1.0/10400600.0
01743         };
01744     };
01745     template< >
01746     struct pade_log_number<float>
01747     {
01748         using array = std::array<float, 10>;
01749         static const array number;
01750     };
01751     const typename pade_log_number<float>::array pade_log_number<float>::number =
01752     {
01753         {
01754             0.0,          1.0,          4.0,          1337.0/204.0,
01755             385.0/68.0,  1879.0/680.0,  193.0/255.0,  197.0/1820.0,
01756             419.0/61880.0,  7129.0/61261200.0
01757         };
01758     };
01759     template< >
01760     struct pade_log_denom<float>
01761     {
01762         using array = std::array<float, 10>;
01763         static const array denom;
01764     };
01765     const typename pade_log_denom<float>::array pade_log_denom<float>::denom =
01766     {
01767         {
01768             1.0,          9.0/2.0,          144.0/17.0,  147.0/17.0,
01769             441.0/85.0,  63.0/34.0,          84.0/221.0,  9.0/221.0,
01770             9.0/4862.0,  1.0/48620.0
01771         };
01772     };
01773     template< >
01774     struct pade_log_number<long double>
01775     {
01776         using array = std::array<long double, 18>;
01777         static const array number;
01778     };
01779     const typename pade_log_number<long double>::array pade_log_number<long double>::number =
01780     {

```

```

01778         0.0L,                                1.0L,                                8.0L,
01779         3835.0L/132.0L,                        11363807.0L/122760.0L,                162981.0L/1705.0L,
01780         9036157.0L/125860.0L,
01780         18009875.0L/453096.0L,                44211925.0L/2718576.0L,                4149566.0L/849555.0L,
01781         16973929.0L/16020180.0L,
01781         172459.0L/1068012.0L,                116317061.0L/7025382936.0L,                19679783.0L/18441630207.0L,
01782         23763863.0L/614721006900.0L,
01782         50747.0L/79318839600.0L, 42142223.0L/14295951736466400.0L
01783     };
01784     template< >
01785     struct pade_log_denom<long double>
01786     {
01787         using array = std::array<long double, 18>;
01788         static const array denom;
01789     };
01790     const typename pade_log_denom<long double>::array pade_log_denom<long double>::denom =
01791     {
01792         1.0L,                                17.0L/2.0L,                                1088.0L/33.0L,
01793         850.0L/11.0L,                        41650.0L/341.0L,                        140777.0L/1023.0L,                1126216.0L/9889.0L,
01794         63206.0L/899.0L,
01794         790075.0L/24273.0L,                60775.0L/5394.0L,                        38896.0L/13485.0L,
01795         21658.0L/40455.0L,
01795         21658.0L/310155.0L,                4165.0L/682341.0L,                        680.0L/2047023.0L,
01796         34.0L/3411705.0L,
01796         17.0L/129644790.0L,                1.0L/2333606220
01797     };
01798 #if defined(_GLUCAT_USE_QD)
01799     template< >
01800     struct pade_log_numer<dd_real>
01801     {
01802         using array = std::array<dd_real, 22>;
01803         static const array numer;
01804     };
01805     const typename pade_log_numer<dd_real>::array pade_log_numer<dd_real>::numer =
01806     {
01807         dd_real("0"),                        dd_real("1"),
01808         dd_real("10"),                        dd_real("22781")/dd_real("492"),
01809         dd_real("21603")/dd_real("164"),      dd_real("5492649")/dd_real("21320"),
01810         dd_real("978724")/dd_real("2665"),    dd_real("4191605")/dd_real("10619"),
01811         dd_real("12874933")/dd_real("39442"), dd_real("11473457")/dd_real("54612"),
01812         dd_real("2406734")/dd_real("22755"),  dd_real("166770367")/dd_real("4004880"),
01813         dd_real("30653165")/dd_real("2402928"), dd_real("647746389")/dd_real("215195552"),
01814         dd_real("25346331")/dd_real("47074027"), dd_real("278270613")/dd_real("3900419380"),
01815         dd_real("105689791")/dd_real("15601677520"), dd_real("606046475")/dd_real("1379188292768"),
01816         dd_real("969715")/dd_real("53502994116"), dd_real("11098301")/dd_real("26204577562592"),
01817         dd_real("118999")/dd_real("26204577562592"), dd_real("18858053")/dd_real("1392249205900512960")
01818     };
01819     template< >
01820     struct pade_log_denom<dd_real>
01821     {
01822         using array = std::array<dd_real, 22>;
01823         static const array denom;
01824     };
01825     const typename pade_log_denom<dd_real>::array pade_log_denom<dd_real>::denom =
01826     {
01827         dd_real("1"),                        dd_real("21")/dd_real("2"),
01828         dd_real("2100")/dd_real("41"),        dd_real("12635")/dd_real("82"),
01829         dd_real("341145")/dd_real("1066"),    dd_real("1037799")/dd_real("2132"),
01830         dd_real("11069856")/dd_real("19721"), dd_real("9883800")/dd_real("19721"),
01831         dd_real("6918660")/dd_real("19721"),  dd_real("293930")/dd_real("1517"),
01832         dd_real("1410864")/dd_real("16687"),  dd_real("88179")/dd_real("3034"),
01833         dd_real("734825")/dd_real("94054"),    dd_real("305235")/dd_real("188108"),
01834         dd_real("348840")/dd_real("1363783"), dd_real("40698")/dd_real("1363783"),
01835         dd_real("6783")/dd_real("2727566"),    dd_real("9975")/dd_real("70916716"),
01836         dd_real("266")/dd_real("53187537"),    dd_real("7")/dd_real("70916716"),
01837         dd_real("7")/dd_real("8155422340"),    dd_real("1")/dd_real("538257874440")
01838     };
01839     template< >
01840     struct pade_log_numer<qd_real>
01841     {
01842         using array = std::array<qd_real, 34>;
01843         static const array numer;
01844     };
01845     const typename pade_log_numer<qd_real>::array pade_log_numer<qd_real>::numer =
01846     {
01847         qd_real("0"),                        qd_real("1"),
01848         qd_real("16"),
01849         qd_real("95201")/qd_real("780"),
01850         qd_real("30721")/qd_real("52"),
01851         qd_real("7416257")/qd_real("3640"),
01852         qd_real("1039099")/qd_real("195"),
01853         qd_real("6097772319")/qd_real("555100"),
01854         qd_real("1564058073")/qd_real("85400"),
01855         qd_real("30404640205")/qd_real("1209264"),

```

```

01853         qd_real("725351278")/qd_real("25193"),
qd_real("4092322670789")/qd_real("147429436"),
01854         qd_real("4559713849589")/qd_real("201040140"),
qd_real("5049361751189")/qd_real("320023080"),
01855         qd_real("74979677195")/qd_real("8000577"),
qd_real("16569850691873")/qd_real("3481514244"),
01856         qd_real("1065906022369")/qd_real("515779888"),
qd_real("335956770855841")/qd_real("438412904800"),
01857         qd_real("1462444287585964")/qd_real("6041877844275"),
qd_real("397242326339851")/qd_real("6122436215532"),
01858         qd_real("64211291334131")/qd_real("4373168725380"),
qd_real("142322343550859")/qd_real("51080680851480"),
01859         qd_real("154355972958659")/qd_real("351179680853925"),
qd_real("167483568676259")/qd_real("2937139148960100"),
01860         qd_real("4230788929433")/qd_real("704913395750424"),
qd_real("197968763176019")/qd_real("392923948371995600"),
01861         qd_real("10537522306718")/qd_real("319250708052246425"),
qd_real("236648286272519")/qd_real("144249197475035425500"),
01862         qd_real("260715545088119")/qd_real("4375558990076074573500"),
qd_real("289596255666839")/qd_real("192874640282553367199880"),
01863         qd_real("8802625510547")/qd_real("361639950529787563499775"),
qd_real("373831661521439")/qd_real("1659204093030665341336967700"),
01864         qd_real("446033437968239")/qd_real("464577146048586295574350956000"),
qd_real("53676090078349")/qd_real("47386868896955802148583797512000")
01865     };
01866     template< >
01867     struct pade_log_denom<qd_real>
01868     {
01869         using array = std::array<qd_real, 34>;
01870         static const array denom;
01871     };
01872     const typename pade_log_denom<qd_real>::array pade_log_denom<qd_real>::denom =
01873     {
01874         qd_real("1"),
qd_real("33")/qd_real("2"),
01875         qd_real("8448")/qd_real("65"),
qd_real("42284")/qd_real("65"),
01876         qd_real("211420")/qd_real("91"),
qd_real("573562")/qd_real("91"),
01877         qd_real("32119472")/qd_real("2379"),
qd_real("92917044")/qd_real("3965"),
01878         qd_real("603960786")/qd_real("17995"),
qd_real("144626625")/qd_real("3599"),
01879         qd_real("2776831200")/qd_real("68381"),
qd_real("16692542100")/qd_real("478667"),
01880         qd_real("12241197540")/qd_real("478667"),
qd_real("1098569010")/qd_real("68381"),
01881         qd_real("31387686000")/qd_real("3624193"),
qd_real("9939433900")/qd_real("2479711"),
01882         qd_real("67091178825")/qd_real("42155087"),
qd_real("2683647153")/qd_real("4959422"),
01883         qd_real("19083713088")/qd_real("121505839"),
qd_real("4708152900")/qd_real("121505839"),
01884         qd_real("941630580")/qd_real("116546417"),
qd_real("88704330")/qd_real("62755763"),
01885         qd_real("12902448")/qd_real("62755763"),
qd_real("1542684")/qd_real("62755763"),
01886         qd_real("6427850")/qd_real("2698497809"),
qd_real("3471039")/qd_real("18889484663"),
01887         qd_real("8544096")/qd_real("774468871183"),
qd_real("39556")/qd_real("79027435835"),
01888         qd_real("118668")/qd_real("7191496660985"),
qd_real("10230")/qd_real("27327687311743"),
01889         qd_real("5456")/qd_real("1011124430534491"),
qd_real("44")/qd_real("1011124430534491"),
01890         qd_real("11")/qd_real("70778710137414370"),
qd_real("1")/qd_real("7219428434016265740")
01891     };
01892 #endif
01893 }
01894
01895 namespace glucat{
01897     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01898     static
01899     auto
01900     pade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
matrix_multi<Scalar_T,LO,HI,Tune_P>
01901     {
01902         // Reference: [GW], Section 4.3, pp318-322
01903         // Reference: [CHKL]
01904         // Reference: [GL], Section 11.3, p572-576
01905         // Reference: [Z], Padel
01906
01907         using traits_t = numeric_traits<Scalar_T>;
01908         if (val == Scalar_T(0) || val.isnan())
01909             return traits_t::NaN();
01910         else

```

```

01911         return pade_approx(pade::pade_log_numer<Scalar_T>::numer,
01912                             pade::pade_log_denom<Scalar_T>::denom,
01913                             val - Scalar_T(1));
01914     }
01915
01916     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01917     static
01918     auto
01919     cascade_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const
01920     matrix_multi<Scalar_T,LO,HI,Tune_P>
01921     {
01922         // Reference: [CHKL]
01923         using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
01924         using traits_t = numeric_traits<Scalar_T>;
01925         if (val == Scalar_T(0) || val.isnan())
01926             return traits_t::NaN();
01927
01928         using limits_t = std::numeric_limits<Scalar_T>;
01929         static const auto epsilon = limits_t::epsilon();
01930         static const auto max_inner_norm = traits_t::pow(epsilon, 2);
01931         static const auto max_outer_norm = Scalar_T(6.0/limits_t::digits);
01932         auto Y = val;
01933         auto E = multivector_t(Scalar_T(0));
01934         Scalar_T norm_Y_1;
01935         auto pow_2_outer_step = Scalar_T(1);
01936         auto pow_4_outer_step = Scalar_T(1);
01937         int outer_step;
01938         for (outer_step = 0, norm_Y_1 = norm(Y - Scalar_T(1));
01939             outer_step != Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm;
01940             ++outer_step, norm_Y_1 = norm(Y - Scalar_T(1)))
01941         {
01942             if (Y == Scalar_T(0) || Y.isnan())
01943                 return traits_t::NaN();
01944
01945             // Incomplete product form of Denman-Beavers square root iteration
01946             auto M = Y;
01947             for (auto
01948                 inner_step = 0;
01949                 inner_step != Tune_P::log_max_inner_steps &&
01950                 norm(M - Scalar_T(1)) * pow_4_outer_step > max_inner_norm;
01951                 ++inner_step)
01952                 db_step(M, Y);
01953
01954             E += (M - Scalar_T(1)) * pow_2_outer_step;
01955             pow_2_outer_step *= Scalar_T(2);
01956             pow_4_outer_step *= Scalar_T(4);
01957         }
01958         if (outer_step == Tune_P::log_max_outer_steps && norm_Y_1 * pow_2_outer_step > max_outer_norm)
01959             return traits_t::NaN();
01960         else
01961             return pade_log(Y) * pow_2_outer_step - E;
01962     }
01963
01964     template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
01965     auto
01966     matrix_log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val,
01967               const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
01968               const index_t level) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
01969     {
01970         // Scaled incomplete square root cascade and scaled Pade' approximation of log
01971         // Reference: [CHKL]
01972
01973         using traits_t = numeric_traits<Scalar_T>;
01974         if (val == Scalar_T(0) || val.isnan())
01975             return traits_t::NaN();
01976
01977         static const auto pi = traits_t::pi();
01978         const auto scr_val = val.scalar();
01979         if (val == scr_val)
01980         {
01981             if (scr_val < Scalar_T(0))
01982                 return i * pi + traits_t::log(-scr_val);
01983             else
01984                 return traits_t::log(scr_val);
01985         }
01986
01987         // Scale val towards abs(A) == 1 or towards A == 1 as appropriate
01988         const auto max_norm = Scalar_T(1.0/9.0);
01989         const auto scale =
01990             (scr_val != Scalar_T(0) && norm(val/scr_val - Scalar_T(1)) < max_norm)
01991             ? scr_val
01992             : (scr_val < Scalar_T(0))
01993               ? -abs(val)
01994               : abs(val);
01995         if (scale == Scalar_T(0))
01996             return traits_t::NaN();
01997     }
01998

```

```

01999     using multivector_t = matrix_multi<Scalar_T,LO,HI,Tune_P>;
02000     const auto log_scale = traits_t::log(traits_t::abs(scale));
02001     auto rescale = multivector_t(log_scale);
02002     if (scale < Scalar_T(0))
02003         rescale = i * pi + log_scale;
02004     const auto unitval = val/scale;
02005     if (inv(unitval).isnan())
02006         return traits_t::NaN();
02007
02008 #if defined(_GLUCAT_USE_EIGENVALUES)
02009     auto scaled_result = multivector_t();
02010     if (level == 0)
02011     {
02012         using matrix_t = typename multivector_t::matrix_t;
02013
02014         // What kind of eigenvalues does the matrix contain?
02015         auto genus = matrix::classify_eigenvalues(unitval.m_matrix);
02016         switch (genus.m_eig_case)
02017         {
02018             case matrix::neg_real_eigs:
02019                 scaled_result = matrix_log(-i * unitval, i, level + 1) + i * pi/Scalar_T(2);
02020                 break;
02021             case matrix::both_eigs:
02022             {
02023                 const Scalar_T safe_arg = genus.m_safe_arg;
02024                 scaled_result = matrix_log(exp(i*safe_arg) * unitval, i, level + 1) - i * safe_arg;
02025             }
02026             break;
02027             default:
02028                 scaled_result = cascade_log(unitval);
02029                 break;
02030         }
02031     }
02032     else
02033         scaled_result = cascade_log(unitval);
02034 #else
02035     auto scaled_result = cascade_log(unitval);
02036 #endif
02037     return (scaled_result.isnan())
02038         ? traits_t::NaN()
02039         : scaled_result + rescale;
02040 }
02041
02042 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02043 auto
02044 log(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val, const matrix_multi<Scalar_T,LO,HI,Tune_P>& i,
02045 bool prechecked) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>
02046 {
02047     using traits_t = numeric_traits<Scalar_T>;
02048
02049     if (val == Scalar_T(0) || val.isnan())
02050         return traits_t::NaN();
02051
02052     check_complex(val, i, prechecked);
02053
02054     switch (Tune_P::function_precision)
02055     {
02056     case precision_demoted:
02057     {
02058         using demoted_scalar_t = typename traits_t::demoted::type;
02059         using demoted_multivector_t = matrix_multi<demoted_scalar_t,LO,HI,Tune_P>;
02060
02061         const auto& demoted_val = demoted_multivector_t(val);
02062         const auto& demoted_i = demoted_multivector_t(i);
02063
02064         return matrix_log(demoted_val, demoted_i, 0);
02065     }
02066     break;
02067     case precision_promoted:
02068     {
02069         using promoted_scalar_t = typename traits_t::promoted::type;
02070         using promoted_multivector_t = matrix_multi<promoted_scalar_t,LO,HI,Tune_P>;
02071
02072         const auto& promoted_val = promoted_multivector_t(val);
02073         const auto& promoted_i = promoted_multivector_t(i);
02074
02075         return matrix_log(promoted_val, promoted_i, 0);
02076     }
02077     break;
02078     default:
02079         return matrix_log(val, i, 0);
02080     }
02081 }
02082
02083 template< typename Scalar_T, const index_t LO, const index_t HI, typename Tune_P >
02084 auto
02085 exp(const matrix_multi<Scalar_T,LO,HI,Tune_P>& val) -> const matrix_multi<Scalar_T,LO,HI,Tune_P>

```

```

02087 {
02088     using traits_t = numeric_traits<Scalar_T>;
02089     if (val.isnan())
02090         return traits_t::NaN();
02091
02092     const auto scr_val = val.scalar();
02093     if (val == scr_val)
02094         return traits_t::exp(scr_val);
02095
02096     switch (Tune_P::function_precision)
02097     {
02098     case precision_demoted:
02099     {
02100         using demoted_scalar_t = typename traits_t::demoted::type;
02101         using demoted_multivector_t = matrix_multi<demoted_scalar_t, LO, HI, Tune_P>;
02102
02103         const auto& demoted_val = demoted_multivector_t(val);
02104         return clifford_exp(demoted_val);
02105     }
02106     break;
02107     case precision_promoted:
02108     {
02109         using promoted_scalar_t = typename traits_t::promoted::type;
02110         using promoted_multivector_t = matrix_multi<promoted_scalar_t, LO, HI, Tune_P>;
02111
02112         const auto& promoted_val = promoted_multivector_t(val);
02113         return clifford_exp(promoted_val);
02114     }
02115     break;
02116     default:
02117         return clifford_exp(val);
02118     }
02119 }
02120 }
02121 #endif // _GLUCAT_MATRIX_MULTI_IMP_H

```

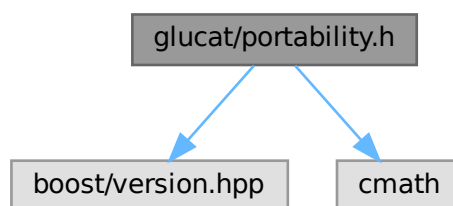
## 7.39 glucat/portability.h File Reference

```

#include <boost/version.hpp>
#include <cmath>

```

Include dependency graph for portability.h:



[illegible]

- `#define _GLUCAT_ISNAN(x) (x != x)`
- `#define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))`
- `#define UBLAS_ABS abs`
- `#define UBLAS_SQRT sqrt`

#### 7.39.1.1 \_GLUCAT\_ISINF

Definition at line 43 of file portability.h.

Referenced by `glucat::numeric_traits< Scalar_T >::isInf()`.

Definition at line 42 of file portability.h.

Referenced by `glucat::numeric_traits< Scalar_T >::isNaN()`.

Definition at line 51 of file portability.h.

### 7.39.1.4 UBLAS\_SQRT

```
#define UBLAS_SQRT sqrt
```

Definition at line 52 of file [portability.h](#).

## 7.40 portability.h

[Go to the documentation of this file.](#)

```
00001 #ifndef _GLUCAT_PORTABILITY_H
00002 #define _GLUCAT_PORTABILITY_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   portability.h : Work around non-standard compilers and libraries
00006   -----
00007   begin                : Sun 2001-08-18
00008   copyright            : (C) 2001-2016 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030   *****/
00031   See also Arvind Raja's original header comments in glucat.h
00032   *****/
00033
00034 #include <boost/version.hpp>
00035 #include <cmath>
00036
00037 // Workaround for isnan and isinf
00038 #if __cplusplus > 199711L
00039 # define _GLUCAT_ISNAN(x) (std::isnan(x))
00040 # define _GLUCAT_ISINF(x) (std::isinf(x))
00041 #else
00042 # define _GLUCAT_ISNAN(x) (x != x)
00043 # define _GLUCAT_ISINF(x) (!_GLUCAT_ISNAN(x) && _GLUCAT_ISNAN(x-x))
00044 #endif
00045
00046 // Workaround for abs and sqrt
00047 #if BOOST_VERSION >= 103400
00048 # define UBLAS_ABS type_abs
00049 # define UBLAS_SQRT type_sqrt
00050 #else
00051 # define UBLAS_ABS abs
00052 # define UBLAS_SQRT sqrt
00053 #endif
00054
00055 // Use with Cygwin gcc to obtain __WORDSIZE
00056 #if defined(HAVE_BITS_WORDSIZE_H)
00057 # include <bits/wordsize.h>
00058 #endif
00059
00060 #endif // _GLUCAT_PORTABILITY_H
```

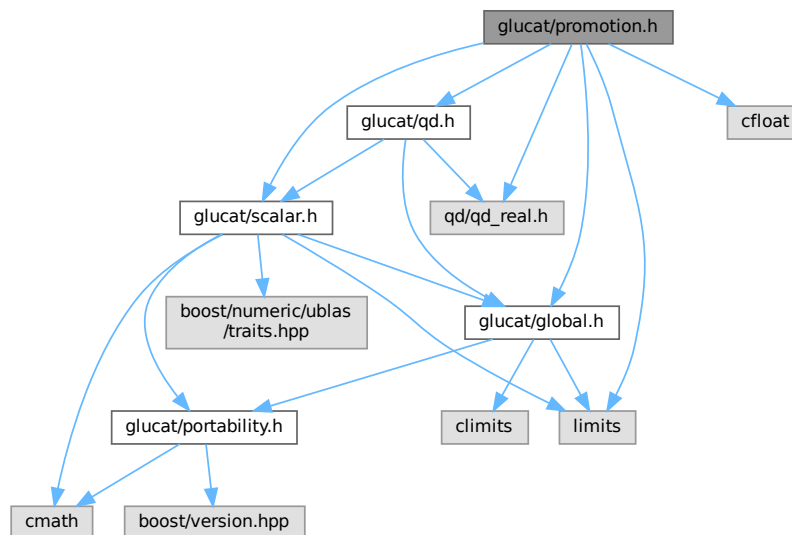
## 7.41 glucat/promotion.h File Reference

```
#include "glucat/global.h"
#include "glucat/scalar.h"
```

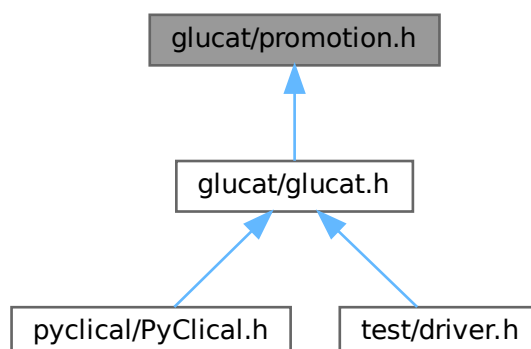


```
#include "glucat/qd.h"
#include <cfloat>
#include <limits>
#include <qd/qd_real.h>
```

Include dependency graph for promotion.h:



This graph shows which files directly or indirectly include this file:



## Classes

- struct [glucat::numeric\\_traits< Scalar\\_T >::promoted](#)  
*Extra traits which extend numeric limits.*
- struct [glucat::numeric\\_traits< Scalar\\_T >::demoted](#)  
*Demoted type for long double.*

## Namespaces

- namespace [glucat](#)

## 7.42 promotion.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_PROMOTION_H
00002 #define _GLUCAT_PROMOTION_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     promotion.h : Define promotion and demotion for specific scalar types
00006     -----
00007     begin                : 2021-11-13
00008     copyright            : (C) 2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036 #include "glucat/qd.h"
00037
00038 #include <float>
00039 #include <limits>
00040
00041 #if defined(_GLUCAT_USE_QD)
00042 #include <qd/qd_real.h>
00043 #endif
00044
00045 namespace glucat
00046 {
00047     // Reference: [AA], 2.4, p. 30-31
00048
00049     #if !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00050     #if DBL_MANT_DIG < LDBL_MANT_DIG
00051
00052         template<>
00053         struct
00054             numeric_traits<double>::
00055             promoted {using type = long double;};
00056
00057         template<>
00058         struct
00059             numeric_traits<long double>::
00060             demoted {using type = double;};
00061     #else
00062
00063         template<>
00064         struct
00065             numeric_traits<double>::
00066             promoted {using type = double;};
00067
00068         template<>
00069         struct
00070             numeric_traits<long double>::
00071             demoted {using type = float;};
00072     #endif
00073
00074     }
00075
00076 }
```

```

00079
00080 # endif // DBL_MANT_DIG < LDBL_MANT_DIG
00081
00082 template<>
00083 struct
00084     numeric_traits<long double>::
00085     promoted {using type = long double;};
00086
00087 #else
00088
00089 # if (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00090
00091     template<>
00092     struct
00093         numeric_traits<double>::
00094         promoted {using type = long double;};
00095
00096     template<>
00097     struct
00098         numeric_traits<long double>::
00099         demoted {using type = double;};
00100
00101     template<>
00102     struct
00103         numeric_traits<long double>::
00104         promoted {using type = dd_real;};
00105
00106     template<>
00107     struct
00108         numeric_traits<dd_real>::
00109         demoted {using type = long double;};
00110
00111     template<>
00112     struct
00113         numeric_traits<dd_real>::
00114         promoted {using type = qd_real;};
00115
00116     template<>
00117     struct
00118         numeric_traits<qd_real>::
00119         demoted {using type = dd_real;};
00120
00121 # elif (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00122
00123     template<>
00124     struct
00125         numeric_traits<double>::
00126         promoted {using type = dd_real;};
00127
00128     template<>
00129     struct
00130         numeric_traits<long double>::
00131         demoted {using type = float;};
00132
00133     template<>
00134     struct
00135         numeric_traits<long double>::
00136         promoted {using type = dd_real;};
00137
00138     template<>
00139     struct
00140         numeric_traits<dd_real>::
00141         demoted {using type = double;};
00142
00143     template<>
00144     struct
00145         numeric_traits<dd_real>::
00146         promoted {using type = qd_real;};
00147
00148     template<>
00149     struct
00150         numeric_traits<qd_real>::
00151         demoted {using type = dd_real;};
00152
00153 # else
00154
00155     template<>
00156     struct
00157         numeric_traits<double>::
00158         promoted {using type = dd_real;};
00159
00160     template<>
00161     struct
00162         numeric_traits<dd_real>::
00163         demoted {using type = double;};
00164
00165     template<>
00166     struct
00167         numeric_traits<dd_real>::
00168         demoted {using type = double;};
00169
00170     template<>
00171     struct
00172         numeric_traits<dd_real>::
00173         demoted {using type = double;};
00174
00175     template<>
00176     struct
00177         numeric_traits<dd_real>::
00178         demoted {using type = double;};
00179
00180     template<>

```

```

00182 struct
00183 numeric_traits<dd_real>::
00184 promoted {using type = long double;};
00185
00187 template<>
00188 struct
00189 numeric_traits<long double>::
00190 demoted {using type = dd_real;};
00191
00193 template<>
00194 struct
00195 numeric_traits<long double>::
00196 promoted {using type = qd_real;};
00197
00199 template<>
00200 struct
00201 numeric_traits<qd_real>::
00202 demoted {using type = long double;};
00203
00204 # endif // (DBL_MANT_DIG < LDBL_MANT_DIG) && (LDBL_MANT_DIG < DBL_MANT_DIG*2)
00205
00207 template<>
00208 struct
00209 numeric_traits<qd_real>::
00210 promoted {using type = qd_real;};
00211
00212 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00213
00214 } // namespace glucat
00215
00216 #endif // _GLUCAT_PROMOTION_H

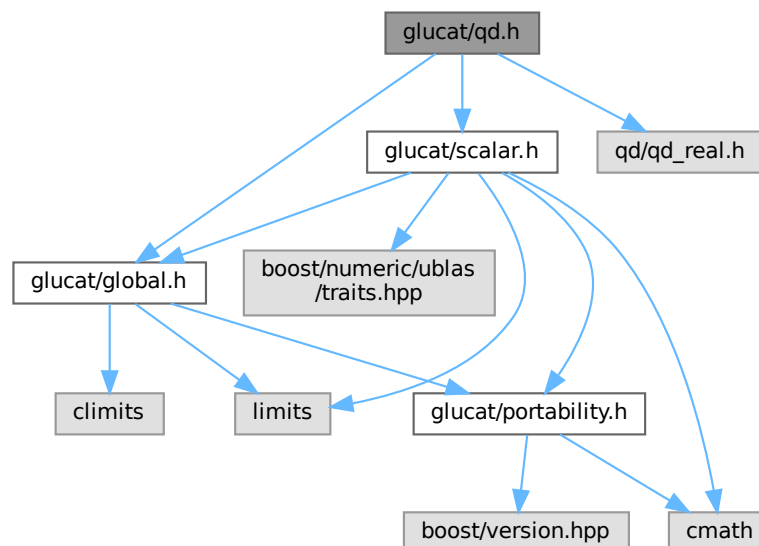
```

## 7.43 glucat/qd.h File Reference

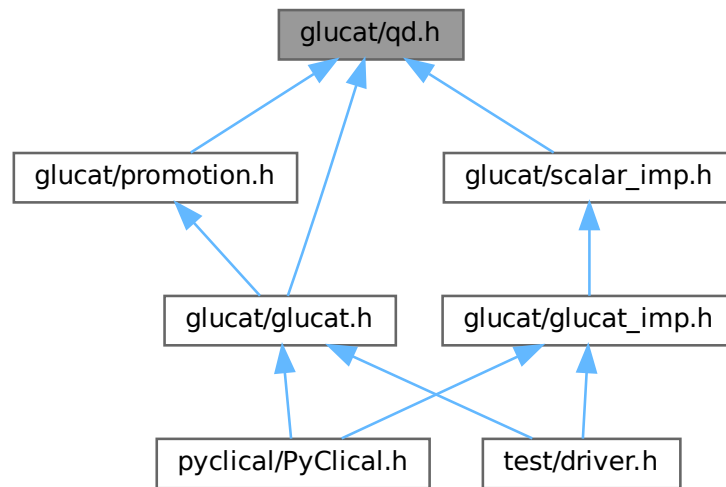
```

#include "glucat/global.h"
#include "glucat/scalar.h"
#include <qd/qd_real.h>
Include dependency graph for qd.h:

```



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace `glucat`

## 7.44 qd.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_QD_H
00002 #define _GLUCAT_QD_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   qd.h : Define functions for dd_real and qd_real as scalar_t
00006   -----
00007   begin           : 2010-03-23
00008   copyright       : (C) 2010-2016 by Paul C. Leopardi
00009   *****/
00010
00011   This library is free software: you can redistribute it and/or modify
00012   it under the terms of the GNU Lesser General Public License as published
00013   by the Free Software Foundation, either version 3 of the License, or
00014   (at your option) any later version.
00015
00016   This library is distributed in the hope that it will be useful,
00017   but WITHOUT ANY WARRANTY; without even the implied warranty of
00018   MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019   GNU Lesser General Public License for more details.
00020
00021   You should have received a copy of the GNU Lesser General Public License
00022   along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024   *****/
00025   This library is based on a prototype written by Arvind Raja and was
00026   licensed under the LGPL with permission of the author. See Arvind Raja,
00027   "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028   in Ablamowicz, Lounesto and Parra (eds.)
00029   "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030   *****/
00031   See also Arvind Raja's original header comments and references in glucat.h
00032   *****/
00033

```

```

00034 #include "glucat/global.h"
00035 #include "glucat/scalar.h"
00036
00037 #if defined(_GLUCAT_USE_QD)
00038 # include <qd/qd_real.h>
00039 #endif
00040
00041 namespace glucat
00042 {
00043     // Reference: [AA], 2.4, p. 30-31
00044
00045     #if defined(_GLUCAT_USE_QD) && defined(QD_API)
00046     # define _GLUCAT_QD_F(_T, _F) \
00047     template<> \
00048     inline \
00049     auto \
00050     numeric_traits<_T>:: \
00051     _F(const _T& val) -> _T \
00052     { return ::_F(val); }
00053
00054     template<>
00055     inline
00056     auto
00057     numeric_traits<dd_real>::
00058     isNaN(const dd_real& val) -> bool
00059     { return val.isnan(); }
00060
00061     template<>
00062     inline
00063     auto
00064     numeric_traits<dd_real>::
00065     isInf(const dd_real& val) -> bool
00066     { return val.isinf(); }
00067
00068     template<>
00069     inline
00070     auto
00071     numeric_traits<dd_real>::
00072     isNaN_or_isInf(const dd_real& val) -> bool
00073     { return val.isnan() || val.isinf(); }
00074
00075     template<>
00076     inline
00077     auto
00078     numeric_traits<dd_real>::
00079     to_int(const dd_real& val) -> int
00080     { return ::to_int(val); }
00081
00082     template<>
00083     inline
00084     auto
00085     numeric_traits<dd_real>::
00086     to_double(const dd_real& val) -> double
00087     { return ::to_double(val); }
00088
00089     template<>
00090     inline
00091     auto
00092     numeric_traits<dd_real>::
00093     fmod(const dd_real& lhs, const dd_real& rhs) -> dd_real
00094     { return ::fmod(lhs, rhs); }
00095
00096     template<>
00097     inline
00098     auto
00099     numeric_traits<dd_real>::
00100     pow(const dd_real& val, int n) -> dd_real
00101     {
00102         if (val == dd_real(0))
00103         {
00104             return
00105                 (n < 0)
00106                 ? NaN()
00107                 : (n == 0)
00108                 ? dd_real(1)
00109                 : dd_real(0);
00110         }
00111         auto result = dd_real(1);
00112         auto power =
00113             (n < 0)
00114             ? dd_real(1)/val
00115             : val;
00116         for (auto
00117             k = std::abs(n);
00118             k != 0;
00119             k /= 2)

```

```

00130     {
00131         if (k % 2)
00132             result *= power;
00133         power *= power;
00134     }
00135     return result;
00136 }
00137
00138 template<>
00139 inline
00140 auto
00141 numeric_traits<dd_real>::
00142 pi() -> dd_real
00143 { return dd_real::_pi; }
00144
00145 template<>
00146 inline
00147 auto
00148 numeric_traits<dd_real>::
00149 ln_2() -> dd_real
00150 { return dd_real::_log2; }
00151
00152 _GLUCAT_QD_F(dd_real, exp)
00153
00154 _GLUCAT_QD_F(dd_real, log)
00155
00156 _GLUCAT_QD_F(dd_real, cos)
00157
00158 _GLUCAT_QD_F(dd_real, acos)
00159
00160 _GLUCAT_QD_F(dd_real, cosh)
00161
00162 _GLUCAT_QD_F(dd_real, sinh)
00163
00164 _GLUCAT_QD_F(dd_real, atan)
00165
00166 _GLUCAT_QD_F(dd_real, tanh)
00167
00168 template<>
00169 inline
00170 auto
00171 numeric_traits<qd_real>::
00172 isNaN(const qd_real& val) -> bool
00173 { return val.isnan(); }
00174
00175 template<>
00176 inline
00177 auto
00178 numeric_traits<qd_real>::
00179 isInf(const qd_real& val) -> bool
00180 { return val.isinf(); }
00181
00182 template<>
00183 inline
00184 auto
00185 numeric_traits<qd_real>::
00186 isNaN_or_isInf(const qd_real& val) -> bool
00187 { return val.isnan() || val.isinf(); }
00188
00189 template<>
00190 inline
00191 auto
00192 numeric_traits<qd_real>::
00193 to_int(const qd_real& val) -> int
00194 { return ::to_int(val); }
00195
00196 template<>
00197 inline
00198 auto
00199 numeric_traits<qd_real>::

```

```

00224 to_double(const qd_real& val) -> double
00225 { return ::to_double(val); }
00226
00228 template<>
00229 inline
00230 auto
00231 numeric_traits<qd_real>::
00232 fmod(const qd_real& lhs, const qd_real& rhs) -> qd_real
00233 { return ::fmod(lhs, rhs); }
00234
00236 template<>
00237 inline
00238 auto
00239 numeric_traits<qd_real>::
00240 pow(const qd_real& val, int n) -> qd_real
00241 {
00242     if (val == qd_real(0))
00243     {
00244         return
00245             (n < 0)
00246             ? NaN()
00247             : (n == 0)
00248             ? qd_real(1)
00249             : qd_real(0);
00250     }
00251     auto result = qd_real(1);
00252     auto power =
00253         (n < 0)
00254         ? qd_real(1)/val
00255         : val;
00256     for (auto
00257         k = std::abs(n);
00258         k != 0;
00259         k /= 2)
00260     {
00261         if (k % 2)
00262             result *= power;
00263         power *= power;
00264     }
00265     return result;
00266 }
00267
00269 template<>
00270 inline
00271 auto
00272 numeric_traits<qd_real>::
00273 pi() -> qd_real
00274 { return qd_real::_pi; }
00275
00277 template<>
00278 inline
00279 auto
00280 numeric_traits<qd_real>::
00281 ln_2() -> qd_real
00282 { return qd_real::_log2; }
00283
00285 _GLUCAT_QD_F(qd_real, exp)
00286
00287 _GLUCAT_QD_F(qd_real, log)
00288
00289 _GLUCAT_QD_F(qd_real, cos)
00290
00291 _GLUCAT_QD_F(qd_real, acos)
00292
00293 _GLUCAT_QD_F(qd_real, acos)
00294
00295 _GLUCAT_QD_F(qd_real, cosh)
00296
00297 _GLUCAT_QD_F(qd_real, cosh)
00298
00299 _GLUCAT_QD_F(qd_real, sin)
00300
00301 _GLUCAT_QD_F(qd_real, sin)
00302
00303 _GLUCAT_QD_F(qd_real, asin)
00304
00305 _GLUCAT_QD_F(qd_real, asin)
00306
00307 _GLUCAT_QD_F(qd_real, sinh)
00308
00309 _GLUCAT_QD_F(qd_real, tan)
00310
00311 _GLUCAT_QD_F(qd_real, atan)
00312
00313 _GLUCAT_QD_F(qd_real, tanh)
00314
00315 _GLUCAT_QD_F(qd_real, tanh)

```



```

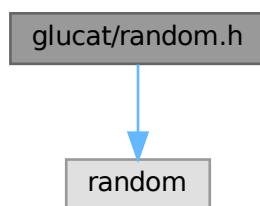
00316
00317 #endif // !defined(_GLUCAT_USE_QD) || !defined(QD_API)
00318
00319 } // namespace glucat
00320
00321 #endif // _GLUCAT_QD_H

```

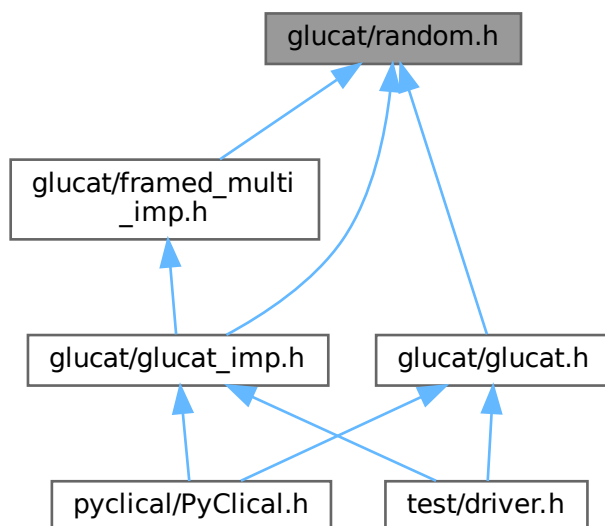
## 7.45 glucat/random.h File Reference

```
#include <random>
```

Include dependency graph for random.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class `glucat::random_generator< Scalar_T >`  
Random number generator with single instance per `Scalar_T`.

## Namespaces

- namespace [glucat](#)

## 7.46 random.h

[Go to the documentation of this file.](#)

```

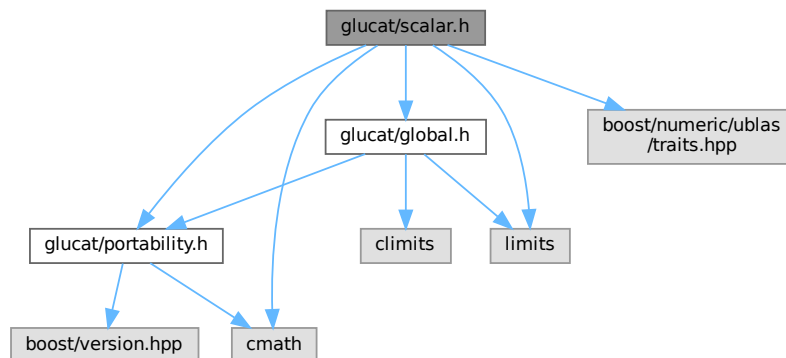
00001 #ifndef _GLUCAT_RANDOM_H
00002 #define _GLUCAT_RANDOM_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     random.h : Random number generator with single instance per Scalar_T
00006     -----
00007     begin                : 2010-03-28
00008     copyright            : (C) 2001-2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include <random>
00035
00036 namespace glucat
00037 {
00038     // Enforce singleton
00039     // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00040     template< typename Scalar_T >
00041     class random_generator
00042     {
00043     private:
00044         friend class friend_for_private_destructor;
00045     public:
00046         static auto generator() -> random_generator& { static random_generator g; return g; }
00047         random_generator(const random_generator&) = delete;
00048         auto operator= (const random_generator&) -> random_generator& = delete;
00049     private:
00050         static const unsigned long seed = 19590921UL;
00051
00052         std::mt19937 uint_gen;
00053         std::uniform_real_distribution<double> uniform_dist;
00054         std::normal_distribution<double> normal_dist;
00055
00056         random_generator() :
00057             uint_gen(), uniform_dist(0.0, 1.0), normal_dist(0.0, 1.0)
00058         { this->uint_gen.seed(seed); }
00059
00060         ~random_generator() = default;
00061
00062     public:
00063         auto uniform() -> Scalar_T
00064         { return Scalar_T(this->uniform_dist(this->uint_gen)); }
00065         auto normal() -> Scalar_T
00066         { return Scalar_T(this->normal_dist(this->uint_gen)); }
00067     };
00068 }
00069
00070 #endif // _GLUCAT_RANDOM_H

```

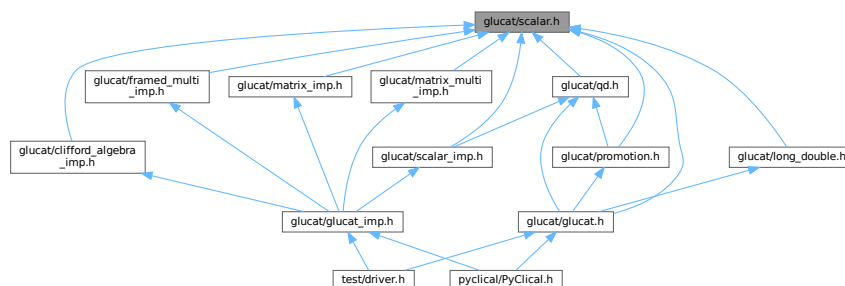
## 7.47 glucat/scalar.h File Reference

```
#include "glucat/portability.h"
#include "glucat/global.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>
```

Include dependency graph for scalar.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class `glucat::numeric_traits< Scalar_T >`  
Extra traits which extend numeric limits.
- struct `glucat::numeric_traits< Scalar_T >::promoted`  
Extra traits which extend numeric limits.
- struct `glucat::numeric_traits< Scalar_T >::demoted`  
Demoted type for long double.

### Namespaces

- namespace `glucat`

## Functions

- `template<typename Scalar_T>`  
`auto glucat::log2 (const Scalar_T &x) -> Scalar_T`  
*Log base 2 of scalar.*

## 7.48 `scalar.h`

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_H
00002 #define _GLUCAT_SCALAR_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     scalar.h : Define functions for scalar_t
00006     -----
00007     begin                : 2001-12-20
00008     copyright            : (C) 2001-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030     *****/
00031     See also Arvind Raja's original header comments and references in glucat.h
00032     *****/
00033
00034 #include "glucat/portability.h"
00035 #include "glucat/global.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044     // Reference: [AA], 2.4, p. 30-31
00045     template< typename Scalar_T >
00046     class numeric_traits
00047     {
00048     private:
00049         inline
00050         static
00051         auto
00052         isInf(const Scalar_T& val, bool_to_type<false>) -> bool
00053         { return false; }
00054
00055         inline
00056         static
00057         auto
00058         isInf(const Scalar_T& val, bool_to_type<true>) -> bool
00059         { return _GLUCAT_ISINF(val); }
00060
00061         inline
00062         static
00063         auto
00064         isNaN(const Scalar_T& val, bool_to_type<false>) -> bool
00065         { return false; }
00066
00067         inline
00068         static
00069         auto
00070         isNaN(const Scalar_T& val, bool_to_type<true>) -> bool
00071         { return false; }
00072
00073         inline
00074         static
00075         auto
00076         isNaN(const Scalar_T& val, bool_to_type<true>) -> bool
00077         { return false; }
00078     };

```

```

00076     { return _GLUCAT_ISNAN(val); }
00077
00078 public:
00080     inline
00081     static
00082     auto
00083     isInf(const Scalar_T& val) -> bool
00084     {
00085         return isInf(val,
00086             bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00087     }
00088
00090     inline
00091     static
00092     auto
00093     isNaN(const Scalar_T& val) -> bool
00094     {
00095         return isNaN(val,
00096             bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() );
00097     }
00098
00100     inline
00101     static
00102     auto
00103     isNaN_or_isInf(const Scalar_T& val) -> bool
00104     {
00105         return isNaN(val,
00106             bool_to_type< std::numeric_limits<Scalar_T>::has_quiet_NaN >() )
00107             || isInf(val,
00108                 bool_to_type< std::numeric_limits<Scalar_T>::has_infinity >() );
00109     }
00110
00112     inline
00113     static
00114     auto
00115     NaN() -> Scalar_T
00116     {
00117         return std::numeric_limits<Scalar_T>::has_quiet_NaN
00118             ? std::numeric_limits<Scalar_T>::quiet_NaN()
00119             : Scalar_T(std::log(0.0));
00120     }
00121
00123     inline
00124     static
00125     auto
00126     to_int(const Scalar_T& val) -> int
00127     { return static_cast<int>(val); }
00128
00130     inline
00131     static
00132     auto
00133     to_double(const Scalar_T& val) -> double
00134     { return static_cast<double>(val); }
00135
00137     template <typename Other_Scalar_T >
00138     inline
00139     static
00140     auto
00141     to_scalar_t(const Other_Scalar_T& val) -> Scalar_T
00142     { return static_cast<Scalar_T>(val); }
00143
00145     struct promoted {using type = double;};
00146
00148     struct demoted {using type = float;};
00149
00151     inline
00152     static
00153     auto
00154     fmod(const Scalar_T& lhs, const Scalar_T& rhs) -> Scalar_T
00155     { return std::fmod(lhs, rhs); }
00156
00158     inline
00159     static
00160     auto
00161     conj(const Scalar_T& val) -> Scalar_T
00162     { return val; }
00163
00165     inline
00166     static
00167     auto
00168     real(const Scalar_T& val) -> Scalar_T
00169     { return val; }
00170
00172     inline
00173     static
00174     auto
00175     imag(const Scalar_T& val) -> Scalar_T

```

```
00176     { return Scalar_T(0); }
00177
00179     inline
00180     static
00181     auto
00182     abs(const Scalar_T& val) -> Scalar_T
00183     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_ABS(val); }
00184
00186     inline
00187     static
00188     auto
00189     pi() -> Scalar_T
00190     { return Scalar_T(3.14159265358979323); }
00191
00193     inline
00194     static
00195     auto
00196     ln_2() -> Scalar_T
00197     { return Scalar_T(0.693147180559945309); }
00198
00200     inline
00201     static
00202     auto
00203     pow(const Scalar_T& val, int n) -> Scalar_T
00204     { return std::pow(val, n); }
00205
00207     inline
00208     static
00209     auto
00210     sqrt(const Scalar_T& val) -> Scalar_T
00211     { return boost::numeric::ublas::type_traits<Scalar_T>::UBLAS_SQRT(val); }
00212
00214     inline
00215     static
00216     auto
00217     exp(const Scalar_T& val) -> Scalar_T
00218     { return std::exp(val); }
00219
00221     inline
00222     static
00223     auto
00224     log(const Scalar_T& val) -> Scalar_T
00225     { return std::log(val); }
00226
00228     inline
00229     static
00230     auto
00231     log2(const Scalar_T& val) -> Scalar_T
00232     { return log(val)/ln_2(); }
00233
00235     inline
00236     static
00237     auto
00238     cos(const Scalar_T& val) -> Scalar_T
00239     { return std::cos(val); }
00240
00242     inline
00243     static
00244     auto
00245     acos(const Scalar_T& val) -> Scalar_T
00246     { return std::acos(val); }
00247
00249     inline
00250     static
00251     auto
00252     cosh(const Scalar_T& val) -> Scalar_T
00253     { return std::cosh(val); }
00254
00256     inline
00257     static
00258     auto
00259     sin(const Scalar_T& val) -> Scalar_T
00260     { return std::sin(val); }
00261
00263     inline
00264     static
00265     auto
00266     asin(const Scalar_T& val) -> Scalar_T
00267     { return std::asin(val); }
00268
00270     inline
00271     static
00272     auto
00273     sinh(const Scalar_T& val) -> Scalar_T
00274     { return std::sinh(val); }
00275
00277     inline
```

```

00278     static
00279     auto
00280     tan(const Scalar_T& val) -> Scalar_T
00281     { return std::tan(val); }
00282
00283     inline
00284     static
00285     auto
00286     atan(const Scalar_T& val) -> Scalar_T
00287     { return std::atan(val); }
00288
00289     inline
00290     static
00291     auto
00292     tanh(const Scalar_T& val) -> Scalar_T
00293     { return std::tanh(val); }
00294
00295     };
00296
00297     template< typename Scalar_T >
00298     inline
00299     auto
00300     log2(const Scalar_T& x) -> Scalar_T
00301     { return numeric_traits<Scalar_T>::log2(x); }
00302 }
00303 #endif // _GLUCAT_SCALAR_H

```

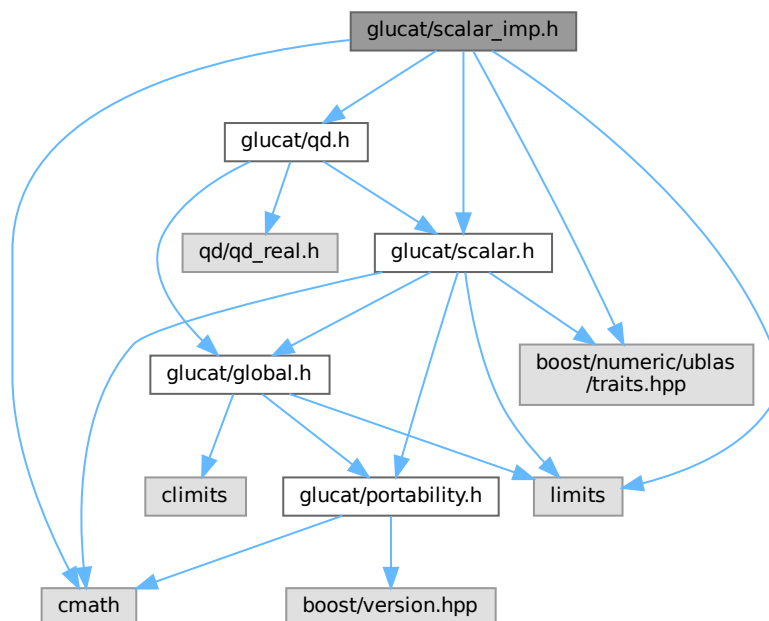
## 7.49 glucat/scalar\_imp.h File Reference

```

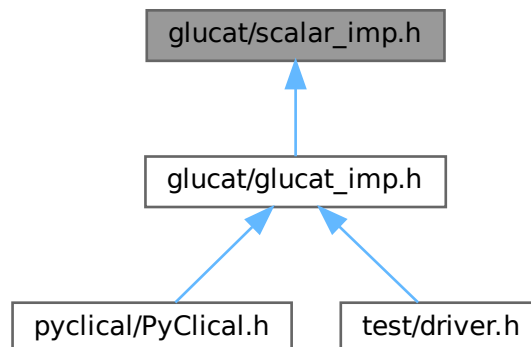
#include "glucat/scalar.h"
#include "glucat/qd.h"
#include <boost/numeric/ublas/traits.hpp>
#include <cmath>
#include <limits>

```

Include dependency graph for scalar\_imp.h:



This graph shows which files directly or indirectly include this file:



## Namespaces

- namespace [glucat](#)

## Functions

- `template<typename Scalar_T >`  
`auto glucat::to\_promote (const Scalar_T &val) -> typename numeric\_traits< Scalar_T >::promoted::type`  
*Cast to promote.*
- `template<typename Scalar_T >`  
`auto glucat::to\_demote (const Scalar_T &val) -> typename numeric\_traits< Scalar_T >::demoted::type`  
*Cast to demote.*

## 7.50 scalar\_imp.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_SCALAR_IMP_H
00002 #define _GLUCAT_SCALAR_IMP_H
00003 /*****
00004  GluCat : Generic library of universal Clifford algebra templates
00005  scalar_imp.h : Define functions for scalar_t
00006  -----
00007  begin                : 2001-12-20
00008  copyright            : (C) 2001-2014 by Paul C. Leopardi
00009  *****/
00010
00011  This library is free software: you can redistribute it and/or modify
00012  it under the terms of the GNU Lesser General Public License as published
00013  by the Free Software Foundation, either version 3 of the License, or
00014  (at your option) any later version.
00015
00016  This library is distributed in the hope that it will be useful,
00017  but WITHOUT ANY WARRANTY; without even the implied warranty of
00018  MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019  GNU Lesser General Public License for more details.
00020
00021  You should have received a copy of the GNU Lesser General Public License
00022  along with this library. If not, see <http://www.gnu.org/licenses/>.
00023

```



```

00024 *****
00025 This library is based on a prototype written by Arvind Raja and was
00026 licensed under the LGPL with permission of the author. See Arvind Raja,
00027 "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028 in Ablamowicz, Lounesto and Parra (eds.)
00029 "Clifford algebras with numeric and symbolic computations, Birkhauser, 1996."
00030 *****
00031 See also Arvind Raja's original header comments and references in glucat.h
00032 *****/
00033
00034 #include "glucat/scalar.h"
00035 #include "glucat/qd.h"
00036
00037 #include <boost/numeric/ublas/traits.hpp>
00038
00039 #include <cmath>
00040 #include <limits>
00041
00042 namespace glucat
00043 {
00044     // Reference: [AA], 2.4, p. 30-31
00045
00046     template< >
00047     template< typename Other_Scalar_T >
00048     inline
00049     auto
00050     numeric_traits<float>::
00051     to_scalar_t(const Other_Scalar_T& val) -> float
00052     { return static_cast<float>(numeric_traits<Other_Scalar_T>::to_double(val)); }
00053
00054     template< >
00055     template< typename Other_Scalar_T >
00056     inline
00057     auto
00058     numeric_traits<double>::
00059     to_scalar_t(const Other_Scalar_T& val) -> double
00060     { return numeric_traits<Other_Scalar_T>::to_double(val); }
00061
00062     #if defined(_GLUCAT_USE_QD)
00063     template< >
00064     template< >
00065     inline
00066     auto
00067     numeric_traits<long double>::
00068     to_scalar_t(const dd_real& val) -> long double
00069     { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00070
00071     template< >
00072     template< >
00073     inline
00074     auto
00075     numeric_traits<long double>::
00076     to_scalar_t(const qd_real& val) -> long double
00077     { return static_cast<long double>(val.x[0]) + static_cast<long double>(val.x[1]); }
00078
00079     template< >
00080     template< >
00081     inline
00082     auto
00083     numeric_traits<dd_real>::
00084     to_scalar_t(const long double& val) -> dd_real
00085     { return {double(val), double(val - static_cast<long double>(double(val)))}; }
00086
00087     template< >
00088     template< >
00089     inline
00090     auto
00091     numeric_traits<dd_real>::
00092     to_scalar_t(const qd_real& val) -> dd_real
00093     { return {val.x[0], val.x[1]}; }
00094
00095     template< >
00096     template< >
00097     inline
00098     auto
00099     numeric_traits<qd_real>::
00100     to_scalar_t(const long double& val) -> qd_real
00101     { return {double(val), double(val - static_cast<long double>(double(val))), 0.0, 0.0}; }
00102
00103     template< >
00104     template< >
00105     inline
00106     auto
00107     numeric_traits<qd_real>::
00108     to_scalar_t(const dd_real& val) -> qd_real
00109     { return {val.x[0], val.x[1], 0.0, 0.0}; }
00110
00111 #endif
00112

```

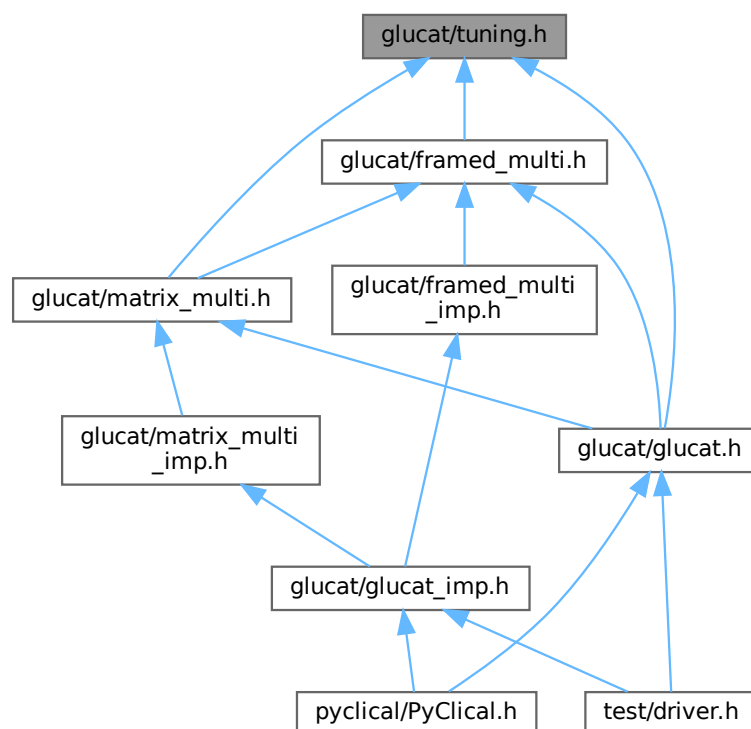
```

00120
00122 template< typename Scalar_T >
00123 inline
00124 auto
00125 to_promote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::promoted::type
00126 {
00127     using promoted_scalar_t = typename numeric_traits<Scalar_T>::promoted::type;
00128     return numeric_traits<promoted_scalar_t>::to_scalar_t(val);
00129 }
00130
00132 template< typename Scalar_T >
00133 inline
00134 auto
00135 to_demote(const Scalar_T& val) -> typename numeric_traits<Scalar_T>::demoted::type
00136 {
00137     using demoted_scalar_t = typename numeric_traits<Scalar_T>::demoted::type;
00138     return numeric_traits<demoted_scalar_t>::to_scalar_t(val);
00139 }
00140 }
00141
00142 #endif // _GLUCAT_SCALAR_IMP_H

```

## 7.51 glucat/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



### Functions

- `_GLUCAT_CTAssert` (std::numeric\_limits< unsigned int >::radix==2, CannotSetThresholds) namespace glucat

## 7.51.1 Function Documentation

### 7.51.1.1 `_GLUCAT_CTAssert()`

```
_GLUCAT_CTAssert (
    std::numeric_limits< unsigned int >::radix == 2,
    CannotSetThresholds )
```

Base class for policies

Precision policy

Tuning policy

Minimum index count needed to invoke matrix multiplication algorithm

Maximum steps of iterative refinement in division algorithm

Maximum number of steps in cyclic reduction square root iteration

Maximum number of steps in Denman-Beavers square root iteration

Maximum number of incomplete square roots in cascade log algorithm

Maximum number of steps in incomplete square root within cascade log algorithm

Maximum index count of folded frames in basis cache

Minimum map size needed to invoke generalized FFT

Minimum matrix dimension needed to invoke inverse generalized FFT

Minimum size needed for to invoke faster products algorithms

Denominator of proportion of different bits allowed in approximate equality

Extra number of different bits allowed in approximate equality

Precision used for exp, log and sqrt functions

Definition at line 35 of file [tuning.h](#).

## 7.52 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TUNING_H
00002 #define GLUCAT_TUNING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Policy classes to control tuning
00006               -----
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 // If radix of int is not 2, we can't easily set thresholds
00035 _GLUCAT_CTAssert(std::numeric_limits<unsigned int>::radix == 2, CannotSetThresholds)
00036
00037 namespace glucat
00038 {
00039     struct policy{};
00040
00041     enum precision_t
00042     {
00043         precision_demoted,
00044         precision_same,
00045         precision_promoted
00046     };
00047
00048 // Tuning policy default constants
00049
00050     const unsigned int Tuning_Default_Mult_Matrix_Threshold = 8;
00051     const unsigned int Tuning_Default_Div_Max_Steps = 4;
00052     const unsigned int Tuning_Default_CR_Sqrt_Max_Steps = 256;
00053     const unsigned int Tuning_Default_DB_Sqrt_Max_Steps = 256;
00054     const unsigned int Tuning_Default_Log_Max_Outer_Steps = 256;
00055     const unsigned int Tuning_Default_Log_Max_Inner_Steps = 32;
00056     const unsigned int Tuning_Default_Basis_Max_Count = 12;
00057     const unsigned int Tuning_Default_Fast_Size_Threshold = 1 << 6;
00058     const unsigned int Tuning_Default_Inv_Fast_Dim_Threshold = 1 << 3;
00059     const unsigned int Tuning_Default_Products_Size_Threshold = 1 << 22;
00060     const unsigned int Tuning_Default_Denom_Different_Bits = 8;
00061     const unsigned int Tuning_Default_Extra_Different_Bits = 8;
00062     const precision_t Tuning_Default_Function_Precision = precision_same;
00063
00064     template
00065     <
00066         unsigned int Mult_Matrix_Threshold = Tuning_Default_Mult_Matrix_Threshold,
00067         unsigned int Div_Max_Steps = Tuning_Default_Div_Max_Steps,
00068         unsigned int CR_Sqrt_Max_Steps = Tuning_Default_CR_Sqrt_Max_Steps,
00069         unsigned int DB_Sqrt_Max_Steps = Tuning_Default_DB_Sqrt_Max_Steps,
00070         unsigned int Log_Max_Outer_Steps = Tuning_Default_Log_Max_Outer_Steps,
00071         unsigned int Log_Max_Inner_Steps = Tuning_Default_Log_Max_Inner_Steps,
00072         unsigned int Basis_Max_Count = Tuning_Default_Basis_Max_Count,
00073         unsigned int Fast_Size_Threshold = Tuning_Default_Fast_Size_Threshold,
00074         unsigned int Inv_Fast_Dim_Threshold = Tuning_Default_Inv_Fast_Dim_Threshold,
00075         unsigned int Products_Size_Threshold = Tuning_Default_Products_Size_Threshold,
00076         unsigned int Denom_Different_Bits = Tuning_Default_Denom_Different_Bits,
00077         unsigned int Extra_Different_Bits = Tuning_Default_Extra_Different_Bits,
00078         precision_t Function_Precision = Tuning_Default_Function_Precision
00079     >
00080     struct tuning : policy
00081     {
00082         using tune_p = tuning
00083     <

```

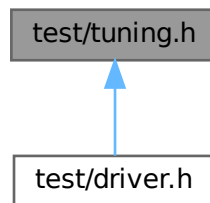
```

00086     Mult_Matrix_Threshold,
00087     Div_Max_Steps,
00088     CR_Sqrt_Max_Steps,
00089     DB_Sqrt_Max_Steps,
00090     Log_Max_Outer_Steps,
00091     Log_Max_Inner_Steps,
00092     Basis_Max_Count,
00093     Fast_Size_Threshold,
00094     Inv_Fast_Dim_Threshold,
00095     Products_Size_Threshold,
00096     Denom_Different_Bits,
00097     Extra_Different_Bits,
00098     Function_Precision
00099 >;
00100 // Tuning for multiplication
00102     enum { mult_matrix_threshold = Mult_Matrix_Threshold };
00103 // Tuning for division
00105     enum { div_max_steps = Div_Max_Steps };
00106 // Tuning for sqrt
00108     enum { cr_sqrt_max_steps = CR_Sqrt_Max_Steps };
00110     enum { db_sqrt_max_steps = DB_Sqrt_Max_Steps };
00111 // Tuning for log
00113     enum { log_max_outer_steps = Log_Max_Outer_Steps };
00115     enum { log_max_inner_steps = Log_Max_Inner_Steps };
00116 // Tuning for basis cache
00118     enum { basis_max_count = Basis_Max_Count };
00119 // Tuning for FFT
00121     enum { fast_size_threshold = Fast_Size_Threshold };
00123     enum { inv_fast_dim_threshold = Inv_Fast_Dim_Threshold };
00124 // Tuning for products (other than geometric product)
00126     enum { products_size_threshold = Products_Size_Threshold };
00127 // Tuning for precision of exp, log and sqrt functions
00129     enum { denom_different_bits = Denom_Different_Bits };
00131     enum { extra_different_bits = Extra_Different_Bits };
00133     static const precision_t function_precision = Function_Precision;
00134 };
00135
00136 using tuning_demoted = tuning
00137 <
00138     Tuning_Default_Mult_Matrix_Threshold,
00139     Tuning_Default_Div_Max_Steps,
00140     Tuning_Default_CR_Sqrt_Max_Steps,
00141     Tuning_Default_DB_Sqrt_Max_Steps,
00142     Tuning_Default_Log_Max_Outer_Steps,
00143     Tuning_Default_Log_Max_Inner_Steps,
00144     Tuning_Default_Basis_Max_Count,
00145     Tuning_Default_Fast_Size_Threshold,
00146     Tuning_Default_Inv_Fast_Dim_Threshold,
00147     Tuning_Default_Products_Size_Threshold,
00148     Tuning_Default_Denom_Different_Bits,
00149     Tuning_Default_Extra_Different_Bits,
00150     precision_demoted
00151 >;
00152
00153 using tuning_promoted = tuning
00154 <
00155     Tuning_Default_Mult_Matrix_Threshold,
00156     Tuning_Default_Div_Max_Steps,
00157     Tuning_Default_CR_Sqrt_Max_Steps,
00158     Tuning_Default_DB_Sqrt_Max_Steps,
00159     Tuning_Default_Log_Max_Outer_Steps,
00160     Tuning_Default_Log_Max_Inner_Steps,
00161     Tuning_Default_Basis_Max_Count,
00162     Tuning_Default_Fast_Size_Threshold,
00163     Tuning_Default_Inv_Fast_Dim_Threshold,
00164     Tuning_Default_Products_Size_Threshold,
00165     Tuning_Default_Denom_Different_Bits,
00166     Tuning_Default_Extra_Different_Bits,
00167     precision_promoted
00168 >;
00169 }
00170
00171 #endif // GLUCAT_TUNING_H

```

## 7.53 test/tuning.h File Reference

This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Typedefs

- using [glucat::tuning\\_slow](#)
- using [glucat::tuning\\_naive](#)
- using [glucat::tuning\\_fast](#)

### Variables

- const unsigned int [glucat::Tuning\\_Int\\_Digits](#) = std::numeric\_limits<int>::digits
- const unsigned int [glucat::Tuning\\_Max\\_Threshold](#) = 1 << [Tuning\\_Int\\_Digits](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Mult\\_Matrix\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Basis\\_Max\\_Count](#) = 0
- const unsigned int [glucat::Tuning\\_Slow\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Slow\\_Products\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Naive\\_Basis\\_Max\\_Count](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Fast\\_Size\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Naive\\_Inv\\_Fast\\_Dim\\_Threshold](#) = [Tuning\\_Max\\_Threshold](#)
- const unsigned int [glucat::Tuning\\_Fast\\_Mult\\_Matrix\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Div\\_Max\\_Steps](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_CR\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_DB\\_Sqrt\\_Max\\_Steps](#) = 256
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Outer\\_Steps](#) = 16
- const unsigned int [glucat::Tuning\\_Fast\\_Log\\_Max\\_Inner\\_Steps](#) = 8
- const unsigned int [glucat::Tuning\\_Fast\\_Basis\\_Max\\_Count](#) = 1
- const unsigned int [glucat::Tuning\\_Fast\\_Fast\\_Size\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Inv\\_Fast\\_Dim\\_Threshold](#) = 0
- const unsigned int [glucat::Tuning\\_Fast\\_Products\\_Size\\_Threshold](#) = 0

## 7.54 tuning.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_TUNING_H
00002 #define GLUCAT_TEST_TUNING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     tuning.h : Class definitions to control test tuning
00006
00007     begin                : Sun 2001-12-09
00008     copyright            : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 namespace glucat
00035 {
00036     const unsigned int Tuning_Int_Digits = std::numeric_limits<int>::digits;
00037     const unsigned int Tuning_Max_Threshold = 1 « Tuning_Int_Digits;
00038
00039     // Specific tuning policy constants and tuning policies
00040
00041     const unsigned int Tuning_Slow_Mult_Matrix_Threshold = Tuning_Max_Threshold;
00042     const unsigned int Tuning_Slow_Basis_Max_Count = 0;
00043     const unsigned int Tuning_Slow_Fast_Size_Threshold = Tuning_Max_Threshold;
00044     const unsigned int Tuning_Slow_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00045     const unsigned int Tuning_Slow_Products_Size_Threshold = Tuning_Max_Threshold;
00046
00047     using tuning_slow = tuning
00048     <
00049         Tuning_Slow_Mult_Matrix_Threshold,
00050         Tuning_Default_Div_Max_Steps,
00051         Tuning_Default_CR_Sqrt_Max_Steps,
00052         Tuning_Default_DB_Sqrt_Max_Steps,
00053         Tuning_Default_Log_Max_Outer_Steps,
00054         Tuning_Default_Log_Max_Inner_Steps,
00055         Tuning_Slow_Basis_Max_Count,
00056         Tuning_Slow_Fast_Size_Threshold,
00057         Tuning_Slow_Inv_Fast_Dim_Threshold,
00058         Tuning_Slow_Products_Size_Threshold,
00059         Tuning_Default_Denom_Different_Bits,
00060         Tuning_Default_Extra_Different_Bits,
00061         Tuning_Default_Function_Precision
00062     >;
00063
00064     const unsigned int Tuning_Naive_Mult_Matrix_Threshold = 0;
00065     const unsigned int Tuning_Naive_Basis_Max_Count = Tuning_Max_Threshold;
00066     const unsigned int Tuning_Naive_Fast_Size_Threshold = Tuning_Max_Threshold;
00067     const unsigned int Tuning_Naive_Inv_Fast_Dim_Threshold = Tuning_Max_Threshold;
00068
00069     using tuning_naive = tuning
00070     <
00071         Tuning_Naive_Mult_Matrix_Threshold,
00072         Tuning_Default_Div_Max_Steps,
00073         Tuning_Default_CR_Sqrt_Max_Steps,
00074         Tuning_Default_DB_Sqrt_Max_Steps,
00075         Tuning_Default_Log_Max_Outer_Steps,
00076         Tuning_Default_Log_Max_Inner_Steps,
00077         Tuning_Naive_Basis_Max_Count,
00078         Tuning_Naive_Fast_Size_Threshold,
00079         Tuning_Naive_Inv_Fast_Dim_Threshold,
00080         Tuning_Default_Products_Size_Threshold,
00081         Tuning_Default_Denom_Different_Bits,
00082         Tuning_Default_Extra_Different_Bits,

```

```

00083     Tuning_Default_Function_Precision
00084     >;
00085
00086     const unsigned int Tuning_Fast_Mult_Matrix_Threshold = 0;
00087     const unsigned int Tuning_Fast_Div_Max_Steps = 0;
00088     const unsigned int Tuning_Fast_CR_Sqrt_Max_Steps = 256;
00089     const unsigned int Tuning_Fast_DB_Sqrt_Max_Steps = 256;
00090     const unsigned int Tuning_Fast_Log_Max_Outer_Steps = 16;
00091     const unsigned int Tuning_Fast_Log_Max_Inner_Steps = 8;
00092     const unsigned int Tuning_Fast_Basis_Max_Count = 1;
00093     const unsigned int Tuning_Fast_Fast_Size_Threshold = 0;
00094     const unsigned int Tuning_Fast_Inv_Fast_Dim_Threshold = 0;
00095     const unsigned int Tuning_Fast_Products_Size_Threshold = 0;
00096
00097     using tuning_fast = tuning
00098     <
00099         Tuning_Fast_Mult_Matrix_Threshold,
00100         Tuning_Fast_Div_Max_Steps,
00101         Tuning_Fast_CR_Sqrt_Max_Steps,
00102         Tuning_Fast_DB_Sqrt_Max_Steps,
00103         Tuning_Fast_Log_Max_Outer_Steps,
00104         Tuning_Fast_Log_Max_Inner_Steps,
00105         Tuning_Fast_Basis_Max_Count,
00106         Tuning_Fast_Fast_Size_Threshold,
00107         Tuning_Fast_Inv_Fast_Dim_Threshold,
00108         Tuning_Fast_Products_Size_Threshold,
00109         Tuning_Default_Denom_Different_Bits,
00110         Tuning_Default_Extra_Different_Bits,
00111         Tuning_Default_Function_Precision
00112     >;
00113 }
00114 #endif // GLUCAT_TEST_TUNING_H

```

## 7.55 pyclical/glucat.pxd File Reference

### Namespaces

- namespace [glucat](#)

## 7.56 glucat.pxd

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClical: Python interface to GluCat:
00005 #     Generic library of universal Clifford algebra templates
00006 #
00007 # glucat.pxd: Basic Cython definitions
00008 #     corresponding to C++ definitions from PyClical.h.
00009 # Kept as a separate module from PyClical.pxd to avoid namespace clashes.
00010 #
00011 #     copyright           : (C) 2008-2012 by Paul C. Leopardi
00012 #
00013 #     This library is free software: you can redistribute it and/or modify
00014 #     it under the terms of the GNU Lesser General Public License as published
00015 #     by the Free Software Foundation, either version 3 of the License, or
00016 #     (at your option) any later version.
00017 #
00018 #     This library is distributed in the hope that it will be useful,
00019 #     but WITHOUT ANY WARRANTY; without even the implied warranty of
00020 #     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00021 #     GNU Lesser General Public License for more details.
00022 #
00023 #     You should have received a copy of the GNU Lesser General Public License
00024 #     along with this library. If not, see <http://www.gnu.org/licenses/>.
00025
00026 from libcpp.vector cimport vector
00027
00028 cdef extern from "PyClical.h":
00029
00030     cdef cppclass String:
00031         char* c_str()
00032

```



```

00033     cdef cppclass IndexSet:
00034         IndexSet ()
00035         IndexSet (IndexSet Ist) except+
00036         IndexSet (int idx) except+
00037         IndexSet (char* str) except+
00038         inline bint operator==(IndexSet Rhs)
00039         inline bint operator!=(IndexSet Rhs)
00040         inline bint operator<(IndexSet Rhs)
00041         inline IndexSet invert "operator~"()
00042         inline bint getitem "operator[]"(int idx)
00043         inline IndexSet set()
00044         inline IndexSet set(int idx) except+
00045         inline IndexSet set(int idx, int val) except+
00046         inline IndexSet reset()
00047         inline IndexSet reset(int idx) except+
00048         int count()
00049         int count_pos()
00050         int count_neg()
00051         int min()
00052         int max()
00053         int sign_of_mult(IndexSet Rhs)
00054         int sign_of_square()
00055         int hash_fn()
00056
00057         int compare(IndexSet Lhs, IndexSet Rhs)
00058         int min_neg(IndexSet Ist)
00059         int max_pos(IndexSet Ist)
00060
00061     ctypedef double scalar_t
00062
00063     cdef cppclass Clifford:
00064         Clifford ()
00065         Clifford (Clifford Clf) except+
00066         Clifford (Clifford Clf, IndexSet ist) except+
00067         Clifford (scalar_t scr) except+
00068         Clifford (char* str) except+
00069         Clifford (IndexSet ist, scalar_t scr) except+
00070         Clifford (vector[scalar_t] vec, IndexSet ist) except+
00071         bint operator==(Clifford Rhs)
00072         bint operator!=(Clifford Rhs)
00073         Clifford neg "operator-"()
00074         scalar_t getitem "operator[]"(IndexSet Ist)
00075         Clifford call "operator()"(int grade)
00076         scalar_t scalar()
00077         Clifford pure()
00078         Clifford even()
00079         Clifford odd()
00080         vector[scalar_t] vector_part()
00081         vector[scalar_t] vector_part(IndexSet frm) except+
00082         Clifford involute()
00083         Clifford reverse()
00084         Clifford conj()
00085         Clifford random(IndexSet Ist, scalar_t fill)
00086         scalar_t norm()
00087         scalar_t quad()
00088         IndexSet frame()
00089         scalar_t max_abs()
00090         Clifford inv()
00091         Clifford pow(int m)
00092         Clifford outer_pow(int m)
00093         Clifford truncated(scalar_t limit)
00094         bint isinf()
00095         bint isnan()
00096         void write(char* msg)
00097
00098         scalar_t error_squared_tol(Clipford Clf)
00099         scalar_t error_squared(Clipford Lhs, Clifford Rhs, scalar_t threshold)
00100         bint approx_equal(Clipford Lhs, Clifford Rhs, scalar_t threshold, scalar_t tol)
00101         scalar_t scalar(Clipford Clf)
00102         scalar_t real(Clipford Clf)
00103         scalar_t imag(Clipford Clf)
00104         Clifford pure(Clipford Clf)
00105         Clifford even(Clipford Clf)
00106         Clifford odd(Clipford Clf)
00107         Clifford involute(Clipford Clf)
00108         Clifford reverse(Clipford Clf)
00109         Clifford conj(Clipford Clf)
00110         scalar_t norm(Clipford Clf)
00111         scalar_t abs(Clipford Clf)
00112         scalar_t max_abs(Clipford Clf)
00113         scalar_t quad(Clipford Clf)
00114         Clifford inv(Clipford Clf)
00115         Clifford pow(Clipford Clf, int m)
00116         Clifford outer_pow(Clipford Clf, int m)
00117
00118         Clifford complexifier(Clipford Clf)
00119         Clifford sqrt(Clipford Clf, Clifford I) except+

```

```

00120 Clifford sqrt(Clifford Clf)
00121 Clifford exp(Clifford Clf)
00122 Clifford log(Clifford Clf, Clifford I) except+
00123 Clifford log(Clifford Clf)
00124 Clifford cos(Clifford Clf, Clifford I) except+
00125 Clifford cos(Clifford Clf)
00126 Clifford acos(Clifford Clf, Clifford I) except+
00127 Clifford acos(Clifford Clf)
00128 Clifford cosh(Clifford Clf)
00129 Clifford acosh(Clifford Clf, Clifford I) except+
00130 Clifford acosh(Clifford Clf)
00131 Clifford sin(Clifford Clf, Clifford I) except+
00132 Clifford sin(Clifford Clf)
00133 Clifford asin(Clifford Clf, Clifford I) except+
00134 Clifford asin(Clifford Clf)
00135 Clifford sinh(Clifford Clf)
00136 Clifford asinh(Clifford Clf, Clifford I) except+
00137 Clifford asinh(Clifford Clf)
00138 Clifford tan(Clifford Clf, Clifford I) except+
00139 Clifford tan(Clifford Clf)
00140 Clifford atan(Clifford Clf, Clifford I) except+
00141 Clifford atan(Clifford Clf)
00142 Clifford tanh(Clifford Clf)
00143 Clifford atanh(Clifford Clf, Clifford I) except+
00144 Clifford atanh(Clifford Clf)
00145
00146 cdef extern from "PyClical.h" namespace "cga3":
00147 Clifford agc3(Clifford Clf)
00148 Clifford cga3(Clifford Clf)
00149 Clifford cga3std(Clifford Clf)

```

## 7.57 pyclical/PyClical.h File Reference

```

#include "glucat/glucat_config.h"
#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include <iostream>
#include <sstream>
#include <iomanip>
#include <limits>

```

Include dependency graph for PyClical.h:



### Namespaces

- namespace `cga3`  
*Definitions for 3D Conformal Geometric Algebra [DL].*

### Typedefs

- using `String` = `std::string`
- using `IndexSet` = `index_set<lo_ndx, hi_ndx>`
- using `scalar_t` = `double`
- using `Clifford` = `matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>`

## Functions

- `template<typename Scalar_T >`  
`PyObject * PyFloat_FromDouble (Scalar_T v)`
- `template<typename Index_Set_T >`  
`String index_set_to_repr (const Index_Set_T &ist)`  
*The "official" string representation of Index\_Set\_T ist.*
- `template<typename Index_Set_T >`  
`String index_set_to_str (const Index_Set_T &ist)`  
*The "informal" string representation of Index\_Set\_T ist.*
- `template<typename Multivector_T >`  
`String clifford_to_repr (const Multivector_T &mv)`  
*The "official" string representation of Multivector\_T mv.*
- `template<typename Multivector_T >`  
`String clifford_to_str (const Multivector_T &mv)`  
*The "informal" string representation of Multivector\_T mv.*
- `template<typename Multivector_T >`  
`Multivector_T cga3::cga3 (const Multivector_T &x)`  
*Convert Euclidean 3D vector to Conformal Geometric Algebra null vector [DL (10.50)].*
- `template<typename Multivector_T >`  
`Multivector_T cga3::cga3std (const Multivector_T &X)`  
*Convert CGA3 null vector to standard Conformal Geometric Algebra null vector [DL (10.52)].*
- `template<typename Multivector_T >`  
`Multivector_T cga3::agc3 (const Multivector_T &X)`  
*Convert CGA3 null vector to Euclidean 3D vector [DL (10.50)].*

## Variables

- `String glucat_package_version = GLUCAT_PACKAGE_VERSION`
- `const index_t lo_ndx = DEFAULT_LO`
- `const index_t hi_ndx = DEFAULT_HI`
- `const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()`

## 7.57.1 Typedef Documentation

### 7.57.1.1 Clifford

```
using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>
```

Definition at line 148 of file [PyClical.h](#).

### 7.57.1.2 IndexSet

```
using IndexSet = index_set<lo_ndx, hi_ndx>
```

Definition at line 145 of file [PyClical.h](#).

### 7.57.1.3 scalar\_t

```
using scalar_t = double
```

Definition at line 147 of file [PyClical.h](#).

### 7.57.1.4 String

```
using String = std::string
```

Definition at line 51 of file [PyClical.h](#).

## 7.57.2 Function Documentation

### 7.57.2.1 clifford\_to\_repr()

```
template<typename Multivector_T >  
String clifford_to_repr (  
    const Multivector_T & mv ) [inline]
```

The “official” string representation of Multivector\_T mv.

Definition at line 75 of file [PyClical.h](#).

Referenced by [PyClical.clifford::\\_\\_repr\\_\\_\(\)](#).

### 7.57.2.2 clifford\_to\_str()

```
template<typename Multivector_T >  
String clifford_to_str (  
    const Multivector_T & mv ) [inline]
```

The “informal” string representation of Multivector\_T mv.

Definition at line 86 of file [PyClical.h](#).

References [glucat::abs\(\)](#).

Referenced by [PyClical.clifford::\\_\\_str\\_\\_\(\)](#).

### 7.57.2.3 index\_set\_to\_repr()

```
template<typename Index_Set_T >  
String index_set_to_repr (  
    const Index_Set_T & ist ) [inline]
```

The “official” string representation of Index\_Set\_T ist.

Definition at line 57 of file [PyClical.h](#).

Referenced by [PyClical.index\\_set::\\_\\_repr\\_\\_\(\)](#).

#### 7.57.2.4 index\_set\_to\_str()

```
template<typename Index_Set_T >
String index_set_to_str (
    const Index_Set_T & ist ) [inline]
```

The "informal" string representation of Index\_Set\_T ist.

Definition at line 66 of file [PyClical.h](#).

Referenced by [PyClical.index\\_set::\\_\\_str\\_\\_\(\)](#).

#### 7.57.2.5 PyFloat\_FromDouble()

```
template<typename Scalar_T >
PyObject * PyFloat_FromDouble (
    Scalar_T v ) [inline]
```

Create a PyFloatObject object from Scalar\_T v. Needed because Scalar\_T might not be the same as double.

Definition at line 45 of file [PyClical.h](#).

### 7.57.3 Variable Documentation

#### 7.57.3.1 epsilon

```
const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon()
```

Definition at line 150 of file [PyClical.h](#).

Referenced by [glucat::cascade\\_log\(\)](#), and [glucat::matrix::classify\\_eigenvalues\(\)](#).

#### 7.57.3.2 glucat\_package\_version

```
String glucat_package_version = GLUCAT_PACKAGE_VERSION
```

Definition at line 53 of file [PyClical.h](#).

#### 7.57.3.3 hi\_ndx

```
const index_t hi_ndx = DEFAULT_HI
```

Definition at line 144 of file [PyClical.h](#).

#### 7.57.3.4 lo\_ndx

```
const index_t lo_ndx = DEFAULT_LO
```

Definition at line 143 of file [PyClical.h](#).

## 7.58 PyClicl.h

Go to the documentation of this file.

```

00001 /*****
00002     GluCat : Generic library of universal Clifford algebra templates
00003     PyClicl.h : C++ definitions needed by PyClicl
00004     -----
00005     copyright      : (C) 2008-2021 by Paul C. Leopardi
00006     ****
00007
00008     This library is free software: you can redistribute it and/or modify
00009     it under the terms of the GNU Lesser General Public License as published
00010     by the Free Software Foundation, either version 3 of the License, or
00011     (at your option) any later version.
00012
00013     This library is distributed in the hope that it will be useful,
00014     but WITHOUT ANY WARRANTY; without even the implied warranty of
00015     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00016     GNU Lesser General Public License for more details.
00017
00018     You should have received a copy of the GNU Lesser General Public License
00019     along with this library. If not, see <http://www.gnu.org/licenses/>.
00020
00021     ****
00022     This library is based on a prototype written by Arvind Raja and was
00023     licensed under the LGPL with permission of the author. See Arvind Raja,
00024     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00025     in Ablamowicz, Lounesto and Parra (eds.)
00026     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00027     ****
00028     See also Arvind Raja's original header comments in glucat/glucat.h
00029     ****
00030 // References for algorithms:
00031 // [DL]:
00032 // C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00033
00034 #include "glucat/glucat_config.h"
00035 #include "glucat/glucat.h"
00036 #include "glucat/glucat_imp.h"
00037 #include <iostream>
00038 #include <sstream>
00039 #include <iomanip>
00040 #include <limits>
00041
00042 template<typename Scalar_T>
00043 inline PyObject* PyFloat_FromDouble(Scalar_T v)
00044 { return ::PyFloat_FromDouble(glucat::numeric_traits<Scalar_T>::to_double(v)); }
00045
00046 // String representations for use by PyClicl Python classes.
00047
00048 using String = std::string;
00049
00050 String glucat_package_version = GLUCAT_PACKAGE_VERSION;
00051
00052 template<typename Index_Set_T>
00053 inline String index_set_to_repr(const Index_Set_T& ist)
00054 {
00055     std::ostringstream os;
00056     os << "index_set(" << ist << ")";
00057     return os.str();
00058 }
00059
00060 template<typename Index_Set_T>
00061 inline String index_set_to_str(const Index_Set_T& ist)
00062 {
00063     std::ostringstream os;
00064     os << ist;
00065     return os.str();
00066 }
00067
00068 template<typename Multivector_T>
00069 inline String clifford_to_repr(const Multivector_T& mv)
00070 {
00071     using scalar_t = typename Multivector_T::scalar_t;
00072     std::ostringstream os;
00073     os << std::setprecision(std::numeric_limits<scalar_t>::digits10 + 1);
00074     os << "clifford(\"" << mv << "\")";
00075     return os.str();
00076 }
00077
00078 template<typename Multivector_T>
00079 inline String clifford_to_str(const Multivector_T& mv)
00080 {
00081     using scalar_t = typename Multivector_T::scalar_t;

```

```

00089     std::ostringstream os;
00090     if (abs(mv) < std::numeric_limits<scalar_t>::epsilon())
00091         os << 0.0;
00092     else
00093         os << std::setprecision(4) << mv.truncated(scalar_t(1.0e-4));
00094     return os.str();
00095 }
00096
00097
00098 namespace cga3
00099 {
00100     template<typename Multivector_T>
00101     inline Multivector_T cga3(const Multivector_T& x)
00102     {
00103         using cl = Multivector_T;
00104         using ist = typename cl::index_set_t;
00105         static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00106
00107         return (cl(ist(4)) - x) * ninf3 * (x - cl(ist(4)));
00108     }
00109
00110     template<typename Multivector_T>
00111     inline Multivector_T cga3std(const Multivector_T& X)
00112     {
00113         using cl = Multivector_T;
00114         using ist = typename cl::index_set_t;
00115         using scalar_t = typename cl::scalar_t;
00116         static const cl ninf3 = cl(ist(4)) + cl(ist(-1));
00117
00118         return scalar_t(-2.0) * X / (X & ninf3);
00119     }
00120
00121     template<typename Multivector_T>
00122     inline Multivector_T agc3(const Multivector_T& X)
00123     {
00124         using cl = Multivector_T;
00125         using ist = typename cl::index_set_t;
00126         using scalar_t = typename cl::scalar_t;
00127
00128         const cl& cga3stdX = cga3std(X);
00129         return (cl(ist(1))*cga3stdX[ist(1)] +
00130                cl(ist(2))*cga3stdX[ist(2)] +
00131                cl(ist(3))*cga3stdX[ist(3)]) / scalar_t(2.0);
00132     }
00133 }
00134
00135 // Specifications of the IndexSet and Clifford C++ classes for use with PyClical.
00136
00137 using namespace glucat;
00138 const index_t lo_ndx = DEFAULT_LO;
00139 const index_t hi_ndx = DEFAULT_HI;
00140 using IndexSet = index_set<lo_ndx, hi_ndx>;
00141
00142 using scalar_t = double;
00143 using Clifford = matrix_multi<scalar_t, lo_ndx, hi_ndx, tuning_promoted>;
00144
00145 const scalar_t epsilon = std::numeric_limits<scalar_t>::epsilon();
00146
00147 // Do not warn about unused values. This affects clang++ as well as g++.
00148 #pragma GCC diagnostic ignored "-Wunused-value"
00149 #if defined(__clang__)
00150 // Do not warn about unused functions. The affects clang++ only.
00151 #pragma clang diagnostic ignored "-Wunused-function"
00152 // Do not warn about unneeded internal declarations. The affects clang++ only.
00153 #pragma clang diagnostic ignored "-Wunneeded-internal-declaration"
00154 #endif

```

## 7.59 pyclical/PyClical.pxd File Reference

### Namespaces

- namespace [PyClical](#)

## 7.60 PyClicl.pxd

[Go to the documentation of this file.](#)

```
00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 #
00004 # PyClicl: Python interface to GluCat:
00005 #         Generic library of universal Clifford algebra templates
00006 #
00007 # PyClicl.pxd: Basic Cython definitions for PyClicl
00008 #         corresponding to C++ definitions from PyClicl.h.
00009 #
00010 #         copyright          : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #         This library is free software: you can redistribute it and/or modify
00013 #         it under the terms of the GNU Lesser General Public License as published
00014 #         by the Free Software Foundation, either version 3 of the license, or
00015 #         (at your option) any later version.
00016 #
00017 #         This library is distributed in the hope that it will be useful,
00018 #         but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #         MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 #         GNU Lesser General Public License for more details.
00021 #
00022 #         You should have received a copy of the GNU Lesser General Public License
00023 #         along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 cimport glucat
00026 from glucat cimport IndexSet, String, Clifford, scalar_t, vector
00027 from libcpp.string cimport string
00028
00029 cdef extern from "PyClicl.h":
00030     string glucat_package_version
00031
00032     IndexSet operator&(IndexSet Lhs, IndexSet Rhs)
00033     IndexSet operator|(IndexSet Lhs, IndexSet Rhs)
00034     IndexSet operator^(IndexSet Lhs, IndexSet Rhs)
00035
00036     string index_set_to_repr(IndexSet& Ist)
00037     string index_set_to_str(IndexSet& Ist)
00038
00039     Clifford operator+(Clifford Lhs, Clifford Rhs)
00040     Clifford operator-(Clifford Lhs, Clifford Rhs)
00041     Clifford operator*(Clifford Lhs, Clifford Rhs)
00042     Clifford operator&(Clifford Lhs, Clifford Rhs)
00043     Clifford operator%(Clifford Lhs, Clifford Rhs)
00044     Clifford operator^(Clifford Lhs, Clifford Rhs)
00045     Clifford operator/(Clifford Lhs, Clifford Rhs)
00046     Clifford operator|(Clifford Lhs, Clifford Rhs)
00047
00048     string clifford_to_repr(Clifford& Clf)
00049     string clifford_to_str(Clifford& Clf)
00050
00051     const scalar_t epsilon
```

## 7.61 pyclicl/PyClicl.pyx File Reference

### Classes

- class [PyClicl.index\\_set](#)
- class [PyClicl.clifford](#)

### Namespaces

- namespace [PyClicl](#)

### Functions

- [PyClicl.index\\_set\\_hidden\\_doctests](#) ()
- [PyClicl.clifford\\_hidden\\_doctests](#) ()
- [PyClicl.e](#) (obj)
- [PyClicl.istpq](#) (p, q)
- [PyClicl.\\_test](#) ()



## Variables

- `PyClical.__version__` = `str(glucat_package_version, 'utf-8')`
- `PyClical.lhs`
- `PyClical.rhs`
- `PyClical.threshold` = `error_squared_tol(rhs)` if threshold is `None` else threshold
- `PyClical.None`
- `PyClical.tol` = `error_squared_tol(rhs)` if tol is `None` else tol
- `PyClical.obj`
- `PyClical.i`
- `PyClical.ixt`
- `PyClical.fill`
- `PyClical.scalar_epsilon` = `epsilon`
- `float PyClical.pi` = `atan(clifford(1.0)) * 4.0`
- `float PyClical.tau` = `atan(clifford(1.0)) * 8.0`
- `PyClical.cl` = `clifford`
- `PyClical.ist` = `index_set`
- `PyClical.ninf3` = `e(4) + e(-1)`
- `PyClical.nbar3` = `e(4) - e(-1)`

## 7.62 PyClical.pyx

[Go to the documentation of this file.](#)

```

00001 # -*- coding: utf-8 -*-
00002 # cython: language_level=3
00003 # distutils: language = c++
00004 #
00005 # PyClical: Python interface to GluCat:
00006 #         Generic library of universal Clifford algebra templates
00007 #
00008 # PyClical.pyx: Cython definitions visible from Python.
00009 #
00010 #         copyright          : (C) 2008-2021 by Paul C. Leopardi
00011 #
00012 #         This library is free software: you can redistribute it and/or modify
00013 #         it under the terms of the GNU Lesser General Public License as published
00014 #         by the Free Software Foundation, either version 3 of the License, or
00015 #         (at your option) any later version.
00016 #
00017 #         This library is distributed in the hope that it will be useful,
00018 #         but WITHOUT ANY WARRANTY; without even the implied warranty of
00019 #         MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00020 #         GNU Lesser General Public License for more details.
00021 #
00022 #         You should have received a copy of the GNU Lesser General Public License
00023 #         along with this library. If not, see <http://www.gnu.org/licenses/>.
00024 #
00025 # References for definitions:
00026 # [DL]:
00027 # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
00028 #
00029 import math
00030 import numbers
00031 import collections
00032 #
00033 from PyClical cimport *
00034 #
00035 __version__ = str(glucat_package_version, 'utf-8')
00036 #
00037 # Forward reference
00038 cdef class index_set
00039 #
00040 cdef inline IndexSet toIndexSet(obj):
00041     """
00042     Return the C++ IndexSet instance wrapped by index_set(obj).
00043     """
00044     return index_set(obj).instance[0]
00045 #
00046 cdef class index_set:
00047     """
00048     Python class index_set wraps C++ class IndexSet.

```

```

00049     """
00050     cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
00051
00052     cdef inline wrap(index_set self, IndexSet other):
00053         """
00054         Wrap an instance of the C++ class IndexSet.
00055         """
00056         self.instance[0] = other
00057         return self
00058
00059     cdef inline IndexSet unwrap(index_set self):
00060         """
00061         Return the wrapped C++ IndexSet instance.
00062         """
00063         return self.instance[0]
00064
00065     cpdef copy(index_set self):
00066         """
00067         Copy this index_set object.
00068
00069         >> s=index_set(1); t=s.copy(); print(t)
00070         {1}
00071         """
00072         return index_set(self)
00073
00074     def __cinit__(self, other = 0):
00075         """
00076         Construct an object of type index_set.
00077
00078         >> print(index_set(1))
00079         {1}
00080         >> print(index_set({1,2}))
00081         {1,2}
00082         >> print(index_set(index_set({1,2})))
00083         {1,2}
00084         >> print(index_set({1,2}))
00085         {1,2}
00086         >> print(index_set({1,2,1}))
00087         {1,2}
00088         >> print(index_set("{1,2,1}"))
00089         {1,2}
00090         >> print(index_set(""))
00091         {}
00092         """
00093         error_msg_prefix = "Cannot initialize index_set object from"
00094         if isinstance(other, index_set):
00095             self.instance = new IndexSet((<index_set>other).unwrap())
00096         elif isinstance(other, numbers.Integral):
00097             self.instance = new IndexSet(<int>other)
00098         elif isinstance(other, (set, frozenset)):
00099             try:
00100                 self.instance = new IndexSet()
00101                 for idx in other:
00102                     self[idx] = True
00103             except IndexError:
00104                 raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
00105             except (RuntimeError, TypeError):
00106                 raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
00107         elif isinstance(other, str):
00108             try:
00109                 bother = other.encode("UTF-8")
00110                 self.instance = new IndexSet(<char *>bother)
00111             except RuntimeError:
00112                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
00113         else:
00114             raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00115
00116     def __dealloc__(self):
00117         """
00118         Clean up by deallocating the instance of C++ class IndexSet.
00119         """
00120         del self.instance
00121
00122     def __richcmp__(lhs, rhs, int op):
00123         """
00124         Compare two objects of class index_set.
00125
00126         >> index_set(1) == index_set({1})
00127         True
00128         >> index_set({1}) != index_set({1})
00129         False
00130         >> index_set({1}) != index_set({2})
00131         True
00132         >> index_set({1}) == index_set({2})
00133         False
00134         >> index_set({1}) < index_set({2})
00135         True

```

```

00136         >> index_set({1}) <= index_set({2})
00137         True
00138         >> index_set({1}) > index_set({2})
00139         False
00140         >> index_set({1}) >= index_set({2})
00141         False
00142         """
00143         if (lhs is None) or (rhs is None):
00144             eq = bool(lhs is rhs)
00145             if op == 2: # ==
00146                 return eq
00147             elif op == 3: # !=
00148                 return not eq
00149             else:
00150                 if op == 0: # <
00151                     return False
00152                 elif op == 1: # <=
00153                     return eq
00154                 elif op == 4: # >
00155                     return False
00156                 elif op == 5: # >=
00157                     return eq
00158                 else:
00159                     return NotImplemented
00160         else:
00161             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
00162             if op == 2: # ==
00163                 return eq
00164             elif op == 3: # !=
00165                 return not eq
00166             else:
00167                 lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
00168                 if op == 0: # <
00169                     return lt
00170                 elif op == 1: # <=
00171                     return lt or eq
00172                 elif op == 4: # >
00173                     return not (lt or eq)
00174                 elif op == 5: # >=
00175                     return not lt
00176                 else:
00177                     return NotImplemented
00178
00179     def __setitem__(self, idx, val):
00180         """
00181         Set the value of an index_set object at index idx to value val.
00182
00183         >> s=index_set({1}); s[2] = True; print(s)
00184         {1,2}
00185         >> s=index_set({1,2}); s[1] = False; print(s)
00186         {2}
00187         """
00188         self.instance.set(idx, val)
00189         return
00190
00191     def __getitem__(self, idx):
00192         """
00193         Get the value of an index_set object at an index.
00194
00195         >> index_set({1})[1]
00196         True
00197         >> index_set({1})[2]
00198         False
00199         >> index_set({2})[-1]
00200         False
00201         >> index_set({2})[1]
00202         False
00203         >> index_set({2})[2]
00204         True
00205         >> index_set({2})[33]
00206         False
00207         """
00208         return self.instance.getitem(idx)
00209
00210     def __contains__(self, idx):
00211         """
00212         Check that an index_set object contains the index idx: idx in self.
00213
00214         >> 1 in index_set({1})
00215         True
00216         >> 2 in index_set({1})
00217         False
00218         >> -1 in index_set({2})
00219         False
00220         >> 1 in index_set({2})
00221         False
00222         >> 2 in index_set({2})

```

```

00223         True
00224         >> 33 in index_set({2})
00225         False
00226         """
00227         return self.instance.getitem(idx)
00228
00229     def __iter__(self):
00230         """
00231         Iterate over the indices of an index_set.
00232
00233         >> for i in index_set({-3,4,7}):print(i, end=",")
00234         -3,4,7,
00235         """
00236         for idx in range(self.min(), self.max()+1):
00237             if idx in self:
00238                 yield idx
00239
00240     def __invert__(self):
00241         """
00242         Set complement: not.
00243
00244         >>
00245         print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))
00246         {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}
00247         """
00248         return index_set().wrap( self.instance.invert() )
00249
00250     def __xor__(lhs, rhs):
00251         """
00252         Symmetric set difference: exclusive or.
00253
00254         >> print(index_set({1}) ^ index_set({2}))
00255         {1,2}
00256         >> print(index_set({1,2}) ^ index_set({2}))
00257         {1}
00258         """
00259         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
00260
00261     def __ixor__(self, rhs):
00262         """
00263         Symmetric set difference: exclusive or.
00264
00265         >> x = index_set({1}); x ^= index_set({2}); print(x)
00266         {1,2}
00267         >> x = index_set({1,2}); x ^= index_set({2}); print(x)
00268         {1}
00269         """
00270         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
00271
00272     def __and__(lhs, rhs):
00273         """
00274         Set intersection: and.
00275
00276         >> print(index_set({1}) & index_set({2}))
00277         {}
00278         >> print(index_set({1,2}) & index_set({2}))
00279         {2}
00280         """
00281         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
00282
00283     def __iand__(self, rhs):
00284         """
00285         Set intersection: and.
00286
00287         >> x = index_set({1}); x &= index_set({2}); print(x)
00288         {}
00289         >> x = index_set({1,2}); x &= index_set({2}); print(x)
00290         {2}
00291         """
00292         return self.wrap( self.unwrap() & toIndexSet(rhs) )
00293
00294     def __or__(lhs, rhs):
00295         """
00296         Set union: or.
00297
00298         >> print(index_set({1}) | index_set({2}))
00299         {1,2}
00300         >> print(index_set({1,2}) | index_set({2}))
00301         {1,2}
00302         """
00303         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
00304
00305     def __ior__(self, rhs):
00306         """
00307         Set union: or.

```

```

00308         >> x = index_set({1}); x |= index_set({2}); print(x)
00309         {1,2}
00310         >> x = index_set({1,2}); x |= index_set({2}); print(x)
00311         {1,2}
00312         """
00313         return self.wrap( self.unwrap() | toIndexSet(rhs) )
00314
00315     def count(self):
00316         """
00317         Cardinality: Number of indices included in set.
00318
00319         >> index_set({-1,1,2}).count()
00320         3
00321         """
00322         return self.instance.count()
00323
00324     def count_neg(self):
00325         """
00326         Number of negative indices included in set.
00327
00328         >> index_set({-1,1,2}).count_neg()
00329         1
00330         """
00331         return self.instance.count_neg()
00332
00333     def count_pos(self):
00334         """
00335         Number of positive indices included in set.
00336
00337         >> index_set({-1,1,2}).count_pos()
00338         2
00339         """
00340         return self.instance.count_pos()
00341
00342     def min(self):
00343         """
00344         Minimum member.
00345
00346         >> index_set({-1,1,2}).min()
00347         -1
00348         """
00349         return self.instance.min()
00350
00351     def max(self):
00352         """
00353         Maximum member.
00354
00355         >> index_set({-1,1,2}).max()
00356         2
00357         """
00358         return self.instance.max()
00359
00360     def hash_fn(self):
00361         """
00362         Hash function.
00363         """
00364         return self.instance.hash_fn()
00365
00366     def sign_of_mult(self, rhs):
00367         """
00368         Sign of geometric product of two Clifford basis elements.
00369
00370         >> s = index_set({1,2}); t=index_set({-1}); s.sign_of_mult(t)
00371         1
00372         """
00373         return self.instance.sign_of_mult(toIndexSet(rhs))
00374
00375     def sign_of_square(self):
00376         """
00377         Sign of geometric square of a Clifford basis element.
00378
00379         >> s = index_set({1,2}); s.sign_of_square()
00380         -1
00381         """
00382         return self.instance.sign_of_square()
00383
00384     def __repr__(self):
00385         """
00386         The "official" string representation of self.
00387
00388         >> index_set({1,2}).__repr__()
00389         'index_set({1,2})'
00390         >> repr(index_set({1,2}))
00391         'index_set({1,2})'
00392         """
00393         return index_set_to_repr( self.unwrap() ).decode()
00394

```

```

00395     def __str__(self):
00396         """
00397         The "informal" string representation of self.
00398
00399         >> index_set({1,2}).__str__()
00400         '{1,2}'
00401         >> str(index_set({1,2}))
00402         '{1,2}'
00403         """
00404         return index_set_to_str( self.unwrap() ).decode()
00405
00406 def index_set_hidden_doctests():
00407     """
00408     Tests for functions that Doctest cannot see.
00409
00410     For index_set.__cinit__: Construct index_set.
00411
00412     >> print(index_set(1))
00413     {1}
00414     >> print(index_set({1,2}))
00415     {1,2}
00416     >> print(index_set(index_set({1,2})))
00417     {1,2}
00418     >> print(index_set({1,2}))
00419     {1,2}
00420     >> print(index_set({1,2,1}))
00421     {1,2}
00422     >> print(index_set({1,2,1}))
00423     {1,2}
00424     >> print(index_set(""))
00425     {}
00426     >> print(index_set("{}"))
00427     Traceback (most recent call last):
00428     ...
00429     ValueError: Cannot initialize index_set object from invalid string '{}'.
00430     >> print(index_set("{1}"))
00431     Traceback (most recent call last):
00432     ...
00433     ValueError: Cannot initialize index_set object from invalid string '{1}'.
00434     >> print(index_set("{1,2,100}"))
00435     Traceback (most recent call last):
00436     ...
00437     ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.
00438     >> print(index_set({1,2,100}))
00439     Traceback (most recent call last):
00440     ...
00441     IndexError: Cannot initialize index_set object from invalid {1, 2, 100}.
00442     >> print(index_set([1,2]))
00443     Traceback (most recent call last):
00444     ...
00445     TypeError: Cannot initialize index_set object from <class 'list'>.
00446
00447     For index_set.__richcmp__: Compare two objects of class index_set.
00448
00449     >> index_set(1) == index_set({1})
00450     True
00451     >> index_set({1}) != index_set({1})
00452     False
00453     >> index_set({1}) != index_set({2})
00454     True
00455     >> index_set({1}) == index_set({2})
00456     False
00457     >> index_set({1}) < index_set({2})
00458     True
00459     >> index_set({1}) <= index_set({2})
00460     True
00461     >> index_set({1}) > index_set({2})
00462     False
00463     >> index_set({1}) >= index_set({2})
00464     False
00465     >> None == index_set({1,2})
00466     False
00467     >> None != index_set({1,2})
00468     True
00469     >> None < index_set({1,2})
00470     False
00471     >> None <= index_set({1,2})
00472     False
00473     >> None > index_set({1,2})
00474     False
00475     >> None >= index_set({1,2})
00476     False
00477     >> index_set({1,2}) == None
00478     False
00479     >> index_set({1,2}) != None
00480     True
00481     >> index_set({1,2}) < None

```

```

00482     False
00483     >> index_set({1,2}) <= None
00484     False
00485     >> index_set({1,2}) > None
00486     False
00487     >> index_set({1,2}) >= None
00488     False
00489     """
00490     return
00491
00492 cpdef inline compare(lhs,rhs):
00493     """
00494     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
00495     -1 if a<b, +1 if a>b, 0 if a==b.
00496
00497     >> compare(index_set({1,2}),index_set({-1,3}))
00498     -1
00499     >> compare(index_set({-1,4}),index_set({-1,3}))
00500     1
00501     """
00502     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
00503
00504 cpdef inline min_neg(obj):
00505     """
00506     Minimum negative index, or 0 if none.
00507
00508     >> min_neg(index_set({1,2}))
00509     0
00510     """
00511     return glucat.min_neg( toIndexSet(obj) )
00512
00513 cpdef inline max_pos(obj):
00514     """
00515     Maximum positive index, or 0 if none.
00516
00517     >> max_pos(index_set({1,2}))
00518     2
00519     """
00520     return glucat.max_pos( toIndexSet(obj) )
00521
00522 cdef inline vector[scalar_t] list_to_vector(lst):
00523     """
00524     Create a C++ std::vector[scalar_t] from an iterable Python object.
00525     """
00526     cdef vector[scalar_t] v
00527     for s in lst:
00528         v.push_back(<scalar_t>s)
00529     return v
00530
00531 # Forward reference.
00532 cdef class clifford
00533
00534 cdef inline Clifford toClifford(obj):
00535     return clifford(obj).instance[0]
00536
00537 cdef class clifford:
00538     """
00539     Python class clifford wraps C++ class Clifford.
00540     """
00541     cdef Clifford *instance # Wrapped instance of C++ class Clifford.
00542
00543     cdef inline wrap(clifford self, Clifford other):
00544         """
00545         Wrap an instance of the C++ class Clifford.
00546         """
00547         self.instance[0] = other
00548         return self
00549
00550     cdef inline Clifford unwrap(clifford self):
00551         """
00552         Return the wrapped C++ Clifford instance.
00553         """
00554         return self.instance[0]
00555
00556     cpdef copy(clifford self):
00557         """
00558         Copy this clifford object.
00559
00560         >> x=clifford("1{2}"); y=x.copy(); print(y)
00561         {2}
00562         """
00563         return clifford(self)
00564
00565     def __cinit__(self, other = 0, ixt = None):
00566         """
00567         Construct an object of type clifford.
00568

```

```

00569     >> print(clifford(2))
00570     2
00571     >> print(clifford(2.0))
00572     2
00573     >> print(clifford(1.0e-1))
00574     0.1
00575     >> print(clifford("2"))
00576     2
00577     >> print(clifford("2{1,2,3}"))
00578     2{1,2,3}
00579     >> print(clifford(clifford("2{1,2,3}")))
00580     2{1,2,3}
00581     >> print(clifford("-{1}"))
00582     -{1}
00583     >> print(clifford(2, index_set({1,2})))
00584     2{1,2}
00585     >> print(clifford([2,3], index_set({1,2})))
00586     2{1}+3{2}
00587     """
00588     error_msg_prefix = "Cannot initialize clifford object from"
00589     if ixt is None:
00590         try:
00591             if isinstance(other, clifford):
00592                 self.instance = new Clifford((<clifford>other).unwrap())
00593             elif isinstance(other, index_set):
00594                 self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
00595             elif isinstance(other, numbers.Real):
00596                 self.instance = new Clifford(<scalar_t>other)
00597             elif isinstance(other, str):
00598                 try:
00599                     bother = other.encode("UTF-8")
00600                     self.instance = new Clifford(<char *>bother)
00601                 except RuntimeError:
00602                     raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
00603             else:
00604                 raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
00605         except RuntimeError as err:
00606             raise ValueError(error_msg_prefix + " " + str(type(other))
00607                             + " value " + repr(other) + ":"
00608                             + "\n\t" + str(err))
00609     elif isinstance(ixt, index_set):
00610         if isinstance(other, numbers.Real):
00611             self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
00612         elif isinstance(other, collections.abc.Sequence):
00613             self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
00614         else:
00615             raise TypeError(error_msg_prefix + " (" + str(type(other))
00616                             + ", " + repr(ixt) + ").")
00617     else:
00618         raise TypeError(error_msg_prefix + " (" + str(type(other))
00619                             + ", " + str(type(ixt)) + ").")
00620
00621     def __dealloc__(self):
00622         """
00623         Clean up by deallocating the instance of C++ class Clifford.
00624         """
00625         del self.instance
00626
00627     def __contains__(self, x):
00628         """
00629         Not applicable.
00630
00631         >> x=clifford(index_set({-3,4,7})); -3 in x
00632         Traceback (most recent call last):
00633         ...
00634         TypeError: Not applicable.
00635         """
00636         raise TypeError("Not applicable.")
00637
00638     def __iter__(self):
00639         """
00640         Not applicable.
00641
00642         >> for a in clifford(index_set({-3,4,7})):print(a, end=",")
00643         Traceback (most recent call last):
00644         ...
00645         TypeError: Not applicable.
00646         """
00647         raise TypeError("Not applicable.")
00648
00649     def reframe(self, ixt):
00650         """
00651         Put self into a larger frame, containing the union of self.frame() and index set ixt.
00652         This can be used to make multiplication faster, by multiplying within a common frame.
00653
00654         >> clifford("2+3{1}").reframe(index_set({1,2,3}))
00655         clifford("2+3{1}")

```



```

00656         >> s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() ==
(s|t);
00657         True
00658         """
00659         error_msg_prefix = "Cannot reframe"
00660         if isinstance(ixt, index_set):
00661             try:
00662                 result = clifford()
00663                 result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
00664             except RuntimeError as err:
00665                 raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
00666                                     + str(ixt) + ";"
00667                                     + "\n\t" + str(err))
00668         else:
00669             raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
00670         return result
00671
00672     def __richcmp__(lhs, rhs, int op):
00673         """
00674         Compare objects of type clifford.
00675
00676         >> clifford("{1}") == clifford("1{1}")
00677         True
00678         >> clifford("{1}") != clifford("1.0{1}")
00679         False
00680         >> clifford("{1}") != clifford("1.0")
00681         True
00682         >> clifford("{1,2}") == None
00683         False
00684         >> clifford("{1,2}") != None
00685         True
00686         >> None == clifford("{1,2}")
00687         False
00688         >> None != clifford("{1,2}")
00689         True
00690         """
00691         if op == 2: # ==
00692             if (lhs is None) or (rhs is None):
00693                 return bool(lhs is rhs)
00694             else:
00695                 return bool( toClifford(lhs) == toClifford(rhs) )
00696         elif op == 3: # !=
00697             if (lhs is None) or (rhs is None):
00698                 return not bool(lhs is rhs)
00699             else:
00700                 return bool( toClifford(lhs) != toClifford(rhs) )
00701         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
00702             raise TypeError("This comparison operator is not implemented for "
00703                             + str(type(lhs)) + ", " + str(type(rhs)) + ".")
00704         else:
00705             return NotImplemented
00706
00707     def __getitem__(self, ixt):
00708         """
00709         Subscripting: map from index set to scalar coordinate.
00710
00711         >> clifford("{1}")[index_set(1)]
00712         1.0
00713         >> clifford("{1}")[index_set({1})]
00714         1.0
00715         >> clifford("{1}")[index_set({1,2})]
00716         0.0
00717         >> clifford("2{1,2}")[index_set({1,2})]
00718         2.0
00719         """
00720         return self.instance.getitem(toIndexSet(ixt))
00721
00722     def __neg__(self):
00723         """
00724         Unary -.
00725
00726         >> print(-clifford("{1}"))
00727         -{1}
00728         """
00729         return clifford().wrap( self.instance.neg() )
00730
00731     def __pos__(self):
00732         """
00733         Unary +.
00734
00735         >> print(+clifford("{1}"))
00736         {1}
00737         """
00738         return clifford(self)
00739
00740     def __add__(lhs, rhs):
00741         """

```

```

00742         Geometric sum.
00743
00744         >> print(clifford(1) + clifford("{2}"))
00745         1+{2}
00746         >> print(clifford("{1}") + clifford("{2}"))
00747         {1}+{2}
00748         """
00749         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
00750
00751     def __iadd__(self, rhs):
00752         """
00753         Geometric sum.
00754
00755         >> x = clifford(1); x += clifford("{2}"); print(x)
00756         1+{2}
00757         """
00758         return self.wrap( self.unwrap() + toClifford(rhs) )
00759
00760     def __sub__(lhs, rhs):
00761         """
00762         Geometric difference.
00763
00764         >> print(clifford(1) - clifford("{2}"))
00765         1-{2}
00766         >> print(clifford("{1}") - clifford("{2}"))
00767         {1}-{2}
00768         """
00769         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
00770
00771     def __isub__(self, rhs):
00772         """
00773         Geometric difference.
00774
00775         >> x = clifford(1); x -= clifford("{2}"); print(x)
00776         1-{2}
00777         """
00778         return self.wrap( self.unwrap() - toClifford(rhs) )
00779
00780     def __mul__(lhs, rhs):
00781         """
00782         Geometric product.
00783
00784         >> print(clifford("{1}") * clifford("{2}"))
00785         {1,2}
00786         >> print(clifford(2) * clifford("{2}"))
00787         2{2}
00788         >> print(clifford("{1}") * clifford("{1,2}"))
00789         {2}
00790         """
00791         return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
00792
00793     def __imul__(self, rhs):
00794         """
00795         Geometric product.
00796
00797         >> x = clifford(2); x *= clifford("{2}"); print(x)
00798         2{2}
00799         >> x = clifford("{1}"); x *= clifford("{2}"); print(x)
00800         {1,2}
00801         >> x = clifford("{1}"); x *= clifford("{1,2}"); print(x)
00802         {2}
00803         """
00804         return self.wrap( self.unwrap() * toClifford(rhs) )
00805
00806     def __mod__(lhs, rhs):
00807         """
00808         Contraction.
00809
00810         >> print(clifford("{1}") % clifford("{2}"))
00811         0
00812         >> print(clifford(2) % clifford("{2}"))
00813         2{2}
00814         >> print(clifford("{1}") % clifford("{1}"))
00815         1
00816         >> print(clifford("{1}") % clifford("{1,2}"))
00817         {2}
00818         """
00819         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
00820
00821     def __imod__(self, rhs):
00822         """
00823         Contraction.
00824
00825         >> x = clifford("{1}"); x %= clifford("{2}"); print(x)
00826         0
00827         >> x = clifford(2); x %= clifford("{2}"); print(x)
00828         2{2}

```

```

00829         >> x = clifford("{1}"); x %= clifford("{1}"); print(x)
00830         1
00831         >> x = clifford("{1}"); x %= clifford("{1,2}"); print(x)
00832         {2}
00833         """
00834         return self.wrap( self.unwrap() % toClifford(rhs) )
00835
00836     def __and__(lhs, rhs):
00837         """
00838         Inner product.
00839
00840         >> print(clifford("{1}") & clifford("{2}"))
00841         0
00842         >> print(clifford(2) & clifford("{2}"))
00843         0
00844         >> print(clifford("{1}") & clifford("{1}"))
00845         1
00846         >> print(clifford("{1}") & clifford("{1,2}"))
00847         {2}
00848         """
00849         return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
00850
00851     def __iand__(self, rhs):
00852         """
00853         Inner product.
00854
00855         >> x = clifford("{1}"); x &= clifford("{2}"); print(x)
00856         0
00857         >> x = clifford(2); x &= clifford("{2}"); print(x)
00858         0
00859         >> x = clifford("{1}"); x &= clifford("{1}"); print(x)
00860         1
00861         >> x = clifford("{1}"); x &= clifford("{1,2}"); print(x)
00862         {2}
00863         """
00864         return self.wrap( self.unwrap() & toClifford(rhs) )
00865
00866     def __xor__(lhs, rhs):
00867         """
00868         Outer product.
00869
00870         >> print(clifford("{1}") ^ clifford("{2}"))
00871         {1,2}
00872         >> print(clifford(2) ^ clifford("{2}"))
00873         2{2}
00874         >> print(clifford("{1}") ^ clifford("{1}"))
00875         0
00876         >> print(clifford("{1}") ^ clifford("{1,2}"))
00877         0
00878         """
00879         return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
00880
00881     def __ixor__(self, rhs):
00882         """
00883         Outer product.
00884
00885         >> x = clifford("{1}"); x ^= clifford("{2}"); print(x)
00886         {1,2}
00887         >> x = clifford(2); x ^= clifford("{2}"); print(x)
00888         2{2}
00889         >> x = clifford("{1}"); x ^= clifford("{1}"); print(x)
00890         0
00891         >> x = clifford("{1}"); x ^= clifford("{1,2}"); print(x)
00892         0
00893         """
00894         return self.wrap( self.unwrap() ^ toClifford(rhs) )
00895
00896     def __truediv__(lhs, rhs):
00897         """
00898         Geometric quotient.
00899
00900         >> print(clifford("{1}") / clifford("{2}"))
00901         {1,2}
00902         >> print(clifford(2) / clifford("{2}"))
00903         2{2}
00904         >> print(clifford("{1}") / clifford("{1}"))
00905         1
00906         >> print(clifford("{1}") / clifford("{1,2}"))
00907         -{2}
00908         """
00909         return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
00910
00911     def __idiv__(self, rhs):
00912         """
00913         Geometric quotient.
00914
00915         >> x = clifford("{1}"); x /= clifford("{2}"); print(x)

```

```

00916         {1,2}
00917         >> x = clifford(2); x /= clifford("{2}"); print(x)
00918         2{2}
00919         >> x = clifford("{1}"); x /= clifford("{1}"); print(x)
00920         1
00921         >> x = clifford("{1}"); x /= clifford("{1,2}"); print(x)
00922         -{2}
00923         """
00924         return self.wrap( self.unwrap() / toClifford(rhs) )
00925
00926     def inv(self):
00927         """
00928         Geometric multiplicative inverse.
00929
00930         >> x = clifford("{1}"); print(x.inv())
00931         {1}
00932         >> x = clifford(2); print(x.inv())
00933         0.5
00934         >> x = clifford("{1,2}"); print(x.inv())
00935         -{1,2}
00936         """
00937         return clifford().wrap( self.instance.inv() )
00938
00939     def __or__(lhs, rhs):
00940         """
00941         Transform left hand side, using right hand side as a transformation.
00942
00943         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|x)
00944         -{1}
00945         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); print(y|exp(x))
00946         -{1}
00947         """
00948         return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
00949
00950     def __ior__(self, rhs):
00951         """
00952         Transform left hand side, using right hand side as a transformation.
00953
00954         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=x; print(y)
00955         -{1}
00956         >> x=clifford("{1,2}") * pi/2; y=clifford("{1}"); y|=exp(x); print(y)
00957         -{1}
00958         """
00959         return self.wrap( self.unwrap() | toClifford(rhs) )
00960
00961     def __pow__(self, m, dummy):
00962         """
00963         Power: self to the m.
00964
00965         >> x=clifford("{1}"); print(x ** 2)
00966         1
00967         >> x=clifford("2"); print(x ** 2)
00968         4
00969         >> x=clifford("2+{1}"); print(x ** 0)
00970         1
00971         >> x=clifford("2+{1}"); print(x ** 1)
00972         2+{1}
00973         >> x=clifford("2+{1}"); print(x ** 2)
00974         5+4{1}
00975         >> i=clifford("{1,2}"); print(exp(pi/2) * (i ** i))
00976         1
00977         """
00978         return pow(self, m)
00979
00980     def pow(self, m):
00981         """
00982         Power: self to the m.
00983
00984         >> x=clifford("{1}"); print(x.pow(2))
00985         1
00986         >> x=clifford("2"); print(x.pow(2))
00987         4
00988         >> x=clifford("2+{1}"); print(x.pow(0))
00989         1
00990         >> x=clifford("2+{1}"); print(x.pow(1))
00991         2+{1}
00992         >> x=clifford("2+{1}"); print(x.pow(2))
00993         5+4{1}
00994         >> print(clifford("1+{1}+{1,2}").pow(3))
00995         1+3{1}+3{1,2}
00996         >> i=clifford("{1,2}"); print(exp(pi/2) * i.pow(i))
00997         1
00998         """
00999         if isinstance(m, numbers.Integral):
01000             return clifford().wrap( self.instance.pow(m) )
01001         else:
01002             return exp(m * log(self))

```

```

01003
01004     def outer_pow(self, m):
01005         """
01006         Outer product power.
01007
01008         >> x=clifford("2+{1}"); print(x.outer_pow(0))
01009         1
01010         >> x=clifford("2+{1}"); print(x.outer_pow(1))
01011         2+{1}
01012         >> x=clifford("2+{1}"); print(x.outer_pow(2))
01013         4+4{1}
01014         >> print(clifford("1+{1}+{1,2}").outer_pow(3))
01015         1+3{1}+3{1,2}
01016
01017         """
01018         return clifford().wrap( self.instance.outer_pow(m) )
01019
01020     def __call__(self, grade):
01021         """
01022         Pure grade-vector part.
01023
01024         >> print(clifford("{1}") (1))
01025         {1}
01026         >> print(clifford("{1}") (0))
01027         0
01028         >> print(clifford("1+{1}+{1,2}") (0))
01029         1
01030         >> print(clifford("1+{1}+{1,2}") (1))
01031         {1}
01032         >> print(clifford("1+{1}+{1,2}") (2))
01033         {1,2}
01034         >> print(clifford("1+{1}+{1,2}") (3))
01035         0
01036         """
01037         return clifford().wrap( self.instance.call(grade) )
01038
01039     def scalar(self):
01040         """
01041         Scalar part.
01042
01043         >> clifford("1+{1}+{1,2}").scalar()
01044         1.0
01045         >> clifford("{1,2}").scalar()
01046         0.0
01047         """
01048         return self.instance.scalar()
01049
01050     def pure(self):
01051         """
01052         Pure part.
01053
01054         >> print(clifford("1+{1}+{1,2}").pure())
01055         {1}+{1,2}
01056         >> print(clifford("{1,2}").pure() )
01057         {1,2}
01058         """
01059         return clifford().wrap( self.instance.pure() )
01060
01061     def even(self):
01062         """
01063         Even part of multivector, sum of even grade terms.
01064
01065         >> print(clifford("1+{1}+{1,2}").even())
01066         1+{1,2}
01067         """
01068         return clifford().wrap( self.instance.even() )
01069
01070     def odd(self):
01071         """
01072         Odd part of multivector, sum of odd grade terms.
01073
01074         >> print(clifford("1+{1}+{1,2}").odd())
01075         {1}
01076         """
01077         return clifford().wrap( self.instance.odd() )
01078
01079     def vector_part(self, frm = None):
01080         """
01081         Vector part of multivector, as a Python list, with respect to frm.
01082
01083         >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part())
01084         [2.0, 3.0]
01085         >> print(clifford("1+2{1}+3{2}+4{1,2}").vector_part(index_set({-1,1,2})))
01086         [0.0, 2.0, 3.0]
01087         """
01088         error_msg_prefix = "Cannot take vector part of "
01089         cdef vector[scalar_t] vec

```

```

01090         cdef int n
01091         cdef int i
01092     try:
01093         if frm is None:
01094             vec = self.instance.vector_part()
01095         else:
01096             vec = self.instance.vector_part((<index_set>frm).unwrap())
01097         n = vec.size()
01098         lst = [0.0]*n
01099         for i in xrange(n):
01100             lst[i] = vec[i]
01101         return lst
01102     except RuntimeError as err:
01103         raise ValueError(error_msg_prefix + str(self) + " using invalid "
01104                           + repr(frm) + " as frame:\n\t"
01105                           + str(err))
01106
01107 def involute(self):
01108     """
01109     Main involution, each {i} is replaced by -{i} in each term,
01110     eg. clifford("{1}") -> -clifford("{1}").
01111
01112     >> print(clifford("{1}").involute())
01113     -{1}
01114     >> print((clifford("{2}") * clifford("{1}")).involute())
01115     -{1,2}
01116     >> print((clifford("{1}") * clifford("{2}")).involute())
01117     {1,2}
01118     >> print(clifford("1+{1}+{1,2}").involute())
01119     1-{1}+{1,2}
01120     """
01121     return clifford().wrap( self.instance.involute() )
01122
01123 def reverse(self):
01124     """
01125     Reversion, eg. clifford("{1})*clifford("{2}") -> clifford("{2})*clifford("{1}").
01126
01127     >> print(clifford("{1}").reverse())
01128     {1}
01129     >> print((clifford("{2}") * clifford("{1}")).reverse())
01130     {1,2}
01131     >> print((clifford("{1}") * clifford("{2}")).reverse())
01132     -{1,2}
01133     >> print(clifford("1+{1}+{1,2}").reverse())
01134     1+{1}-{1,2}
01135     """
01136     return clifford().wrap( self.instance.reverse() )
01137
01138 def conj(self):
01139     """
01140     Conjugation, reverse o involute == involute o reverse.
01141
01142     >> print((clifford("{1}")).conj())
01143     -{1}
01144     >> print((clifford("{2}") * clifford("{1}")).conj())
01145     {1,2}
01146     >> print((clifford("{1}") * clifford("{2}")).conj())
01147     -{1,2}
01148     >> print(clifford("1+{1}+{1,2}").conj())
01149     1-{1}-{1,2}
01150     """
01151     return clifford().wrap( self.instance.conj() )
01152
01153 def quad(self):
01154     """
01155     Quadratic form == (rev(x)*x)(0).
01156
01157     >> print(clifford("1+{1}+{1,2}").quad())
01158     3.0
01159     >> print(clifford("1+{-1}+{1,2}+{1,2,3}").quad())
01160     2.0
01161     """
01162     return self.instance.quad()
01163
01164 def norm(self):
01165     """
01166     Norm == sum of squares of coordinates.
01167
01168     >> clifford("1+{1}+{1,2}").norm()
01169     3.0
01170     >> clifford("1+{-1}+{1,2}+{1,2,3}").norm()
01171     4.0
01172     """
01173     return self.instance.norm()
01174
01175 def abs(self):
01176     """

```

```

01177         Absolute value: square root of norm.
01178
01179         »> clifford("1+{-1}+{1,2}+{1,2,3}").abs()
01180         2.0
01181         """
01182         return glucat.abs( self.unwrap() )
01183
01184     def max_abs(self):
01185         """
01186         Maximum of absolute values of components of multivector: multivector infinity norm.
01187
01188         »> clifford("1+{-1}+{1,2}+{1,2,3}").max_abs()
01189         1.0
01190         »> clifford("3+2{1}+{1,2}").max_abs()
01191         3.0
01192         """
01193         return self.instance.max_abs()
01194
01195     def truncated(self, limit):
01196         """
01197         Remove all terms of self with relative size smaller than limit.
01198
01199         »> clifford("1e8+{1}+1e-8{1,2}").truncated(1.0e-6)
01200         clifford("100000000")
01201         »> clifford("1e4+{1}+1e-4{1,2}").truncated(1.0e-6)
01202         clifford("10000+{1}")
01203         """
01204         return clifford().wrap( self.instance.truncated(limit) )
01205
01206     def isinf(self):
01207         """
01208         Check if a multivector contains any infinite values.
01209
01210         »> clifford().isinf()
01211         False
01212         """
01213         return self.instance.isnan()
01214
01215     def isnan(self):
01216         """
01217         Check if a multivector contains any IEEE NaN values.
01218
01219         »> clifford().isnan()
01220         False
01221         """
01222         return self.instance.isnan()
01223
01224     def frame(self):
01225         """
01226         Subalgebra generated by all generators of terms of given multivector.
01227
01228         »> print(clifford("1+3{-1}+2{1,2}+4{-2,7}").frame())
01229         {-2,-1,1,2,7}
01230         »> s=clifford("1+3{-1}+2{1,2}+4{-2,7}").frame(); type(s)
01231         <class 'PyClical.index_set'>
01232         """
01233         return index_set().wrap( self.instance.frame() )
01234
01235     def __repr__(self):
01236         """
01237         The "official" string representation of self.
01238
01239         »> clifford("1+3{-1}+2{1,2}+4{-2,7}").__repr__()
01240         'clifford("1+3{-1}+2{1,2}+4{-2,7}")'
01241         """
01242         return clifford_to_repr( self.unwrap() ).decode()
01243
01244     def __str__(self):
01245         """
01246         The "informal" string representation of self.
01247
01248         »> clifford("1+3{-1}+2{1,2}+4{-2,7}").__str__()
01249         '1+3{-1}+2{1,2}+4{-2,7}'
01250         """
01251         return clifford_to_str( self.unwrap() ).decode()
01252
01253     def clifford_hidden_doctests():
01254         """
01255         Tests for functions that Doctest cannot see.
01256
01257         For clifford.__cinit__: Construct an object of type clifford.
01258
01259         »> print(clifford(2))
01260         2
01261         »> print(clifford(2.0))
01262         2
01263         »> print(clifford(1.0e-1))

```

```

01264     0.1
01265     >> print(clifford("2"))
01266     2
01267     >> print(clifford("2{1,2,3}"))
01268     2{1,2,3}
01269     >> print(clifford(clifford("2{1,2,3}")))
01270     2{1,2,3}
01271     >> print(clifford("-{1}"))
01272     -{1}
01273     >> print(clifford(2,index_set({1,2})))
01274     2{1,2}
01275     >> print(clifford([2,3],index_set({1,2})))
01276     2{1}+3{2}
01277     >> print(clifford([1,2]))
01278     Traceback (most recent call last):
01279     ...
01280     TypeError: Cannot initialize clifford object from <class 'list'>.
01281     >> print(clifford(None))
01282     Traceback (most recent call last):
01283     ...
01284     TypeError: Cannot initialize clifford object from <class 'NoneType'>.
01285     >> print(clifford(None,[1,2]))
01286     Traceback (most recent call last):
01287     ...
01288     TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).
01289     >> print(clifford([1,2],[1,2]))
01290     Traceback (most recent call last):
01291     ...
01292     TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'list'>).
01293     >> print(clifford(""))
01294     Traceback (most recent call last):
01295     ...
01296     ValueError: Cannot initialize clifford object from invalid string "".
01297     >> print(clifford("{}"))
01298     Traceback (most recent call last):
01299     ...
01300     ValueError: Cannot initialize clifford object from invalid string '{}'.
01301     >> print(clifford("{1}"))
01302     Traceback (most recent call last):
01303     ...
01304     ValueError: Cannot initialize clifford object from invalid string '{1}'.
01305     >> print(clifford("{}+"))
01306     Traceback (most recent call last):
01307     ...
01308     ValueError: Cannot initialize clifford object from invalid string '+'.
01309     >> print(clifford("{}-"))
01310     Traceback (most recent call last):
01311     ...
01312     ValueError: Cannot initialize clifford object from invalid string '-'.
01313     >> print(clifford("{1}+"))
01314     Traceback (most recent call last):
01315     ...
01316     ValueError: Cannot initialize clifford object from invalid string '{1}+'.
01317
01318     For clifford.__richcmp__: Compare objects of type clifford.
01319
01320     >> clifford("{1}") == clifford("1{1}")
01321     True
01322     >> clifford("{1}") != clifford("1.0{1}")
01323     False
01324     >> clifford("{1}") != clifford("1.0")
01325     True
01326     >> clifford("{1,2}") == None
01327     False
01328     >> clifford("{1,2}") != None
01329     True
01330     >> None == clifford("{1,2}")
01331     False
01332     >> None != clifford("{1,2}")
01333     True
01334     """
01335     return
01336
01337 cpdef inline error_squared_tol(obj):
01338     """
01339     Quadratic norm error tolerance relative to a specific multivector.
01340
01341     >> print(error_squared_tol(clifford("{1}")) * 3.0 - error_squared_tol(clifford("1{1}-2{2}+3{3}")))
01342     0.0
01343     """
01344     return glucat.error_squared_tol(toClifford(obj))
01345
01346 cpdef inline error_squared(lhs, rhs, threshold):
01347     """
01348     Relative or absolute error using the quadratic norm.
01349
01350     >> err2=scalar_epsilon*scalar_epsilon

```



```

01351
01352     >> print(error_squared(clifford("{1}"), clifford("1{1}"), err2))
01353     0.0
01354     >> print(error_squared(clifford("1{1}-3{2}+4{3}"), clifford("{1}"), err2))
01355     25.0
01356     """
01357     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
01358
01359 cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
01360     """
01361     Test for approximate equality of multivectors.
01362
01363     >> err2=scalar_epsilon*scalar_epsilon
01364
01365     >> print(approx_equal(clifford("{1}"), clifford("1{1}")))
01366     True
01367     >> print(approx_equal(clifford("1{1}-3{2}+4{3}"), clifford("{1}")))
01368     False
01369     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+0.001"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01370     False
01371     >> print(approx_equal(clifford("1{1}-3{2}+4{3}+1.0e-30"), clifford("1{1}-3{2}+4{3}"), err2, err2))
01372     True
01373     """
01374     threshold = error_squared_tol(rhs) if threshold is None else threshold
01375     tol = error_squared_tol(rhs) if tol is None else tol
01376     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold, <scalar_t>tol)
01377
01378 cpdef inline inv(obj):
01379     """
01380     Geometric multiplicative inverse.
01381
01382     >> print(inv(clifford("{1}")))
01383     {1}
01384     >> print(inv(clifford("{-1}")))
01385     -{-1}
01386     >> print(inv(clifford("{-2,-1}")))
01387     -{-2,-1}
01388     >> print(inv(clifford("{-1}+{1}")))
01389     nan
01390     """
01391     return clifford(obj).inv()
01392
01393 cpdef inline scalar(obj):
01394     """
01395     Scalar part.
01396
01397     >> scalar(clifford("1+{1}+{1,2}"))
01398     1.0
01399     >> scalar(clifford("{1,2}"))
01400     0.0
01401     """
01402     return clifford(obj).scalar()
01403
01404 cpdef inline real(obj):
01405     """
01406     Real part: synonym for scalar part.
01407
01408     >> real(clifford("1+{1}+{1,2}"))
01409     1.0
01410     >> real(clifford("{1,2}"))
01411     0.0
01412     """
01413     return clifford(obj).scalar()
01414
01415 cpdef inline imag(obj):
01416     """
01417     Imaginary part: deprecated (always 0).
01418
01419     >> imag(clifford("1+{1}+{1,2}"))
01420     0.0
01421     >> imag(clifford("{1,2}"))
01422     0.0
01423     """
01424     return 0.0
01425
01426 cpdef inline pure(obj):
01427     """
01428     Pure part
01429
01430     >> print(pure(clifford("1+{1}+{1,2}")))
01431     {1}+{1,2}
01432     >> print(pure(clifford("{1,2}")))
01433     {1,2}
01434     """
01435     return clifford(obj).pure()
01436
01437 cpdef inline even(obj):

```

```

01438     """
01439     Even part of multivector, sum of even grade terms.
01440
01441     >> print(even(clifford("1+{1}+{1,2}")))
01442     1+{1,2}
01443     """
01444     return clifford(obj).even()
01445
01446 cpdef inline odd(obj):
01447     """
01448     Odd part of multivector, sum of odd grade terms.
01449
01450     >> print(odd(clifford("1+{1}+{1,2}")))
01451     {1}
01452     """
01453     return clifford(obj).odd()
01454
01455 cpdef inline involute(obj):
01456     """
01457     Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
01458
01459     >> print(involute(clifford("{1}")))
01460     -{1}
01461     >> print(involute(clifford("{2}") * clifford("{1}")))
01462     -{1,2}
01463     >> print(involute(clifford("{1}") * clifford("{2}")))
01464     {1,2}
01465     >> print(involute(clifford("1+{1}+{1,2}")))
01466     1-{1}+{1,2}
01467     """
01468     return clifford(obj).involute()
01469
01470 cpdef inline reverse(obj):
01471     """
01472     Reversion, eg. {1}*{2} -> {2}*{1}
01473
01474     >> print(reverse(clifford("{1}")))
01475     {1}
01476     >> print(reverse(clifford("{2}") * clifford("{1}")))
01477     {1,2}
01478     >> print(reverse(clifford("{1}") * clifford("{2}")))
01479     -{1,2}
01480     >> print(reverse(clifford("1+{1}+{1,2}")))
01481     1+{1}-{1,2}
01482     """
01483     return clifford(obj).reverse()
01484
01485 cpdef inline conj(obj):
01486     """
01487     Conjugation, reverse o involute == involute o reverse.
01488
01489     >> print(conj(clifford("{1}")))
01490     -{1}
01491     >> print(conj(clifford("{2}") * clifford("{1}")))
01492     {1,2}
01493     >> print(conj(clifford("{1}") * clifford("{2}")))
01494     -{1,2}
01495     >> print(conj(clifford("1+{1}+{1,2}")))
01496     1-{1}-{1,2}
01497     """
01498     return clifford(obj).conj()
01499
01500 cpdef inline quad(obj):
01501     """
01502     Quadratic form == (rev(x)*x)(0).
01503
01504     >> print(quad(clifford("1+{1}+{1,2}")))
01505     3.0
01506     >> print(quad(clifford("1+{-1}+{1,2}+{1,2,3}")))
01507     2.0
01508     """
01509     return clifford(obj).quad()
01510
01511 cpdef inline norm(obj):
01512     """
01513     norm == sum of squares of coordinates.
01514
01515     >> norm(clifford("1+{1}+{1,2}")))
01516     3.0
01517     >> norm(clifford("1+{-1}+{1,2}+{1,2,3}")))
01518     4.0
01519     """
01520     return clifford(obj).norm()
01521
01522 cpdef inline abs(obj):
01523     """
01524     Absolute value of multivector: multivector 2-norm.

```

```

01525
01526     >> abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01527     2.0
01528     """
01529     return glucat.abs(toClifford(obj))
01530
01531 cpdef inline max_abs(obj):
01532     """
01533     Maximum absolute value of coordinates multivector: multivector infinity-norm.
01534
01535     >> max_abs(clifford("1+{-1}+{1,2}+{1,2,3}"))
01536     1.0
01537     >> max_abs(clifford("3+2{1}+{1,2}"))
01538     3.0
01539     """
01540
01541     return glucat.max_abs(toClifford(obj))
01542
01543 cpdef inline pow(obj, m):
01544     """
01545     Integer power of multivector: obj to the m.
01546
01547     >> x=clifford("{1}"); print(pow(x,2))
01548     1
01549     >> x=clifford("2"); print(pow(x,2))
01550     4
01551     >> x=clifford("2+{1}"); print(pow(x,0))
01552     1
01553     >> x=clifford("2+{1}"); print(pow(x,1))
01554     2+{1}
01555     >> x=clifford("2+{1}"); print(pow(x,2))
01556     5+4{1}
01557     >> print(pow(clifford("1+{1}+{1,2}"),3))
01558     1+3{1}+3{1,2}
01559     >> i=clifford("{1,2}"); print(exp(pi/2) * pow(i, i))
01560     1
01561     """
01562     try:
01563         math.pow(obj, m)
01564     except:
01565         return clifford(obj).pow(m)
01566
01567 cpdef inline outer_pow(obj, m):
01568     """
01569     Outer product power of multivector.
01570
01571     >> print(outer_pow(clifford("1+{1}+{1,2}"),3))
01572     1+3{1}+3{1,2}
01573     """
01574     return clifford(obj).outer_pow(m)
01575
01576 cpdef inline complexifier(obj):
01577     """
01578     Square root of -1 which commutes with all members of the frame of the given multivector.
01579
01580     >> print(complexifier(clifford(index_set({1}))))
01581     {1,2,3}
01582     >> print(complexifier(clifford(index_set({-1}))))
01583     {-1}
01584     >> print(complexifier(index_set({1})))
01585     {1,2,3}
01586     >> print(complexifier(index_set({-1})))
01587     {-1}
01588     """
01589     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
01590
01591 cpdef inline sqrt(obj, i = None):
01592     """
01593     Square root of multivector with optional complexifier.
01594
01595     >> print(sqrt(-1))
01596     {-1}
01597     >> print(sqrt(clifford("2{-1}")))
01598     1+{-1}
01599     >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)
01600     {1,2,3}
01601     -1
01602     >> j=sqrt(-1,"{1,2,3}"); print(j); print(j*j)
01603     {1,2,3}
01604     -1
01605     """
01606     if not (i is None):
01607         return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
01608     else:
01609         try:
01610             return math.sqrt(obj)
01611         except:

```

```

01612         return clifford().wrap( glucat.sqrt(toClifford(obj)) )
01613
01614 cpdef inline exp(obj):
01615     """
01616     Exponential of multivector.
01617
01618     >> x=clifford("{1,2}") * pi/4; print(exp(x))
01619     0.7071+0.7071{1,2}
01620     >> x=clifford("{1,2}") * pi/2; print(exp(x))
01621     {1,2}
01622     """
01623     try:
01624         return math.exp(obj)
01625     except:
01626         return clifford().wrap( glucat.exp(toClifford(obj)) )
01627
01628 cpdef inline log(obj,i = None):
01629     """
01630     Natural logarithm of multivector with optional complexifier.
01631
01632     >> x=clifford("{-1}"); print((log(x,"{-1}") * 2/pi))
01633     {-1}
01634     >> x=clifford("{1,2}"); print((log(x,"{1,2,3}") * 2/pi))
01635     {1,2}
01636     >> x=clifford("{1,2}"); print((log(x) * 2/pi))
01637     {1,2}
01638     >> x=clifford("{1,2}"); print((log(x,"{1,2}") * 2/pi))
01639     Traceback (most recent call last):
01640     ...
01641     RuntimeError: check_complex(val, i): i is not a valid complexifier for val
01642     """
01643     if not (i is None):
01644         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
01645     else:
01646         try:
01647             return math.log(obj)
01648         except:
01649             return clifford().wrap( glucat.log(toClifford(obj)) )
01650
01651 cpdef inline cos(obj,i = None):
01652     """
01653     Cosine of multivector with optional complexifier.
01654
01655     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01656     {1,2}
01657     >> x=clifford("{1,2}"); print(cos(acos(x)))
01658     {1,2}
01659     """
01660     if not (i is None):
01661         return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
01662     else:
01663         try:
01664             return math.cos(obj)
01665         except:
01666             return clifford().wrap( glucat.cos(toClifford(obj)) )
01667
01668 cpdef inline acos(obj,i = None):
01669     """
01670     Inverse cosine of multivector with optional complexifier.
01671
01672     >> x=clifford("{1,2}"); print(cos(acos(x),"{1,2,3}"))
01673     {1,2}
01674     >> x=clifford("{1,2}"); print(cos(acos(x),"{-1,1,2,3,4}"))
01675     {1,2}
01676     >> print(acos(0) / pi)
01677     0.5
01678     >> x=clifford("{1,2}"); print(cos(acos(x)))
01679     {1,2}
01680     """
01681     if not (i is None):
01682         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
01683     else:
01684         try:
01685             return math.acos(obj)
01686         except:
01687             return clifford().wrap( glucat.acos(toClifford(obj)) )
01688
01689 cpdef inline cosh(obj):
01690     """
01691     Hyperbolic cosine of multivector.
01692
01693     >> x=clifford("{1,2}") * pi; print(cosh(x))
01694     -1
01695     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01696     {1,2,3}
01697     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01698     {1,2}

```

```

01699     """
01700     try:
01701         return math.cosh(obj)
01702     except:
01703         return clifford().wrap( glucat.cosh(toClifford(obj)) )
01704
01705 cpdef inline acosh(obj,i = None):
01706     """
01707     Inverse hyperbolic cosine of multivector with optional complexifier.
01708
01709     >> print(acosh(0,"{-2,-1,1}"))
01710     1.571{-2,-1,1}
01711     >> x=clifford("{1,2,3}"); print(cosh(acosh(x,"{-1,1,2,3,4}")))
01712     {1,2,3}
01713     >> print(acosh(0))
01714     1.571{-1}
01715     >> x=clifford("{1,2,3}"); print(cosh(acosh(x)))
01716     {1,2,3}
01717     >> x=clifford("{1,2}"); print(cosh(acosh(x)))
01718     {1,2}
01719     """
01720     if not (i is None):
01721         return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
01722     else:
01723         try:
01724             return math.acosh(obj)
01725         except:
01726             return clifford().wrap( glucat.acosh(toClifford(obj)) )
01727
01728 cpdef inline sin(obj,i = None):
01729     """
01730     Sine of multivector with optional complexifier.
01731
01732     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))
01733     {-1}
01734     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01735     {-1}
01736     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01737     {1,2,3}
01738     """
01739     if not (i is None):
01740         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
01741     else:
01742         try:
01743             return math.sin(obj)
01744         except:
01745             return clifford().wrap( glucat.sin(toClifford(obj)) )
01746
01747 cpdef inline asin(obj,i = None):
01748     """
01749     Inverse sine of multivector with optional complexifier.
01750
01751     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),s))
01752     {-1}
01753     >> s="{1}"; x=clifford(s); print(asin(sin(x,s),"{-2,-1,1}"))
01754     {-1}
01755     >> print(asin(1) / pi)
01756     0.5
01757     >> x=clifford("{1,2,3}"); print(asin(sin(x)))
01758     {1,2,3}
01759     """
01760     if not (i is None):
01761         return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
01762     else:
01763         try:
01764             return math.asin(obj)
01765         except:
01766             return clifford().wrap( glucat.asin(toClifford(obj)) )
01767
01768 cpdef inline sinh(obj):
01769     """
01770     Hyperbolic sine of multivector.
01771
01772     >> x=clifford("{1,2}") * pi/2; print(sinh(x))
01773     {1,2}
01774     >> x=clifford("{1,2}") * pi/6; print(sinh(x))
01775     0.5{1,2}
01776     """
01777     try:
01778         return math.sinh(obj)
01779     except:
01780         return clifford().wrap( glucat.sinh(toClifford(obj)) )
01781
01782 cpdef inline asinh(obj,i = None):
01783     """
01784     Inverse hyperbolic sine of multivector with optional complexifier.
01785

```

```

01786     >> x=clifford("{1,2}"); print(asinh(x,"{1,2,3}") * 2/pi)
01787     {1,2}
01788     >> x=clifford("{1,2}"); print(asinh(x) * 2/pi)
01789     {1,2}
01790     >> x=clifford("{1,2}") / 2; print(asinh(x) * 6/pi)
01791     {1,2}
01792     """
01793     if not (i is None):
01794         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
01795     else:
01796         try:
01797             return math.asinh(obj)
01798         except:
01799             return clifford().wrap( glucat.asinh(toClifford(obj)) )
01800
01801 cpdef inline tan(obj,i = None):
01802     """
01803     Tangent of multivector with optional complexifier.
01804
01805     >> x=clifford("{1,2}"); print(tan(x,"{1,2,3}"))
01806     0.7616{1,2}
01807     >> x=clifford("{1,2}"); print(tan(x))
01808     0.7616{1,2}
01809     """
01810     if not (i is None):
01811         return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
01812     else:
01813         try:
01814             return math.tan(obj)
01815         except:
01816             return clifford().wrap( glucat.tan(toClifford(obj)) )
01817
01818 cpdef inline atan(obj,i = None):
01819     """
01820     Inverse tangent of multivector with optional complexifier.
01821
01822     >> s=index_set({1,2,3}); x=clifford("{1}"); print(tan(atan(x,s),s))
01823     {1}
01824     >> x=clifford("{1}"); print(tan(atan(x)))
01825     {1}
01826     """
01827     if not (i is None):
01828         return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
01829     else:
01830         try:
01831             return math.atan(obj)
01832         except:
01833             return clifford().wrap( glucat.atan(toClifford(obj)) )
01834
01835 cpdef inline tanh(obj):
01836     """
01837     Hyperbolic tangent of multivector.
01838
01839     >> x=clifford("{1,2}") * pi/4; print(tanh(x))
01840     {1,2}
01841     """
01842     try:
01843         return math.tanh(obj)
01844     except:
01845         return clifford().wrap( glucat.tanh(toClifford(obj)) )
01846
01847 cpdef inline atanh(obj,i = None):
01848     """
01849     Inverse hyperbolic tangent of multivector with optional complexifier.
01850
01851     >> s=index_set({1,2,3}); x=clifford("{1,2}"); print(tanh(atanh(x,s)))
01852     {1,2}
01853     >> x=clifford("{1,2}"); print(tanh(atanh(x)))
01854     {1,2}
01855     """
01856     if not (i is None):
01857         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
01858     else:
01859         try:
01860             return math.atanh(obj)
01861         except:
01862             return clifford().wrap( glucat.atanh(toClifford(obj)) )
01863
01864 cpdef inline random_clifford(index_set ixt, fill = 1.0):
01865     """
01866     Random multivector within a frame.
01867
01868     >> print(random_clifford(index_set({-3,-1,2})).frame())
01869     {-3,-1,2}
01870     """
01871     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
01872

```

```

01873 cpdef inline cga3(obj):
01874     """
01875     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
    definition.
01876
01877     >> x=clifford("2{1}+9{2}+{3}"); print(cga3(x))
01878     87{-1}+4{1}+18{2}+2{3}+85{4}
01879     """
01880     return clifford().wrap( glucat.cga3(toClifford(obj)) )
01881
01882 cpdef inline cga3std(obj):
01883     """
01884     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
01885
01886     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x)))
01887     87{-1}+4{1}+18{2}+2{3}+85{4}
01888     >> x=clifford("2{1}+9{2}+{3}"); print(cga3std(cga3(x))-cga3(x))
01889     0
01890     """
01891     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
01892
01893 cpdef inline agc3(obj):
01894     """
01895     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
01896
01897     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x)))
01898     2{1}+9{2}+{3}
01899     >> x=clifford("2{1}+9{2}+{3}"); print(agc3(cga3(x))-x)
01900     0
01901     """
01902     return clifford().wrap( glucat.agc3(toClifford(obj)) )
01903
01904 # Some abbreviations.
01905 scalar_epsilon = epsilon
01906
01907 pi = atan(clifford(1.0)) * 4.0
01908 tau = atan(clifford(1.0)) * 8.0
01909
01910 cl = clifford
01911 """
01912 Abbreviation for clifford.
01913
01914 >> print(cl(2))
01915 2
01916 >> print(cl(2.0))
01917 2
01918 >> print(cl(5.0e-1))
01919 0.5
01920 >> print(cl("2"))
01921 2
01922 >> print(cl("2{1,2,3}"))
01923 2{1,2,3}
01924 >> print(cl(cl("2{1,2,3}")))
01925 2{1,2,3}
01926 """
01927
01928 ist = index_set
01929 """
01930 Abbreviation for index_set.
01931
01932 >> print(ist("{1,2,3}"))
01933 {1,2,3}
01934 """
01935
01936 def e(obj):
01937     """
01938     Abbreviation for clifford(index_set(obj)).
01939
01940     >> print(e(1))
01941     {1}
01942     >> print(e(-1))
01943     {-1}
01944     >> print(e(0))
01945     1
01946     """
01947     return clifford(index_set(obj))
01948
01949 def istpq(p, q):
01950     """
01951     Abbreviation for index_set({-q,...p}).
01952
01953     >> print(istpq(2,3))
01954     {-3,-2,-1,1,2}
01955     """
01956     return index_set(set(range(-q,p+1)))
01957
01958 ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].

```

```

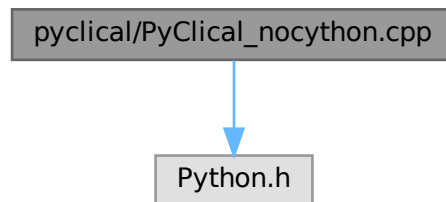
01959 nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
01960
01961 # Doctest interface.
01962 def _test():
01963     import PyClical, doctest
01964     return doctest.testmod(PyClical)
01965
01966 if __name__ == "__main__":
01967     _test()

```

## 7.63 pyclical/PyClical\_nocython.cpp File Reference

```
#include "Python.h"
```

Include dependency graph for PyClical\_nocython.cpp:



### Macros

- `#define PY_SSIZE_T_CLEAN`

### 7.63.1 Macro Definition Documentation

#### 7.63.1.1 PY\_SSIZE\_T\_CLEAN

```
#define PY_SSIZE_T_CLEAN
```

Definition at line 23 of file [PyClical\\_nocython.cpp](#).

## 7.64 PyClical\_nocython.cpp

[Go to the documentation of this file.](#)

```

00001 /* Generated by Cython 0.29.28 */
00002
00003 /* BEGIN: Cython Metadata
00004 {
00005     "distutils": {
00006         "depends": [
00007             "PyClical.h"
00008         ],
00009         "include_dirs": [
00010             "."
00011         ],

```



```

00012         "language": "c++",
00013         "name": "PyClical",
00014         "sources": [
00015             "PyClical.pyx"
00016         ]
00017     },
00018     "module_name": "PyClical"
00019 }
00020 END: Cython Metadata */
00021
00022 #ifndef PY_SSIZE_T_CLEAN
00023 #define PY_SSIZE_T_CLEAN
00024 #endif /* PY_SSIZE_T_CLEAN */
00025 #include "Python.h"
00026 #ifndef Py_PYTHON_H
00027     #error Python headers needed to compile C extensions, please install development version of
    Python.
00028 #elif PY_VERSION_HEX < 0x02060000 || (0x03000000 <= PY_VERSION_HEX && PY_VERSION_HEX < 0x03030000)
00029     #error Cython requires Python 2.6+ or Python 3.3+.
00030 #else
00031 #define CYTHON_ABI "0_29_28"
00032 #define CYTHON_HEX_VERSION 0x001D1CF0
00033 #define CYTHON_FUTURE_DIVISION 1
00034 #include <stddef.h>
00035 #ifndef offsetof
00036     #define offsetof(type, member) ( (size_t) & ((type*)0) -> member )
00037 #endif
00038 #if !defined(WIN32) && !defined(MS_WINDOWS)
00039     #ifndef __stdcall
00040         #define __stdcall
00041     #endif
00042     #ifndef __cdecl
00043         #define __cdecl
00044     #endif
00045     #ifndef __fastcall
00046         #define __fastcall
00047     #endif
00048 #endif
00049 #ifndef DL_IMPORT
00050     #define DL_IMPORT(t) t
00051 #endif
00052 #ifndef DL_EXPORT
00053     #define DL_EXPORT(t) t
00054 #endif
00055 #define __PYX_COMMA ,
00056 #ifndef HAVE_LONG_LONG
00057     #if PY_VERSION_HEX >= 0x02070000
00058         #define HAVE_LONG_LONG
00059     #endif
00060 #endif
00061 #ifndef PY_LONG_LONG
00062     #define PY_LONG_LONG LONG_LONG
00063 #endif
00064 #ifndef Py_HUGE_VAL
00065     #define Py_HUGE_VAL HUGE_VAL
00066 #endif
00067 #ifdef PYPY_VERSION
00068     #define CYTHON_COMPILING_IN_PYPY 1
00069     #define CYTHON_COMPILING_IN_PYSTON 0
00070     #define CYTHON_COMPILING_IN_CPYTHON 0
00071     #undef CYTHON_USE_TYPE_SLOTS
00072     #define CYTHON_USE_TYPE_SLOTS 0
00073     #undef CYTHON_USE_PYTYPE_LOOKUP
00074     #define CYTHON_USE_PYTYPE_LOOKUP 0
00075     #if PY_VERSION_HEX < 0x03050000
00076         #undef CYTHON_USE_ASYNC_SLOTS
00077         #define CYTHON_USE_ASYNC_SLOTS 0
00078     #elif !defined(CYTHON_USE_ASYNC_SLOTS)
00079         #define CYTHON_USE_ASYNC_SLOTS 1
00080     #endif
00081     #undef CYTHON_USE_PYLIST_INTERNALS
00082     #define CYTHON_USE_PYLIST_INTERNALS 0
00083     #undef CYTHON_USE_UNICODE_INTERNALS
00084     #define CYTHON_USE_UNICODE_INTERNALS 0
00085     #undef CYTHON_USE_UNICODE_WRITER
00086     #define CYTHON_USE_UNICODE_WRITER 0
00087     #undef CYTHON_USE_PYLONG_INTERNALS
00088     #define CYTHON_USE_PYLONG_INTERNALS 0
00089     #undef CYTHON_AVOID_BORROWED_REFS
00090     #define CYTHON_AVOID_BORROWED_REFS 1
00091     #undef CYTHON_ASSUME_SAFE_MACROS
00092     #define CYTHON_ASSUME_SAFE_MACROS 0
00093     #undef CYTHON_UNPACK_METHODS
00094     #define CYTHON_UNPACK_METHODS 0
00095     #undef CYTHON_FAST_THREAD_STATE
00096     #define CYTHON_FAST_THREAD_STATE 0
00097     #undef CYTHON_FAST_PYCALL

```

```

00098 #define CYTHON_FAST_PYCALL 0
00099 #undef CYTHON_PEP489_MULTI_PHASE_INIT
00100 #define CYTHON_PEP489_MULTI_PHASE_INIT 0
00101 #undef CYTHON_USE_TP_FINALIZE
00102 #define CYTHON_USE_TP_FINALIZE 0
00103 #undef CYTHON_USE_DICT_VERSIONS
00104 #define CYTHON_USE_DICT_VERSIONS 0
00105 #undef CYTHON_USE_EXC_INFO_STACK
00106 #define CYTHON_USE_EXC_INFO_STACK 0
00107 #elif defined(PYSTON_VERSION)
00108 #define CYTHON_COMPILING_IN_PYPY 0
00109 #define CYTHON_COMPILING_IN_PYSTON 1
00110 #define CYTHON_COMPILING_IN_CPYTHON 0
00111 #ifndef CYTHON_USE_TYPE_SLOTS
00112 #define CYTHON_USE_TYPE_SLOTS 1
00113 #endif
00114 #undef CYTHON_USE_PYTYPE_LOOKUP
00115 #define CYTHON_USE_PYTYPE_LOOKUP 0
00116 #undef CYTHON_USE_ASYNC_SLOTS
00117 #define CYTHON_USE_ASYNC_SLOTS 0
00118 #undef CYTHON_USE_PYLIST_INTERNALS
00119 #define CYTHON_USE_PYLIST_INTERNALS 0
00120 #ifndef CYTHON_USE_UNICODE_INTERNALS
00121 #define CYTHON_USE_UNICODE_INTERNALS 1
00122 #endif
00123 #undef CYTHON_USE_UNICODE_WRITER
00124 #define CYTHON_USE_UNICODE_WRITER 0
00125 #undef CYTHON_USE_PYLONG_INTERNALS
00126 #define CYTHON_USE_PYLONG_INTERNALS 0
00127 #ifndef CYTHON_AVOID_BORROWED_REFS
00128 #define CYTHON_AVOID_BORROWED_REFS 0
00129 #endif
00130 #ifndef CYTHON_ASSUME_SAFE_MACROS
00131 #define CYTHON_ASSUME_SAFE_MACROS 1
00132 #endif
00133 #ifndef CYTHON_UNPACK_METHODS
00134 #define CYTHON_UNPACK_METHODS 1
00135 #endif
00136 #undef CYTHON_FAST_THREAD_STATE
00137 #define CYTHON_FAST_THREAD_STATE 0
00138 #undef CYTHON_FAST_PYCALL
00139 #define CYTHON_FAST_PYCALL 0
00140 #undef CYTHON_PEP489_MULTI_PHASE_INIT
00141 #define CYTHON_PEP489_MULTI_PHASE_INIT 0
00142 #undef CYTHON_USE_TP_FINALIZE
00143 #define CYTHON_USE_TP_FINALIZE 0
00144 #undef CYTHON_USE_DICT_VERSIONS
00145 #define CYTHON_USE_DICT_VERSIONS 0
00146 #undef CYTHON_USE_EXC_INFO_STACK
00147 #define CYTHON_USE_EXC_INFO_STACK 0
00148 #else
00149 #define CYTHON_COMPILING_IN_PYPY 0
00150 #define CYTHON_COMPILING_IN_PYSTON 0
00151 #define CYTHON_COMPILING_IN_CPYTHON 1
00152 #ifndef CYTHON_USE_TYPE_SLOTS
00153 #define CYTHON_USE_TYPE_SLOTS 1
00154 #endif
00155 #if PY_VERSION_HEX < 0x02070000
00156 #undef CYTHON_USE_PYTYPE_LOOKUP
00157 #define CYTHON_USE_PYTYPE_LOOKUP 0
00158 #elif !defined(CYTHON_USE_PYTYPE_LOOKUP)
00159 #define CYTHON_USE_PYTYPE_LOOKUP 1
00160 #endif
00161 #if PY_MAJOR_VERSION < 3
00162 #undef CYTHON_USE_ASYNC_SLOTS
00163 #define CYTHON_USE_ASYNC_SLOTS 0
00164 #elif !defined(CYTHON_USE_ASYNC_SLOTS)
00165 #define CYTHON_USE_ASYNC_SLOTS 1
00166 #endif
00167 #if PY_VERSION_HEX < 0x02070000
00168 #undef CYTHON_USE_PYLONG_INTERNALS
00169 #define CYTHON_USE_PYLONG_INTERNALS 0
00170 #elif !defined(CYTHON_USE_PYLONG_INTERNALS)
00171 #define CYTHON_USE_PYLONG_INTERNALS 1
00172 #endif
00173 #ifndef CYTHON_USE_PYLIST_INTERNALS
00174 #define CYTHON_USE_PYLIST_INTERNALS 1
00175 #endif
00176 #ifndef CYTHON_USE_UNICODE_INTERNALS
00177 #define CYTHON_USE_UNICODE_INTERNALS 1
00178 #endif
00179 #if PY_VERSION_HEX < 0x030300F0 || PY_VERSION_HEX >= 0x030B00A2
00180 #undef CYTHON_USE_UNICODE_WRITER
00181 #define CYTHON_USE_UNICODE_WRITER 0
00182 #elif !defined(CYTHON_USE_UNICODE_WRITER)
00183 #define CYTHON_USE_UNICODE_WRITER 1
00184 #endif

```

```

00185 #ifndef CYTHON_AVOID_BORROWED_REFS
00186 #define CYTHON_AVOID_BORROWED_REFS 0
00187 #endif
00188 #ifndef CYTHON_ASSUME_SAFE_MACROS
00189 #define CYTHON_ASSUME_SAFE_MACROS 1
00190 #endif
00191 #ifndef CYTHON_UNPACK_METHODS
00192 #define CYTHON_UNPACK_METHODS 1
00193 #endif
00194 #if PY_VERSION_HEX >= 0x030B00A4
00195 #undef CYTHON_FAST_THREAD_STATE
00196 #define CYTHON_FAST_THREAD_STATE 0
00197 #elif !defined(CYTHON_FAST_THREAD_STATE)
00198 #define CYTHON_FAST_THREAD_STATE 1
00199 #endif
00200 #ifndef CYTHON_FAST_PYCALL
00201 #define CYTHON_FAST_PYCALL (PY_VERSION_HEX < 0x030B00A1)
00202 #endif
00203 #ifndef CYTHON_PEP489_MULTI_PHASE_INIT
00204 #define CYTHON_PEP489_MULTI_PHASE_INIT (PY_VERSION_HEX >= 0x03050000)
00205 #endif
00206 #ifndef CYTHON_USE_TP_FINALIZE
00207 #define CYTHON_USE_TP_FINALIZE (PY_VERSION_HEX >= 0x030400a1)
00208 #endif
00209 #ifndef CYTHON_USE_DICT_VERSIONS
00210 #define CYTHON_USE_DICT_VERSIONS (PY_VERSION_HEX >= 0x030600B1)
00211 #endif
00212 #if PY_VERSION_HEX >= 0x030B00A4
00213 #undef CYTHON_USE_EXC_INFO_STACK
00214 #define CYTHON_USE_EXC_INFO_STACK 0
00215 #elif !defined(CYTHON_USE_EXC_INFO_STACK)
00216 #define CYTHON_USE_EXC_INFO_STACK (PY_VERSION_HEX >= 0x030700A3)
00217 #endif
00218 #endif
00219 #if !defined(CYTHON_FAST_PYCCALL)
00220 #define CYTHON_FAST_PYCCALL (CYTHON_FAST_PYCALL && PY_VERSION_HEX >= 0x030600B1)
00221 #endif
00222 #if CYTHON_USE_PYLONG_INTERNALS
00223 #if PY_MAJOR_VERSION < 3
00224 #include "longintrepr.h"
00225 #endif
00226 #undef SHIFT
00227 #undef BASE
00228 #undef MASK
00229 #ifdef SIZEOF_VOID_P
00230 enum { __pyx_check_sizeof_voidp = 1 / (int) (sizeof(void*) == sizeof(void*)) };
00231 #endif
00232 #endif
00233 #ifndef __has_attribute
00234 #define __has_attribute(x) 0
00235 #endif
00236 #ifndef __has_cpp_attribute
00237 #define __has_cpp_attribute(x) 0
00238 #endif
00239 #ifndef CYTHON_RESTRICT
00240 #if defined(__GNUC__)
00241 #define CYTHON_RESTRICT __restrict__
00242 #elif defined(_MSC_VER) && _MSC_VER >= 1400
00243 #define CYTHON_RESTRICT __restrict
00244 #elif defined(__STDC_VERSION__) && __STDC_VERSION__ >= 199901L
00245 #define CYTHON_RESTRICT restrict
00246 #else
00247 #define CYTHON_RESTRICT
00248 #endif
00249 #endif
00250 #ifndef CYTHON_UNUSED
00251 #if defined(__GNUC__)
00252 # if !defined(__cplusplus) || (__GNUC__ > 3 || (__GNUC__ == 3 && __GNUC_MINOR__ >= 4))
00253 #   define CYTHON_UNUSED __attribute__((__unused__))
00254 # else
00255 #   define CYTHON_UNUSED
00256 # endif
00257 # elif defined(__ICC) || (defined(__INTEL_COMPILER) && !defined(_MSC_VER))
00258 #   define CYTHON_UNUSED __attribute__((__unused__))
00259 # else
00260 #   define CYTHON_UNUSED
00261 # endif
00262 #endif
00263 #ifndef CYTHON_MAYBE_UNUSED_VAR
00264 # if defined(__cplusplus)
00265     template<class T> void CYTHON_MAYBE_UNUSED_VAR( const T& ) { }
00266 # else
00267 #   define CYTHON_MAYBE_UNUSED_VAR(x) (void)(x)
00268 # endif
00269 #endif
00270 #ifndef CYTHON_NCP_UNUSED
00271 # if CYTHON_COMPILING_IN_CPYTHON

```

```

00272 # define CYTHON_NCP_UNUSED
00273 # else
00274 # define CYTHON_NCP_UNUSED CYTHON_UNUSED
00275 # endif
00276 #endif
00277 #define __Pyx_void_to_None(void_result) ((void)(void_result), Py_INCREF(Py_None), Py_None)
00278 #ifndef _MSC_VER
00279     #ifndef _MSC_STDINT_H_
00280         #if _MSC_VER < 1300
00281             typedef unsigned char    uint8_t;
00282             typedef unsigned int     uint32_t;
00283         #else
00284             typedef unsigned __int8  uint8_t;
00285             typedef unsigned __int32  uint32_t;
00286         #endif
00287     #endif
00288 #else
00289     #include <stdint.h>
00290 #endif
00291 #ifndef CYTHON_FALLTHROUGH
00292     #if defined(__cplusplus) && __cplusplus >= 201103L
00293         #if __has_cpp_attribute(fallthrough)
00294             #define CYTHON_FALLTHROUGH [[fallthrough]]
00295         #elif __has_cpp_attribute(clang::fallthrough)
00296             #define CYTHON_FALLTHROUGH [[clang::fallthrough]]
00297         #elif __has_cpp_attribute(gnu::fallthrough)
00298             #define CYTHON_FALLTHROUGH [[gnu::fallthrough]]
00299         #endif
00300     #endif
00301 #ifndef CYTHON_FALLTHROUGH
00302     #if __has_attribute(fallthrough)
00303         #define CYTHON_FALLTHROUGH __attribute__((fallthrough))
00304     #else
00305         #define CYTHON_FALLTHROUGH
00306     #endif
00307 #endif
00308 #if defined(__clang__) && defined(__apple_build_version__)
00309     #if __apple_build_version__ < 7000000
00310         #undef CYTHON_FALLTHROUGH
00311         #define CYTHON_FALLTHROUGH
00312     #endif
00313 #endif
00314 #endif
00315
00316 #ifndef __cplusplus
00317     #error "Cython files generated with the C++ option must be compiled with a C++ compiler."
00318 #endif
00319 #ifndef CYTHON_INLINE
00320     #if defined(__clang__)
00321         #define CYTHON_INLINE __inline__ __attribute__((unused))
00322     #else
00323         #define CYTHON_INLINE inline
00324     #endif
00325 #endif
00326 template<typename T>
00327 void __Pyx_call_destructor(T& x) {
00328     x.~T();
00329 }
00330 template<typename T>
00331 class __Pyx_FakeReference {
00332 public:
00333     __Pyx_FakeReference() : ptr(NULL) { }
00334     __Pyx_FakeReference(const T& ref) : ptr(const_cast<T*>(&ref)) { }
00335     T *operator->() { return ptr; }
00336     T *operator&() { return ptr; }
00337     operator T&() { return *ptr; }
00338     template<typename U> bool operator == (U other) { return *ptr == other; }
00339     template<typename U> bool operator != (U other) { return *ptr != other; }
00340 private:
00341     T *ptr;
00342 };
00343
00344 #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x02070600 && !defined(Py_OptimizeFlag)
00345     #define Py_OptimizeFlag 0
00346 #endif
00347 #define __PYX_BUILD_PY_SSIZE_T "n"
00348 #define CYTHON_FORMAT_SSIZE_T "z"
00349 #if PY_MAJOR_VERSION < 3
00350     #define __Pyx_BUILTIN_MODULE_NAME "__builtin__"
00351     #define __Pyx_PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)\
00352         PyCode_New(a+k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)
00353     #define __Pyx_DefaultClassType PyClass_Type
00354 #else
00355     #define __Pyx_BUILTIN_MODULE_NAME "builtins"
00356     #define __Pyx_DefaultClassType PyType_Type
00357 #if PY_VERSION_HEX >= 0x030B00A1
00358     static CYTHON_INLINE PyCodeObject* __Pyx_PyCode_New(int a, int k, int l, int s, int f,

```

```

00359                                     PyObject *code, PyObject *c, PyObject* n, PyObject
00360 *v,
00361                                     PyObject *fv, PyObject *cell, PyObject* fn,
00362                                     PyObject *name, int fline, PyObject *lnos) {
00363     PyObject *kwds=NULL, *argcount=NULL, *posonlyargcount=NULL, *kwnonlyargcount=NULL;
00364     PyObject *nlocals=NULL, *stacksize=NULL, *flags=NULL, *replace=NULL, *call_result=NULL,
00365     *empty=NULL;
00366     const char *fn_cstr=NULL;
00367     const char *name_cstr=NULL;
00368     PyCodeObject* co=NULL;
00369     PyObject *type, *value, *traceback;
00370     PyErr_Fetch(&type, &value, &traceback);
00371     if (!(kwds=PyDict_New())) goto end;
00372     if (!(argcount=PyLong_FromLong(a))) goto end;
00373     if (PyDict_SetItemString(kwds, "co_argcount", argcount) != 0) goto end;
00374     if (!(posonlyargcount=PyLong_FromLong(0))) goto end;
00375     if (PyDict_SetItemString(kwds, "co_posonlyargcount", posonlyargcount) != 0) goto end;
00376     if (!(kwnonlyargcount=PyLong_FromLong(k))) goto end;
00377     if (PyDict_SetItemString(kwds, "co_kwnonlyargcount", kwnonlyargcount) != 0) goto end;
00378     if (!(nlocals=PyLong_FromLong(l))) goto end;
00379     if (PyDict_SetItemString(kwds, "co_nlocals", nlocals) != 0) goto end;
00380     if (!(stacksize=PyLong_FromLong(s))) goto end;
00381     if (PyDict_SetItemString(kwds, "co_stacksize", stacksize) != 0) goto end;
00382     if (!(flags=PyLong_FromLong(f))) goto end;
00383     if (PyDict_SetItemString(kwds, "co_flags", flags) != 0) goto end;
00384     if (PyDict_SetItemString(kwds, "co_code", code) != 0) goto end;
00385     if (PyDict_SetItemString(kwds, "co_consts", c) != 0) goto end;
00386     if (PyDict_SetItemString(kwds, "co_names", n) != 0) goto end;
00387     if (PyDict_SetItemString(kwds, "co_varnames", v) != 0) goto end;
00388     if (PyDict_SetItemString(kwds, "co_freevars", fv) != 0) goto end;
00389     if (PyDict_SetItemString(kwds, "co_cellvars", cell) != 0) goto end;
00390     if (PyDict_SetItemString(kwds, "co_linetable", lnos) != 0) goto end;
00391     if (!(fn_cstr=PyUnicode_AsUTF8AndSize(fn, NULL))) goto end;
00392     if (!(name_cstr=PyUnicode_AsUTF8AndSize(name, NULL))) goto end;
00393     if (!(co = PyCode_NewEmpty(fn_cstr, name_cstr, fline))) goto end;
00394     if (!(replace = PyObject_GetAttrString((PyObject*)co, "replace"))) goto cleanup_code_too;
00395     if (!(empty = PyTuple_New(0))) goto cleanup_code_too; // unfortunately __pyx_empty_tuple isn't
00396     available here
00397     if (!(call_result = PyObject_Call(replace, empty, kwds))) goto cleanup_code_too;
00398     Py_XDECREF((PyObject*)co);
00399     co = (PyCodeObject*)call_result;
00400     call_result = NULL;
00401     if (0) {
00402         cleanup_code_too:
00403         Py_XDECREF((PyObject*)co);
00404         co = NULL;
00405     }
00406     end:
00407     Py_XDECREF(kwds);
00408     Py_XDECREF(argcount);
00409     Py_XDECREF(posonlyargcount);
00410     Py_XDECREF(kwnonlyargcount);
00411     Py_XDECREF(nlocals);
00412     Py_XDECREF(stacksize);
00413     Py_XDECREF(replace);
00414     Py_XDECREF(call_result);
00415     Py_XDECREF(empty);
00416     if (type) {
00417         PyErr_Restore(type, value, traceback);
00418     }
00419     return co;
00420 }
00421 #else
00422 #define __Pyx_PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)\
00423     PyCode_New(a, k, l, s, f, code, c, n, v, fv, cell, fn, name, fline, lnos)
00424 #endif
00425 #define __Pyx_DefaultClassType PyType_Type
00426 #endif
00427 #ifndef Py_TPFLAGS_CHECKTYPES
00428 #define Py_TPFLAGS_CHECKTYPES 0
00429 #endif
00430 #ifndef Py_TPFLAGS_HAVE_INDEX
00431 #define Py_TPFLAGS_HAVE_INDEX 0
00432 #endif
00433 #ifndef Py_TPFLAGS_HAVE_NEWBUFFER
00434 #define Py_TPFLAGS_HAVE_NEWBUFFER 0
00435 #endif
00436 #ifndef Py_TPFLAGS_HAVE_FINALIZE
00437 #define Py_TPFLAGS_HAVE_FINALIZE 0
00438 #endif
00439 #ifndef METH_STACKLESS
00440 #define METH_STACKLESS 0
00441 #endif
00442 #if PY_VERSION_HEX <= 0x030700A3 || !defined(METH_FASTCALL)
00443     #ifndef METH_FASTCALL
00444         #define METH_FASTCALL 0x80
00445     #endif
00446 #endif

```

```

00443     typedef PyObject *(*__Pyx_PyCFunctionFast) (PyObject *self, PyObject *const *args, Py_ssize_t
nargs);
00444     typedef PyObject *(*__Pyx_PyCFunctionFastWithKeywords) (PyObject *self, PyObject *const *args,
00445  Py_ssize_t nargs, PyObject *kwnames);
00446 #else
00447 #define __Pyx_PyCFunctionFast _PyCFunctionFast
00448 #define __Pyx_PyCFunctionFastWithKeywords _PyCFunctionFastWithKeywords
00449 #endif
00450 #if CYTHON_FAST_PYCCALL
00451 #define __Pyx_PyFastCFunction_Check(func) \
00452     ((PyCFunction_Check(func) && (METH_FASTCALL == (PyCFunction_GET_FLAGS(func) & ~(METH_CLASS |
METH_STATIC | METH_COEXIST | METH_KEYWORDS | METH_STACKLESS))))))
00453 #else
00454 #define __Pyx_PyFastCFunction_Check(func) 0
00455 #endif
00456 #if CYTHON_COMPILING_IN_PYPY && !defined(PyObject_Malloc)
00457 #define PyObject_Malloc(s)    PyMem_Malloc(s)
00458 #define PyObject_Free(p)      PyMem_Free(p)
00459 #define PyObject_Realloc(p)    PyMem_Realloc(p)
00460 #endif
00461 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX < 0x030400A1
00462 #define PyMem_RawMalloc(n)      PyMem_Malloc(n)
00463 #define PyMem_RawRealloc(p, n) PyMem_Realloc(p, n)
00464 #define PyMem_RawFree(p)       PyMem_Free(p)
00465 #endif
00466 #if CYTHON_COMPILING_IN_PYSTON
00467 #define __Pyx_PyCode_HasFreeVars(co) PyCode_HasFreeVars(co)
00468 #define __Pyx_PyFrame_SetLineNumber(frame, lineno) PyFrame_SetLineNumber(frame, lineno)
00469 #else
00470 #define __Pyx_PyCode_HasFreeVars(co) (PyCode_GetNumFree(co) > 0)
00471 #define __Pyx_PyFrame_SetLineNumber(frame, lineno) (frame)->f_lineno = (lineno)
00472 #endif
00473 #if !CYTHON_FAST_THREAD_STATE || PY_VERSION_HEX < 0x02070000
00474 #define __Pyx_PyThreadState_Current PyThreadState_GET()
00475 #elif PY_VERSION_HEX >= 0x03060000
00476 #define __Pyx_PyThreadState_Current _PyThreadState_UncheckedGet()
00477 #elif PY_VERSION_HEX >= 0x03000000
00478 #define __Pyx_PyThreadState_Current PyThreadState_GET()
00479 #else
00480 #define __Pyx_PyThreadState_Current _PyThreadState_Current
00481 #endif
00482 #if PY_VERSION_HEX < 0x030700A2 && !defined(PyThread_tss_create) && !defined(Py_tss_NEEDS_INIT)
00483 #include "pythread.h"
00484 #define Py_tss_NEEDS_INIT 0
00485 typedef int Py_tss_t;
00486 static CYTHON_INLINE int PyThread_tss_create(Py_tss_t *key) {
00487     *key = PyThread_create_key();
00488     return 0;
00489 }
00490 static CYTHON_INLINE Py_tss_t * PyThread_tss_alloc(void) {
00491     Py_tss_t *key = (Py_tss_t *)PyObject_Malloc(sizeof(Py_tss_t));
00492     *key = Py_tss_NEEDS_INIT;
00493     return key;
00494 }
00495 static CYTHON_INLINE void PyThread_tss_free(Py_tss_t *key) {
00496     PyObject_Free(key);
00497 }
00498 static CYTHON_INLINE int PyThread_tss_is_created(Py_tss_t *key) {
00499     return *key != Py_tss_NEEDS_INIT;
00500 }
00501 static CYTHON_INLINE void PyThread_tss_delete(Py_tss_t *key) {
00502     PyThread_delete_key(*key);
00503     *key = Py_tss_NEEDS_INIT;
00504 }
00505 static CYTHON_INLINE int PyThread_tss_set(Py_tss_t *key, void *value) {
00506     return PyThread_set_key_value(*key, value);
00507 }
00508 static CYTHON_INLINE void * PyThread_tss_get(Py_tss_t *key) {
00509     return PyThread_get_key_value(*key);
00510 }
00511 #endif
00512 #if CYTHON_COMPILING_IN_CPYTHON || defined(_PyDict_NewPresized)
00513 #define __Pyx_PyDict_NewPresized(n) ((n <= 8) ? PyDict_New() : _PyDict_NewPresized(n))
00514 #else
00515 #define __Pyx_PyDict_NewPresized(n) PyDict_New()
00516 #endif
00517 #if PY_MAJOR_VERSION >= 3 || CYTHON_FUTURE_DIVISION
00518 #define __Pyx_PyNumber_Divide(x, y)      PyNumber_TrueDivide(x, y)
00519 #define __Pyx_PyNumber_InPlaceDivide(x, y) PyNumber_InPlaceTrueDivide(x, y)
00520 #else
00521 #define __Pyx_PyNumber_Divide(x, y)      PyNumber_Divide(x, y)
00522 #define __Pyx_PyNumber_InPlaceDivide(x, y) PyNumber_InPlaceDivide(x, y)
00523 #endif
00524 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x030500A1 && CYTHON_USE_UNICODE_INTERNALS
00525 #define __Pyx_PyDict_GetItemStr(dict, name) _PyDict_GetItem_KnownHash(dict, name, ((PyASCIIObject *)
name)->hash)
00526 #else

```

```

00527 #define __Pyx_PyDict_GetItemStr(dict, name) PyDict_GetItem(dict, name)
00528 #endif
00529 #if PY_VERSION_HEX > 0x03030000 && defined(PyUnicode_KIND)
00530     #define CYTHON_PEP393_ENABLED 1
00531     #if defined(PyUnicode_IS_READY)
00532         #define __Pyx_PyUnicode_READY(op)          (likely(PyUnicode_IS_READY(op)) ?\
00533             0 : _PyUnicode_Ready((PyObject *) (op)))
00534     #else
00535         #define __Pyx_PyUnicode_READY(op)          (0)
00536     #endif
00537     #define __Pyx_PyUnicode_GET_LENGTH(u)          PyUnicode_GET_LENGTH(u)
00538     #define __Pyx_PyUnicode_READ_CHAR(u, i)        PyUnicode_READ_CHAR(u, i)
00539     #define __Pyx_PyUnicode_MAX_CHAR_VALUE(u)      PyUnicode_MAX_CHAR_VALUE(u)
00540     #define __Pyx_PyUnicode_KIND(u)                PyUnicode_KIND(u)
00541     #define __Pyx_PyUnicode_DATA(u)                PyUnicode_DATA(u)
00542     #define __Pyx_PyUnicode_READ(k, d, i)          PyUnicode_READ(k, d, i)
00543     #define __Pyx_PyUnicode_WRITE(k, d, i, ch)     PyUnicode_WRITE(k, d, i, ch)
00544     #if defined(PyUnicode_IS_READY) && defined(PyUnicode_GET_SIZE)
00545     #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03090000
00546         #define __Pyx_PyUnicode_IS_TRUE(u)         (0 != (likely(PyUnicode_IS_READY(u)) ?
PyUnicode_GET_LENGTH(u) : ((PyCompactUnicodeObject *) (u))->wstr_length))
00547     #else
00548         #define __Pyx_PyUnicode_IS_TRUE(u)         (0 != (likely(PyUnicode_IS_READY(u)) ?
PyUnicode_GET_LENGTH(u) : PyUnicode_GET_SIZE(u)))
00549     #endif
00550     #else
00551         #define __Pyx_PyUnicode_IS_TRUE(u)         (0 != PyUnicode_GET_LENGTH(u))
00552     #endif
00553 #else
00554     #define CYTHON_PEP393_ENABLED 0
00555     #define PyUnicode_1BYTE_KIND 1
00556     #define PyUnicode_2BYTE_KIND 2
00557     #define PyUnicode_4BYTE_KIND 4
00558     #define __Pyx_PyUnicode_READY(op)              (0)
00559     #define __Pyx_PyUnicode_GET_LENGTH(u)          PyUnicode_GET_SIZE(u)
00560     #define __Pyx_PyUnicode_READ_CHAR(u, i)        ((Py_UCS4) (PyUnicode_AS_UNICODE(u)[i]))
00561     #define __Pyx_PyUnicode_MAX_CHAR_VALUE(u)      ((sizeof(Py_UNICODE) == 2) ? 65535 : 1114111)
00562     #define __Pyx_PyUnicode_KIND(u)                (sizeof(Py_UNICODE))
00563     #define __Pyx_PyUnicode_DATA(u)                ((void*) PyUnicode_AS_UNICODE(u))
00564     #define __Pyx_PyUnicode_READ(k, d, i)          ((void) (k), (Py_UCS4) (((Py_UNICODE*) d)[i]))
00565     #define __Pyx_PyUnicode_WRITE(k, d, i, ch)     (((void) (k)), ((Py_UNICODE*) d)[i] = ch)
00566     #define __Pyx_PyUnicode_IS_TRUE(u)             (0 != PyUnicode_GET_SIZE(u))
00567 #endif
00568 #if CYTHON_COMPILING_IN_PYPY
00569     #define __Pyx_PyUnicode_Concat(a, b)           PyNumber_Add(a, b)
00570     #define __Pyx_PyUnicode_ConcatSafe(a, b)        PyNumber_Add(a, b)
00571 #else
00572     #define __Pyx_PyUnicode_Concat(a, b)           PyUnicode_Concat(a, b)
00573     #define __Pyx_PyUnicode_ConcatSafe(a, b)        ((unlikely((a) == Py_None) || unlikely((b) == Py_None)) ?\
00574         PyNumber_Add(a, b) : __Pyx_PyUnicode_Concat(a, b))
00575 #endif
00576 #if CYTHON_COMPILING_IN_PYPY && !defined(PyUnicode_Contains)
00577     #define PyUnicode_Contains(u, s)               PySequence_Contains(u, s)
00578 #endif
00579 #if CYTHON_COMPILING_IN_PYPY && !defined(PyByteArray_Check)
00580     #define PyByteArray_Check(obj)                  PyObject_TypeCheck(obj, &PyByteArray_Type)
00581 #endif
00582 #if CYTHON_COMPILING_IN_PYPY && !defined(PyObject_Format)
00583     #define PyObject_Format(obj, fmt)               PyObject_CallMethod(obj, "__format__", "O", fmt)
00584 #endif
00585 #define __Pyx_PyString_FormatSafe(a, b)            ((unlikely((a) == Py_None || (PyString_Check(b) &&
!PyString_CheckExact(b)))) ? PyNumber_Remainder(a, b) : __Pyx_PyString_Format(a, b))
00586 #define __Pyx_PyUnicode_FormatSafe(a, b)           ((unlikely((a) == Py_None || (PyUnicode_Check(b) &&
!PyUnicode_CheckExact(b)))) ? PyNumber_Remainder(a, b) : PyUnicode_Format(a, b))
00587 #if PY_MAJOR_VERSION >= 3
00588     #define __Pyx_PyString_Format(a, b)            PyUnicode_Format(a, b)
00589 #else
00590     #define __Pyx_PyString_Format(a, b)            PyString_Format(a, b)
00591 #endif
00592 #if PY_MAJOR_VERSION < 3 && !defined(PyObject_ASCII)
00593     #define PyObject_ASCII(o)                      PyObject_Repr(o)
00594 #endif
00595 #if PY_MAJOR_VERSION >= 3
00596     #define PyBaseString_Type                      PyUnicode_Type
00597     #define PyStringObject                          PyUnicodeObject
00598     #define PyString_Type                          PyUnicode_Type
00599     #define PyString_Check                          PyUnicode_Check
00600     #define PyString_CheckExact                     PyUnicode_CheckExact
00601 #ifndef PyObject_Unicode
00602     #define PyObject_Unicode                        PyObject_Str
00603 #endif
00604 #endif
00605 #if PY_MAJOR_VERSION >= 3
00606     #define __Pyx_PyBaseString_Check(obj)          PyUnicode_Check(obj)
00607     #define __Pyx_PyBaseString_CheckExact(obj)     PyUnicode_CheckExact(obj)
00608 #else
00609     #define __Pyx_PyBaseString_Check(obj)          (PyString_Check(obj) || PyUnicode_Check(obj))

```



```

00610 #define __Pyx_PyBaseString_CheckExact(obj) (PyString_CheckExact(obj) || PyUnicode_CheckExact(obj))
00611 #endif
00612 #ifndef PySet_CheckExact
00613 #define PySet_CheckExact(obj) (Py_TYPE(obj) == &PySet_Type)
00614 #endif
00615 #if PY_VERSION_HEX >= 0x030900A4
00616 #define __Pyx_SET_REFCNT(obj, refcnt) Py_SET_REFCNT(obj, refcnt)
00617 #define __Pyx_SET_SIZE(obj, size) Py_SET_SIZE(obj, size)
00618 #else
00619 #define __Pyx_SET_REFCNT(obj, refcnt) Py_REFCNT(obj) = (refcnt)
00620 #define __Pyx_SET_SIZE(obj, size) Py_SIZE(obj) = (size)
00621 #endif
00622 #if CYTHON_ASSUME_SAFE_MACROS
00623 #define __Pyx_PySequence_SIZE(seq) Py_SIZE(seq)
00624 #else
00625 #define __Pyx_PySequence_SIZE(seq) PySequence_Size(seq)
00626 #endif
00627 #if PY_MAJOR_VERSION >= 3
00628 #define PyIntObject PyLongObject
00629 #define PyInt_Type PyLong_Type
00630 #define PyInt_Check(op) PyLong_Check(op)
00631 #define PyInt_CheckExact(op) PyLong_CheckExact(op)
00632 #define PyInt_FromString PyLong_FromString
00633 #define PyInt_FromUnicode PyLong_FromUnicode
00634 #define PyInt_FromLong PyLong_FromLong
00635 #define PyInt_FromSize_t PyLong_FromSize_t
00636 #define PyInt_FromSsize_t PyLong_FromSsize_t
00637 #define PyInt_AsLong PyLong_AsLong
00638 #define PyInt_AS_LONG PyLong_AS_LONG
00639 #define PyInt_AsSsize_t PyLong_AsSsize_t
00640 #define PyInt_AsUnsignedLongMask PyLong_AsUnsignedLongMask
00641 #define PyInt_AsUnsignedLongLongMask PyLong_AsUnsignedLongLongMask
00642 #define PyNumber_Int PyNumber_Long
00643 #endif
00644 #if PY_MAJOR_VERSION >= 3
00645 #define PyBoolObject PyLongObject
00646 #endif
00647 #if PY_MAJOR_VERSION >= 3 && CYTHON_COMPILING_IN_PYPY
00648 #ifndef PyUnicode_InternFromString
00649 #define PyUnicode_InternFromString(s) PyUnicode_FromString(s)
00650 #endif
00651 #endif
00652 #if PY_VERSION_HEX < 0x030200A4
00653 typedef long Py_hash_t;
00654 #define __Pyx_PyInt_FromHash_t PyInt_FromLong
00655 #define __Pyx_PyInt_AsHash_t __Pyx_PyIndex_AsHash_t
00656 #else
00657 #define __Pyx_PyInt_FromHash_t PyInt_FromSsize_t
00658 #define __Pyx_PyInt_AsHash_t __Pyx_PyIndex_AsSsize_t
00659 #endif
00660 #if PY_MAJOR_VERSION >= 3
00661 #define __Pyx_PyMethod_New(func, self, klass) ((self) ? ((void)(klass), PyMethod_New(func, self)) :
__Pyx_NewRef(func))
00662 #else
00663 #define __Pyx_PyMethod_New(func, self, klass) PyMethod_New(func, self, klass)
00664 #endif
00665 #if CYTHON_USE_ASYNC_SLOTS
00666 #if PY_VERSION_HEX >= 0x030500B1
00667 #define __Pyx_PyAsyncMethodsStruct PyAsyncMethods
00668 #define __Pyx_PyType_AsAsync(obj) (Py_TYPE(obj)->tp_as_async)
00669 #else
00670 #define __Pyx_PyType_AsAsync(obj) ((__Pyx_PyAsyncMethodsStruct*) (Py_TYPE(obj)->tp_reserved))
00671 #endif
00672 #else
00673 #define __Pyx_PyType_AsAsync(obj) NULL
00674 #endif
00675 #ifndef __Pyx_PyAsyncMethodsStruct
00676 typedef struct {
00677     unaryfunc am_await;
00678     unaryfunc am_aiter;
00679     unaryfunc am_anext;
00680 } __Pyx_PyAsyncMethodsStruct;
00681 #endif
00682
00683 #if defined(WIN32) || defined(MS_WINDOWS)
00684 #define _USE_MATH_DEFINES
00685 #endif
00686 #include <math.h>
00687 #ifndef NAN
00688 #define __PYX_NAN() ((float) NAN)
00689 #else
00690 static CYTHON_INLINE float __PYX_NAN() {
00691     float value;
00692     memset(&value, 0xFF, sizeof(value));
00693     return value;
00694 }
00695 #endif

```



```

00696 #if defined(__CYGWIN__) && defined(_LDBL_EQ_DBL)
00697 #define __Pyx_trunc1 trunc
00698 #else
00699 #define __Pyx_trunc1 trunc1
00700 #endif
00701
00702 #define __PYX_MARK_ERR_POS(f_index, lineno) \
00703 { __pyx_filename = __pyx_f[f_index]; (void)__pyx_filename; __pyx_lineno = lineno;
00704 (void)__pyx_lineno; __pyx_clineno = __LINE__; (void)__pyx_clineno; }
00704 #define __PYX_ERR(f_index, lineno, Ln_error) \
00705 { __PYX_MARK_ERR_POS(f_index, lineno) goto Ln_error; }
00706
00707 #ifndef __PYX_EXTERN_C
00708 #ifdef __cplusplus
00709 #define __PYX_EXTERN_C extern "C"
00710 #else
00711 #define __PYX_EXTERN_C extern
00712 #endif
00713 #endif
00714
00715 #define __PYX_HAVE__PyClical
00716 #define __PYX_HAVE_API__PyClical
00717 /* Early includes */
00718 #include "ios"
00719 #include "new"
00720 #include "stdexcept"
00721 #include "typeinfo"
00722 #include <vector>
00723 #include "PyClical.h"
00724 #include <string.h>
00725 #include <string>
00726 #ifdef _OPENMP
00727 #include <omp.h>
00728 #endif /* _OPENMP */
00729
00730 #if defined(PYREX_WITHOUT_ASSERTIONS) && !defined(CYTHON_WITHOUT_ASSERTIONS)
00731 #define CYTHON_WITHOUT_ASSERTIONS
00732 #endif
00733
00734 typedef struct {PyObject **p; const char *s; const Py_ssize_t n; const char* encoding;
00735                const char is_unicode; const char is_str; const char intern; } __Pyx_StringTabEntry;
00736
00737 #define __PYX_DEFAULT_STRING_ENCODING_IS_ASCII 0
00738 #define __PYX_DEFAULT_STRING_ENCODING_IS_UTF8 0
00739 #define __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT (PY_MAJOR_VERSION >= 3 &&
__PYX_DEFAULT_STRING_ENCODING_IS_UTF8)
00740 #define __PYX_DEFAULT_STRING_ENCODING ""
00741 #define __Pyx_PyObject_FromString __Pyx_PyBytes_FromString
00742 #define __Pyx_PyObject_FromStringAndSize __Pyx_PyBytes_FromStringAndSize
00743 #define __Pyx_uchar_cast(c) ((unsigned char)c)
00744 #define __Pyx_long_cast(x) ((long)x)
00745 #define __Pyx_fits_Py_ssize_t(v, type, is_signed) ( \
00746     (sizeof(type) < sizeof(Py_ssize_t)) || \
00747     (sizeof(type) > sizeof(Py_ssize_t) && \
00748      likely(v < (type)PY_SSIZE_T_MAX || \
00749       v == (type)PY_SSIZE_T_MAX) && \
00750     (!is_signed || likely(v > (type)PY_SSIZE_T_MIN || \
00751      v == (type)PY_SSIZE_T_MIN))) || \
00752     (sizeof(type) == sizeof(Py_ssize_t) && \
00753      (is_signed || likely(v < (type)PY_SSIZE_T_MAX || \
00754       v == (type)PY_SSIZE_T_MAX))) )
00755 static CYTHON_INLINE int __Pyx_is_valid_index(Py_ssize_t i, Py_ssize_t limit) {
00756     return (size_t) i < (size_t) limit;
00757 }
00758 #if defined (__cplusplus) && __cplusplus >= 201103L
00759 #include <cstdlib>
00760 #define __Pyx_sst_abs(value) std::abs(value)
00761 #elif SIZEOF_INT >= SIZEOF_SIZE_T
00762 #define __Pyx_sst_abs(value) abs(value)
00763 #elif SIZEOF_LONG >= SIZEOF_SIZE_T
00764 #define __Pyx_sst_abs(value) labs(value)
00765 #elif defined (_MSC_VER)
00766 #define __Pyx_sst_abs(value) ((Py_ssize_t)_abs64(value))
00767 #elif defined (__STDC_VERSION__) && __STDC_VERSION__ >= 199901L
00768 #define __Pyx_sst_abs(value) llabs(value)
00769 #elif defined (__GNUC__)
00770 #define __Pyx_sst_abs(value) __builtin_llabs(value)
00771 #else
00772 #define __Pyx_sst_abs(value) ((value<0) ? -value : value)
00773 #endif
00774 static CYTHON_INLINE const char* __Pyx_PyObject_AsString(PyObject*);
00775 static CYTHON_INLINE const char* __Pyx_PyObject_AsStringAndSize(PyObject*, Py_ssize_t* length);
00776 #define __Pyx_PyByteArray_FromString(s) PyByteArray_FromStringAndSize((const char*)s, strlen((const
char*)s))
00777 #define __Pyx_PyByteArray_FromStringAndSize(s, l) PyByteArray_FromStringAndSize((const char*)s, l)
00778 #define __Pyx_PyBytes_FromString PyBytes_FromString
00779 #define __Pyx_PyBytes_FromStringAndSize PyBytes_FromStringAndSize

```

```

00780 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_FromString(const char*);
00781 #if PY_MAJOR_VERSION < 3
00782     #define __Pyx_PyStr_FromString      __Pyx_PyBytes_FromString
00783     #define __Pyx_PyStr_FromStringAndSize __Pyx_PyBytes_FromStringAndSize
00784 #else
00785     #define __Pyx_PyStr_FromString      __Pyx_PyUnicode_FromString
00786     #define __Pyx_PyStr_FromStringAndSize __Pyx_PyUnicode_FromStringAndSize
00787 #endif
00788 #define __Pyx_PyBytes_AsWritableString(s)    ((char*) PyBytes_AS_STRING(s))
00789 #define __Pyx_PyBytes_AsWritableSString(s)    ((signed char*) PyBytes_AS_STRING(s))
00790 #define __Pyx_PyBytes_AsWritableUString(s)    ((unsigned char*) PyBytes_AS_STRING(s))
00791 #define __Pyx_PyBytes_AsString(s)            ((const char*) PyBytes_AS_STRING(s))
00792 #define __Pyx_PyBytes_AsSString(s)           ((const signed char*) PyBytes_AS_STRING(s))
00793 #define __Pyx_PyBytes_AsUString(s)           ((const unsigned char*) PyBytes_AS_STRING(s))
00794 #define __Pyx_PyObject_AsWritableString(s)    ((char*) __Pyx_PyObject_AsString(s))
00795 #define __Pyx_PyObject_AsWritableSString(s)    ((signed char*) __Pyx_PyObject_AsString(s))
00796 #define __Pyx_PyObject_AsWritableUString(s)    ((unsigned char*) __Pyx_PyObject_AsString(s))
00797 #define __Pyx_PyObject_AsSString(s)           ((const signed char*) __Pyx_PyObject_AsString(s))
00798 #define __Pyx_PyObject_AsUString(s)           ((const unsigned char*) __Pyx_PyObject_AsString(s))
00799 #define __Pyx_PyObject_FromCString(s)         __Pyx_PyObject_FromString((const char*)s)
00800 #define __Pyx_PyBytes_FromCString(s)          __Pyx_PyBytes_FromString((const char*)s)
00801 #define __Pyx_PyByteArray_FromCString(s)      __Pyx_PyByteArray_FromString((const char*)s)
00802 #define __Pyx_PyStr_FromCString(s)            __Pyx_PyStr_FromString((const char*)s)
00803 #define __Pyx_PyUnicode_FromCString(s)        __Pyx_PyUnicode_FromString((const char*)s)
00804 static CYTHON_INLINE size_t __Pyx_Py_UNICODE_strlen(const Py_UNICODE *u) {
00805     const Py_UNICODE *u_end = u;
00806     while (*u_end++) ;
00807     return (size_t)(u_end - u - 1);
00808 }
00809 #define __Pyx_PyUnicode_FromUnicode(u)         PyUnicode_FromUnicode(u, __Pyx_Py_UNICODE_strlen(u))
00810 #define __Pyx_PyUnicode_FromUnicodeAndLength PyUnicode_FromUnicode
00811 #define __Pyx_PyUnicode_AsUnicode             PyUnicode_AsUnicode
00812 #define __Pyx_NewRef(obj) (Py_INCREF(obj), obj)
00813 #define __Pyx_Owned_Py_None(b) __Pyx_NewRef(Py_None)
00814 static CYTHON_INLINE PyObject * __Pyx_PyBool_FromLong(long b);
00815 static CYTHON_INLINE int __Pyx_PyObject_IsTrue(PyObject*);
00816 static CYTHON_INLINE int __Pyx_PyObject_IsTrueAndDecref(PyObject*);
00817 static CYTHON_INLINE PyObject* __Pyx_PyNumber_IntOrLong(PyObject* x);
00818 #define __Pyx_PySequence_Tuple(obj)\
00819     (likely(PyTuple_CheckExact(obj)) ? __Pyx_NewRef(obj) : PySequence_Tuple(obj))
00820 static CYTHON_INLINE Py_ssize_t __Pyx_PyIndex_AsSsize_t(PyObject*);
00821 static CYTHON_INLINE PyObject * __Pyx_PyInt_FromSize_t(size_t);
00822 static CYTHON_INLINE Py_hash_t __Pyx_PyIndex_AsHash_t(PyObject*);
00823 #if CYTHON_ASSUME_SAFE_MACROS
00824 #define __pyx_PyFloat_AsDouble(x) (PyFloat_CheckExact(x) ? PyFloat_AS_DOUBLE(x) : PyFloat_AsDouble(x))
00825 #else
00826 #define __pyx_PyFloat_AsDouble(x) PyFloat_AsDouble(x)
00827 #endif
00828 #define __pyx_PyFloat_AsFloat(x) ((float) __pyx_PyFloat_AsDouble(x))
00829 #if PY_MAJOR_VERSION >= 3
00830 #define __Pyx_PyNumber_Int(x) (PyLong_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Long(x))
00831 #else
00832 #define __Pyx_PyNumber_Int(x) (PyInt_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Int(x))
00833 #endif
00834 #define __Pyx_PyNumber_Float(x) (PyFloat_CheckExact(x) ? __Pyx_NewRef(x) : PyNumber_Float(x))
00835 #if PY_MAJOR_VERSION < 3 && __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
00836 static int __Pyx_sys_getdefaultencoding_not_ascii;
00837 static int __Pyx_init_sys_getdefaultencoding_params(void) {
00838     PyObject* sys;
00839     PyObject* default_encoding = NULL;
00840     PyObject* ascii_chars_u = NULL;
00841     PyObject* ascii_chars_b = NULL;
00842     const char* default_encoding_c;
00843     sys = PyImport_ImportModule("sys");
00844     if (!sys) goto bad;
00845     default_encoding = PyObject_CallMethod(sys, (char*) "getdefaultencoding", NULL);
00846     Py_DECREF(sys);
00847     if (!default_encoding) goto bad;
00848     default_encoding_c = PyBytes_AsString(default_encoding);
00849     if (!default_encoding_c) goto bad;
00850     if (strcmp(default_encoding_c, "ascii") == 0) {
00851         __Pyx_sys_getdefaultencoding_not_ascii = 0;
00852     } else {
00853         char ascii_chars[128];
00854         int c;
00855         for (c = 0; c < 128; c++) {
00856             ascii_chars[c] = c;
00857         }
00858         __Pyx_sys_getdefaultencoding_not_ascii = 1;
00859         ascii_chars_u = PyUnicode_DecodeASCII(ascii_chars, 128, NULL);
00860         if (!ascii_chars_u) goto bad;
00861         ascii_chars_b = PyUnicode_AsEncodedString(ascii_chars_u, default_encoding_c, NULL);
00862         if (!ascii_chars_b || !PyBytes_Check(ascii_chars_b) || memcmp(ascii_chars,
PyBytes_AS_STRING(ascii_chars_b), 128) != 0) {
00863             PyErr_Format(
00864                 PyExc_ValueError,
00865                 "This module compiled with c_string_encoding=ascii, but default encoding '%.200s' is

```

```

        not a superset of ascii.",
00866         default_encoding_c);
00867         goto bad;
00868     }
00869     Py_DECREF(ascii_chars_u);
00870     Py_DECREF(ascii_chars_b);
00871 }
00872 Py_DECREF(default_encoding);
00873 return 0;
00874 bad:
00875     Py_XDECREF(default_encoding);
00876     Py_XDECREF(ascii_chars_u);
00877     Py_XDECREF(ascii_chars_b);
00878     return -1;
00879 }
00880 #endif
00881 #if __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT && PY_MAJOR_VERSION >= 3
00882 #define __Pyx_PyUnicode_FromStringAndSize(c_str, size) PyUnicode_DecodeUTF8(c_str, size, NULL)
00883 #else
00884 #define __Pyx_PyUnicode_FromStringAndSize(c_str, size) PyUnicode_Decode(c_str, size,
__PYX_DEFAULT_STRING_ENCODING, NULL)
00885 #if __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
00886 static char* __PYX_DEFAULT_STRING_ENCODING;
00887 static int __Pyx_init_sys_getdefaultencoding_params(void) {
00888     PyObject* sys;
00889     PyObject* default_encoding = NULL;
00890     char* default_encoding_c;
00891     sys = PyImport_ImportModule("sys");
00892     if (!sys) goto bad;
00893     default_encoding = PyObject_CallMethod(sys, (char*) (const char*) "getdefaultencoding", NULL);
00894     Py_DECREF(sys);
00895     if (!default_encoding) goto bad;
00896     default_encoding_c = PyBytes_AsString(default_encoding);
00897     if (!default_encoding_c) goto bad;
00898     __PYX_DEFAULT_STRING_ENCODING = (char*) malloc(strlen(default_encoding_c) + 1);
00899     if (!__PYX_DEFAULT_STRING_ENCODING) goto bad;
00900     strcpy(__PYX_DEFAULT_STRING_ENCODING, default_encoding_c);
00901     Py_DECREF(default_encoding);
00902     return 0;
00903 bad:
00904     Py_XDECREF(default_encoding);
00905     return -1;
00906 }
00907 #endif
00908 #endif
00909
00910
00911 /* Test for GCC > 2.95 */
00912 #if defined(__GNUC__) && (__GNUC__ > 2 || (__GNUC__ == 2 && (__GNUC_MINOR__ > 95)))
00913     #define likely(x) __builtin_expect(!!(x), 1)
00914     #define unlikely(x) __builtin_expect(!!(x), 0)
00915 #else /* !__GNUC__ or GCC < 2.95 */
00916     #define likely(x) (x)
00917     #define unlikely(x) (x)
00918 #endif /* __GNUC__ */
00919 static CYTHON_INLINE void __Pyx_pretend_to_initialize(void* ptr) { (void)ptr; }
00920
00921 static PyObject * __pyx_m = NULL;
00922 static PyObject * __pyx_d;
00923 static PyObject * __pyx_b;
00924 static PyObject * __pyx_cython_runtime = NULL;
00925 static PyObject * __pyx_empty_tuple;
00926 static PyObject * __pyx_empty_bytes;
00927 static PyObject * __pyx_empty_unicode;
00928 static int __pyx_lineno;
00929 static int __pyx_clineno = 0;
00930 static const char * __pyx_cfilenm= __FILE__;
00931 static const char * __pyx_filename;
00932
00933
00934 static const char * __pyx_f[] = {
00935     "PyClical.pyx",
00936     "stringsource",
00937 };
00938
00939 /*--- Type declarations ---*/
00940 struct __pyx_obj_8PyClical_index_set;
00941 struct __pyx_obj_8PyClical_clifford;
00942 struct __pyx_obj_8PyClical__pyx_scope_struct____iter____;
00943 struct __pyx_opt_args_8PyClical_approx_equal;
00944 struct __pyx_opt_args_8PyClical_sqrt;
00945 struct __pyx_opt_args_8PyClical_log;
00946 struct __pyx_opt_args_8PyClical_cos;
00947 struct __pyx_opt_args_8PyClical_acos;
00948 struct __pyx_opt_args_8PyClical_acosh;
00949 struct __pyx_opt_args_8PyClical_sin;
00950 struct __pyx_opt_args_8PyClical_asin;

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00951 struct __pyx_opt_args_8PyClical_asinh;
00952 struct __pyx_opt_args_8PyClical_tan;
00953 struct __pyx_opt_args_8PyClical_atan;
00954 struct __pyx_opt_args_8PyClical_atanh;
00955 struct __pyx_opt_args_8PyClical_random_clifford;
00956
00957 /* "PyClical.pyx":1359
00958 *     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
00959 *
00960 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):          # ««««««««
00961 *     """
00962 *         Test for approximate equality of multivectors.
00963 */
00964 struct __pyx_opt_args_8PyClical_approx_equal {
00965     int __pyx_n;
00966     PyObject *threshold;
00967     PyObject *tol;
00968 };
00969
00970 /* "PyClical.pyx":1591
00971 *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
00972 *
00973 * cpdef inline sqrt(obj, i = None):          # ««««««««
00974 *     """
00975 *         Square root of multivector with optional complexifier.
00976 */
00977 struct __pyx_opt_args_8PyClical_sqrt {
00978     int __pyx_n;
00979     PyObject *i;
00980 };
00981
00982 /* "PyClical.pyx":1628
00983 *     return clifford().wrap( glucat.exp(toClifford(obj)) )
00984 *
00985 * cpdef inline log(obj,i = None):          # ««««««««
00986 *     """
00987 *         Natural logarithm of multivector with optional complexifier.
00988 */
00989 struct __pyx_opt_args_8PyClical_log {
00990     int __pyx_n;
00991     PyObject *i;
00992 };
00993
00994 /* "PyClical.pyx":1651
00995 *     return clifford().wrap( glucat.log(toClifford(obj)) )
00996 *
00997 * cpdef inline cos(obj,i = None):          # ««««««««
00998 *     """
00999 *         Cosine of multivector with optional complexifier.
01000 */
01001 struct __pyx_opt_args_8PyClical_cos {
01002     int __pyx_n;
01003     PyObject *i;
01004 };
01005
01006 /* "PyClical.pyx":1668
01007 *     return clifford().wrap( glucat.cos(toClifford(obj)) )
01008 *
01009 * cpdef inline acos(obj,i = None):          # ««««««««
01010 *     """
01011 *         Inverse cosine of multivector with optional complexifier.
01012 */
01013 struct __pyx_opt_args_8PyClical_acos {
01014     int __pyx_n;
01015     PyObject *i;
01016 };
01017
01018 /* "PyClical.pyx":1705
01019 *     return clifford().wrap( glucat.cosh(toClifford(obj)) )
01020 *
01021 * cpdef inline acosh(obj,i = None):          # ««««««««
01022 *     """
01023 *         Inverse hyperbolic cosine of multivector with optional complexifier.
01024 */
01025 struct __pyx_opt_args_8PyClical_acosh {
01026     int __pyx_n;
01027     PyObject *i;
01028 };
01029
01030 /* "PyClical.pyx":1728
01031 *     return clifford().wrap( glucat.acosh(toClifford(obj)) )
01032 *
01033 * cpdef inline sin(obj,i = None):          # ««««««««
01034 *     """
01035 *         Sine of multivector with optional complexifier.
01036 */
01037 struct __pyx_opt_args_8PyClical_sin {

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01038     int __pyx_n;
01039     PyObject *i;
01040 };
01041
01042 /* "PyClical.pyx":1747
01043 *         return clifford().wrap( glucat.sin(toClifford(obj)) )
01044 *
01045 * cpdef inline asin(obj,i = None):          # ««««««««
01046 *     """
01047 *     Inverse sine of multivector with optional complexifier.
01048 */
01049 struct __pyx_opt_args_8PyClical_asin {
01050     int __pyx_n;
01051     PyObject *i;
01052 };
01053
01054 /* "PyClical.pyx":1782
01055 *         return clifford().wrap( glucat.sinh(toClifford(obj)) )
01056 *
01057 * cpdef inline asinh(obj,i = None):          # ««««««««
01058 *     """
01059 *     Inverse hyperbolic sine of multivector with optional complexifier.
01060 */
01061 struct __pyx_opt_args_8PyClical_asinh {
01062     int __pyx_n;
01063     PyObject *i;
01064 };
01065
01066 /* "PyClical.pyx":1801
01067 *         return clifford().wrap( glucat.asinh(toClifford(obj)) )
01068 *
01069 * cpdef inline tan(obj,i = None):            # ««««««««
01070 *     """
01071 *     Tangent of multivector with optional complexifier.
01072 */
01073 struct __pyx_opt_args_8PyClical_tan {
01074     int __pyx_n;
01075     PyObject *i;
01076 };
01077
01078 /* "PyClical.pyx":1818
01079 *         return clifford().wrap( glucat.tan(toClifford(obj)) )
01080 *
01081 * cpdef inline atan(obj,i = None):           # ««««««««
01082 *     """
01083 *     Inverse tangent of multivector with optional complexifier.
01084 */
01085 struct __pyx_opt_args_8PyClical_atan {
01086     int __pyx_n;
01087     PyObject *i;
01088 };
01089
01090 /* "PyClical.pyx":1847
01091 *         return clifford().wrap( glucat.tanh(toClifford(obj)) )
01092 *
01093 * cpdef inline atanh(obj,i = None):          # ««««««««
01094 *     """
01095 *     Inverse hyperbolic tangent of multivector with optional complexifier.
01096 */
01097 struct __pyx_opt_args_8PyClical_atanh {
01098     int __pyx_n;
01099     PyObject *i;
01100 };
01101
01102 /* "PyClical.pyx":1864
01103 *         return clifford().wrap( glucat.atanh(toClifford(obj)) )
01104 *
01105 * cpdef inline random_clifford(index_set ixt, fill = 1.0):          # ««««««««
01106 *     """
01107 *     Random multivector within a frame.
01108 */
01109 struct __pyx_opt_args_8PyClical_random_clifford {
01110     int __pyx_n;
01111     PyObject *fill;
01112 };
01113
01114 /* "PyClical.pyx":38
01115 *
01116 * # Forward reference
01117 * cdef class index_set          # ««««««««
01118 *
01119 * cdef inline IndexSet toIndexSet(obj):
01120 */
01121 struct __pyx_obj_8PyClical_index_set {
01122     PyObject_HEAD
01123     struct __pyx_vtabstruct_8PyClical_index_set *__pyx_vtab;
01124     IndexSet *instance;

```

```

01125 };
01126
01127
01128 /* "PyClicl.pyx":532
01129 *
01130 * # Forward reference.
01131 * cdef class clifford # ««««««««
01132 *
01133 * cdef inline Clifford toClifford(obj):
01134 */
01135 struct __pyx_obj_8PyClicl_clifford {
01136     PyObject_HEAD
01137     struct __pyx_vtabstruct_8PyClicl_clifford *__pyx_vtab;
01138     Clifford *instance;
01139 };
01140
01141
01142 /* "PyClicl.pyx":229
01143 *     return self.instance.getitem(idx)
01144 *
01145 *     def __iter__(self): # ««««««««
01146 *         """
01147 *         Iterate over the indices of an index_set.
01148 */
01149 struct __pyx_obj_8PyClicl__pyx_scope_struct__iter__ {
01150     PyObject_HEAD
01151     PyObject *__pyx_v_idx;
01152     struct __pyx_obj_8PyClicl_index_set *__pyx_v_self;
01153     PyObject *__pyx_t_0;
01154     Py_ssize_t __pyx_t_1;
01155     PyObject *(*__pyx_t_2)(PyObject *);
01156 };
01157
01158
01159
01160 /* "PyClicl.pyx":46
01161 *     return index_set(obj).instance[0]
01162 *
01163 * cdef class index_set: # ««««««««
01164 *     """
01165 *     Python class index_set wraps C++ class IndexSet.
01166 */
01167
01168 struct __pyx_vtabstruct_8PyClicl_index_set {
01169     PyObject *(*wrap)(struct __pyx_obj_8PyClicl_index_set *, IndexSet);
01170     IndexSet (*unwrap)(struct __pyx_obj_8PyClicl_index_set *);
01171     PyObject *(*copy)(struct __pyx_obj_8PyClicl_index_set *, int __pyx_skip_dispatch);
01172 };
01173 static struct __pyx_vtabstruct_8PyClicl_index_set *__pyx_vtabptr_8PyClicl_index_set;
01174 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_9index_set_wrap(struct __pyx_obj_8PyClicl_index_set
*, IndexSet);
01175 static CYTHON_INLINE IndexSet __pyx_f_8PyClicl_9index_set_unwrap(struct __pyx_obj_8PyClicl_index_set
*);
01176
01177
01178 /* "PyClicl.pyx":537
01179 *     return clifford(obj).instance[0]
01180 *
01181 * cdef class clifford: # ««««««««
01182 *     """
01183 *     Python class clifford wraps C++ class Clifford.
01184 */
01185
01186 struct __pyx_vtabstruct_8PyClicl_clifford {
01187     PyObject *(*wrap)(struct __pyx_obj_8PyClicl_clifford *, Clifford);
01188     Clifford (*unwrap)(struct __pyx_obj_8PyClicl_clifford *);
01189     PyObject *(*copy)(struct __pyx_obj_8PyClicl_clifford *, int __pyx_skip_dispatch);
01190 };
01191 static struct __pyx_vtabstruct_8PyClicl_clifford *__pyx_vtabptr_8PyClicl_clifford;
01192 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_8clifford_wrap(struct __pyx_obj_8PyClicl_clifford *,
Clifford);
01193 static CYTHON_INLINE Clifford __pyx_f_8PyClicl_8clifford_unwrap(struct __pyx_obj_8PyClicl_clifford
*);
01194
01195 /* --- Runtime support code (head) --- */
01196 /* Refnanny.proto */
01197 #ifndef CYTHON_REFNANNY
01198     #define CYTHON_REFNANNY 0
01199 #endif
01200 #if CYTHON_REFNANNY
01201     typedef struct {
01202         void (*INCREf)(void*, PyObject*, int);
01203         void (*DECREf)(void*, PyObject*, int);
01204         void (*GOTREf)(void*, PyObject*, int);
01205         void (*GIVEREF)(void*, PyObject*, int);
01206         void* (*SetupContext)(const char*, int, const char*);
01207         void (*FinishContext)(void**);
01208     }

```

```

01208 } __Pyx_RefNannyAPIStruct;
01209 static __Pyx_RefNannyAPIStruct *__Pyx_RefNanny = NULL;
01210 static __Pyx_RefNannyAPIStruct *__Pyx_RefNannyImportAPI(const char *modname);
01211 #define __Pyx_RefNannyDeclarations void *__pyx_refnanny = NULL;
01212 #ifndef WITH_THREAD
01213 #define __Pyx_RefNannySetupContext(name, acquire_gil)\
01214     if (acquire_gil) {\
01215         PyGILState_STATE __pyx_gilstate_save = PyGILState_Ensure();\
01216         __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__);\
01217         PyGILState_Release(__pyx_gilstate_save);\
01218     } else {\
01219         __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__);\
01220     }\
01221 #else
01222 #define __Pyx_RefNannySetupContext(name, acquire_gil)\
01223     __pyx_refnanny = __Pyx_RefNanny->SetupContext((name), __LINE__, __FILE__)
01224 #endif
01225 #define __Pyx_RefNannyFinishContext()\
01226     __Pyx_RefNanny->FinishContext(&__pyx_refnanny)
01227 #define __Pyx_INCREF(r) __Pyx_RefNanny->INCRREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01228 #define __Pyx_DECREF(r) __Pyx_RefNanny->DECRREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01229 #define __Pyx_GOTREF(r) __Pyx_RefNanny->GOTREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01230 #define __Pyx_GIVEREF(r) __Pyx_RefNanny->GIVEREF(__pyx_refnanny, (PyObject *) (r), __LINE__)
01231 #define __Pyx_XINCRREF(r) do { if((r) != NULL) {__Pyx_INCREF(r); }} while(0)
01232 #define __Pyx_XDECREF(r) do { if((r) != NULL) {__Pyx_DECREF(r); }} while(0)
01233 #define __Pyx_XGOTREF(r) do { if((r) != NULL) {__Pyx_GOTREF(r); }} while(0)
01234 #define __Pyx_XGIVEREF(r) do { if((r) != NULL) {__Pyx_GIVEREF(r); }} while(0)
01235 #else
01236 #define __Pyx_RefNannyDeclarations
01237 #define __Pyx_RefNannySetupContext(name, acquire_gil)
01238 #define __Pyx_RefNannyFinishContext()
01239 #define __Pyx_INCREF(r) Py_INCREF(r)
01240 #define __Pyx_DECREF(r) Py_DECREF(r)
01241 #define __Pyx_GOTREF(r)
01242 #define __Pyx_GIVEREF(r)
01243 #define __Pyx_XINCRREF(r) Py_XINCRREF(r)
01244 #define __Pyx_XDECREF(r) Py_XDECREF(r)
01245 #define __Pyx_XGOTREF(r)
01246 #define __Pyx_XGIVEREF(r)
01247 #endif
01248 #define __Pyx_XDECREF_SET(r, v) do {\
01249     PyObject *tmp = (PyObject *) r;\
01250     r = v; __Pyx_XDECREF(tmp);\
01251 } while (0)
01252 #define __Pyx_DECREF_SET(r, v) do {\
01253     PyObject *tmp = (PyObject *) r;\
01254     r = v; __Pyx_DECREF(tmp);\
01255 } while (0)
01256 #define __Pyx_CLEAR(r) do { PyObject* tmp = ((PyObject*)(r)); r = NULL; __Pyx_DECREF(tmp);} while(0)
01257 #define __Pyx_XCLEAR(r) do { if((r) != NULL) {PyObject* tmp = ((PyObject*)(r)); r = NULL; __Pyx_DECREF(tmp);} } while(0)
01258
01259 /* PyObjectGetAttrStr.proto */
01260 #if CYTHON_USE_TYPE_SLOTS
01261 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStr(PyObject* obj, PyObject* attr_name);
01262 #else
01263 #define __Pyx_PyObject_GetAttrStr(o,n) PyObject_GetAttr(o,n)
01264 #endif
01265
01266 /* GetBuiltinName.proto */
01267 static PyObject *__Pyx_GetBuiltinName(PyObject *name);
01268
01269 /* PyCFunctionFastCall.proto */
01270 #if CYTHON_FAST_PYCALL
01271 static CYTHON_INLINE PyObject* __Pyx_PyCFunction_FastCall(PyObject *func, PyObject **args, Py_ssize_t nargs);
01272 #else
01273 #define __Pyx_PyCFunction_FastCall(func, args, nargs) (assert(0), NULL)
01274 #endif
01275
01276 /* PyFunctionFastCall.proto */
01277 #if CYTHON_FAST_PYCALL
01278 #define __Pyx_PyFunction_FastCall(func, args, nargs)\
01279     __Pyx_PyFunction_FastCallDict((func), (args), (nargs), NULL)
01280 #if 1 || PY_VERSION_HEX < 0x030600B1
01281 static PyObject* __Pyx_PyFunction_FastCallDict(PyObject *func, PyObject **args, Py_ssize_t nargs, PyObject *kwargs);
01282 #else
01283 #define __Pyx_PyFunction_FastCallDict(func, args, nargs, kwargs) _PyFunction_FastCallDict(func, args, nargs, kwargs)
01284 #endif
01285 #define __Pyx_BUILD_ASSERT_EXPR(cond)\
01286     (sizeof(char [1 - 2*(cond)]) - 1)
01287 #ifndef Py_MEMBER_SIZE
01288 #define Py_MEMBER_SIZE(type, member) sizeof(((type *)0)->member)
01289 #endif

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01290 #if CYTHON_FAST_PYCALL
01291     static size_t __pyx_pyframe_localsplus_offset = 0;
01292     #include "frameobject.h"
01293     #define __Pyx_PyFrame_Initialize_Offsets()\
01294     ((void)__Pyx_BUILD_ASSERT_EXPR(sizeof(PyFrameObject) == offsetof(PyFrameObject, f_localsplus) +
Py_MEMBER_SIZE(PyFrameObject, f_localsplus)),\
01295     (void)(__pyx_pyframe_localsplus_offset = ((size_t)PyFrame_Type.tp_basicsize) -
Py_MEMBER_SIZE(PyFrameObject, f_localsplus)))
01296     #define __Pyx_PyFrame_GetLocalsplus(frame)\
01297     (assert(__pyx_pyframe_localsplus_offset), (PyObject *)((char *) (frame)) +
__pyx_pyframe_localsplus_offset))
01298 #endif // CYTHON_FAST_PYCALL
01299 #endif
01300
01301 /* PyObjectCall.proto */
01302 #if CYTHON_COMPILING_IN_CPYTHON
01303 static CYTHON_INLINE PyObject* __Pyx_PyObject_Call(PyObject *func, PyObject *arg, PyObject *kw);
01304 #else
01305 #define __Pyx_PyObject_Call(func, arg, kw) PyObject_Call(func, arg, kw)
01306 #endif
01307
01308 /* PyObjectCallMeth0.proto */
01309 #if CYTHON_COMPILING_IN_CPYTHON
01310 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallMeth0(PyObject *func, PyObject *arg);
01311 #endif
01312
01313 /* PyObjectCallOneArg.proto */
01314 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg);
01315
01316 /* PyThreadStateGet.proto */
01317 #if CYTHON_FAST_THREAD_STATE
01318 #define __Pyx_PyThreadState_declare PyThreadState *__pyx_tstate;
01319 #define __Pyx_PyThreadState_assign __pyx_tstate = __Pyx_PyThreadState_Current;
01320 #define __Pyx_PyErr_Occurred() __pyx_tstate->curexc_type
01321 #else
01322 #define __Pyx_PyThreadState_declare
01323 #define __Pyx_PyThreadState_assign
01324 #define __Pyx_PyErr_Occurred() PyErr_Occurred()
01325 #endif
01326
01327 /* PyErrFetchRestore.proto */
01328 #if CYTHON_FAST_THREAD_STATE
01329 #define __Pyx_PyErr_Clear() __Pyx_ErrRestore(NULL, NULL, NULL)
01330 #define __Pyx_ErrRestoreWithState(type, value, tb) __Pyx_ErrRestoreInState(PyThreadState_GET(), type,
value, tb)
01331 #define __Pyx_ErrFetchWithState(type, value, tb) __Pyx_ErrFetchInState(PyThreadState_GET(), type,
value, tb)
01332 #define __Pyx_ErrRestore(type, value, tb) __Pyx_ErrRestoreInState(__pyx_tstate, type, value, tb)
01333 #define __Pyx_ErrFetch(type, value, tb) __Pyx_ErrFetchInState(__pyx_tstate, type, value, tb)
01334 static CYTHON_INLINE void __Pyx_ErrRestoreInState(PyThreadState *tstate, PyObject *type, PyObject
**value, PyObject **tb);
01335 static CYTHON_INLINE void __Pyx_ErrFetchInState(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb);
01336 #if CYTHON_COMPILING_IN_CPYTHON
01337 #define __Pyx_PyErr_SetNone(exc) (Py_INCREF(exc), __Pyx_ErrRestore((exc), NULL, NULL))
01338 #else
01339 #define __Pyx_PyErr_SetNone(exc) PyErr_SetNone(exc)
01340 #endif
01341 #else
01342 #define __Pyx_PyErr_Clear() PyErr_Clear()
01343 #define __Pyx_PyErr_SetNone(exc) PyErr_SetNone(exc)
01344 #define __Pyx_ErrRestoreWithState(type, value, tb) PyErr_Restore(type, value, tb)
01345 #define __Pyx_ErrFetchWithState(type, value, tb) PyErr_Fetch(type, value, tb)
01346 #define __Pyx_ErrRestoreInState(tstate, type, value, tb) PyErr_Restore(type, value, tb)
01347 #define __Pyx_ErrFetchInState(tstate, type, value, tb) PyErr_Fetch(type, value, tb)
01348 #define __Pyx_ErrRestore(type, value, tb) PyErr_Restore(type, value, tb)
01349 #define __Pyx_ErrFetch(type, value, tb) PyErr_Fetch(type, value, tb)
01350 #endif
01351
01352 /* WriteUnraisableException.proto */
01353 static void __Pyx_WriteUnraisable(const char *name, int clineno,
01354                                 int lineno, const char *filename,
01355                                 int full_traceback, int nogil);
01356
01357 /* PyDictVersioning.proto */
01358 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_TYPE_SLOTS
01359 #define __PYX_DICT_VERSION_INIT ((PY_UINT64_T) -1)
01360 #define __PYX_GET_DICT_VERSION(dict) (((PyDictObject*)(dict))->ma_version_tag)
01361 #define __PYX_UPDATE_DICT_CACHE(dict, value, cache_var, version_var)\
01362     (version_var) = __PYX_GET_DICT_VERSION(dict);\
01363     (cache_var) = (value);
01364 #define __PYX_PY_DICT_LOOKUP_IF_MODIFIED(VAR, DICT, LOOKUP) {\
01365     static PY_UINT64_T __pyx_dict_version = 0;\
01366     static PyObject *__pyx_dict_cached_value = NULL;\
01367     if (likely(__PYX_GET_DICT_VERSION(DICT) == __pyx_dict_version)) {\
01368         (VAR) = __pyx_dict_cached_value;\
01369     } else {\

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01370         (VAR) = __pyx_dict_cached_value = (LOOKUP);\
01371         __pyx_dict_version = __PYX_GET_DICT_VERSION(DICT);\
01372     }\
01373 }
01374 static CYTHON_INLINE PY_UINT64_T __Pyx_get_tp_dict_version(PyObject *obj);
01375 static CYTHON_INLINE PY_UINT64_T __Pyx_get_object_dict_version(PyObject *obj);
01376 static CYTHON_INLINE int __Pyx_object_dict_version_matches(PyObject* obj, PY_UINT64_T tp_dict_version,
PY_UINT64_T obj_dict_version);
01377 #else
01378 #define __PYX_GET_DICT_VERSION(dict) (0)
01379 #define __PYX_UPDATE_DICT_CACHE(dict, value, cache_var, version_var)
01380 #define __PYX_PY_DICT_LOOKUP_IF_MODIFIED(VAR, DICT, LOOKUP) (VAR) = (LOOKUP);
01381 #endif
01382
01383 /* PyObjectCallNoArg.proto */
01384 #if CYTHON_COMPILING_IN_CPYTHON
01385 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallNoArg(PyObject *func);
01386 #else
01387 #define __Pyx_PyObject_CallNoArg(func) __Pyx_PyObject_Call(func, __pyx_empty_tuple, NULL)
01388 #endif
01389
01390 /* RaiseDoubleKeywords.proto */
01391 static void __Pyx_RaiseDoubleKeywordsError(const char* func_name, PyObject* kw_name);
01392
01393 /* ParseKeywords.proto */
01394 static int __Pyx_ParseOptionalKeywords(PyObject *kwds, PyObject **argnames[],\
PyObject *kwds2, PyObject *values[], Py_ssize_t num_pos_args,\
const char* function_name);
01397
01398 /* RaiseArgTupleInvalid.proto */
01399 static void __Pyx_RaiseArgtupleInvalid(const char* func_name, int exact,
Py_ssize_t num_min, Py_ssize_t num_max, Py_ssize_t num_found);
01401
01402 /* GetModuleGlobalName.proto */
01403 #if CYTHON_USE_DICT_VERSIONS
01404 #define __Pyx_GetModuleGlobalName(var, name) {\
01405     static PY_UINT64_T __pyx_dict_version = 0;\
01406     static PyObject * __pyx_dict_cached_value = NULL;\
01407     (var) = (likely(__pyx_dict_version == __PYX_GET_DICT_VERSION(__pyx_d))) ?\
01408         (likely(__pyx_dict_cached_value) ? __Pyx_NewRef(__pyx_dict_cached_value) :\
__Pyx_GetBuiltinName(name)) :\
__Pyx_GetModuleGlobalName(name, &__pyx_dict_version, &__pyx_dict_cached_value);\
01410 }
01411 #define __Pyx_GetModuleGlobalNameUncached(var, name) {\
01412     PY_UINT64_T __pyx_dict_version;\
01413     PyObject * __pyx_dict_cached_value;\
01414     (var) = __Pyx_GetModuleGlobalName(name, &__pyx_dict_version, &__pyx_dict_cached_value);\
01415 }
01416 static PyObject * __Pyx_GetModuleGlobalName(PyObject *name, PY_UINT64_T *dict_version, PyObject
**dict_cached_value);
01417 #else
01418 #define __Pyx_GetModuleGlobalName(var, name) (var) = __Pyx_GetModuleGlobalName(name)
01419 #define __Pyx_GetModuleGlobalNameUncached(var, name) (var) = __Pyx_GetModuleGlobalName(name)
01420 static CYTHON_INLINE PyObject * __Pyx_GetModuleGlobalName(PyObject *name);
01421 #endif
01422
01423 /* GetTopmostException.proto */
01424 #if CYTHON_USE_EXC_INFO_STACK
01425 static _PyErr_StackItem * __Pyx_PyErr_GetTopmostException(PyThreadState *tstate);
01426 #endif
01427
01428 /* SaveResetException.proto */
01429 #if CYTHON_FAST_THREAD_STATE
01430 #define __Pyx_ExceptionSave(type, value, tb) __Pyx_ExceptionSave(__pyx_tstate, type, value, tb)
01431 static CYTHON_INLINE void __Pyx_ExceptionSave(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb);
01432 #define __Pyx_ExceptionReset(type, value, tb) __Pyx_ExceptionReset(__pyx_tstate, type, value, tb)
01433 static CYTHON_INLINE void __Pyx_ExceptionReset(PyThreadState *tstate, PyObject *type, PyObject
*value, PyObject *tb);
01434 #else
01435 #define __Pyx_ExceptionSave(type, value, tb) PyErr_GetExcInfo(type, value, tb)
01436 #define __Pyx_ExceptionReset(type, value, tb) PyErr_SetExcInfo(type, value, tb)
01437 #endif
01438
01439 /* PyErrExceptionMatches.proto */
01440 #if CYTHON_FAST_THREAD_STATE
01441 #define __Pyx_PyErr_ExceptionMatches(err) __Pyx_PyErr_ExceptionMatchesInState(__pyx_tstate, err)
01442 static CYTHON_INLINE int __Pyx_PyErr_ExceptionMatchesInState(PyThreadState* tstate, PyObject* err);
01443 #else
01444 #define __Pyx_PyErr_ExceptionMatches(err) PyErr_ExceptionMatches(err)
01445 #endif
01446
01447 /* GetException.proto */
01448 #if CYTHON_FAST_THREAD_STATE
01449 #define __Pyx_GetException(type, value, tb) __Pyx_GetException(__pyx_tstate, type, value, tb)
01450 static int __Pyx_GetException(PyThreadState *tstate, PyObject **type, PyObject **value, PyObject
**tb);

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01451 #else
01452 static int __Pyx_GetException(PyObject **type, PyObject **value, PyObject **tb);
01453 #endif
01454
01455 /* RaiseException.proto */
01456 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb, PyObject *cause);
01457
01458 /* PyObjectCall2Args.proto */
01459 static CYTHON_UNUSED PyObject* __Pyx_PyObject_Call2Args(PyObject* function, PyObject* arg1, PyObject*
arg2);
01460
01461 /* PyIntBinop.proto */
01462 #if !CYTHON_COMPILING_IN_PYPY
01463 static PyObject* __Pyx_PyInt_AddObjC(PyObject *op1, PyObject *op2, long intval, int inplace, int
zerodivision_check);
01464 #else
01465 #define __Pyx_PyInt_AddObjC(op1, op2, intval, inplace, zerodivision_check)\
01466     (inplace ? PyNumber_InPlaceAdd(op1, op2) : PyNumber_Add(op1, op2))
01467 #endif
01468
01469 /* PySequenceContains.proto */
01470 static CYTHON_INLINE int __Pyx_PySequence_ContainsTF(PyObject* item, PyObject* seq, int eq) {
01471     int result = PySequence_Contains(seq, item);
01472     return unlikely(result < 0) ? result : (result == (eq == Py_EQ));
01473 }
01474
01475 /* IncludeCppStringH.proto */
01476 #include <string>
01477
01478 /* decode_c_string_utf16.proto */
01479 static CYTHON_INLINE PyObject *__Pyx_PyUnicode_DecodeUTF16(const char *s, Py_ssize_t size, const char
*errors) {
01480     int byteorder = 0;
01481     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01482 }
01483 static CYTHON_INLINE PyObject *__Pyx_PyUnicode_DecodeUTF16LE(const char *s, Py_ssize_t size, const
char *errors) {
01484     int byteorder = -1;
01485     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01486 }
01487 static CYTHON_INLINE PyObject *__Pyx_PyUnicode_DecodeUTF16BE(const char *s, Py_ssize_t size, const
char *errors) {
01488     int byteorder = 1;
01489     return PyUnicode_DecodeUTF16(s, size, errors, &byteorder);
01490 }
01491
01492 /* decode_c_bytes.proto */
01493 static CYTHON_INLINE PyObject* __Pyx_decode_c_bytes(
01494     const char* cstring, Py_ssize_t length, Py_ssize_t start, Py_ssize_t stop,
01495     const char* encoding, const char* errors,
01496     PyObject* (*decode_func)(const char *s, Py_ssize_t size, const char *errors));
01497
01498 /* decode_cpp_string.proto */
01499 static CYTHON_INLINE PyObject* __Pyx_decode_cpp_string(
01500     std::string cppstring, Py_ssize_t start, Py_ssize_t stop,
01501     const char* encoding, const char* errors,
01502     PyObject* (*decode_func)(const char *s, Py_ssize_t size, const char *errors)) {
01503     return __Pyx_decode_c_bytes(
01504         cppstring.data(), cppstring.size(), start, stop, encoding, errors, decode_func);
01505 }
01506
01507 /* SwapException.proto */
01508 #if CYTHON_FAST_THREAD_STATE
01509 #define __Pyx_ExceptionSwap(type, value, tb) __Pyx__ExceptionSwap(__pyx_tstate, type, value, tb)
01510 static CYTHON_INLINE void __Pyx__ExceptionSwap(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb);
01511 #else
01512 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyObject **type, PyObject **value, PyObject **tb);
01513 #endif
01514
01515 /* SetItemInt.proto */
01516 #define __Pyx_SetItemInt(o, i, v, type, is_signed, to_py_func, is_list, wraparound, boundscheck)\
01517     (__Pyx_fits_Py_ssize_t(i, type, is_signed) ?\
01518     __Pyx_SetItemInt_Fast(o, (Py_ssize_t)i, v, is_list, wraparound, boundscheck) : \
01519     (is_list ? (PyErr_SetString(PyExc_IndexError, "list assignment index out of range"), -1) : \
01520     __Pyx_SetItemInt_Generic(o, to_py_func(i), v))
01521 static int __Pyx_SetItemInt_Generic(PyObject *o, PyObject *j, PyObject *v);
01522 static CYTHON_INLINE int __Pyx_SetItemInt_Fast(PyObject *o, Py_ssize_t i, PyObject *v,
01523     int is_list, int wraparound, int boundscheck);
01524
01525 /* ArgTypeTest.proto */
01526 #define __Pyx_ArgTypeTest(obj, type, none_allowed, name, exact)\
01527     ((likely((Py_TYPE(obj) == type) | (none_allowed && (obj == Py_None)))) ? 1 : \
01528     __Pyx_ArgTypeTest(obj, type, name, exact))
01529 static int __Pyx_ArgTypeTest(PyObject *obj, PyTypeObject *type, const char *name, int exact);
01530
01531 /* Import.proto */

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01532 static PyObject * __Pyx_Import(PyObject *name, PyObject *from_list, int level);
01533
01534 /* IncludeStringH.proto */
01535 #include <string.h>
01536
01537 /* PyObject_GenericGetAttrNoDict.proto */
01538 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
01539 static CYTHON_INLINE PyObject* __Pyx_PyObject_GenericGetAttrNoDict(PyObject* obj, PyObject*
    attr_name);
01540 #else
01541 #define __Pyx_PyObject_GenericGetAttrNoDict PyObject_GenericGetAttr
01542 #endif
01543
01544 /* PyObject_GenericGetAttr.proto */
01545 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
01546 static PyObject* __Pyx_PyObject_GenericGetAttr(PyObject* obj, PyObject* attr_name);
01547 #else
01548 #define __Pyx_PyObject_GenericGetAttr PyObject_GenericGetAttr
01549 #endif
01550
01551 /* SetVTable.proto */
01552 static int __Pyx_SetVtable(PyObject *dict, void *vtable);
01553
01554 /* PyObjectGetAttrStrNoError.proto */
01555 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStrNoError(PyObject* obj, PyObject* attr_name);
01556
01557 /* SetupReduce.proto */
01558 static int __Pyx_setup_reduce(PyObject* type_obj);
01559
01560 /* BytesEquals.proto */
01561 static CYTHON_INLINE int __Pyx_PyBytes_Equals(PyObject* s1, PyObject* s2, int equals);
01562
01563 /* UnicodeEquals.proto */
01564 static CYTHON_INLINE int __Pyx_PyUnicode_Equals(PyObject* s1, PyObject* s2, int equals);
01565
01566 /* CLineInTraceback.proto */
01567 #ifdef CYTHON_CLINE_IN_TRACEBACK
01568 #define __Pyx_CLineForTraceback(tstate, c_line) (((CYTHON_CLINE_IN_TRACEBACK)) ? c_line : 0)
01569 #else
01570 static int __Pyx_CLineForTraceback(PyThreadState *tstate, int c_line);
01571 #endif
01572
01573 /* CodeObjectCache.proto */
01574 typedef struct {
01575     PyCodeObject* code_object;
01576     int code_line;
01577 } __Pyx_CodeObjectCacheEntry;
01578 struct __Pyx_CodeObjectCache {
01579     int count;
01580     int max_count;
01581     __Pyx_CodeObjectCacheEntry* entries;
01582 };
01583 static struct __Pyx_CodeObjectCache __pyx_code_cache = {0,0,NULL};
01584 static int __pyx_bisect_code_objects(__Pyx_CodeObjectCacheEntry* entries, int count, int code_line);
01585 static PyCodeObject* __pyx_find_code_object(int code_line);
01586 static void __pyx_insert_code_object(int code_line, PyCodeObject* code_object);
01587
01588 /* AddTraceback.proto */
01589 static void __Pyx_AddTraceback(const char *funcname, int c_line,
    int py_line, const char *filename);
01590
01591
01592 /* GCCDiagnostics.proto */
01593 #if defined(__GNUC__) && (__GNUC__ > 4 || (__GNUC__ == 4 && __GNUC_MINOR__ >= 6))
01594 #define __Pyx_HAS_GCC_DIAGNOSTIC
01595 #endif
01596
01597 /* CppExceptionConversion.proto */
01598 #ifndef __Pyx_CppExn2PyErr
01599 #include <new>
01600 #include <typeinfo>
01601 #include <stdexcept>
01602 #include <ios>
01603 static void __Pyx_CppExn2PyErr() {
01604     try {
01605         if (PyErr_Occurred())
01606             ; // let the latest Python exn pass through and ignore the current one
01607         else
01608             throw;
01609     } catch (const std::bad_alloc& exn) {
01610         PyErr_SetString(PyExc_MemoryError, exn.what());
01611     } catch (const std::bad_cast& exn) {
01612         PyErr_SetString(PyExc_TypeError, exn.what());
01613     } catch (const std::bad_typeid& exn) {
01614         PyErr_SetString(PyExc_TypeError, exn.what());
01615     } catch (const std::domain_error& exn) {
01616         PyErr_SetString(PyExc_ValueError, exn.what());
01617     } catch (const std::invalid_argument& exn) {

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01618     PyErr_SetString(PyExc_ValueError, exn.what());
01619 } catch (const std::ios_base::failure& exn) {
01620     PyErr_SetString(PyExc_IOError, exn.what());
01621 } catch (const std::out_of_range& exn) {
01622     PyErr_SetString(PyExc_IndexError, exn.what());
01623 } catch (const std::overflow_error& exn) {
01624     PyErr_SetString(PyExc_OverflowError, exn.what());
01625 } catch (const std::range_error& exn) {
01626     PyErr_SetString(PyExc_ArithmeticError, exn.what());
01627 } catch (const std::underflow_error& exn) {
01628     PyErr_SetString(PyExc_ArithmeticError, exn.what());
01629 } catch (const std::exception& exn) {
01630     PyErr_SetString(PyExc_RuntimeError, exn.what());
01631 }
01632 catch (...)
01633 {
01634     PyErr_SetString(PyExc_RuntimeError, "Unknown exception");
01635 }
01636 }
01637 #endif
01638
01639 /* CIntFromPy.proto */
01640 static CYTHON_INLINE int __Pyx_PyInt_As_int(PyObject *);
01641
01642 /* CIntToPy.proto */
01643 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_int(int value);
01644
01645 /* CIntToPy.proto */
01646 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_long(long value);
01647
01648 /* CIntFromPy.proto */
01649 static CYTHON_INLINE long __Pyx_PyInt_As_long(PyObject *);
01650
01651 /* FastTypeChecks.proto */
01652 #if CYTHON_COMPILING_IN_CPYTHON
01653 #define __Pyx_TypeCheck(obj, type) __Pyx_IsSubtype(Py_TYPE(obj), (PyTypeObject *)type)
01654 static CYTHON_INLINE int __Pyx_IsSubtype(PyTypeObject *a, PyTypeObject *b);
01655 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches(PyObject *err, PyObject *type);
01656 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches2(PyObject *err, PyObject *typel, PyObject
    *type2);
01657 #else
01658 #define __Pyx_TypeCheck(obj, type) PyObject_TypeCheck(obj, (PyTypeObject *)type)
01659 #define __Pyx_PyErr_GivenExceptionMatches(err, type) PyErr_GivenExceptionMatches(err, type)
01660 #define __Pyx_PyErr_GivenExceptionMatches2(err, typel, type2) (PyErr_GivenExceptionMatches(err, typel)
    || PyErr_GivenExceptionMatches(err, type2))
01661 #endif
01662 #define __Pyx_PyException_Check(obj) __Pyx_TypeCheck(obj, PyExc_Exception)
01663
01664 /* FetchCommonType.proto */
01665 static PyTypeObject* __Pyx_FetchCommonType(PyTypeObject* type);
01666
01667 /* PyObjectGetMethod.proto */
01668 static int __Pyx_PyObject_GetMethod(PyObject *obj, PyObject *name, PyObject **method);
01669
01670 /* PyObjectCallMethod1.proto */
01671 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* obj, PyObject* method_name, PyObject* arg);
01672
01673 /* CoroutineBase.proto */
01674 typedef PyObject *(*__pyx_coroutine_body_t)(PyObject *, PyThreadState *, PyObject *);
01675 #if CYTHON_USE_EXC_INFO_STACK
01676 #define __Pyx_ExcInfoStruct __PyErr_StackItem
01677 #else
01678 typedef struct {
01679     PyObject *exc_type;
01680     PyObject *exc_value;
01681     PyObject *exc_traceback;
01682 } __Pyx_ExcInfoStruct;
01683 #endif
01684 typedef struct {
01685     PyObject_HEAD
01686     __pyx_coroutine_body_t body;
01687     PyObject *closure;
01688     __Pyx_ExcInfoStruct gi_exc_state;
01689     PyObject *gi_weakreflist;
01690     PyObject *classobj;
01691     PyObject *yieldfrom;
01692     PyObject *gi_name;
01693     PyObject *gi_qualname;
01694     PyObject *gi_modulename;
01695     PyObject *gi_code;
01696     PyObject *gi_frame;
01697     int resume_label;
01698     char is_running;
01699 } __pyx_CoroutineObject;
01700 static __pyx_CoroutineObject * __Pyx__Coroutine_New(
01701     PyTypeObject *type, __pyx_coroutine_body_t body, PyObject *code, PyObject *closure,
01702     PyObject *name, PyObject *qualname, PyObject *module_name);

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01703 static __pyx_CoroutineObject *__Pyx__Coroutine_NewInit (
01704     __pyx_CoroutineObject *gen, __pyx_coroutine_body_t body, PyObject *code, PyObject
    *closure,
01705     PyObject *name, PyObject *qualname, PyObject *module_name);
01706 static CYTHON_INLINE void __Pyx_Coroutine_ExceptionClear(__Pyx_ExcInfoStruct *self);
01707 static int __Pyx_Coroutine_Clear(PyObject *self);
01708 static PyObject *__Pyx_Coroutine_Send(PyObject *self, PyObject *value);
01709 static PyObject *__Pyx_Coroutine_Close(PyObject *self);
01710 static PyObject *__Pyx_Coroutine_Throw(PyObject *gen, PyObject *args);
01711 #if CYTHON_USE_EXC_INFO_STACK
01712 #define __Pyx_Coroutine_SwapException(self)
01713 #define __Pyx_Coroutine_ResetAndClearException(self)
    __Pyx_Coroutine_ExceptionClear(&(self)->gi_exc_state)
01714 #else
01715 #define __Pyx_Coroutine_SwapException(self) {\
01716     __Pyx_ExceptionSwap(&(self)->gi_exc_state.exc_type, &(self)->gi_exc_state.exc_value,
    &(self)->gi_exc_state.exc_traceback);\
01717     __Pyx_Coroutine_ResetFrameBackpointer(&(self)->gi_exc_state);\
01718 }
01719 #define __Pyx_Coroutine_ResetAndClearException(self) {\
01720     __Pyx_ExceptionReset((self)->gi_exc_state.exc_type, (self)->gi_exc_state.exc_value,
    (self)->gi_exc_state.exc_traceback);\
01721     (self)->gi_exc_state.exc_type = (self)->gi_exc_state.exc_value =
    (self)->gi_exc_state.exc_traceback = NULL;\
01722 }
01723 #endif
01724 #if CYTHON_FAST_THREAD_STATE
01725 #define __Pyx_PyGen_FetchStopIterationValue(pvalue)\
01726     __Pyx_PyGen__FetchStopIterationValue(__pyx_tstate, pvalue)
01727 #else
01728 #define __Pyx_PyGen_FetchStopIterationValue(pvalue)\
01729     __Pyx_PyGen__FetchStopIterationValue(__Pyx_PyThreadState_Current, pvalue)
01730 #endif
01731 static int __Pyx_PyGen__FetchStopIterationValue(PyThreadState *tstate, PyObject **pvalue);
01732 static CYTHON_INLINE void __Pyx_Coroutine_ResetFrameBackpointer(__Pyx_ExcInfoStruct *exc_state);
01733
01734 /* PatchModuleWithCoroutine.proto */
01735 static PyObject* __Pyx_Coroutine_patch_module(PyObject* module, const char* py_code);
01736
01737 /* PatchGeneratorABC.proto */
01738 static int __Pyx_patch_abc(void);
01739
01740 /* Generator.proto */
01741 #define __Pyx_Generator_USED
01742 static PyTypeObject *__pyx_GeneratorType = 0;
01743 #define __Pyx_Generator_CheckExact(obj) (Py_TYPE(obj) == __pyx_GeneratorType)
01744 #define __Pyx_Generator_New(body, code, closure, name, qualname, module_name)\
01745     __Pyx__Coroutine_New(__pyx_GeneratorType, body, code, closure, name, qualname, module_name)
01746 static PyObject *__Pyx_Generator_Next(PyObject *self);
01747 static int __pyx_Generator_init(void);
01748
01749 /* CheckBinaryVersion.proto */
01750 static int __Pyx_check_binary_version(void);
01751
01752 /* InitStrings.proto */
01753 static int __Pyx_InitStrings(__Pyx_StringTabEntry *t);
01754
01755 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, IndexSet __pyx_v_other); /* proto*/
01756 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self); /* proto*/
01757 static PyObject *__pyx_f_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set *__pyx_v_self,
    int __pyx_skip_dispatch); /* proto*/
01758 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, Clifford __pyx_v_other); /* proto*/
01759 static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto*/
01760 static PyObject *__pyx_f_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
    int __pyx_skip_dispatch); /* proto*/
01761
01762 /* Module declarations from 'libcpp.vector' */
01763
01764 /* Module declarations from 'glucat' */
01765
01766 /* Module declarations from 'libc.string' */
01767
01768 /* Module declarations from 'libcpp.string' */
01769
01770 /* Module declarations from 'PyClical' */
01771 static PyTypeObject *__pyx_ptype_8PyClical_index_set = 0;
01772 static PyTypeObject *__pyx_ptype_8PyClical_clifford = 0;
01773 static PyTypeObject *__pyx_ptype_8PyClical__pyx_scope_struct__iter__ = 0;
01774 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_toIndexSet(PyObject *); /*proto*/
01775 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_compare(PyObject *, PyObject *, int
    __pyx_skip_dispatch); /*proto*/
01776 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_min_neg(PyObject *, int __pyx_skip_dispatch);
    /*proto*/

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01777 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_max_pos(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01778 static CYTHON_INLINE std::vector<scalar_t> __pyx_f_8PyClical_list_to_vector(PyObject *); /*proto*/
01779 static CYTHON_INLINE Clifford __pyx_f_8PyClical_toClifford(PyObject *); /*proto*/
01780 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_error_squared_tol(PyObject *, int
__pyx_skip_dispatch); /*proto*/
01781 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_error_squared(PyObject *, PyObject *, PyObject *, int
__pyx_skip_dispatch); /*proto*/
01782 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_approx_equal(PyObject *, PyObject *, int
__pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_approx_equal * __pyx_optional_args); /*proto*/
01783 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_inv(PyObject *, int __pyx_skip_dispatch); /*proto*/
01784 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_scalar(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01785 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_real(PyObject *, int __pyx_skip_dispatch); /*proto*/
01786 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_imag(PyObject *, int __pyx_skip_dispatch); /*proto*/
01787 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_pure(PyObject *, int __pyx_skip_dispatch); /*proto*/
01788 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_even(PyObject *, int __pyx_skip_dispatch); /*proto*/
01789 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_odd(PyObject *, int __pyx_skip_dispatch); /*proto*/
01790 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_involute(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01791 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_reverse(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01792 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_conj(PyObject *, int __pyx_skip_dispatch); /*proto*/
01793 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_quad(PyObject *, int __pyx_skip_dispatch); /*proto*/
01794 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_norm(PyObject *, int __pyx_skip_dispatch); /*proto*/
01795 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_abs(PyObject *, int __pyx_skip_dispatch); /*proto*/
01796 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_max_abs(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01797 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_pow(PyObject *, PyObject *, int __pyx_skip_dispatch);
/*proto*/
01798 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_outer_pow(PyObject *, PyObject *, int
__pyx_skip_dispatch); /*proto*/
01799 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_complexifier(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01800 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_sqrt(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_sqrt * __pyx_optional_args); /*proto*/
01801 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_exp(PyObject *, int __pyx_skip_dispatch); /*proto*/
01802 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_log(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_log * __pyx_optional_args); /*proto*/
01803 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cos(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_cos * __pyx_optional_args); /*proto*/
01804 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_acos(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_acos * __pyx_optional_args); /*proto*/
01805 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cosh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01806 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_acosh(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_acosh * __pyx_optional_args); /*proto*/
01807 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_sin(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_sin * __pyx_optional_args); /*proto*/
01808 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_asin(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_asin * __pyx_optional_args); /*proto*/
01809 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_tanh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01810 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_asinh(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_asinh * __pyx_optional_args); /*proto*/
01811 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_tan(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_tan * __pyx_optional_args); /*proto*/
01812 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_atan(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_atan * __pyx_optional_args); /*proto*/
01813 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_tanh(PyObject *, int __pyx_skip_dispatch); /*proto*/
01814 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_atanh(PyObject *, int __pyx_skip_dispatch, struct
__pyx_opt_args_8PyClical_atanh * __pyx_optional_args); /*proto*/
01815 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_random_clifford(struct __pyx_obj_8PyClical_index_set
*, int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_random_clifford * __pyx_optional_args);
/*proto*/
01816 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cga3(PyObject *, int __pyx_skip_dispatch); /*proto*/
01817 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cga3std(PyObject *, int __pyx_skip_dispatch);
/*proto*/
01818 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_agc3(PyObject *, int __pyx_skip_dispatch); /*proto*/
01819 static CYTHON_INLINE PyObject * __pyx_convert_PyObject_string_to_py_std_in_string(std::string const
&); /*proto*/
01820 static CYTHON_INLINE PyObject * __pyx_convert_PyUnicode_string_to_py_std_in_string(std::string const
&); /*proto*/
01821 static CYTHON_INLINE PyObject * __pyx_convert_PyStr_string_to_py_std_in_string(std::string const &);
/*proto*/
01822 static CYTHON_INLINE PyObject * __pyx_convert_PyBytes_string_to_py_std_in_string(std::string const &);
/*proto*/
01823 static CYTHON_INLINE PyObject * __pyx_convert_PyByteArray_string_to_py_std_in_string(std::string const
&); /*proto*/
01824 #define __Pyx_MODULE_NAME "PyClical"
01825 extern int __pyx_module_is_main_PyClical;
01826 int __pyx_module_is_main_PyClical = 0;
01827
01828 /* Implementation of 'PyClical' */
01829 static PyObject * __pyx_builtin_IndexError;
01830 static PyObject * __pyx_builtin_RuntimeError;
01831 static PyObject * __pyx_builtin_TypeError;
01832 static PyObject * __pyx_builtin_ValueError;
01833 static PyObject * __pyx_builtin_NotImplemented;

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01834 static PyObject *__pyx_builtin_range;
01835 static PyObject *__pyx_builtin_xrange;
01836 static const char __pyx_k_[] = ".";
01837 static const char __pyx_k_e[] = "e";
01838 static const char __pyx_k_i[] = "i";
01839 static const char __pyx_k_m[] = "m";
01840 static const char __pyx_k_p[] = "p";
01841 static const char __pyx_k_q[] = "q";
01842 static const char __pyx_k_2[] = " ";
01843 static const char __pyx_k_5[] = ":";
01844 static const char __pyx_k_6[] = "\n\t";
01845 static const char __pyx_k_7[] = "(";
01846 static const char __pyx_k_8[] = ", ";
01847 static const char __pyx_k_9[] = ").";
01848 static const char __pyx_k_cl[] = "cl";
01849 static const char __pyx_k_pi[] = "pi";
01850 static const char __pyx_k_abc[] = "abc";
01851 static const char __pyx_k_cos[] = "cos";
01852 static const char __pyx_k_exp[] = "exp";
01853 static const char __pyx_k_frm[] = "frm";
01854 static const char __pyx_k_inv[] = "inv";
01855 static const char __pyx_k_ist[] = "ist";
01856 static const char __pyx_k_ixt[] = "ixt";
01857 static const char __pyx_k_lhs[] = "lhs";
01858 static const char __pyx_k_log[] = "log";
01859 static const char __pyx_k_max[] = "max";
01860 static const char __pyx_k_min[] = "min";
01861 static const char __pyx_k_obj[] = "obj";
01862 static const char __pyx_k_odd[] = "odd";
01863 static const char __pyx_k_pow[] = "pow";
01864 static const char __pyx_k_rhs[] = "rhs";
01865 static const char __pyx_k_sin[] = "sin";
01866 static const char __pyx_k_tan[] = "tan";
01867 static const char __pyx_k_tau[] = "tau";
01868 static const char __pyx_k_tol[] = "tol";
01869 static const char __pyx_k_Real[] = "Real";
01870 static const char __pyx_k_acos[] = "acos";
01871 static const char __pyx_k_args[] = "args";
01872 static const char __pyx_k_asin[] = "asin";
01873 static const char __pyx_k_atan[] = "atan";
01874 static const char __pyx_k_conj[] = "conj";
01875 static const char __pyx_k_copy[] = "copy";
01876 static const char __pyx_k_cosh[] = "cosh";
01877 static const char __pyx_k_even[] = "even";
01878 static const char __pyx_k_fill[] = "fill";
01879 static const char __pyx_k_from[] = " from ";
01880 static const char __pyx_k_iter[] = "__iter__";
01881 static const char __pyx_k_main[] = "__main__";
01882 static const char __pyx_k_math[] = "math";
01883 static const char __pyx_k_name[] = "__name__";
01884 static const char __pyx_k_norm[] = "norm";
01885 static const char __pyx_k_pure[] = "pure";
01886 static const char __pyx_k_quad[] = "quad";
01887 static const char __pyx_k_send[] = "send";
01888 static const char __pyx_k_sinh[] = "sinh";
01889 static const char __pyx_k_sqrt[] = "sqrt";
01890 static const char __pyx_k_tanh[] = "tanh";
01891 static const char __pyx_k_test[] = "_test";
01892 static const char __pyx_k_UTF_8[] = "UTF-8";
01893 static const char __pyx_k_acosh[] = "acosh";
01894 static const char __pyx_k_asinh[] = "asinh";
01895 static const char __pyx_k_atanh[] = "atanh";
01896 static const char __pyx_k_close[] = "close";
01897 static const char __pyx_k_grade[] = "grade";
01898 static const char __pyx_k_istpq[] = "istpq";
01899 static const char __pyx_k_nbar3[] = "nbar3";
01900 static const char __pyx_k_ninf3[] = "ninf3";
01901 static const char __pyx_k_other[] = "other";
01902 static const char __pyx_k_range[] = "range";
01903 static const char __pyx_k_throw[] = "throw";
01904 static const char __pyx_k_using[] = " using (";
01905 static const char __pyx_k_utf_8[] = "utf-8";
01906 static const char __pyx_k_value[] = " value ";
01907 static const char __pyx_k_encode[] = "encode";
01908 static const char __pyx_k_import[] = "__import__";
01909 static const char __pyx_k_reduce[] = "__reduce__";
01910 static const char __pyx_k_scalar[] = "scalar";
01911 static const char __pyx_k_test_2[] = "__test__";
01912 static const char __pyx_k_xrange[] = "xrange";
01913 static const char __pyx_k_doctest[] = "doctest";
01914 static const char __pyx_k_invalid[] = " invalid ";
01915 static const char __pyx_k_numbers[] = "numbers";
01916 static const char __pyx_k_reverse[] = "reverse";
01917 static const char __pyx_k_testmod[] = "testmod";
01918 static const char __pyx_k_version[] = "__version__";
01919 static const char __pyx_k_Integral[] = "Integral";
01920 static const char __pyx_k_PyClical[] = "PyClical";
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01921 static const char __pyx_k_Sequence[] = "Sequence";
01922 static const char __pyx_k_as_frame[] = " as frame:\n\t";
01923 static const char __pyx_k_clifford[] = "clifford";
01924 static const char __pyx_k_getstate[] = "__getstate__";
01925 static const char __pyx_k_involute[] = "involute";
01926 static const char __pyx_k_setstate[] = "__setstate__";
01927 static const char __pyx_k_to_frame[] = " to frame ";
01928 static const char __pyx_k_TypeError[] = "TypeError";
01929 static const char __pyx_k_index_set[] = "index_set";
01930 static const char __pyx_k_outer_pow[] = "outer_pow";
01931 static const char __pyx_k_reduce_ex[] = "__reduce_ex__";
01932 static const char __pyx_k_threshold[] = "threshold";
01933 static const char __pyx_k_IndexError[] = "IndexError";
01934 static const char __pyx_k_ValueError[] = "ValueError";
01935 static const char __pyx_k_pyx_vtable[] = "__pyx_vtable__";
01936 static const char __pyx_k_collections[] = "collections";
01937 static const char __pyx_k_e_line_1936[] = "e (line 1936)";
01938 static const char __pyx_k_PyClical_pyx[] = "PyClical.pyx";
01939 static const char __pyx_k_RuntimeError[] = "RuntimeError";
01940 static const char __pyx_k_abs_line_1522[] = "abs (line 1522)";
01941 static const char __pyx_k_cos_line_1651[] = "cos (line 1651)";
01942 static const char __pyx_k_exp_line_1614[] = "exp (line 1614)";
01943 static const char __pyx_k_inv_line_1378[] = "inv (line 1378)";
01944 static const char __pyx_k_log_line_1628[] = "log (line 1628)";
01945 static const char __pyx_k_odd_line_1446[] = "odd (line 1446)";
01946 static const char __pyx_k_pow_line_1543[] = "pow (line 1543)";
01947 static const char __pyx_k_reduce_cython[] = "__reduce_cython__";
01948 static const char __pyx_k_sin_line_1728[] = "sin (line 1728)";
01949 static const char __pyx_k_tan_line_1801[] = "tan (line 1801)";
01950 static const char __pyx_k_using_invalid[] = " using invalid ";
01951 static const char __pyx_k_Cannot_reframe[] = "Cannot reframe";
01952 static const char __pyx_k_NotImplemented[] = "NotImplemented";
01953 static const char __pyx_k_Not_applicable[] = "Not applicable.";
01954 static const char __pyx_k_acos_line_1668[] = "acos (line 1668)";
01955 static const char __pyx_k_agc3_line_1893[] = "agc3 (line 1893)";
01956 static const char __pyx_k_asin_line_1747[] = "asin (line 1747)";
01957 static const char __pyx_k_atan_line_1818[] = "atan (line 1818)";
01958 static const char __pyx_k_cga3_line_1873[] = "cga3 (line 1873)";
01959 static const char __pyx_k_conj_line_1485[] = "conj (line 1485)";
01960 static const char __pyx_k_cosh_line_1689[] = "cosh (line 1689)";
01961 static const char __pyx_k_even_line_1437[] = "even (line 1437)";
01962 static const char __pyx_k_imag_line_1415[] = "imag (line 1415)";
01963 static const char __pyx_k_invalid_string[] = " invalid string ";
01964 static const char __pyx_k_norm_line_1511[] = "norm (line 1511)";
01965 static const char __pyx_k_pure_line_1426[] = "pure (line 1426)";
01966 static const char __pyx_k_quad_line_1500[] = "quad (line 1500)";
01967 static const char __pyx_k_real_line_1404[] = "real (line 1404)";
01968 static const char __pyx_k_scalar_epsilon[] = "scalar_epsilon";
01969 static const char __pyx_k_sinh_line_1768[] = "sinh (line 1768)";
01970 static const char __pyx_k_sqrt_line_1591[] = "sqrt (line 1591)";
01971 static const char __pyx_k_tanh_line_1835[] = "tanh (line 1835)";
01972 static const char __pyx_k_acosh_line_1705[] = "acosh (line 1705)";
01973 static const char __pyx_k_asinh_line_1782[] = "asinh (line 1782)";
01974 static const char __pyx_k_atanh_line_1847[] = "atanh (line 1847)";
01975 static const char __pyx_k_istpq_line_1949[] = "istpq (line 1949)";
01976 static const char __pyx_k_setstate_cython[] = "__setstate_cython__";
01977 static const char __pyx_k_compare_line_492[] = "compare (line 492)";
01978 static const char __pyx_k_index_set__iter[] = "index_set.__iter__";
01979 static const char __pyx_k_max_pos_line_513[] = "max_pos (line 513)";
01980 static const char __pyx_k_min_neg_line_504[] = "min_neg (line 504)";
01981 static const char __pyx_k_scalar_line_1393[] = "scalar (line 1393)";
01982 static const char __pyx_k_cga3std_line_1882[] = "cga3std (line 1882)";
01983 static const char __pyx_k_max_abs_line_1531[] = "max_abs (line 1531)";
01984 static const char __pyx_k_reverse_line_1470[] = "reverse (line 1470)";
01985 static const char __pyx_k_cline_in_traceback[] = "cline_in_traceback";
01986 static const char __pyx_k_involute_line_1455[] = "involute (line 1455)";
01987 static const char __pyx_k_outer_pow_line_1567[] = "outer_pow (line 1567)";
01988 static const char __pyx_k_clifford_inv_line_926[] = "clifford.inv (line 926)";
01989 static const char __pyx_k_clifford_pow_line_980[] = "clifford.pow (line 980)";
01990 static const char __pyx_k_approx_equal_line_1359[] = "approx_equal (line 1359)";
01991 static const char __pyx_k_clifford_abs_line_1175[] = "clifford.abs (line 1175)";
01992 static const char __pyx_k_clifford_copy_line_556[] = "clifford.copy (line 556)";
01993 static const char __pyx_k_clifford_odd_line_1070[] = "clifford.odd (line 1070)";
01994 static const char __pyx_k_complexifier_line_1576[] = "complexifier (line 1576)";
01995 static const char __pyx_k_index_set_copy_line_65[] = "index_set.copy (line 65)";
01996 static const char __pyx_k_index_set_max_line_351[] = "index_set.max (line 351)";
01997 static const char __pyx_k_index_set_min_line_342[] = "index_set.min (line 342)";
01998 static const char __pyx_k_clifford_conj_line_1138[] = "clifford.conj (line 1138)";
01999 static const char __pyx_k_clifford_even_line_1061[] = "clifford.even (line 1061)";
02000 static const char __pyx_k_clifford_norm_line_1164[] = "clifford.norm (line 1164)";
02001 static const char __pyx_k_clifford_pure_line_1050[] = "clifford.pure (line 1050)";
02002 static const char __pyx_k_clifford_quad_line_1153[] = "clifford.quad (line 1153)";
02003 static const char __pyx_k_error_squared_line_1346[] = "error_squared (line 1346)";
02004 static const char __pyx_k_Unary_print_clifford_1_1[] = "\n          Unary -. \n\n          >>
print(-clifford(\n\n          -. \n\n          ");
02005 static const char __pyx_k_clifford__or__line_939[] = "clifford.__or__ (line 939)";
02006 static const char __pyx_k_clifford_frame_line_1224[] = "clifford.frame (line 1224)";

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02007 static const char __pyx_k_clifford_hidden_doctests[] = "clifford_hidden_doctests";
02008 static const char __pyx_k_clifford_isinf_line_1206[] = "clifford.isinf (line 1206)";
02009 static const char __pyx_k_clifford_isnan_line_1215[] = "clifford.isnan (line 1215)";
02010 static const char __pyx_k_index_set_count_line_315[] = "index_set.count (line 315)";
02011 static const char __pyx_k_clifford__add_line_740[] = "clifford.__add__ (line 740)";
02012 static const char __pyx_k_clifford__and_line_836[] = "clifford.__and__ (line 836)";
02013 static const char __pyx_k_clifford__ior_line_950[] = "clifford.__ior__ (line 950)";
02014 static const char __pyx_k_clifford__mod_line_806[] = "clifford.__mod__ (line 806)";
02015 static const char __pyx_k_clifford__mul_line_780[] = "clifford.__mul__ (line 780)";
02016 static const char __pyx_k_clifford__neg_line_722[] = "clifford.__neg__ (line 722)";
02017 static const char __pyx_k_clifford__pos_line_731[] = "clifford.__pos__ (line 731)";
02018 static const char __pyx_k_clifford__pow_line_961[] = "clifford.__pow__ (line 961)";
02019 static const char __pyx_k_clifford__sub_line_760[] = "clifford.__sub__ (line 760)";
02020 static const char __pyx_k_clifford__xor_line_866[] = "clifford.__xor__ (line 866)";
02021 static const char __pyx_k_clifford_reframe_line_649[] = "clifford.reframe (line 649)";
02022 static const char __pyx_k_clifford_scalar_line_1039[] = "clifford.scalar (line 1039)";
02023 static const char __pyx_k_index_set__or_line_293[] = "index_set.__or__ (line 293)";
02024 static const char __pyx_k_index_set_hidden_doctests[] = "index_set_hidden_doctests";
02025 static const char __pyx_k_random_clifford_line_1864[] = "random_clifford (line 1864)";
02026 static const char __pyx_k_Cannot_take_vector_part_of[] = "Cannot take vector part of ";
02027 static const char __pyx_k_Unary_print_clifford_1_1_2[] = "\n          Unary +.\n\n\n" >>>
    print(+clifford("\{1\}"))\n          {1}\n          ";
02028 static const char __pyx_k_clifford__iadd_line_751[] = "clifford.__iadd__ (line 751)";
02029 static const char __pyx_k_clifford__iand_line_851[] = "clifford.__iand__ (line 851)";
02030 static const char __pyx_k_clifford__idiv_line_911[] = "clifford.__idiv__ (line 911)";
02031 static const char __pyx_k_clifford__imod_line_821[] = "clifford.__imod__ (line 821)";
02032 static const char __pyx_k_clifford__imul_line_793[] = "clifford.__imul__ (line 793)";
02033 static const char __pyx_k_clifford__isub_line_771[] = "clifford.__isub__ (line 771)";
02034 static const char __pyx_k_clifford__iter_line_638[] = "clifford.__iter__ (line 638)";
02035 static const char __pyx_k_clifford__ixor_line_881[] = "clifford.__ixor__ (line 881)";
02036 static const char __pyx_k_clifford__str_line_1244[] = "clifford.__str__ (line 1244)";
02037 static const char __pyx_k_clifford_max_abs_line_1184[] = "clifford.max_abs (line 1184)";
02038 static const char __pyx_k_clifford_reverse_line_1123[] = "clifford.reverse (line 1123)";
02039 static const char __pyx_k_index_set__and_line_271[] = "index_set.__and__ (line 271)";
02040 static const char __pyx_k_index_set__ior_line_304[] = "index_set.__ior__ (line 304)";
02041 static const char __pyx_k_index_set__str_line_395[] = "index_set.__str__ (line 395)";
02042 static const char __pyx_k_index_set__xor_line_249[] = "index_set.__xor__ (line 249)";
02043 static const char __pyx_k_clifford__call_line_1020[] = "clifford.__call__ (line 1020)";
02044 static const char __pyx_k_clifford__repr_line_1235[] = "clifford.__repr__ (line 1235)";
02045 static const char __pyx_k_clifford_involute_line_1107[] = "clifford.involute (line 1107)";
02046 static const char __pyx_k_error_squared_tol_line_1337[] = "error_squared_tol (line 1337)";
02047 static const char __pyx_k_index_set__iand_line_282[] = "index_set.__iand__ (line 282)";
02048 static const char __pyx_k_index_set__iter_line_229[] = "index_set.__iter__ (line 229)";
02049 static const char __pyx_k_index_set__ixor_line_260[] = "index_set.__ixor__ (line 260)";
02050 static const char __pyx_k_index_set__repr_line_384[] = "index_set.__repr__ (line 384)";
02051 static const char __pyx_k_clifford_outer_pow_line_1004[] = "clifford.outer_pow (line 1004)";
02052 static const char __pyx_k_clifford_truncated_line_1195[] = "clifford.truncated (line 1195)";
02053 static const char __pyx_k_index_set_count_neg_line_324[] = "index_set.count_neg (line 324)";
02054 static const char __pyx_k_index_set_count_pos_line_333[] = "index_set.count_pos (line 333)";
02055 static const char __pyx_k_clifford_getitem_line_707[] = "clifford.__getitem__ (line 707)";
02056 static const char __pyx_k_clifford_truediv_line_896[] = "clifford.__truediv__ (line 896)";
02057 static const char __pyx_k_index_set__invert_line_240[] = "index_set.__invert__ (line 240)";
02058 static const char __pyx_k_Abbreviation_for_index_set_q_p[] = "\n          Abbreviation for
index_set((-q, ...p)).\n\n          >> print(istpg(2,3))\n          {-3,-2,-1,1,2}\n          ";
02059 static const char __pyx_k_Conjugation_reverse_o_involute[] = "\n          Conjugation, reverse o
involute == involute o reverse.\n\n          >> print((clifford("\{1\}")).conj())\n          {-1}\n\n          >> print((clifford("\{2\}")) * clifford("\{1\}")).conj())\n          {1,2}\n\n          >>
print((clifford("\{1\}")) * clifford("\{2\}")).conj())\n          {-1,2}\n\n          >>
print(clifford("\{1+{1}+{1,2}\}").conj())\n          1-{1}-{1,2}\n          ";
02060 static const char __pyx_k_Geometric_product_x_clifford_2[] = "\n          Geometric product.\n\n
>> x = clifford(2); x *= clifford("\{2\}"); print(x)\n          2{2}\n\n          >> x = clifford("\{1\}");
x *= clifford("\{2\}"); print(x)\n          {1,2}\n\n          >> x = clifford("\{1\}"); x *=
clifford("\{1,2\}"); print(x)\n          {2}\n          ";
02061 static const char __pyx_k_Geometric_sum_print_clifford_1[] = "\n          Geometric sum.\n\n
>> print(clifford(1) + clifford("\{2\}"))\n          1+{2}\n\n          >> print(clifford("\{1\}")) +
clifford("\{2\}"))\n          {1}+{2}\n          ";
02062 static const char __pyx_k_Hyperbolic_sine_of_multivector[] = "\n          Hyperbolic sine of
multivector.\n\n          >> x=clifford("\{1,2\}")) * pi/2; print(sinh(x))\n          {1,2}\n\n          >>
x=clifford("\{1,2\}")) * pi/6; print(sinh(x))\n          0.5{1,2}\n          ";
02063 static const char __pyx_k_Inner_product_print_clifford_1[] = "\n          Inner product.\n\n
>> print(clifford("\{1\}")) & clifford("\{2\}"))\n          0\n\n          >> print(clifford(2) &
clifford("\{2\}"))\n          0\n\n          >> print(clifford("\{1\}")) & clifford("\{1\}"))\n          1\n
>> print(clifford("\{1\}")) & clifford("\{1,2\}"))\n          {2}\n          ";
02064 static const char __pyx_k_Inverse_tangent_of_multivector[] = "\n          Inverse tangent of multivector
with optional complexifier.\n\n          >> s=index_set({1,2,3}); x=clifford("\{1\}");
print(tan(atan(x,s)))\n          {1}\n\n          >> x=clifford("\{1\}"); print(tan(atan(x)))\n          {1}\n          ";
02065 static const char __pyx_k_Iterate_over_the_indices_of_an[] = "\n          Iterate over the indices of an
index_set.\n\n          >> for i in index_set((-3,4,7)):print(i, end=\", \")\n          -3,4,7,\n          ";
02066 static const char __pyx_k_Maximum_member_index_set_1_1_2[] = "\n          Maximum member.\n\n
>> index_set((-1,1,2)).max()\n          2\n          ";
02067 static const char __pyx_k_Maximum_positive_index_or_0_if[] = "\n          Maximum positive index, or 0 if
none.\n\n          >> max_pos(index_set({1,2}))\n          2\n          ";
02068 static const char __pyx_k_Minimum_member_index_set_1_1_2[] = "\n          Minimum member.\n\n
>> index_set((-1,1,2)).min()\n          -1\n          ";
02069 static const char __pyx_k_Minimum_negative_index_or_0_if[] = "\n          Minimum negative index, or 0 if
none.\n\n          >> min_neg(index_set({1,2}))\n          0\n          ";
02070 static const char __pyx_k_Odd_part_of_multivector_sum_of[] = "\n          Odd part of multivector, sum

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of odd grade terms.\n\n          >> print(clifford("\1+{1}+{1,2}\").odd())\n          {1}\n          ";
02071 static const char __pyx_k_Outer_product_power_x_clifford[] = "\n          Outer product power.\n\n
>> x=clifford("\2+{1}\"); print(x.outer_pow(0))\n          1\n          >> x=clifford("\2+{1}\");
print(x.outer_pow(1))\n          2+{1}\n          >> x=clifford("\2+{1}\"); print(x.outer_pow(2))\n
4+4{1}\n          >> print(clifford("\1+{1}+{1,2}\").outer_pow(3))\n          1+3{1}+3{1,2}\n\n          ";
02072 static const char __pyx_k_Outer_product_print_clifford_1[] = "\n          Outer product.\n\n
print(clifford("\{1}\") ^ clifford("\{2}\"))\n          {1,2}\n          >> print(clifford(2) ^
clifford("\{2}\"))\n          2{2}\n          >> print(clifford("\{1}\") ^ clifford("\{1}\"))\n          0\n
>> print(clifford("\{1}\") ^ clifford("\{1,2}\"))\n          0\n          ";
02073 static const char __pyx_k_Power_self_to_the_m_x_clifford[] = "\n          Power: self to the m.\n\n
>> x=clifford("\{1}\"); print(x ** 2)\n          1\n          >> x=clifford("\2\"); print(x ** 2)\n
4\n          >> x=clifford("\2+{1}\"); print(x ** 0)\n          1\n          >> x=clifford("\2+{1}\");
print(x ** 1)\n          2+{1}\n          >> x=clifford("\2+{1}\"); print(x ** 2)\n          5+4{1}\n
>> i=clifford("\{1,2}\"); print(exp(pi/2) * (i ** i))\n          1\n          ";
02074 static const char __pyx_k_Pure_part_print_clifford_1_1[] = "\n          Pure part.\n\n          >>
print(clifford("\1+{1}+{1,2}\").pure())\n          {1}+{1,2}\n          >>
print(clifford("\{1,2}\").pure())\n          {1,2}\n          ";
02075 static const char __pyx_k_Quadratic_form_rev_x_x_0_print[] = "\n          Quadratic form ==
(rev(x)*x)(0).\n\n          >> print(clifford("\1+{1}+{1,2}\").quad())\n          3.0\n          >>
print(clifford("\1+{-1}+{1,2}+{1,2,3}\").quad())\n          2.0\n          ";
02076 static const char __pyx_k_Quadratic_norm_error_tolerance[] = "\n          Quadratic norm error tolerance
relative to a specific multivector.\n\n          >> print(error_squared_tol(clifford("\{1}\")) * 3.0 -
error_squared_tol(clifford("\{1,1}-2{2}+3{3}\"))) \n          0.0\n          ";
02077 static const char __pyx_k_Set_complement_not_print_index[] = "\n          Set complement: not.\n\n
>>
print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))\n
{-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32}\n
";
02078 static const char __pyx_k_Set_union_or_print_index_set_1[] = "\n          Set union: or.\n\n          >>
print(index_set({1}) | index_set({2}))\n          {1,2}\n          >> print(index_set({1,2}) |
index_set({2}))\n          {1,2}\n          ";
02079 static const char __pyx_k_Transform_left_hand_side_using[] = "\n          Transform left hand side,
using right hand side as a transformation.\n\n          >> x=clifford("\{1,2}\") * pi/2;
y=clifford("\{1}\"); print(y|x)\n          {-1}\n          >> x=clifford("\{1,2}\") * pi/2;
y=clifford("\{1}\"); print(y|exp(x))\n          {-1}\n          ";
02080 static const char __pyx_k_clifford_vector_part_line_1079[] = "clifford.vector_part (line 1079)";
02081 static const char __pyx_k_index_set__getitem__line_191[] = "index_set.__getitem__ (line 191)";
02082 static const char __pyx_k_index_set__setitem__line_179[] = "index_set.__setitem__ (line 179)";
02083 static const char __pyx_k_lexicographic_compare_eg_3_4_5[] = "\n          \"lexicographic compare\" eg.
{3,4,5} is less than {3,7,8};\n          -1 if a<b, +1 if a>b, 0 if a==b.\n\n          >>
compare(index_set({1,2}),index_set({-1,3}))\n          -1\n          >>
compare(index_set({-1,4}),index_set({-1,3}))\n          1\n          ";
02084 static const char __pyx_k_Abbreviation_for_clifford_index[] = "\n          Abbreviation for
clifford(index_set(obj)).\n\n          >> print(e(1))\n          {1}\n          >> print(e(-1))\n          {-1}\n          >>
print(e(0))\n          1\n          ";
02085 static const char __pyx_k_Absolute_value_of_multivector_m[] = "\n          Absolute value of multivector:
multivector 2-norm.\n\n          >> abs(clifford("\1+{-1}+{1,2}+{1,2,3}\"))\n          2.0\n          ";
02086 static const char __pyx_k_Absolute_value_square_root_of_norm[] = "\n          Absolute value: square root
of norm.\n\n          >> clifford("\1+{-1}+{1,2}+{1,2,3}\").abs()\n          2.0\n          ";
02087 static const char __pyx_k_Cardinality_Number_of_indices_i[] = "\n          Cardinality: Number of
indices included in set.\n\n          >> index_set({-1,1,2}).count()\n          3\n          ";
02088 static const char __pyx_k_Check_if_a_multivector_contains[] = "\n          Check if a multivector
contains any infinite values.\n\n          >> clifford().isinf()\n          False\n          ";
02089 static const char __pyx_k_Contraction_print_clifford_1_cl[] = "\n          Contraction.\n\n          >>
print(clifford("\{1}\") % clifford("\{2}\"))\n          0\n          >> print(clifford(2) %
clifford("\{2}\"))\n          2{2}\n          >> print(clifford("\{1}\") % clifford("\{1}\"))\n          1\n
>> print(clifford("\{1}\") % clifford("\{1,2}\"))\n          {2}\n          ";
02090 static const char __pyx_k_Contraction_x_clifford_1_x_clif[] = "\n          Contraction.\n\n          >> x
= clifford("\{1}\"); x %= clifford("\{2}\"); print(x)\n          0\n          >> x = clifford(2); x %=
clifford("\{2}\"); print(x)\n          2{2}\n          >> x = clifford("\{1}\"); x %= clifford("\{1}\");
print(x)\n          1\n          >> x = clifford("\{1}\"); x %= clifford("\{1,2}\"); print(x)\n
{2}\n          ";
02091 static const char __pyx_k_Convert_CGA3_null_vector_to_Euc[] = "\n          Convert CGA3 null vector to
Euclidean 3D vector using Doran and Lasenby definition.\n\n          >> x=clifford("\2{1}+9{2}+{3}\");
print(agc3(cga3(x)))\n          2{1}+9{2}+{3}\n          >> x=clifford("\2{1}+9{2}+{3}\");
print(agc3(cga3(x))-x)\n          0\n          ";
02092 static const char __pyx_k_Convert_CGA3_null_vector_to_sta[] = "\n          Convert CGA3 null vector to
standard conformal null vector using Doran and Lasenby definition.\n\n          >>
x=clifford("\2{1}+9{2}+{3}\"); print(cga3std(cga3(x)))\n          87{-1}+4{1}+18{2}+2{3}+85{4}\n          >>
x=clifford("\2{1}+9{2}+{3}\"); print(cga3std(cga3(x))-cga3(x))\n          0\n          ";
02093 static const char __pyx_k_Convert_Euclidean_3D_multivector[] = "\n          Convert Euclidean 3D multivector
to Conformal Geometric Algebra using Doran and Lasenby definition.\n\n          >>
x=clifford("\2{1}+9{2}+{3}\"); print(cga3(x))\n          87{-1}+4{1}+18{2}+2{3}+85{4}\n          ";
02094 static const char __pyx_k_Copy_this_clifford_object_x_cli[] = "\n          Copy this clifford
object.\n\n          >> x=clifford("\1{2}\"); y=x.copy(); print(y)\n          {2}\n          ";
02095 static const char __pyx_k_Copy_this_index_set_object_s_in[] = "\n          Copy this index_set
object.\n\n          >> s=index_set(1); t=s.copy(); print(t)\n          {1}\n          ";
02096 static const char __pyx_k_Cosine_of_multivector_with_opti[] = "\n          Cosine of multivector with
optional complexifier.\n\n          >> x=clifford("\{1,2}\"); print(cos(acos(x), "\{1,2,3}\"))\n          {1,2}\n
>> x=clifford("\{1,2}\"); print(cos(acos(x)))\n          {1,2}\n          ";
02097 static const char __pyx_k_Even_part_of_multivector_sum_of[] = "\n          Even part of multivector, sum
of even grade terms.\n\n          >> print(clifford("\1+{1}+{1,2}\").even())\n          1+{1,2}\n          ";
02098 static const char __pyx_k_Exponential_of_multivector_x_cl[] = "\n          Exponential of multivector.\n\n
>> x=clifford("\{1,2}\") * pi/4; print(exp(x))\n          0.7071+0.7071{1,2}\n          >> x=clifford("\{1,2}\") *
pi/2; print(exp(x))\n          {1,2}\n          ";
02099 static const char __pyx_k_Geometric_difference_print_clif[] = "\n          Geometric difference.\n\n

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    >> print(clifford(1) - clifford("{2}\n"))\n          1-{2}\n          >> print(clifford("{1}\n") -
clifford("{2}\n"))\n          {1}-{2}\n          ";
02100 static const char __pyx_k_Geometric_difference_x_clifford[] = "\n          Geometric difference.\n\n
>> x = clifford(1); x -= clifford("{2}\n"); print(x)\n          1-{2}\n          ";
02101 static const char __pyx_k_Geometric_multiplicative_invers[] = "\n          Geometric multiplicative
inverse.\n\n          >> x = clifford("{1}\n"); print(x.inv())\n          {1}\n          >> x = clifford(2);
print(x.inv())\n          0.5\n          >> x = clifford("{1,2}\n"); print(x.inv())\n          -{1,2}\n
";
02102 static const char __pyx_k_Geometric_product_print_cliffor[] = "\n          Geometric product.\n\n
>> print(clifford("{1}\n") * clifford("{2}\n"))\n          {1,2}\n          >> print(clifford(2) *
clifford("{2}\n"))\n          2{2}\n          >> print(clifford("{1}\n") * clifford("{1,2}\n"))\n
{2}\n          ";
02103 static const char __pyx_k_Geometric_quotient_print_cliffo[] = "\n          Geometric quotient.\n\n
>> print(clifford("{1}\n") / clifford("{2}\n"))\n          {1,2}\n          >> print(clifford(2) /
clifford("{2}\n"))\n          2{2}\n          >> print(clifford("{1}\n") / clifford("{1}\n"))\n          1\n
>> print(clifford("{1}\n") / clifford("{1,2}\n"))\n          -{2}\n          ";
02104 static const char __pyx_k_Geometric_quotient_x_clifford_1[] = "\n          Geometric quotient.\n\n
>> x = clifford("{1}\n"); x /= clifford("{2}\n"); print(x)\n          {1,2}\n          >> x = clifford(2);
x /= clifford("{2}\n"); print(x)\n          2{2}\n          >> x = clifford("{1}\n"); x /=
clifford("{1}\n"); print(x)\n          1\n          >> x = clifford("{1}\n"); x /= clifford("{1,2}\n");
print(x)\n          -{2}\n          ";
02105 static const char __pyx_k_Geometric_sum_x_clifford_1_x_cl[] = "\n          Geometric sum.\n\n          >>
x = clifford(1); x += clifford("{2}\n"); print(x)\n          1+{2}\n          ";
02106 static const char __pyx_k_Get_the_value_of_an_index_set_o[] = "\n          Get the value of an index_set
object at an index.\n\n          >> index_set({1})[1]\n          True\n          >> index_set({1})[2]\n
False\n          >> index_set({2})[-1]\n          False\n          >> index_set({2})[1]\n          False\n
>> index_set({2})[2]\n          True\n          >> index_set({2})[33]\n          False\n          ";
02107 static const char __pyx_k_Hyperbolic_cosine_of_multivecto[] = "\n          Hyperbolic cosine of
multivector.\n\n          >> x=clifford("{1,2}\n") * pi; print(cosh(x))\n          -1\n          >>
x=clifford("{1,2,3}\n"); print(cosh(acosh(x))\n          {1,2,3}\n          >> x=clifford("{1,2}\n");
print(cosh(acosh(x))\n          {1,2}\n          ";
02108 static const char __pyx_k_Hyperbolic_tangent_of_multivect[] = "\n          Hyperbolic tangent of
multivector.\n\n          >> x=clifford("{1,2}\n") * pi/4; print(tanh(x))\n          {1,2}\n          ";
02109 static const char __pyx_k_Imaginary_part_deprecated_alway[] = "\n          Imaginary part: deprecated
(always 0).\n\n          >> imag(clifford("{1+{1}+{1,2}\n"))\n          0.0\n          >> imag(clifford("{1,2}\n"))\n
0.0\n          ";
02110 static const char __pyx_k_Inner_product_x_clifford_1_x_cl[] = "\n          Inner product.\n\n          >>
x = clifford("{1}\n"); x &= clifford("{2}\n"); print(x)\n          0\n          >> x = clifford(2); x &=
clifford("{2}\n"); print(x)\n          0\n          >> x = clifford("{1}\n"); x &= clifford("{1}\n");
print(x)\n          1\n          >> x = clifford("{1}\n"); x &= clifford("{1,2}\n"); print(x)\n
{2}\n          ";
02111 static const char __pyx_k_Integer_power_of_multivector_ob[] = "\n          Integer power of multivector: obj
to the m.\n\n          >> x=clifford("{1}\n"); print(pow(x,2))\n          1\n          >> x=clifford("{2}\n");
print(pow(x,2))\n          4\n          >> x=clifford("{2+{1}\n"); print(pow(x,0))\n          1\n          >>
x=clifford("{2+{1}\n"); print(pow(x,1))\n          2+{1}\n          >> x=clifford("{2+{1}\n"); print(pow(x,2))\n
5+4{1}\n          >> print(pow(clifford("{1+{1}+{1,2}\n"),3))\n          1+3{1}+3{1,2}\n          >>
i=clifford("{1,2}\n"); print(exp(pi/2) * pow(i, i))\n          1\n          ";
02112 static const char __pyx_k_Inverse_cosine_of_multivector_w[] = "\n          Inverse cosine of multivector
with optional complexifier.\n\n          >> x=clifford("{1,2}\n"); print(cos(acos(x), "{1,2,3}\n"))\n
{1,2}\n          >> x=clifford("{1,2}\n"); print(cos(acos(x), "{-1,1,2,3,4}\n"))\n          {1,2}\n          >>
print(acos(0) / pi)\n          0.5\n          >> x=clifford("{1,2}\n"); print(cos(acos(x))\n          {1,2}\n          ";
02113 static const char __pyx_k_Inverse_hyperbolic_cosine_of_mu[] = "\n          Inverse hyperbolic cosine of
multivector with optional complexifier.\n\n          >> print(acosh(0, "{-2,-1,1}\n"))\n          1.571{-2,-1,1}\n
>> x=clifford("{1,2,3}\n"); print(cosh(acosh(x), "{-1,1,2,3,4}\n"))\n          {1,2,3}\n          >>
print(acosh(0))\n          1.571{-1}\n          >> x=clifford("{1,2,3}\n"); print(cosh(acosh(x))\n          {1,2,3}\n
>> x=clifford("{1,2}\n"); print(cosh(acosh(x))\n          {1,2}\n          ";
02114 static const char __pyx_k_Inverse_hyperbolic_sine_of_multiv[] = "\n          Inverse hyperbolic sine of
multivector with optional complexifier.\n\n          >> x=clifford("{1,2}\n"); print(asinh(x, "{1,2,3}\n") *
2/pi)\n          {1,2}\n          >> x=clifford("{1,2}\n"); print(asinh(x) * 2/pi)\n          {1,2}\n          >>
x=clifford("{1,2}\n") / 2; print(asinh(x) * 6/pi)\n          {1,2}\n          ";
02115 static const char __pyx_k_Inverse_hyperbolic_tangent_of_m[] = "\n          Inverse hyperbolic tangent of
multivector with optional complexifier.\n\n          >> s=index_set({1,2,3}); x=clifford("{1,2}\n");
print(tanh(atanh(x,s))\n          {1,2}\n          >> x=clifford("{1,2}\n"); print(tanh(atanh(x))\n          {1,2}\n
";
02116 static const char __pyx_k_Inverse_sine_of_multivector_wit[] = "\n          Inverse sine of multivector with
optional complexifier.\n\n          >> s="{1}\n"; x=clifford(s); print(asin(sin(x,s),s))\n          {-1}\n          >>
s="{1}\n"; x=clifford(s); print(asin(sin(x,s), "{-2,-1,1}\n"))\n          {-1}\n          >> print(asin(1) / pi)\n
0.5\n          >> x=clifford("{1,2,3}\n"); print(asin(sin(x))\n          {1,2,3}\n          ";
02117 static const char __pyx_k_Main_involution_each_i_is_repla[] = "\n          Main involution, each {i} is
replaced by -{i} in each term,\n          eg. clifford("{1}\n") -> -clifford("{1}\n").\n\n          >>
print(clifford("{1}\n").involute())\n          -{1}\n          >> print((clifford("{2}\n") *
clifford("{1}\n")).involute())\n          -{1,2}\n          >> print((clifford("{1}\n") *
clifford("{2}\n")).involute())\n          {1,2}\n          >>
print(clifford("{1+{1}+{1,2}\n").involute())\n          1-{1}+{1,2}\n          ";
02118 static const char __pyx_k_Maximum_absolute_value_of_coord[] = "\n          Maximum absolute value of
coordinates multivector: multivector infinity-norm.\n\n          >>
max_abs(clifford("{1+{-1}+{1,2}+{1,2,3}\n"))\n          1.0\n          >> max_abs(clifford("{3+2{1}+{1,2}\n"))\n
3.0\n          ";
02119 static const char __pyx_k_Maximum_of_absolute_values_of_c[] = "\n          Maximum of absolute values of
components of multivector: multivector infinity norm.\n\n          >>
clifford("{1+{-1}+{1,2}+{1,2,3}\n").max_abs()\n          1.0\n          >>
clifford("{3+2{1}+{1,2}\n").max_abs()\n          3.0\n          ";
02120 static const char __pyx_k_Natural_logarithm_of_multivecto[] = "\n          Natural logarithm of multivector
with optional complexifier.\n\n          >> x=clifford("{1}\n"); print((log(x, "{1}\n") * 2/pi))\n
{1}\n          >> x=clifford("{1,2}\n"); print((log(x, "{1,2,3}\n") * 2/pi))\n          {1,2}\n          >>
x=clifford("{1,2}\n"); print((log(x) * 2/pi))\n          {1,2}\n          >> x=clifford("{1,2}\n");
print((log(x, "{1,2}\n") * 2/pi))\n          Traceback (most recent call last):\n          ...
RuntimeError:

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check_complex(val, i): i is not a valid complexifier for val\n
02121 static const char __pyx_k_Norm_sum_of_squares_of_coordina[] = "\n      Norm == sum of squares of\n      coordinates.\n\n      >> clifford(\"1+{1}+{1,2}\").norm()\n      clifford(\"1+{-1}+{1,2}+{1,2,3}\").norm()\n      4.0\n      3.0\n      >>
02122 static const char __pyx_k_Not_applicable_for_a_in_cliffor[] = "\n      Not applicable.\n\n      >>
for a in clifford(index_set({-3,4,7})):print(a, end=\"\\n\\n\")\n      Traceback (most recent call\n      last):\n      ...\\n      TypeError: Not applicable.\n      ";
02123 static const char __pyx_k_Number_of_negative_indices_incl[] = "\n      Number of negative indices\n      included in set.\n\n      >> index_set({-1,1,2}).count_neg()\n      1\n      1\n      ";
02124 static const char __pyx_k_Number_of_positive_indices_incl[] = "\n      Number of positive indices\n      included in set.\n\n      >> index_set({-1,1,2}).count_pos()\n      2\n      2\n      ";
02125 static const char __pyx_k_Outer_product_power_of_multivec[] = "\n      Outer product power of\n      multivector.\n\n      >> print(outer_pow(clifford(\"1+{1}+{1,2}\"),3))\n      1+3{1}+3{1,2}\n      ";
02126 static const char __pyx_k_Outer_product_x_clifford_1_x_cl[] = "\n      Outer product.\n\n      >>
x = clifford(\"{1}\"); x ^= clifford(\"{2}\"); print(x)\n      {1,2}\n      >> x = clifford(2); x\n      ^= clifford(\"{2}\"); print(x)\n      2{2}\n      >> x = clifford(\"{1}\"); x ^=\n      clifford(\"{1}\"); print(x)\n      0\n      >> x = clifford(\"{1}\"); x ^= clifford(\"{1,2}\");\n      print(x)\n      0\n      ";
02127 static const char __pyx_k_Pure_grade_vector_part_print_cl[] = "\n      Pure grade-vector part.\n\n      >> print(clifford(\"{1}\")(1))\n      {1}\n      >> print(clifford(\"{1}\")(0))\n      0\n      >> print(clifford(\"1+{1}+{1,2}\")(0))\n      1\n      >> print(clifford(\"1+{1}+{1,2}\")(1))\n      {1}\n      >> print(clifford(\"1+{1}+{1,2}\")(2))\n      {1,2}\n      >>\n      print(clifford(\"1+{1}+{1,2}\")(3))\n      0\n      ";
02128 static const char __pyx_k_Pure_part_print_pure_clifford_1[] = "\n      Pure part\n\n      >>\n      print(pure(clifford(\"1+{1}+{1,2}\"))) \n      {1}+{1,2}\n      >> print(pure(clifford(\"{1,2}\"))) \n      {1,2}\n      ";
02129 static const char __pyx_k_Put_self_into_a_larger_frame_co[] = "\n      Put self into a larger frame,\n      containing the union of self.frame() and index set ixt.\n      This can be used to make\n      multiplication faster, by multiplying within a common frame.\n\n      >>\n      clifford(\"2+3{1}\").reframe(index_set({1,2,3}))\n      clifford(\"2+3{1}\")\n      >>\n      s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);\n      True\n      ";
02130 static const char __pyx_k_Random_multivector_within_a_fra[] = "\n      Random multivector within a\n      frame.\n\n      >> print(random_clifford(index_set({-3,-1,2})).frame())\n      {-3,-1,2}\n      ";
02131 static const char __pyx_k_Real_part_synonym_for_scalar_pa[] = "\n      Real part: synonym for scalar\n      part.\n\n      >> real(clifford(\"1+{1}+{1,2}\"))\n      1.0\n      >> real(clifford(\"{1,2}\"))\n      0.0\n      ";
02132 static const char __pyx_k_Relative_or_absolute_error_usin[] = "\n      Relative or absolute error using\n      the quadratic norm.\n\n      >> err2=sqrt(epsilon)*sqrt(epsilon)\n      >>\n      print(error_squared(clifford(\"{1}\"), clifford(\"{1}\"), err2))\n      0.0\n      >>\n      print(error_squared(clifford(\"{1}\")+3{2}+4{3}), clifford(\"{1}\"), err2))\n      25.0\n      ";
02133 static const char __pyx_k_Remove_all_terms_of_self_with_r[] = "\n      Remove all terms of self with\n      relative size smaller than limit.\n\n      >> clifford(\"1e8+{1}+1e-8{1,2}\").truncated(1.0e-6)\n      clifford(\"1000000000\")\n      >> clifford(\"1e4+{1}+1e-4{1,2}\").truncated(1.0e-6)\n      clifford(\"10000+{1}\")\n      ";
02134 static const char __pyx_k_Reversion_eg_1_2_2_1_print_reve[] = "\n      Reversion, eg. {1}*{2} ->\n      {2}*{1}\n\n      >> print(reverse(clifford(\"{1}\"))) \n      {1}\n      >> print(reverse(clifford(\"{2}\") * \n      clifford(\"{1}\"))) \n      {1,2}\n      >> print(reverse(clifford(\"{1}\") * clifford(\"{2}\"))) \n      -{1,2}\n      >> print(reverse(clifford(\"1+{1}+{1,2}\"))) \n      1+{1}-{1,2}\n      ";
02135 static const char __pyx_k_Reversion_eg_clifford_1_cliffor[] = "\n      Reversion, eg.\n      clifford(\"{1}\")*clifford(\"{2}\") -> clifford(\"{2}\")*clifford(\"{1}\").\n\n      >>\n      print(clifford(\"{1}\").reverse())\n      {1}\n      >> print((clifford(\"{2}\") * \n      clifford(\"{1}\")).reverse())\n      {1,2}\n      >> print((clifford(\"{1}\") * \n      clifford(\"{2}\")).reverse())\n      -{1,2}\n      >> print(clifford(\"1+{1}+{1,2}\").reverse())\n      1+{1}-{1,2}\n      ";
02136 static const char __pyx_k_Scalar_part_clifford_1_1_2_sc[] = "\n      Scalar part.\n\n      >>\n      clifford(\"1+{1}+{1,2}\").scalar()\n      1.0\n      >> clifford(\"{1,2}\").scalar()\n      0.0\n      ";
02137 static const char __pyx_k_Scalar_part_scalar_clifford_1_1[] = "\n      Scalar part.\n\n      >>\n      scalar(clifford(\"1+{1}+{1,2}\"))\n      1.0\n      >> scalar(clifford(\"{1,2}\"))\n      0.0\n      ";
02138 static const char __pyx_k_Set_intersection_and_print_inde[] = "\n      Set intersection: and.\n\n      >> print(index_set({1}) & index_set({2}))\n      {2}\n      >> print(index_set({1,2}) & \n      index_set({2}))\n      {2}\n      ";
02139 static const char __pyx_k_Set_intersection_and_x_index_se[] = "\n      Set intersection: and.\n\n      >> x = index_set({1}); x &= index_set({2}); print(x)\n      {1}\n      >> x = index_set({1,2}); x\n      &= index_set({2}); print(x)\n      {2}\n      ";
02140 static const char __pyx_k_Set_the_value_of_an_index_set_o[] = "\n      Set the value of an index_set\n      object at index idx to value val.\n\n      >> s=index_set({1}); s[2] = True; print(s)\n      {1,2}\n      >> s=index_set({1,2}); s[1] = False; print(s)\n      {2}\n      ";
02141 static const char __pyx_k_Set_union_or_x_index_set_1_x_in[] = "\n      Set union: or.\n\n      >>\n      x = index_set({1}); x |= index_set({2}); print(x)\n      {1,2}\n      >> x = index_set({1,2}); x\n      |= index_set({2}); print(x)\n      {1,2}\n      ";
02142 static const char __pyx_k_Sign_of_geometric_product_of_tw[] = "\n      Sign of geometric product of\n      two Clifford basis elements.\n\n      >> s = index_set({1,2}); t=index_set({-1});\n      s.sign_of_mult(t)\n      1\n      ";
02143 static const char __pyx_k_Sign_of_geometric_square_of_a_C[] = "\n      Sign of geometric square of a\n      Clifford basis element.\n\n      >> s = index_set({1,2}); s.sign_of_square()\n      -1\n      ";
02144 static const char __pyx_k_Sine_of_multivector_with_option[] = "\n      Sine of multivector with optional\n      complexifier.\n\n      >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n      {-1}\n      >>\n      s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n      {-1}\n      >>\n      x=clifford(\"{1,2,3}\"); print(asin(sin(x)))\n      {1,2,3}\n      ";
02145 static const char __pyx_k_Square_root_of_1_which_commut[] = "\n      Square root of -1 which commutes\n      with all members of the frame of the given multivector.\n\n      >>\n      print(complexifier(clifford(index_set({1})))\n      {1,2,3}\n      >>\n      print(complexifier(clifford(index_set({-1})))\n      {-1}\n      >> print(complexifier(index_set({1})))\n      {1,2,3}\n      >> print(complexifier(index_set({-1})))\n      {-1}\n      ";
02146 static const char __pyx_k_Square_root_of_multivector_with[] = "\n      Square root of multivector with

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(rev(x)*x)(0).\n\n    >> print(quad(clifford("\1+{\1}+{\1,2}\n\n    3.0\n    >>
print(quad(clifford("\1+{-1}+{\1,2}+{\1,2,3}\n\n    2.0\n    ");
02165     static const char __pyx_k_Transform_left_hand_side_using_2[] = "\n        Transform left hand
side, using right hand side as a transformation.\n\n        >> x=clifford("\1,2\n\n    * pi/2;
y=clifford("\1\n\n    "); y|=x; print(y)\n\n        -{\1}\n\n        >> x=clifford("\1,2\n\n    * pi/2;
y=clifford("\1\n\n    "); y|=exp(x); print(y)\n\n        -{\1}\n\n        ";
02166     static const char __pyx_k_clifford_hidden_doctests_line_12[] = "clifford_hidden_doctests (line
1253)";
02167     static const char __pyx_k_index_set_hidden_doctests_line_4[] = "index_set_hidden_doctests
(line 406)";
02168     static const char __pyx_k_index_set_sign_of_square_line_37[] = "index_set.sign_of_square (line
375)";
02169     static const char __pyx_k_no_default__reduce__due_to_non[] = "no default __reduce__ due to
non-trivial __cinit__";
02170     static const char __pyx_k_Check_if_a_multivector_contains_2[] = "\n        Check if a
multivector contains any IEEE NaN values.\n\n        >> clifford().isnan()\n\n        False\n        ";
02171     static const char __pyx_k_Even_part_of_multivector_sum_of_2[] = "\n        Even part of
multivector, sum of even grade terms.\n\n        >> print(even(clifford("\1+{\1}+{\1,2}\n\n    1+{\1,2}\n
");
02172     static const char __pyx_k_Geometric_multiplicative_invers_2[] = "\n        Geometric
multiplicative inverse.\n\n        >> print(inv(clifford("\1\n\n    1\n\n    >>
print(inv(clifford("\{-1\n\n    -{\1}\n\n    >> print(inv(clifford("\{-2,-1\n\n    -{\{-2,-1\n
>> print(inv(clifford("\{-1+{\1\n\n    nan\n    ");
02173     static const char __pyx_k_Main_involution_each_i_is_repla_2[] = "\n        Main involution, each
{i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})\n\n        >>
print(involute(clifford("\1\n\n    -{\1}\n\n        >> print(involute(clifford("\{2\n\n    *
clifford("\1\n\n    -{\1,2}\n\n        >> print(involute(clifford("\1\n\n    * clifford("\{2\n\n    )\n
1,2\n\n        >> print(involute(clifford("\1+{\1}+{\1,2}\n\n    1-{\1}+{\1,2}\n\n        ";
02174     static const char __pyx_k_Symmetric_set_difference_exclus_2[] = "\n        Symmetric set
difference: exclusive or.\n\n        >> x = index_set({1}); x ^= index_set({2}); print(x)\n\n
1,2\n\n        >> x = index_set({1,2}); x ^= index_set({2}); print(x)\n\n        {1}\n\n        ";
02175     static const char __pyx_k_Tests_for_functions_that_Doctes_2[] = "\n        Tests for functions
that Doctest cannot see.\n\n        For clifford.__cinit__: Construct an object of type clifford.\n\n
>> print(clifford(2))\n\n    2\n\n    >> print(clifford(2.0))\n\n    2\n\n    >> print(clifford(1.0e-1))\n\n
0.1\n\n    >> print(clifford("\2\n\n    2\n\n    >> print(clifford("\2{1,2,3}\n\n    2{1,2,3}\n\n
>> print(clifford(clifford("\2{1,2,3}\n\n    2{1,2,3}\n\n    >> print(clifford("\{-1\n\n    -{\1}\n\n
>> print(clifford(2,index_set({1,2}))\n\n    2{1,2}\n\n    >> print(clifford([2,3],index_set({1,2}))\n\n
2{1}+3{2}\n\n    >> print(clifford([1,2]))\n\n    Traceback (most recent call last):\n\n    ...
TypeError: Cannot initialize clifford object from <class 'list'>.\n\n    >> print(clifford(None))\n\n
Traceback (most recent call last):\n\n    ...
TypeError: Cannot initialize clifford object from
<class 'NoneType'>.\n\n    >> print(clifford(None,[1,2]))\n\n    Traceback (most recent call last):\n\n
...
TypeError: Cannot initialize clifford object from (<class 'list'>, <class 'NoneType'>,\n\n
>> print(clifford([1,2],[1,2]))\n\n    Traceback (most recent call last):\n\n    ...
TypeError:
Cannot initialize clifford object from (<class 'list'>, <class 'list'>).\n\n    >>
print(clifford("\n\n    Traceback (most recent call last):\n\n    ...
ValueError: Cannot
initialize clifford object from invalid string ".\n\n    >> print(clifford("\n\n    Traceback (most
recent call last):\n\n    ...
ValueError: Cannot initialize clifford object from invalid string
'{'.\n\n    >> print(clifford("\1\n\n    Traceback (most recent call last):\n\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1'.\n\n    >>
print(clifford("\n\n    Traceback (most recent call last):\n\n    ...
ValueError: Cannot
initialize clifford object from invalid string '+'.n\n    >> print(clifford("\n\n    Traceback
(most recent call last):\n\n    ...
ValueError: Cannot initialize clifford object fro"m invalid
string '-'.n\n    >> print(clifford("\1+\n\n    Traceback (most recent call last):\n\n    ...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.n\n    For
clifford.__richcmp__: Compare objects of type clifford.\n\n    >> clifford("\1\n\n    ==
clifford("\1{1}\n\n    True\n\n    >> clifford("\1\n\n    != clifford("\1.0{1}\n\n    False\n\n    >>
clifford("\1\n\n    != clifford("\1.0\n\n    True\n\n    >> clifford("\1,2\n\n    == None\n\n    False\n\n
>> clifford("\1,2\n\n    != None\n\n    True\n\n    >> None == clifford("\1,2\n\n    False\n\n    >> None
!= clifford("\1,2\n\n    True\n\n    ";
02176     static const char __pyx_k_The_informal_string_representat_2[] = "\n        The
\342\200\234informal\342\200\235 string representation of self.\n\n        >>
clifford("\1+3{-1}+2{1,2}+4{-2,7}\n\n    ).__str__()\n\n        '1+3{-1}+2{1,2}+4{-2,7}\n\n        ";
02177     static const char __pyx_k_The_official_string_representat_2[] = "\n        The
\342\200\234official\342\200\235 string representation of self.\n\n        >>
clifford("\1+3{-1}+2{1,2}+4{-2,7}\n\n    ).__repr__()\n\n        'clifford("\1+3{-1}+2{1,2}+4{-2,7}\n\n    )'\n
";
02178     static PyObject * __pyx_kp_u_;
02179     static PyObject * __pyx_kp_u_Abbreviation_for_clifford_index;
02180     static PyObject * __pyx_kp_u_Abbreviation_for_index_set_q_p;
02181     static PyObject * __pyx_kp_u_Absolute_value_of_multivector_m;
02182     static PyObject * __pyx_kp_u_Absolute_value_square_root_of_n;
02183     static PyObject * __pyx_kp_u_Cannot_initialize_clifford_objec;
02184     static PyObject * __pyx_kp_u_Cannot_initialize_index_set_obje;
02185     static PyObject * __pyx_kp_u_Cannot_reframe;
02186     static PyObject * __pyx_kp_u_Cannot_take_vector_part_of;
02187     static PyObject * __pyx_kp_u_Cardinality_Number_of_indices_i;
02188     static PyObject * __pyx_kp_u_Check_if_a_multivector_contains;
02189     static PyObject * __pyx_kp_u_Check_if_a_multivector_contains_2;
02190     static PyObject * __pyx_kp_u_Conjugation_reverse_o_involute;
02191     static PyObject * __pyx_kp_u_Conjugation_reverse_o_involute_2;
02192     static PyObject * __pyx_kp_u_Contraction_print_clifford_l_cl;
02193     static PyObject * __pyx_kp_u_Contraction_x_clifford_l_x_clif;
02194     static PyObject * __pyx_kp_u_Convert_CGA3_null_vector_to_Euc;
02195     static PyObject * __pyx_kp_u_Convert_CGA3_null_vector_to_sta;
02196     static PyObject * __pyx_kp_u_Convert_Euclidean_3D_multivecto;
02197     static PyObject * __pyx_kp_u_Copy_this_clifford_object_x_cli;
02198     static PyObject * __pyx_kp_u_Copy_this_index_set_object_s_in;

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02199     static PyObject *__pyx_kp_u_Cosine_of_multivector_with_opti;
02200     static PyObject *__pyx_kp_u_Even_part_of_multivector_sum_of;
02201     static PyObject *__pyx_kp_u_Even_part_of_multivector_sum_of_2;
02202     static PyObject *__pyx_kp_u_Exponential_of_multivector_x_cl;
02203     static PyObject *__pyx_kp_u_Geometric_difference_print_clif;
02204     static PyObject *__pyx_kp_u_Geometric_difference_x_clifford;
02205     static PyObject *__pyx_kp_u_Geometric_multiplicative_invers;
02206     static PyObject *__pyx_kp_u_Geometric_multiplicative_invers_2;
02207     static PyObject *__pyx_kp_u_Geometric_product_print_cliffor;
02208     static PyObject *__pyx_kp_u_Geometric_product_x_clifford_2;
02209     static PyObject *__pyx_kp_u_Geometric_quotient_print_cliffo;
02210     static PyObject *__pyx_kp_u_Geometric_quotient_x_clifford_1;
02211     static PyObject *__pyx_kp_u_Geometric_sum_print_clifford_1;
02212     static PyObject *__pyx_kp_u_Geometric_sum_x_clifford_1_x_cl;
02213     static PyObject *__pyx_kp_u_Get_the_value_of_an_index_set_o;
02214     static PyObject *__pyx_kp_u_Hyperbolic_cosine_of_multivecto;
02215     static PyObject *__pyx_kp_u_Hyperbolic_sine_of_multivector;
02216     static PyObject *__pyx_kp_u_Hyperbolic_tangent_of_multivect;
02217     static PyObject *__pyx_kp_u_Imaginary_part_deprecated_alway;
02218     static PyObject *__pyx_n_s_IndexError;
02219     static PyObject *__pyx_kp_u_Inner_product_print_clifford_1;
02220     static PyObject *__pyx_kp_u_Inner_product_x_clifford_1_x_cl;
02221     static PyObject *__pyx_kp_u_Integer_power_of_multivector_ob;
02222     static PyObject *__pyx_n_s_Integral;
02223     static PyObject *__pyx_kp_u_Inverse_cosine_of_multivector_w;
02224     static PyObject *__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu;
02225     static PyObject *__pyx_kp_u_Inverse_hyperbolic_sine_of_mult;
02226     static PyObject *__pyx_kp_u_Inverse_hyperbolic_tangent_of_m;
02227     static PyObject *__pyx_kp_u_Inverse_sine_of_multivector_wit;
02228     static PyObject *__pyx_kp_u_Inverse_tangent_of_multivector;
02229     static PyObject *__pyx_kp_u_Iterate_over_the_indices_of_an;
02230     static PyObject *__pyx_kp_u_Main_involution_each_i_is_repla;
02231     static PyObject *__pyx_kp_u_Main_involution_each_i_is_repla_2;
02232     static PyObject *__pyx_kp_u_Maximum_absolute_value_of_coord;
02233     static PyObject *__pyx_kp_u_Maximum_member_index_set_1_1_2;
02234     static PyObject *__pyx_kp_u_Maximum_of_absolute_values_of_c;
02235     static PyObject *__pyx_kp_u_Maximum_positive_index_or_0_if;
02236     static PyObject *__pyx_kp_u_Minimum_member_index_set_1_1_2;
02237     static PyObject *__pyx_kp_u_Minimum_negative_index_or_0_if;
02238     static PyObject *__pyx_kp_u_Natural_logarithm_of_multivecto;
02239     static PyObject *__pyx_kp_u_Norm_sum_of_squares_of_coordina;
02240     static PyObject *__pyx_n_s_NotImplemented;
02241     static PyObject *__pyx_kp_u_Not_applicable;
02242     static PyObject *__pyx_kp_u_Not_applicable_for_a_in_cliffor;
02243     static PyObject *__pyx_kp_u_Number_of_negative_indices_incl;
02244     static PyObject *__pyx_kp_u_Number_of_positive_indices_incl;
02245     static PyObject *__pyx_kp_u_Odd_part_of_multivector_sum_of;
02246     static PyObject *__pyx_kp_u_Odd_part_of_multivector_sum_of_2;
02247     static PyObject *__pyx_kp_u_Outer_product_power_of_multivec;
02248     static PyObject *__pyx_kp_u_Outer_product_power_x_clifford;
02249     static PyObject *__pyx_kp_u_Outer_product_print_clifford_1;
02250     static PyObject *__pyx_kp_u_Outer_product_x_clifford_1_x_cl;
02251     static PyObject *__pyx_kp_u_Power_self_to_the_m_x_clifford;
02252     static PyObject *__pyx_kp_u_Power_self_to_the_m_x_clifford_2;
02253     static PyObject *__pyx_kp_u_Pure_grade_vector_part_print_cl;
02254     static PyObject *__pyx_kp_u_Pure_part_print_clifford_1_1_1;
02255     static PyObject *__pyx_kp_u_Pure_part_print_pure_clifford_1;
02256     static PyObject *__pyx_kp_u_Put_self_into_a_larger_frame_co;
02257     static PyObject *__pyx_n_s_PyClical;
02258     static PyObject *__pyx_kp_s_PyClical_pyx;
02259     static PyObject *__pyx_kp_u_Quadratic_form_rev_x_x_0_print;
02260     static PyObject *__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2;
02261     static PyObject *__pyx_kp_u_Quadratic_norm_error_tolerance;
02262     static PyObject *__pyx_kp_u_Random_multivector_within_a_fra;
02263     static PyObject *__pyx_n_s_Real;
02264     static PyObject *__pyx_kp_u_Real_part_synonym_for_scalar_pa;
02265     static PyObject *__pyx_kp_u_Relative_or_absolute_error_usin;
02266     static PyObject *__pyx_kp_u_Remove_all_terms_of_self_with_r;
02267     static PyObject *__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve;
02268     static PyObject *__pyx_kp_u_Reversion_eg_clifford_1_cliffor;
02269     static PyObject *__pyx_n_s_RuntimeError;
02270     static PyObject *__pyx_kp_u_Scalar_part_clifford_1_1_1_2_sc;
02271     static PyObject *__pyx_kp_u_Scalar_part_scalar_clifford_1_1;
02272     static PyObject *__pyx_n_s_Sequence;
02273     static PyObject *__pyx_kp_u_Set_complement_not_print_index;
02274     static PyObject *__pyx_kp_u_Set_intersection_and_print_inde;
02275     static PyObject *__pyx_kp_u_Set_intersection_and_x_index_se;
02276     static PyObject *__pyx_kp_u_Set_the_value_of_an_index_set_o;
02277     static PyObject *__pyx_kp_u_Set_union_or_print_index_set_1;
02278     static PyObject *__pyx_kp_u_Set_union_or_x_index_set_1_x_in;
02279     static PyObject *__pyx_kp_u_Sign_of_geometric_product_of_tw;
02280     static PyObject *__pyx_kp_u_Sign_of_geometric_square_of_a_C;
02281     static PyObject *__pyx_kp_u_Sine_of_multivector_with_option;
02282     static PyObject *__pyx_kp_u_Square_root_of_1_which_commutates;
02283     static PyObject *__pyx_kp_u_Square_root_of_multivector_with;
02284     static PyObject *__pyx_kp_u_Subalgebra_generated_by_all_gen;
02285     static PyObject *__pyx_kp_u_Subscripting_map_from_index_set;

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02286     static PyObject *__pyx_kp_u_Symmetric_set_difference_exclus;
02287     static PyObject *__pyx_kp_u_Symmetric_set_difference_exclus_2;
02288     static PyObject *__pyx_kp_u_Tangent_of_multivector_with_opt;
02289     static PyObject *__pyx_kp_u_Test_for_approximate_equality_o;
02290     static PyObject *__pyx_kp_u_Tests_for_functions_that_Doctes;
02291     static PyObject *__pyx_kp_u_Tests_for_functions_that_Doctes_2;
02292     static PyObject *__pyx_kp_u_The_informal_string_representat;
02293     static PyObject *__pyx_kp_u_The_informal_string_representat_2;
02294     static PyObject *__pyx_kp_u_The_official_string_representat;
02295     static PyObject *__pyx_kp_u_The_official_string_representat_2;
02296     static PyObject *__pyx_kp_u_This_comparison_operator_is_not;
02297     static PyObject *__pyx_kp_u_Transform_left_hand_side_using;
02298     static PyObject *__pyx_kp_u_Transform_left_hand_side_using_2;
02299     static PyObject *__pyx_n_s_TypeError;
02300     static PyObject *__pyx_kp_u_UTF_8;
02301     static PyObject *__pyx_kp_u_Unary_print_clifford_l_1;
02302     static PyObject *__pyx_kp_u_Unary_print_clifford_l_1_2;
02303     static PyObject *__pyx_n_s_ValueError;
02304     static PyObject *__pyx_kp_u_Vector_part_of_multivector_as_a;
02305     static PyObject *__pyx_kp_u__2;
02306     static PyObject *__pyx_kp_u__5;
02307     static PyObject *__pyx_kp_u__6;
02308     static PyObject *__pyx_kp_u__7;
02309     static PyObject *__pyx_kp_u__8;
02310     static PyObject *__pyx_kp_u__9;
02311     static PyObject *__pyx_n_s_abc;
02312     static PyObject *__pyx_kp_u_abs_line_1522;
02313     static PyObject *__pyx_n_s_acos;
02314     static PyObject *__pyx_kp_u_acos_line_1668;
02315     static PyObject *__pyx_n_s_acosh;
02316     static PyObject *__pyx_kp_u_acosh_line_1705;
02317     static PyObject *__pyx_kp_u_agc3_line_1893;
02318     static PyObject *__pyx_kp_u_approx_equal_line_1359;
02319     static PyObject *__pyx_n_s_args;
02320     static PyObject *__pyx_kp_u_as_frame;
02321     static PyObject *__pyx_n_s_asin;
02322     static PyObject *__pyx_kp_u_asin_line_1747;
02323     static PyObject *__pyx_n_s_asinh;
02324     static PyObject *__pyx_kp_u_asinh_line_1782;
02325     static PyObject *__pyx_n_s_atan;
02326     static PyObject *__pyx_kp_u_atan_line_1818;
02327     static PyObject *__pyx_n_s_atanh;
02328     static PyObject *__pyx_kp_u_atanh_line_1847;
02329     static PyObject *__pyx_kp_u_cga3_line_1873;
02330     static PyObject *__pyx_kp_u_cga3std_line_1882;
02331     static PyObject *__pyx_n_s_cl;
02332     static PyObject *__pyx_n_s_clifford;
02333     static PyObject *__pyx_kp_u_clifford__add__line_740;
02334     static PyObject *__pyx_kp_u_clifford__and__line_836;
02335     static PyObject *__pyx_kp_u_clifford__call__line_1020;
02336     static PyObject *__pyx_kp_u_clifford__getitem__line_707;
02337     static PyObject *__pyx_kp_u_clifford__iadd__line_751;
02338     static PyObject *__pyx_kp_u_clifford__iand__line_851;
02339     static PyObject *__pyx_kp_u_clifford__idiv__line_911;
02340     static PyObject *__pyx_kp_u_clifford__imod__line_821;
02341     static PyObject *__pyx_kp_u_clifford__imul__line_793;
02342     static PyObject *__pyx_kp_u_clifford__ior__line_950;
02343     static PyObject *__pyx_kp_u_clifford__isub__line_771;
02344     static PyObject *__pyx_kp_u_clifford__iter__line_638;
02345     static PyObject *__pyx_kp_u_clifford__ixor__line_881;
02346     static PyObject *__pyx_kp_u_clifford__mod__line_806;
02347     static PyObject *__pyx_kp_u_clifford__mul__line_780;
02348     static PyObject *__pyx_kp_u_clifford__neg__line_722;
02349     static PyObject *__pyx_kp_u_clifford__or__line_939;
02350     static PyObject *__pyx_kp_u_clifford__pos__line_731;
02351     static PyObject *__pyx_kp_u_clifford__pow__line_961;
02352     static PyObject *__pyx_kp_u_clifford__repr__line_1235;
02353     static PyObject *__pyx_kp_u_clifford__str__line_1244;
02354     static PyObject *__pyx_kp_u_clifford__sub__line_760;
02355     static PyObject *__pyx_kp_u_clifford__truediv__line_896;
02356     static PyObject *__pyx_kp_u_clifford__xor__line_866;
02357     static PyObject *__pyx_kp_u_clifford_abs_line_1175;
02358     static PyObject *__pyx_kp_u_clifford_conj_line_1138;
02359     static PyObject *__pyx_kp_u_clifford_copy_line_556;
02360     static PyObject *__pyx_kp_u_clifford_even_line_1061;
02361     static PyObject *__pyx_kp_u_clifford_frame_line_1224;
02362     static PyObject *__pyx_n_s_clifford_hidden_doctests;
02363     static PyObject *__pyx_kp_u_clifford_hidden_doctests_line_12;
02364     static PyObject *__pyx_kp_u_clifford_inv_line_926;
02365     static PyObject *__pyx_kp_u_clifford_involute_line_1107;
02366     static PyObject *__pyx_kp_u_clifford_isinf_line_1206;
02367     static PyObject *__pyx_kp_u_clifford_isnan_line_1215;
02368     static PyObject *__pyx_kp_u_clifford_max_abs_line_1184;
02369     static PyObject *__pyx_kp_u_clifford_norm_line_1164;
02370     static PyObject *__pyx_kp_u_clifford_odd_line_1070;
02371     static PyObject *__pyx_kp_u_clifford_outer_pow_line_1004;
02372     static PyObject *__pyx_kp_u_clifford_pow_line_980;

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02373 static PyObject *__pyx_kp_u_clifford_pure_line_1050;
02374 static PyObject *__pyx_kp_u_clifford_quad_line_1153;
02375 static PyObject *__pyx_kp_u_clifford_reframe_line_649;
02376 static PyObject *__pyx_kp_u_clifford_reverse_line_1123;
02377 static PyObject *__pyx_kp_u_clifford_scalar_line_1039;
02378 static PyObject *__pyx_kp_u_clifford_truncated_line_1195;
02379 static PyObject *__pyx_kp_u_clifford_vector_part_line_1079;
02380 static PyObject *__pyx_n_s_cline_in_traceback;
02381 static PyObject *__pyx_n_s_close;
02382 static PyObject *__pyx_n_s_collections;
02383 static PyObject *__pyx_kp_u_compare_line_492;
02384 static PyObject *__pyx_kp_u_complexifier_line_1576;
02385 static PyObject *__pyx_n_s_conj;
02386 static PyObject *__pyx_kp_u_conj_line_1485;
02387 static PyObject *__pyx_n_s_copy;
02388 static PyObject *__pyx_n_s_cos;
02389 static PyObject *__pyx_kp_u_cos_line_1651;
02390 static PyObject *__pyx_n_s_cosh;
02391 static PyObject *__pyx_kp_u_cosh_line_1689;
02392 static PyObject *__pyx_n_s_doctest;
02393 static PyObject *__pyx_n_s_e;
02394 static PyObject *__pyx_kp_u_e_line_1936;
02395 static PyObject *__pyx_n_s_encode;
02396 static PyObject *__pyx_kp_u_error_squared_line_1346;
02397 static PyObject *__pyx_kp_u_error_squared_tol_line_1337;
02398 static PyObject *__pyx_n_s_even;
02399 static PyObject *__pyx_kp_u_even_line_1437;
02400 static PyObject *__pyx_n_s_exp;
02401 static PyObject *__pyx_kp_u_exp_line_1614;
02402 static PyObject *__pyx_n_s_fill;
02403 static PyObject *__pyx_n_s_frm;
02404 static PyObject *__pyx_kp_u_from;
02405 static PyObject *__pyx_n_s_getstate;
02406 static PyObject *__pyx_n_s_grade;
02407 static PyObject *__pyx_n_s_i;
02408 static PyObject *__pyx_kp_u_imag_line_1415;
02409 static PyObject *__pyx_n_s_import;
02410 static PyObject *__pyx_n_s_index_set;
02411 static PyObject *__pyx_kp_u_index_set__and__line_271;
02412 static PyObject *__pyx_kp_u_index_set__getitem__line_191;
02413 static PyObject *__pyx_kp_u_index_set__iand__line_282;
02414 static PyObject *__pyx_kp_u_index_set__invert__line_240;
02415 static PyObject *__pyx_kp_u_index_set__ior__line_304;
02416 static PyObject *__pyx_n_s_index_set__iter;
02417 static PyObject *__pyx_kp_u_index_set__iter__line_229;
02418 static PyObject *__pyx_kp_u_index_set__ixor__line_260;
02419 static PyObject *__pyx_kp_u_index_set__or__line_293;
02420 static PyObject *__pyx_kp_u_index_set__repr__line_384;
02421 static PyObject *__pyx_kp_u_index_set__setitem__line_179;
02422 static PyObject *__pyx_kp_u_index_set__str__line_395;
02423 static PyObject *__pyx_kp_u_index_set__xor__line_249;
02424 static PyObject *__pyx_kp_u_index_set_copy_line_65;
02425 static PyObject *__pyx_kp_u_index_set_count_line_315;
02426 static PyObject *__pyx_kp_u_index_set_count_neg_line_324;
02427 static PyObject *__pyx_kp_u_index_set_count_pos_line_333;
02428 static PyObject *__pyx_n_s_index_set_hidden_doctests;
02429 static PyObject *__pyx_kp_u_index_set_hidden_doctests_line_4;
02430 static PyObject *__pyx_kp_u_index_set_max_line_351;
02431 static PyObject *__pyx_kp_u_index_set_min_line_342;
02432 static PyObject *__pyx_kp_u_index_set_sign_of_mult_line_366;
02433 static PyObject *__pyx_kp_u_index_set_sign_of_square_line_37;
02434 static PyObject *__pyx_n_s_inv;
02435 static PyObject *__pyx_kp_u_inv_line_1378;
02436 static PyObject *__pyx_kp_u_invalid;
02437 static PyObject *__pyx_kp_u_invalid_string;
02438 static PyObject *__pyx_n_s_involute;
02439 static PyObject *__pyx_kp_u_involute_line_1455;
02440 static PyObject *__pyx_n_s_ist;
02441 static PyObject *__pyx_n_s_istpq;
02442 static PyObject *__pyx_kp_u_istpq_line_1949;
02443 static PyObject *__pyx_n_s_iter;
02444 static PyObject *__pyx_n_s_ixt;
02445 static PyObject *__pyx_kp_u_lexicographic_compare_eg_3_4_5;
02446 static PyObject *__pyx_n_s_lhs;
02447 static PyObject *__pyx_n_s_log;
02448 static PyObject *__pyx_kp_u_log_line_1628;
02449 static PyObject *__pyx_n_s_m;
02450 static PyObject *__pyx_n_s_main;
02451 static PyObject *__pyx_n_u_main;
02452 static PyObject *__pyx_n_s_math;
02453 static PyObject *__pyx_n_s_max;
02454 static PyObject *__pyx_kp_u_max_abs_line_1531;
02455 static PyObject *__pyx_kp_u_max_pos_line_513;
02456 static PyObject *__pyx_n_s_min;
02457 static PyObject *__pyx_kp_u_min_neg_line_504;
02458 static PyObject *__pyx_n_s_name;
02459 static PyObject *__pyx_n_s_nbar3;
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02460         static PyObject *__pyx_n_s_ninf3;
02461         static PyObject *__pyx_kp_s_no_default_reduce_due_to_non;
02462         static PyObject *__pyx_n_s_norm;
02463         static PyObject *__pyx_kp_u_norm_line_1511;
02464         static PyObject *__pyx_kp_u_norm_sum_of_squares_of_coordina;
02465         static PyObject *__pyx_n_s_numbers;
02466         static PyObject *__pyx_n_s_obj;
02467         static PyObject *__pyx_n_s_odd;
02468         static PyObject *__pyx_kp_u_odd_line_1446;
02469         static PyObject *__pyx_n_s_other;
02470         static PyObject *__pyx_n_s_outer_pow;
02471         static PyObject *__pyx_kp_u_outer_pow_line_1567;
02472         static PyObject *__pyx_n_s_p;
02473         static PyObject *__pyx_n_s_pi;
02474         static PyObject *__pyx_n_s_pow;
02475         static PyObject *__pyx_kp_u_pow_line_1543;
02476         static PyObject *__pyx_n_s_pure;
02477         static PyObject *__pyx_kp_u_pure_line_1426;
02478         static PyObject *__pyx_n_s_pyx_vtable;
02479         static PyObject *__pyx_n_s_q;
02480         static PyObject *__pyx_n_s_quad;
02481         static PyObject *__pyx_kp_u_quad_line_1500;
02482         static PyObject *__pyx_kp_u_random_clifford_line_1864;
02483         static PyObject *__pyx_n_s_range;
02484         static PyObject *__pyx_kp_u_real_line_1404;
02485         static PyObject *__pyx_n_s_reduce;
02486         static PyObject *__pyx_n_s_reduce_cython;
02487         static PyObject *__pyx_n_s_reduce_ex;
02488         static PyObject *__pyx_n_s_reverse;
02489         static PyObject *__pyx_kp_u_reverse_line_1470;
02490         static PyObject *__pyx_n_s_rhs;
02491         static PyObject *__pyx_n_s_scalar;
02492         static PyObject *__pyx_n_s_scalar_epsilon;
02493         static PyObject *__pyx_kp_u_scalar_line_1393;
02494         static PyObject *__pyx_n_s_send;
02495         static PyObject *__pyx_n_s_setstate;
02496         static PyObject *__pyx_n_s_setstate_cython;
02497         static PyObject *__pyx_n_s_sin;
02498         static PyObject *__pyx_kp_u_sin_line_1728;
02499         static PyObject *__pyx_n_s_sinh;
02500         static PyObject *__pyx_kp_u_sinh_line_1768;
02501         static PyObject *__pyx_n_s_sqrt;
02502         static PyObject *__pyx_kp_u_sqrt_line_1591;
02503         static PyObject *__pyx_n_s_tan;
02504         static PyObject *__pyx_kp_u_tan_line_1801;
02505         static PyObject *__pyx_n_s_tanh;
02506         static PyObject *__pyx_kp_u_tanh_line_1835;
02507         static PyObject *__pyx_n_s_tau;
02508         static PyObject *__pyx_n_s_test;
02509         static PyObject *__pyx_n_s_test_2;
02510         static PyObject *__pyx_n_s_testmod;
02511         static PyObject *__pyx_n_s_threshold;
02512         static PyObject *__pyx_n_s_throw;
02513         static PyObject *__pyx_kp_u_to_frame;
02514         static PyObject *__pyx_n_s_tol;
02515         static PyObject *__pyx_kp_u_using;
02516         static PyObject *__pyx_kp_u_using_invalid;
02517         static PyObject *__pyx_kp_u_utf8;
02518         static PyObject *__pyx_kp_u_value;
02519         static PyObject *__pyx_n_s_version;
02520         static PyObject *__pyx_n_s_xrange;
02521         static PyObject *__pyx_pf_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set
02522         *__pyx_v_self); /* proto */
02522         static int __pyx_pf_8PyClical_9index_set_2_cinit__(struct __pyx_obj_8PyClical_index_set
02523         *__pyx_v_self, PyObject *__pyx_v_other); /* proto */
02523         static void __pyx_pf_8PyClical_9index_set_4_dealloc__(struct __pyx_obj_8PyClical_index_set
02524         *__pyx_v_self); /* proto */
02524         static PyObject *__pyx_pf_8PyClical_9index_set_6_richcmp__(struct __pyx_obj_8PyClical_index_set
02525         *__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op); /* proto */
02525         static int __pyx_pf_8PyClical_9index_set_8_setitem__(struct __pyx_obj_8PyClical_index_set
02526         *__pyx_v_self, PyObject *__pyx_v_idx, PyObject *__pyx_v_val); /* proto */
02526         static PyObject *__pyx_pf_8PyClical_9index_set_10_getitem__(struct __pyx_obj_8PyClical_index_set
02527         *__pyx_v_self, PyObject *__pyx_v_idx); /* proto */
02527         static int __pyx_pf_8PyClical_9index_set_12_contains__(struct __pyx_obj_8PyClical_index_set
02528         *__pyx_v_self, PyObject *__pyx_v_idx); /* proto */
02528         static PyObject *__pyx_pf_8PyClical_9index_set_14_iter__(struct __pyx_obj_8PyClical_index_set
02529         *__pyx_v_self); /* proto */
02529         static PyObject *__pyx_pf_8PyClical_9index_set_17_invert__(struct __pyx_obj_8PyClical_index_set
02530         *__pyx_v_self); /* proto */
02530         static PyObject *__pyx_pf_8PyClical_9index_set_19_xor__(PyObject *__pyx_v_lhs, PyObject
02531         *__pyx_v_rhs); /* proto */
02531         static PyObject *__pyx_pf_8PyClical_9index_set_21_ixor__(struct __pyx_obj_8PyClical_index_set
02532         *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02532         static PyObject *__pyx_pf_8PyClical_9index_set_23_and__(PyObject *__pyx_v_lhs, PyObject
02533         *__pyx_v_rhs); /* proto */
02533         static PyObject *__pyx_pf_8PyClical_9index_set_25_iand__(struct __pyx_obj_8PyClical_index_set
02534         *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */

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02534 static PyObject *__pyx_pf_8PyClical_9index_set_27_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02535 static PyObject *__pyx_pf_8PyClical_9index_set_29_iior__(struct __pyx_obj_8PyClical_index_set
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02536 static PyObject *__pyx_pf_8PyClical_9index_set_31count(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02537 static PyObject *__pyx_pf_8PyClical_9index_set_33count_neg(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02538 static PyObject *__pyx_pf_8PyClical_9index_set_35count_pos(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02539 static PyObject *__pyx_pf_8PyClical_9index_set_37min(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02540 static PyObject *__pyx_pf_8PyClical_9index_set_39max(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02541 static PyObject *__pyx_pf_8PyClical_9index_set_41hash_fn(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02542 static PyObject *__pyx_pf_8PyClical_9index_set_43sign_of_mult(struct __pyx_obj_8PyClical_index_set
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02543 static PyObject *__pyx_pf_8PyClical_9index_set_45sign_of_square(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02544 static PyObject *__pyx_pf_8PyClical_9index_set_47_repr__(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02545 static PyObject *__pyx_pf_8PyClical_9index_set_49_str__(struct __pyx_obj_8PyClical_index_set
__pyx_v_self); /* proto */
02546 static PyObject *__pyx_pf_8PyClical_9index_set_51_reduce_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self); /* proto */
02547 static PyObject *__pyx_pf_8PyClical_9index_set_53_setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state); /* proto */
02548 static PyObject *__pyx_pf_8PyClical_index_set_hidden_doctests(CYTHON_UNUSED PyObject *__pyx_self); /*
proto */
02549 static PyObject *__pyx_pf_8PyClical_2compare(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_lhs, PyObject *__pyx_v_rhs); /* proto */
02550 static PyObject *__pyx_pf_8PyClical_4min_neg(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02551 static PyObject *__pyx_pf_8PyClical_6max_pos(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj); /* proto */
02552 static PyObject *__pyx_pf_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford *__pyx_v_self);
/* proto */
02553 static int __pyx_pf_8PyClical_8clifford_2_cinit__(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
PyObject *__pyx_v_other, PyObject *__pyx_v_ixt); /* proto */
02554 static void __pyx_pf_8PyClical_8clifford_4_dealloc__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self); /* proto */
02555 static int __pyx_pf_8PyClical_8clifford_6_contains__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v_x); /* proto */
02556 static PyObject *__pyx_pf_8PyClical_8clifford_8_iter__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self); /* proto */
02557 static PyObject *__pyx_pf_8PyClical_8clifford_10reframe(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_ixt); /* proto */
02558 static PyObject *__pyx_pf_8PyClical_8clifford_12_richcmp__(struct __pyx_obj_8PyClical_clifford
__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op); /* proto */
02559 static PyObject *__pyx_pf_8PyClical_8clifford_14_getitem__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_ixt); /* proto */
02560 static PyObject *__pyx_pf_8PyClical_8clifford_16_neg__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self); /* proto */
02561 static PyObject *__pyx_pf_8PyClical_8clifford_18_pos__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self); /* proto */
02562 static PyObject *__pyx_pf_8PyClical_8clifford_20_add__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02563 static PyObject *__pyx_pf_8PyClical_8clifford_22_iadd__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02564 static PyObject *__pyx_pf_8PyClical_8clifford_24_sub__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02565 static PyObject *__pyx_pf_8PyClical_8clifford_26_isub__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02566 static PyObject *__pyx_pf_8PyClical_8clifford_28_mul__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02567 static PyObject *__pyx_pf_8PyClical_8clifford_30_imul__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02568 static PyObject *__pyx_pf_8PyClical_8clifford_32_mod__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02569 static PyObject *__pyx_pf_8PyClical_8clifford_34_imod__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02570 static PyObject *__pyx_pf_8PyClical_8clifford_36_and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02571 static PyObject *__pyx_pf_8PyClical_8clifford_38_iand__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02572 static PyObject *__pyx_pf_8PyClical_8clifford_40_xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
/* proto */
02573 static PyObject *__pyx_pf_8PyClical_8clifford_42_ixor__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02574 static PyObject *__pyx_pf_8PyClical_8clifford_44_truediv__(PyObject *__pyx_v_lhs, PyObject
__pyx_v_rhs); /* proto */
02575 #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
02576 static PyObject *__pyx_pf_8PyClical_8clifford_46_idiv__(struct __pyx_obj_8PyClical_clifford
__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02577 #endif
02578 static PyObject *__pyx_pf_8PyClical_8clifford_48inv(struct __pyx_obj_8PyClical_clifford

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    __pyx_v_self); /* proto */
02579 static PyObject *__pyx_pf_8PyClical_8clifford_50_or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
    /* proto */
02580 static PyObject *__pyx_pf_8PyClical_8clifford_52_iior__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_rhs); /* proto */
02581 static PyObject *__pyx_pf_8PyClical_8clifford_54_pow__(PyObject *__pyx_v_self, PyObject *__pyx_v_m,
    CYTHON_UNUSED PyObject *__pyx_v_dummy); /* proto */
02582 static PyObject *__pyx_pf_8PyClical_8clifford_56pow(struct __pyx_obj_8PyClical_clifford *__pyx_v_self,
    PyObject *__pyx_v_m); /* proto */
02583 static PyObject *__pyx_pf_8PyClical_8clifford_58outer_pow(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_m); /* proto */
02584 static PyObject *__pyx_pf_8PyClical_8clifford_60_call__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_grade); /* proto */
02585 static PyObject *__pyx_pf_8PyClical_8clifford_62scalar(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02586 static PyObject *__pyx_pf_8PyClical_8clifford_64pure(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02587 static PyObject *__pyx_pf_8PyClical_8clifford_66even(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02588 static PyObject *__pyx_pf_8PyClical_8clifford_68odd(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02589 static PyObject *__pyx_pf_8PyClical_8clifford_70vector_part(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_frm); /* proto */
02590 static PyObject *__pyx_pf_8PyClical_8clifford_72involute(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02591 static PyObject *__pyx_pf_8PyClical_8clifford_74reverse(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02592 static PyObject *__pyx_pf_8PyClical_8clifford_76conj(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02593 static PyObject *__pyx_pf_8PyClical_8clifford_78quad(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02594 static PyObject *__pyx_pf_8PyClical_8clifford_80norm(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02595 static PyObject *__pyx_pf_8PyClical_8clifford_82abs(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02596 static PyObject *__pyx_pf_8PyClical_8clifford_84max_abs(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02597 static PyObject *__pyx_pf_8PyClical_8clifford_86truncated(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self, PyObject *__pyx_v_limit); /* proto */
02598 static PyObject *__pyx_pf_8PyClical_8clifford_88isinf(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02599 static PyObject *__pyx_pf_8PyClical_8clifford_90isnan(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02600 static PyObject *__pyx_pf_8PyClical_8clifford_92frame(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02601 static PyObject *__pyx_pf_8PyClical_8clifford_94_repr__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02602 static PyObject *__pyx_pf_8PyClical_8clifford_96__str__(struct __pyx_obj_8PyClical_clifford
    *__pyx_v_self); /* proto */
02603 static PyObject *__pyx_pf_8PyClical_8clifford_98__reduce_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford *__pyx_v_self); /* proto */
02604 static PyObject *__pyx_pf_8PyClical_8clifford_100__setstate_cython__(CYTHON_UNUSED struct
    __pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state); /* proto */
02605 static PyObject *__pyx_pf_8PyClical_8clifford_hidden_doctests(CYTHON_UNUSED PyObject *__pyx_self); /*
    proto */
02606 static PyObject *__pyx_pf_8PyClical_10error_squared_tol(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02607 static PyObject *__pyx_pf_8PyClical_12error_squared(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold); /* proto */
02608 static PyObject *__pyx_pf_8PyClical_14approx_equal(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, PyObject *__pyx_v_tol); /* proto */
02609 static PyObject *__pyx_pf_8PyClical_16inv(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02610 static PyObject *__pyx_pf_8PyClical_18scalar(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02611 static PyObject *__pyx_pf_8PyClical_20real(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02612 static PyObject *__pyx_pf_8PyClical_22imag(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02613 static PyObject *__pyx_pf_8PyClical_24pure(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02614 static PyObject *__pyx_pf_8PyClical_26even(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02615 static PyObject *__pyx_pf_8PyClical_28odd(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02616 static PyObject *__pyx_pf_8PyClical_30involute(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02617 static PyObject *__pyx_pf_8PyClical_32reverse(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02618 static PyObject *__pyx_pf_8PyClical_34conj(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02619 static PyObject *__pyx_pf_8PyClical_36quad(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02620 static PyObject *__pyx_pf_8PyClical_38norm(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02621 static PyObject *__pyx_pf_8PyClical_40abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */

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02622 static PyObject * __pyx_pf_8PyClical_42max_abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02623 static PyObject * __pyx_pf_8PyClical_44pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_m); /* proto */
02624 static PyObject * __pyx_pf_8PyClical_46outer_pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj, PyObject *__pyx_v_m); /* proto */
02625 static PyObject * __pyx_pf_8PyClical_48complexifier(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02626 static PyObject * __pyx_pf_8PyClical_50sqrt(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02627 static PyObject * __pyx_pf_8PyClical_52exp(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02628 static PyObject * __pyx_pf_8PyClical_54log(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02629 static PyObject * __pyx_pf_8PyClical_56cos(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02630 static PyObject * __pyx_pf_8PyClical_58acos(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02631 static PyObject * __pyx_pf_8PyClical_60cosh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02632 static PyObject * __pyx_pf_8PyClical_62acosh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02633 static PyObject * __pyx_pf_8PyClical_64sin(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02634 static PyObject * __pyx_pf_8PyClical_66asin(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02635 static PyObject * __pyx_pf_8PyClical_68sinh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02636 static PyObject * __pyx_pf_8PyClical_70asinh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02637 static PyObject * __pyx_pf_8PyClical_72tan(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02638 static PyObject * __pyx_pf_8PyClical_74atan(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02639 static PyObject * __pyx_pf_8PyClical_76tanh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02640 static PyObject * __pyx_pf_8PyClical_78atanh(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj,
    PyObject *__pyx_v_i); /* proto */
02641 static PyObject * __pyx_pf_8PyClical_80random_clifford(CYTHON_UNUSED PyObject *__pyx_self, struct
    __pyx_obj_8PyClical_index_set *__pyx_v_ixt, PyObject *__pyx_v_fill); /* proto */
02642 static PyObject * __pyx_pf_8PyClical_82cga3(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02643 static PyObject * __pyx_pf_8PyClical_84cga3std(CYTHON_UNUSED PyObject *__pyx_self, PyObject
    *__pyx_v_obj); /* proto */
02644 static PyObject * __pyx_pf_8PyClical_86agc3(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj);
    /* proto */
02645 static PyObject * __pyx_pf_8PyClical_88e(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_obj); /*
    proto */
02646 static PyObject * __pyx_pf_8PyClical_90istpq(CYTHON_UNUSED PyObject *__pyx_self, PyObject *__pyx_v_p,
    PyObject *__pyx_v_q); /* proto */
02647 static PyObject * __pyx_pf_8PyClical_92_test(CYTHON_UNUSED PyObject *__pyx_self); /* proto */
02648 static PyObject * __pyx_tp_new_8PyClical_index_set(PyTypeObject *t, PyObject *a, PyObject *k);
    /*proto*/
02649 static PyObject * __pyx_tp_new_8PyClical_clifford(PyTypeObject *t, PyObject *a, PyObject *k); /*proto*/
02650 static PyObject * __pyx_tp_new_8PyClical__pyx_scope_struct____iter__(PyTypeObject *t, PyObject *a,
    PyObject *k); /*proto*/
02651 static PyObject * __pyx_float_0_0;
02652 static PyObject * __pyx_float_1_0;
02653 static PyObject * __pyx_float_4_0;
02654 static PyObject * __pyx_float_8_0;
02655 static PyObject * __pyx_int_0;
02656 static PyObject * __pyx_int_1;
02657 static PyObject * __pyx_int_4;
02658 static PyObject * __pyx_int_neg_1;
02659 static PyObject * __pyx_tuple_3;
02660 static PyObject * __pyx_tuple_4;
02661 static PyObject * __pyx_tuple_10;
02662 static PyObject * __pyx_tuple_11;
02663 static PyObject * __pyx_tuple_12;
02664 static PyObject * __pyx_tuple_15;
02665 static PyObject * __pyx_tuple_16;
02666 static PyObject * __pyx_tuple_18;
02667 static PyObject * __pyx_tuple_20;
02668 static PyObject * __pyx_tuple_21;
02669 static PyObject * __pyx_tuple_22;
02670 static PyObject * __pyx_codeobj__13;
02671 static PyObject * __pyx_codeobj__14;
02672 static PyObject * __pyx_codeobj__17;
02673 static PyObject * __pyx_codeobj__19;
02674 static PyObject * __pyx_codeobj__23;
02675 /* Late includes */
02676
02677 /* "PyClical.pyx":40
02678  * cdef class index_set
02679  *
02680  * cdef inline IndexSet toIndexSet(obj): # ««««««
02681  *     """

```

```

02682 *      Return the C++ IndexSet instance wrapped by index_set(obj).
02683 */
02684
02685 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_toIndexSet(PyObject *__pyx_v_obj) {
02686     IndexSet __pyx_r;
02687     __Pyx_RefNannyDeclarations
02688     PyObject *__pyx_t_1 = NULL;
02689     int __pyx_lineno = 0;
02690     const char *__pyx_filename = NULL;
02691     int __pyx_clineno = 0;
02692     __Pyx_RefNannySetupContext("toIndexSet", 0);
02693
02694     /* "PyClical.pyx":44
02695     *      Return the C++ IndexSet instance wrapped by index_set(obj).
02696     *      """
02697     *      return index_set(obj).instance[0]          # ««««««««
02698     *
02699     * cdef class index_set:
02700     */
02701     __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_index_set, __pyx_v_obj);
02702     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 44, __pyx_L1_error)
02703     __Pyx_GOTREF(__pyx_t_1);
02704     __pyx_r = ((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1->instance[0]);
02705     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02706     goto __pyx_L0;
02707
02708     /* "PyClical.pyx":40
02709     * cdef class index_set
02710     *
02711     * cdef inline IndexSet toIndexSet(obj):          # ««««««««
02712     *      """
02713     *      Return the C++ IndexSet instance wrapped by index_set(obj).
02714     */
02715     /* function exit code */
02716     __pyx_L1_error:;
02717     __Pyx_XDECREF(__pyx_t_1);
02718     __Pyx_WriteUnraisable("PyClical.toIndexSet", __pyx_clineno, __pyx_lineno, __pyx_filename, 1, 0);
02719     __Pyx_prepend_to_initialize(&__pyx_r);
02720     __pyx_L0:;
02721     __Pyx_RefNannyFinishContext();
02722     return __pyx_r;
02723 }
02724
02725 /* "PyClical.pyx":52
02726 *      cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
02727 *
02728 *      cdef inline wrap(index_set self, IndexSet other):          # ««««««««
02729 *      """
02730 *      Wrap an instance of the C++ class IndexSet.
02731 */
02732
02733 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_9index_set_wrap(struct __pyx_obj_8PyClical_index_set
__pyx_v_self, IndexSet __pyx_v_other) {
02734     PyObject *__pyx_r = NULL;
02735     __Pyx_RefNannyDeclarations
02736     __Pyx_RefNannySetupContext("wrap", 0);
02737
02738     /* "PyClical.pyx":56
02739     *      Wrap an instance of the C++ class IndexSet.
02740     *      """
02741     *      self.instance[0] = other          # ««««««««
02742     *      return self
02743     */
02744     (__pyx_v_self->instance[0]) = __pyx_v_other;
02745
02746     /* "PyClical.pyx":57
02747     *      """
02748     *      self.instance[0] = other
02749     *      return self          # ««««««««
02750     *
02751     * cdef inline IndexSet unwrap(index_set self):
02752     */
02753     __Pyx_XDECREF(__pyx_r);
02754     __Pyx_INCREF((PyObject *)__pyx_v_self);
02755     __pyx_r = (PyObject *)__pyx_v_self;
02756     goto __pyx_L0;
02757
02758     /* "PyClical.pyx":52
02759     *      cdef IndexSet *instance # Wrapped instance of C++ class IndexSet.
02760     *
02761     *      cdef inline wrap(index_set self, IndexSet other):          # ««««««««
02762     *      """
02763     *      Wrap an instance of the C++ class IndexSet.
02764     */
02765
02766

```



```

02767  /* function exit code */
02768  __pyx_L0:;
02769  __Pyx_XGIVEREF(__pyx_r);
02770  __Pyx_RefNannyFinishContext();
02771  return __pyx_r;
02772 }
02773
02774 /* "PyClical.pyx":59
02775 *      return self
02776 *
02777 *      cdef inline IndexSet unwrap(index_set self):          # ««««««««
02778 *          """
02779 *          Return the wrapped C++ IndexSet instance.
02780 */
02781
02782 static CYTHON_INLINE IndexSet __pyx_f_8PyClical_9index_set_unwrap(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
02783     IndexSet __pyx_r;
02784     __Pyx_RefNannyDeclarations
02785     __Pyx_RefNannySetupContext("unwrap", 0);
02786
02787     /* "PyClical.pyx":63
02788 *          Return the wrapped C++ IndexSet instance.
02789 *          """
02790 *          return self.instance[0]          # ««««««««
02791 *
02792 *          cpdef copy(index_set self):
02793 */
02794     __pyx_r = (__pyx_v_self->instance[0]);
02795     goto __pyx_L0;
02796
02797     /* "PyClical.pyx":59
02798 *      return self
02799 *
02800 *      cdef inline IndexSet unwrap(index_set self):          # ««««««««
02801 *          """
02802 *          Return the wrapped C++ IndexSet instance.
02803 */
02804
02805     /* function exit code */
02806     __pyx_L0:;
02807     __Pyx_RefNannyFinishContext();
02808     return __pyx_r;
02809 }
02810
02811 /* "PyClical.pyx":65
02812 *      return self.instance[0]
02813 *
02814 *      cpdef copy(index_set self):          # ««««««««
02815 *          """
02816 *          Copy this index_set object.
02817 */
02818
02819 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
02820 static PyObject *__pyx_f_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set *__pyx_v_self,
int __pyx_skip_dispatch) {
02821     PyObject *__pyx_r = NULL;
02822     __Pyx_RefNannyDeclarations
02823     PyObject *__pyx_t_1 = NULL;
02824     PyObject *__pyx_t_2 = NULL;
02825     PyObject *__pyx_t_3 = NULL;
02826     PyObject *__pyx_t_4 = NULL;
02827     int __pyx_lineno = 0;
02828     const char *__pyx_filename = NULL;
02829     int __pyx_clineno = 0;
02830     __Pyx_RefNannySetupContext("copy", 0);
02831     /* Check if called by wrapper */
02832     if (unlikely(__pyx_skip_dispatch)) ;
02833     /* Check if overridden in Python */
02834     else if (unlikely((Py_TYPE(((PyObject *)__pyx_v_self))->tp_dictoffset != 0) || (Py_TYPE(((PyObject
*)__pyx_v_self))->tp_flags & (Py_TPFLAGS_IS_ABSTRACT | Py_TPFLAGS_HEAPTYPE)))) {
02835         #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02836         static PY_UINT64_T __pyx_tp_dict_version = __PYX_DICT_VERSION_INIT, __pyx_obj_dict_version =
__PYX_DICT_VERSION_INIT;
02837         if (unlikely(!__Pyx_object_dict_version_matches(((PyObject *)__pyx_v_self), __pyx_tp_dict_version,
__pyx_obj_dict_version))) {
02838             PY_UINT64_T __pyx_type_dict_guard = __Pyx_get_tp_dict_version(((PyObject *)__pyx_v_self));
02839             #endif
02840             __pyx_t_1 = __Pyx_PyObject_GetAttrStr(((PyObject *)__pyx_v_self), __pyx_n_s_copy); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 65, __pyx_L1_error)
02841             __Pyx_GOTREF(__pyx_t_1);
02842             if (!PyCFunction_Check(__pyx_t_1) || (PyCFunction_GET_FUNCTION(__pyx_t_1) !=
(PyCFunction)(void*)__pyx_pw_8PyClical_9index_set_1copy)) {
02843                 __Pyx_XDECREF(__pyx_r);
02844                 __Pyx_INCREF(__pyx_t_1);
02845                 __pyx_t_3 = __pyx_t_1; __pyx_t_4 = NULL;

```

```

02846         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_3))) {
02847             __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
02848             if (likely(__pyx_t_4)) {
02849                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
02850                 __Pyx_INCREF(__pyx_t_4);
02851                 __Pyx_INCREF(function);
02852                 __Pyx_DECREF_SET(__pyx_t_3, function);
02853             }
02854         }
02855         __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
02856         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
02857         if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 65, __pyx_L1_error)
02858         __Pyx_GOTREF(__pyx_t_2);
02859         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
02860         __pyx_r = __pyx_t_2;
02861         __pyx_t_2 = 0;
02862         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02863         goto __pyx_L0;
02864     }
02865     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02866     __pyx_tp_dict_version = __Pyx_get_tp_dict_version(((PyObject *)__pyx_v_self));
02867     __pyx_obj_dict_version = __Pyx_get_obj_dict_version(((PyObject *)__pyx_v_self));
02868     if (unlikely(__pyx_type_dict_guard != __pyx_tp_dict_version)) {
02869         __pyx_tp_dict_version = __pyx_obj_dict_version = __PYX_DICT_VERSION_INIT;
02870     }
02871     #endif
02872     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
02873     #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
02874 }
02875 #endif
02876 }
02877
02878 /* "PyClical.pyx":72
02879 *     {1}
02880 *     """
02881 *     return index_set(self) # ««««««««
02882 *
02883 *     def __cinit__(self, other = 0):
02884 */
02885     __Pyx_XDECREF(__pyx_r);
02886     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_index_set), ((PyObject
*) __pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 72, __pyx_L1_error)
02887     __Pyx_GOTREF(__pyx_t_1);
02888     __pyx_r = __pyx_t_1;
02889     __pyx_t_1 = 0;
02890     goto __pyx_L0;
02891
02892 /* "PyClical.pyx":65
02893 *     return self.instance[0]
02894 *
02895 *     cpdef copy(index_set self): # ««««««««
02896 *     """
02897 *     Copy this index_set object.
02898 */
02899
02900 /* function exit code */
02901 __pyx_L1_error:;
02902     __Pyx_XDECREF(__pyx_t_1);
02903     __Pyx_XDECREF(__pyx_t_2);
02904     __Pyx_XDECREF(__pyx_t_3);
02905     __Pyx_XDECREF(__pyx_t_4);
02906     __Pyx_AddTraceback("PyClical.index_set.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
02907     __pyx_r = 0;
02908     __pyx_L0:;
02909     __Pyx_XGIVEREF(__pyx_r);
02910     __Pyx_RefNannyFinishContext();
02911     return __pyx_r;
02912 }
02913
02914 /* Python wrapper */
02915 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused); /*proto*/
02916 static char __pyx_doc_8PyClical_9index_set_copy[] = "\n        Copy this index_set object.\n\n
>> s=index_set(1); t=s.copy(); print(t)\n        {1}\n        ";
02917 static PyObject *__pyx_pw_8PyClical_9index_set_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
*unused) {
02918     PyObject *__pyx_r = 0;
02919     __Pyx_RefNannyDeclarations
02920     __Pyx_RefNannySetupContext("copy (wrapper)", 0);
02921     __pyx_r = __pyx_pf_8PyClical_9index_set_copy(((struct __pyx_obj_8PyClical_index_set
*) __pyx_v_self));
02922
02923 /* function exit code */
02924     __Pyx_RefNannyFinishContext();
02925     return __pyx_r;
02926 }

```



```

02927
02928 static PyObject *__pyx_pf_8PyClical_9index_set_copy(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self) {
02929     PyObject *__pyx_r = NULL;
02930     __Pyx_RefNannyDeclarations
02931     PyObject *__pyx_t_1 = NULL;
02932     int __pyx_lineno = 0;
02933     const char *__pyx_filename = NULL;
02934     int __pyx_clineno = 0;
02935     __Pyx_RefNannySetupContext("copy", 0);
02936     __Pyx_XDECREF(__pyx_r);
02937     __pyx_t_1 = __pyx_f_8PyClical_9index_set_copy(__pyx_v_self, 1); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 65, __pyx_L1_error)
02938     __Pyx_GOTREF(__pyx_t_1);
02939     __pyx_r = __pyx_t_1;
02940     __pyx_t_1 = 0;
02941     goto __pyx_L0;
02942
02943     /* function exit code */
02944     __pyx_L1_error:;
02945     __Pyx_XDECREF(__pyx_t_1);
02946     __Pyx_AddTraceback("PyClical.index_set.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
02947     __pyx_r = NULL;
02948     __pyx_L0:;
02949     __Pyx_XGIVEREF(__pyx_r);
02950     __Pyx_RefNannyFinishContext();
02951     return __pyx_r;
02952 }
02953
02954 /* "PyClical.pyx":74
02955 *         return index_set(self)
02956 *
02957 *     def __cinit__(self, other = 0):          # ««««««««
02958 *         """
02959 *         Construct an object of type index_set.
02960 */
02961
02962 /* Python wrapper */
02963 static int __pyx_pw_8PyClical_9index_set_3__cinit__(PyObject *__pyx_v_self, PyObject *__pyx_args,
    PyObject *__pyx_kwds); /*proto*/
02964 static int __pyx_pw_8PyClical_9index_set_3__cinit__(PyObject *__pyx_v_self, PyObject *__pyx_args,
    PyObject *__pyx_kwds) {
02965     PyObject *__pyx_v_other = 0;
02966     int __pyx_lineno = 0;
02967     const char *__pyx_filename = NULL;
02968     int __pyx_clineno = 0;
02969     int __pyx_r;
02970     __Pyx_RefNannyDeclarations
02971     __Pyx_RefNannySetupContext("__cinit__ (wrapper)", 0);
02972     {
02973         static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_other,0};
02974         PyObject* values[1] = {0};
02975         values[0] = ((PyObject *)__pyx_int_0);
02976         if (unlikely(__pyx_kwds)) {
02977             Py_ssize_t kw_args;
02978             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
02979             switch (pos_args) {
02980                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
02981                     CYTHON_FALLTHROUGH;
02982                 case 0: break;
02983                 default: goto __pyx_L5_argtuple_error;
02984             }
02985             kw_args = PyDict_Size(__pyx_kwds);
02986             switch (pos_args) {
02987                 case 0:
02988                     if (kw_args > 0) {
02989                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_other);
02990                         if (value) { values[0] = value; kw_args--; }
02991                     }
02992             }
02993             if (unlikely(kw_args > 0)) {
02994                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values, pos_args,
__"__cinit__" < 0)) __PYX_ERR(0, 74, __pyx_L3_error)
02995             }
02996         } else {
02997             switch (PyTuple_GET_SIZE(__pyx_args)) {
02998                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
02999                     CYTHON_FALLTHROUGH;
03000                 case 0: break;
03001                 default: goto __pyx_L5_argtuple_error;
03002             }
03003         }
03004         __pyx_v_other = values[0];
03005     }
03006     goto __pyx_L4_argument_unpacking_done;
03007     __pyx_L5_argtuple_error:;
03008     __Pyx_RaiseArgtupleInvalid("__cinit__", 0, 0, 1, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0, 74,

```

```

__pyx_L3_error)
03009 __pyx_L3_error;;
03010 __Pyx_AddTraceback("PyCliclal.index_set.__cinit__", __pyx_clineno, __pyx_lineno, __pyx_filename);
03011 __Pyx_RefNannyFinishContext();
03012 return -1;
03013 __pyx_L4_argument_unpacking_done;;
03014 __pyx_r = __pyx_pf_8PyCliclal_9index_set_2__cinit__((struct __pyx_obj_8PyCliclal_index_set
*)__pyx_v_self), __pyx_v_other);
03015
03016 /* function exit code */
03017 __Pyx_RefNannyFinishContext();
03018 return __pyx_r;
03019 }
03020
03021 static int __pyx_pf_8PyCliclal_9index_set_2__cinit__(struct __pyx_obj_8PyCliclal_index_set
*__pyx_v_self, PyObject *__pyx_v_other) {
03022     PyObject *__pyx_v_error_msg_prefix = NULL;
03023     PyObject *__pyx_v_idx = NULL;
03024     PyObject *__pyx_v_bother = NULL;
03025     int __pyx_r;
03026     __Pyx_RefNannyDeclarations
03027     int __pyx_t_1;
03028     int __pyx_t_2;
03029     IndexSet *__pyx_t_3;
03030     PyObject *__pyx_t_4 = NULL;
03031     PyObject *__pyx_t_5 = NULL;
03032     int __pyx_t_6;
03033     int __pyx_t_7;
03034     PyObject *__pyx_t_8 = NULL;
03035     PyObject *__pyx_t_9 = NULL;
03036     PyObject *__pyx_t_10 = NULL;
03037     Py_ssize_t __pyx_t_11;
03038     PyObject *(*__pyx_t_12)(PyObject *);
03039     PyObject *__pyx_t_13 = NULL;
03040     PyObject *__pyx_t_14 = NULL;
03041     PyObject *__pyx_t_15 = NULL;
03042     PyObject *__pyx_t_16 = NULL;
03043     char *__pyx_t_17;
03044     int __pyx_lineno = 0;
03045     const char *__pyx_filename = NULL;
03046     int __pyx_clineno = 0;
03047     __Pyx_RefNannySetupContext("__cinit__", 0);
03048
03049     /* "PyCliclal.pyx":93
03050     *     {}
03051     *     """
03052     *     error_msg_prefix = "Cannot initialize index_set object from"
03053     *     if isinstance(other, index_set):
03054     *         self.instance = new IndexSet((<index_set>other).unwrap())
03055     */
03056     __Pyx_INCREF(__pyx_kp_u_Cannot_initialize_index_set_obje);
03057     __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_initialize_index_set_obje;
03058
03059     /* "PyCliclal.pyx":94
03060     *     """
03061     *     error_msg_prefix = "Cannot initialize index_set object from"
03062     *     if isinstance(other, index_set):
03063     *         self.instance = new IndexSet((<index_set>other).unwrap())
03064     *     elif isinstance(other, numbers.Integral):
03065     */
03066     __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyCliclal_index_set);
03067     __pyx_t_2 = (__pyx_t_1 != 0);
03068     if (__pyx_t_2) {
03069
03070         /* "PyCliclal.pyx":95
03071         *     error_msg_prefix = "Cannot initialize index_set object from"
03072         *     if isinstance(other, index_set):
03073         *         self.instance = new IndexSet((<index_set>other).unwrap())
03074         *     elif isinstance(other, numbers.Integral):
03075         *         self.instance = new IndexSet(<int>other)
03076         */
03077         try {
03078             __pyx_t_3 = new IndexSet(__pyx_f_8PyCliclal_9index_set_unwrap(((struct
__pyx_obj_8PyCliclal_index_set *)__pyx_v_other)));
03079         } catch (...) {
03080             __Pyx_CppExn2PyErr();
03081             __PYX_ERR(0, 95, __pyx_L1_error)
03082         }
03083         __pyx_v_self->instance = __pyx_t_3;
03084
03085         /* "PyCliclal.pyx":94
03086         *     """
03087         *     error_msg_prefix = "Cannot initialize index_set object from"
03088         *     if isinstance(other, index_set):
03089         *         self.instance = new IndexSet((<index_set>other).unwrap())
03090         *     elif isinstance(other, numbers.Integral):
03091         */

```

```

03092     goto __pyx_L3;
03093 }
03094
03095 /* "PyClical.pyx":96
03096 *     if isinstance(other, index_set):
03097 *         self.instance = new IndexSet((<index_set>other).unwrap())
03098 *     elif isinstance(other, numbers.Integral):
03099 *         self.instance = new IndexSet(<int>other)
03100 *     elif isinstance(other, (set, frozenset)):
03101 */
03102 __Pyx_GetModuleGlobalName(__pyx_t_4, __pyx_n_s_numbers); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 96,
__pyx_L1_error)
03103 __Pyx_GOTREF(__pyx_t_4);
03104 __pyx_t_5 = __Pyx_PyObject_GetAttrStr(__pyx_t_4, __pyx_n_s_Integral); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 96, __pyx_L1_error)
03105 __Pyx_GOTREF(__pyx_t_5);
03106 __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03107 __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_5); if (unlikely(__pyx_t_2 == ((int)-1)))
__PYX_ERR(0, 96, __pyx_L1_error)
03108 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03109 __pyx_t_1 = (__pyx_t_2 != 0);
03110 if (__pyx_t_1) {
03111
03112     /* "PyClical.pyx":97
03113     *         self.instance = new IndexSet((<index_set>other).unwrap())
03114     *     elif isinstance(other, numbers.Integral):
03115     *         self.instance = new IndexSet(<int>other)
03116     *     elif isinstance(other, (set, frozenset)):
03117     *         try:
03118     */
03119     __pyx_t_6 = __Pyx_PyInt_As_int(__pyx_v_other); if (unlikely((__pyx_t_6 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 97, __pyx_L1_error)
03120     try {
03121         __pyx_t_3 = new IndexSet(((int) __pyx_t_6));
03122     } catch (...) {
03123         __Pyx_CppExn2PyErr();
03124         __PYX_ERR(0, 97, __pyx_L1_error)
03125     }
03126     __pyx_v_self->instance = __pyx_t_3;
03127
03128     /* "PyClical.pyx":96
03129     *     if isinstance(other, index_set):
03130     *         self.instance = new IndexSet((<index_set>other).unwrap())
03131     *     elif isinstance(other, numbers.Integral):
03132     *         self.instance = new IndexSet(<int>other)
03133     *     elif isinstance(other, (set, frozenset)):
03134     */
03135     goto __pyx_L3;
03136 }
03137
03138 /* "PyClical.pyx":98
03139 *     elif isinstance(other, numbers.Integral):
03140 *         self.instance = new IndexSet(<int>other)
03141 *     elif isinstance(other, (set, frozenset)):
03142 *         try:
03143 *             self.instance = new IndexSet()
03144 */
03145 __pyx_t_2 = PySet_Check(__pyx_v_other);
03146 __pyx_t_7 = (__pyx_t_2 != 0);
03147 if (!__pyx_t_7) {
03148 } else {
03149     __pyx_t_1 = __pyx_t_7;
03150     goto __pyx_L4_bool_binop_done;
03151 }
03152 __pyx_t_7 = PyFrozenSet_Check(__pyx_v_other);
03153 __pyx_t_2 = (__pyx_t_7 != 0);
03154 __pyx_t_1 = __pyx_t_2;
03155 __pyx_L4_bool_binop_done;
03156 __pyx_t_2 = (__pyx_t_1 != 0);
03157 if (__pyx_t_2) {
03158
03159     /* "PyClical.pyx":99
03160     *         self.instance = new IndexSet(<int>other)
03161     *     elif isinstance(other, (set, frozenset)):
03162     *         try:
03163     *             self.instance = new IndexSet()
03164     *         for idx in other:
03165     */
03166     {
03167         __Pyx_PyThreadState_declare
03168         __Pyx_PyThreadState_assign
03169         __Pyx_ExceptionSave(&__pyx_t_8, &__pyx_t_9, &__pyx_t_10);
03170         __Pyx_XGOTREF(__pyx_t_8);
03171         __Pyx_XGOTREF(__pyx_t_9);
03172         __Pyx_XGOTREF(__pyx_t_10);
03173         /*try:*/ {
03174

```

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03175         /* "PyClicl.pyx":100
03176         *         elif isinstance(other, (set, frozenset)):
03177         *             try:
03178         *                 self.instance = new IndexSet()           # ««««««««
03179         *                 for idx in other:
03180         *                     self[idx] = True
03181         */
03182         __pyx_t_3 = new IndexSet();
03183         __pyx_v_self->instance = __pyx_t_3;
03184
03185         /* "PyClicl.pyx":101
03186         *             try:
03187         *                 self.instance = new IndexSet()
03188         *                 for idx in other:           # ««««««««
03189         *                     self[idx] = True
03190         *             except IndexError:
03191         */
03192         if (likely(PyList_CheckExact(__pyx_v_other)) || PyTuple_CheckExact(__pyx_v_other)) {
03193             __pyx_t_5 = __pyx_v_other; __Pyx_INCREF(__pyx_t_5); __pyx_t_11 = 0;
03194             __pyx_t_12 = NULL;
03195         } else {
03196             __pyx_t_11 = -1; __pyx_t_5 = PyObject_GetIter(__pyx_v_other); if (unlikely(!__pyx_t_5))
03197             __PYX_ERR(0, 101, __pyx_L6_error)
03198             __Pyx_GOTREF(__pyx_t_5);
03199             __pyx_t_12 = Py_TYPE(__pyx_t_5)->tp_iternext; if (unlikely(!__pyx_t_12)) __PYX_ERR(0, 101,
03200             __pyx_L6_error)
03201         }
03202         for (;;) {
03203             if (likely(!__pyx_t_12)) {
03204                 if (likely(PyList_CheckExact(__pyx_t_5))) {
03205                     if (__pyx_t_11 >= PyList_GET_SIZE(__pyx_t_5)) break;
03206                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
03207                     __pyx_t_4 = PyList_GET_ITEM(__pyx_t_5, __pyx_t_11); __Pyx_INCREF(__pyx_t_4);
03208                     __pyx_t_11++; if (unlikely(0 < 0)) __PYX_ERR(0, 101, __pyx_L6_error)
03209                     #else
03210                     __pyx_t_4 = PySequence_ITEM(__pyx_t_5, __pyx_t_11); __pyx_t_11++; if
03211                     (unlikely(!__pyx_t_4)) __PYX_ERR(0, 101, __pyx_L6_error)
03212                     __Pyx_GOTREF(__pyx_t_4);
03213                     #endif
03214                 } else {
03215                     if (__pyx_t_11 >= PyTuple_GET_SIZE(__pyx_t_5)) break;
03216                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
03217                     __pyx_t_4 = PyTuple_GET_ITEM(__pyx_t_5, __pyx_t_11); __Pyx_INCREF(__pyx_t_4);
03218                     __pyx_t_11++; if (unlikely(0 < 0)) __PYX_ERR(0, 101, __pyx_L6_error)
03219                     #else
03220                     __pyx_t_4 = PySequence_ITEM(__pyx_t_5, __pyx_t_11); __pyx_t_11++; if
03221                     (unlikely(!__pyx_t_4)) __PYX_ERR(0, 101, __pyx_L6_error)
03222                     __Pyx_GOTREF(__pyx_t_4);
03223                     #endif
03224                 }
03225             } else {
03226                 __pyx_t_4 = __pyx_t_12(__pyx_t_5);
03227                 if (unlikely(!__pyx_t_4)) {
03228                     PyObject* exc_type = PyErr_Occurred();
03229                     if (exc_type) {
03230                         if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration)))
03231                             PyErr_Clear();
03232                         else __PYX_ERR(0, 101, __pyx_L6_error)
03233                     }
03234                     break;
03235                 }
03236                 __Pyx_GOTREF(__pyx_t_4);
03237             }
03238             __Pyx_XDECREF_SET(__pyx_v_idx, __pyx_t_4);
03239             __pyx_t_4 = 0;
03240
03241             /* "PyClicl.pyx":102
03242             *                 self.instance = new IndexSet()
03243             *                 for idx in other:
03244             *                     self[idx] = True           # ««««««««
03245             *             except IndexError:
03246             *                 raise IndexError(error_msg_prefix + " invalid " + repr(other) + ".")
03247             */
03248             if (unlikely(PyObject_SetItem((PyObject *)__pyx_v_self, __pyx_v_idx, Py_True) < 0))
03249             __PYX_ERR(0, 102, __pyx_L6_error)
03250
03251             /* "PyClicl.pyx":101
03252             *             try:
03253             *                 self.instance = new IndexSet()
03254             *                 for idx in other:           # ««««««««
03255             *                     self[idx] = True
03256             *             except IndexError:
03257             */
03258             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03259
03260             /* "PyClicl.pyx":99

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03329 *         elif isinstance(other, str):
03330 *             try:
03331 */
03332         __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 106, __pyx_L8_except_error)
03333         __Pyx_GOTREF(__pyx_t_16);
03334         __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 106,
__pyx_L8_except_error)
03335         __Pyx_GOTREF(__pyx_t_15);
03336         __pyx_t_14 = PyNumber_Add(__pyx_t_16, __pyx_t_15); if (unlikely(!__pyx_t_14)) __PYX_ERR(0,
106, __pyx_L8_except_error)
03337         __Pyx_GOTREF(__pyx_t_14);
03338         __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03339         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03340         __pyx_t_15 = PyNumber_Add(__pyx_t_14, __pyx_kp_u); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
106, __pyx_L8_except_error)
03341         __Pyx_GOTREF(__pyx_t_15);
03342         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03343         __pyx_t_14 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_15); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 106, __pyx_L8_except_error)
03344         __Pyx_GOTREF(__pyx_t_14);
03345         __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03346         __Pyx_Raise(__pyx_t_14, 0, 0, 0);
03347         __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03348         __PYX_ERR(0, 106, __pyx_L8_except_error)
03349     }
03350     goto __pyx_L8_except_error;
03351     __pyx_L8_except_error:;
03352
03353     /* "PyClical.pyx":99
03354 *         self.instance = new IndexSet(<int>other)
03355 *         elif isinstance(other, (set, frozenset)):
03356 *             try:
03357 *                 # ««««««««
03358 *                 self.instance = new IndexSet()
03359 *                 for idx in other:
03359 */
03360         __Pyx_XGIVREF(__pyx_t_8);
03361         __Pyx_XGIVREF(__pyx_t_9);
03362         __Pyx_XGIVREF(__pyx_t_10);
03363         __Pyx_ExceptionReset(__pyx_t_8, __pyx_t_9, __pyx_t_10);
03364         goto __pyx_L1_error;
03365         __pyx_L11_try_end:;
03366     }
03367
03368     /* "PyClical.pyx":98
03369 *         elif isinstance(other, numbers.Integral):
03370 *             self.instance = new IndexSet(<int>other)
03371 *         elif isinstance(other, (set, frozenset)):
03372 *             try:
03373 *                 # ««««««««
03374 *                 self.instance = new IndexSet()
03374 */
03375     goto __pyx_L3;
03376 }
03377
03378 /* "PyClical.pyx":107
03379 *         except (RuntimeError, TypeError):
03380 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03381 *         elif isinstance(other, str):
03382 *             try:
03383 *                 # ««««««««
03384 *                 bother = other.encode("UTF-8")
03384 */
03385         __pyx_t_2 = PyUnicode_Check(__pyx_v_other);
03386         __pyx_t_1 = (__pyx_t_2 != 0);
03387         if (likely(__pyx_t_1)) {
03388
03389         /* "PyClical.pyx":108
03390 *             raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03391 *         elif isinstance(other, str):
03392 *             try:
03393 *                 # ««««««««
03394 *                 bother = other.encode("UTF-8")
03395 *                 self.instance = new IndexSet(<char *>bother)
03395 */
03396         {
03397             __Pyx_PyThreadState_declare
03398             __Pyx_PyThreadState_assign
03399             __Pyx_ExceptionSave(&__pyx_t_10, &__pyx_t_9, &__pyx_t_8);
03400             __Pyx_XGOTREF(__pyx_t_10);
03401             __Pyx_XGOTREF(__pyx_t_9);
03402             __Pyx_XGOTREF(__pyx_t_8);
03403             /*try:*/ {
03404
03405             /* "PyClical.pyx":109
03406 *                 elif isinstance(other, str):
03407 *                 try:
03408 *                     # ««««««««
03409 *                     bother = other.encode("UTF-8")
03410 *                     self.instance = new IndexSet(<char *>bother)
03410 *                 except RuntimeError:

```

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03411 */
03412     __pyx_t_4 = __Pyx_PyObject_GetAttrStr(__pyx_v_other, __pyx_n_s_encode); if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 109, __pyx_L18_error)
03413     __Pyx_GOTREF(__pyx_t_4);
03414     __pyx_t_13 = NULL;
03415     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_4))) {
03416         __pyx_t_13 = PyMethod_GET_SELF(__pyx_t_4);
03417         if (likely(__pyx_t_13)) {
03418             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_4);
03419             __Pyx_INCREF(__pyx_t_13);
03420             __Pyx_INCREF(function);
03421             __Pyx_DECREF_SET(__pyx_t_4, function);
03422         }
03423     }
03424     __pyx_t_5 = (__pyx_t_13) ? __Pyx_PyObject_Call2Args(__pyx_t_4, __pyx_t_13, __pyx_kp_u_UTF_8) :
__Pyx_PyObject_CallOneArg(__pyx_t_4, __pyx_kp_u_UTF_8);
03425     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
03426     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 109, __pyx_L18_error)
03427     __Pyx_GOTREF(__pyx_t_5);
03428     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03429     __pyx_v_bother = __pyx_t_5;
03430     __pyx_t_5 = 0;
03431
03432     /* "PyClical.pyx":110
03433     *         try:
03434     *             bother = other.encode("UTF-8")
03435     *             self.instance = new IndexSet(<char *>bother)           # ««««««««
03436     *         except RuntimeError:
03437     *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03438     */
03439     __pyx_t_17 = __Pyx_PyObject_AsWritableString(__pyx_v_bother); if (unlikely(!__pyx_t_17) &&
PyErr_Occurred()) __PYX_ERR(0, 110, __pyx_L18_error)
03440     try {
03441         __pyx_t_3 = new IndexSet(((char *)__pyx_t_17));
03442     } catch (...) {
03443         __Pyx_CppExn2PyErr();
03444         __PYX_ERR(0, 110, __pyx_L18_error)
03445     }
03446     __pyx_v_self->instance = __pyx_t_3;
03447
03448     /* "PyClical.pyx":108
03449     *         raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03450     *     elif isinstance(other, str):
03451     *         try:           # ««««««««
03452     *             bother = other.encode("UTF-8")
03453     *             self.instance = new IndexSet(<char *>bother)
03454     */
03455 }
03456 __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
03457 __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
03458 __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
03459 goto __pyx_L23_try_end;
03460 __pyx_L18_error;;
03461 __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
03462 __Pyx_XDECREF(__pyx_t_14); __pyx_t_14 = 0;
03463 __Pyx_XDECREF(__pyx_t_15); __pyx_t_15 = 0;
03464 __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
03465 __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
03466 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
03467
03468     /* "PyClical.pyx":111
03469     *         bother = other.encode("UTF-8")
03470     *         self.instance = new IndexSet(<char *>bother)
03471     *         except RuntimeError:           # ««««««««
03472     *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03473     *     else:
03474     */
03475     __pyx_t_6 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
03476     if (__pyx_t_6) {
03477         __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
03478         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_4, &__pyx_t_13) < 0) __PYX_ERR(0, 111,
__pyx_L20_except_error)
03479         __Pyx_GOTREF(__pyx_t_5);
03480         __Pyx_GOTREF(__pyx_t_4);
03481         __Pyx_GOTREF(__pyx_t_13);
03482
03483         /* "PyClical.pyx":112
03484         *             self.instance = new IndexSet(<char *>bother)
03485         *         except RuntimeError:
03486         *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03487         *     else:
03488         *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03489         */
03490         __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_invalid_string); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 112, __pyx_L20_except_error)

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03491     __Pyx_GOTREF(__pyx_t_14);
03492     __pyx_t_15 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 112,
__pyx_L20_except_error)
03493     __Pyx_GOTREF(__pyx_t_15);
03494     __pyx_t_16 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
112, __pyx_L20_except_error)
03495     __Pyx_GOTREF(__pyx_t_16);
03496     __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
03497     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03498     __pyx_t_15 = PyNumber_Add(__pyx_t_16, __pyx_kp_u); if (unlikely(!__pyx_t_15)) __PYX_ERR(0,
112, __pyx_L20_except_error)
03499     __Pyx_GOTREF(__pyx_t_15);
03500     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03501     __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_15); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 112, __pyx_L20_except_error)
03502     __Pyx_GOTREF(__pyx_t_16);
03503     __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
03504     __Pyx_Raise(__pyx_t_16, 0, 0, 0);
03505     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
03506     __PYX_ERR(0, 112, __pyx_L20_except_error)
03507 }
03508 goto __pyx_L20_except_error;
03509 __pyx_L20_except_error:;
03510
03511 /* "PyClical.pyx":108
03512 *         raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03513 *     elif isinstance(other, str):
03514 *         try:
03515 *             # ««««««««
03516 *             bother = other.encode("UTF-8")
03517 *             self.instance = new IndexSet(<char *>bother)
03518 */
03518     __Pyx_XGIVEREF(__pyx_t_10);
03519     __Pyx_XGIVEREF(__pyx_t_9);
03520     __Pyx_XGIVEREF(__pyx_t_8);
03521     __Pyx_ExceptionReset(__pyx_t_10, __pyx_t_9, __pyx_t_8);
03522     goto __pyx_L1_error;
03523     __pyx_L23_try_end:;
03524 }
03525
03526 /* "PyClical.pyx":107
03527 *     except (RuntimeError, TypeError):
03528 *         raise ValueError(error_msg_prefix + " invalid " + repr(other) + ".")
03529 *     elif isinstance(other, str):
03530 *         # ««««««««
03531 *         try:
03532 *             bother = other.encode("UTF-8")
03533 */
03533     goto __pyx_L3;
03534 }
03535
03536 /* "PyClical.pyx":114
03537 *         raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
03538 *     else:
03539 *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03540 *         # ««««««««
03541 *     def __dealloc__(self):
03542 */
03542 /*else*/ {
03543     __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 114, __pyx_L1_error)
03544     __Pyx_GOTREF(__pyx_t_13);
03545     __pyx_t_4 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 114, __pyx_L1_error)
03546     __Pyx_GOTREF(__pyx_t_4);
03547     __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0,
114, __pyx_L1_error)
03548     __Pyx_GOTREF(__pyx_t_5);
03549     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
03550     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03551     __pyx_t_4 = __Pyx_PyUnicode_Concat(__pyx_t_5, __pyx_kp_u); if (unlikely(!__pyx_t_4)) __PYX_ERR(0,
114, __pyx_L1_error)
03552     __Pyx_GOTREF(__pyx_t_4);
03553     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03554     __pyx_t_5 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_4); if
(unlikely(!__pyx_t_5)) __PYX_ERR(0, 114, __pyx_L1_error)
03555     __Pyx_GOTREF(__pyx_t_5);
03556     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
03557     __Pyx_Raise(__pyx_t_5, 0, 0, 0);
03558     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
03559     __PYX_ERR(0, 114, __pyx_L1_error)
03560 }
03561 __pyx_L3:;
03562
03563 /* "PyClical.pyx":74
03564 *     return index_set(self)
03565 *
03566 *
03567 *     def __cinit__(self, other = 0):
03568 *         # ««««««««

```



```

03569 *          Construct an object of type index_set.
03570 */
03571
03572 /* function exit code */
03573 __pyx_r = 0;
03574 goto __pyx_L0;
03575 __pyx_L1_error:;
03576 __Pyx_XDECREF(__pyx_t_4);
03577 __Pyx_XDECREF(__pyx_t_5);
03578 __Pyx_XDECREF(__pyx_t_13);
03579 __Pyx_XDECREF(__pyx_t_14);
03580 __Pyx_XDECREF(__pyx_t_15);
03581 __Pyx_XDECREF(__pyx_t_16);
03582 __Pyx_AddTraceback("PyClical.index_set.__cinit__", __pyx_clineno, __pyx_lineno, __pyx_filename);
03583 __pyx_r = -1;
03584 __pyx_L0:;
03585 __Pyx_XDECREF(__pyx_v_error_msg_prefix);
03586 __Pyx_XDECREF(__pyx_v_idx);
03587 __Pyx_XDECREF(__pyx_v_bother);
03588 __Pyx_RefNannyFinishContext();
03589 return __pyx_r;
03590 }
03591
03592 /* "PyClical.pyx":116
03593 *          raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03594 *
03595 *      def __dealloc__(self):
03596 *          """
03597 *          Clean up by deallocating the instance of C++ class IndexSet.
03598 */
03599
03600 /* Python wrapper */
03601 static void __pyx_pw_8PyClical_9index_set_5__dealloc__(PyObject *__pyx_v_self); /*proto*/
03602 static void __pyx_pw_8PyClical_9index_set_5__dealloc__(PyObject *__pyx_v_self) {
03603     __Pyx_RefNannyDeclarations
03604     __Pyx_RefNannySetupContext("__dealloc__ (wrapper)", 0);
03605     __pyx_pf_8PyClical_9index_set_4__dealloc__(((struct __pyx_obj_8PyClical_index_set *)__pyx_v_self));
03606
03607     /* function exit code */
03608     __Pyx_RefNannyFinishContext();
03609 }
03610
03611 static void __pyx_pf_8PyClical_9index_set_4__dealloc__(struct __pyx_obj_8PyClical_index_set
03612 *__pyx_v_self) {
03613     __Pyx_RefNannyDeclarations
03614     __Pyx_RefNannySetupContext("__dealloc__", 0);
03615
03616     /* "PyClical.pyx":120
03617 *          Clean up by deallocating the instance of C++ class IndexSet.
03618 *          """
03619 *          del self.instance
03620 *          # ««««««««
03621 *      def __richcmp__(lhs, rhs, int op):
03622 */
03623     delete __pyx_v_self->instance;
03624
03625     /* "PyClical.pyx":116
03626 *          raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
03627 *
03628 *      def __dealloc__(self):
03629 *          """
03630 *          Clean up by deallocating the instance of C++ class IndexSet.
03631 */
03632
03633     /* function exit code */
03634     __Pyx_RefNannyFinishContext();
03635 }
03636
03637 /* "PyClical.pyx":122
03638 *          del self.instance
03639 *
03640 *      def __richcmp__(lhs, rhs, int op):
03641 *          """
03642 *          Compare two objects of class index_set.
03643 */
03644
03645 /* Python wrapper */
03646 static PyObject *__pyx_pw_8PyClical_9index_set_7__richcmp__(PyObject *__pyx_v_lhs, PyObject
03647 *__pyx_v_rhs, int __pyx_v_op); /*proto*/
03648 static PyObject *__pyx_pw_8PyClical_9index_set_7__richcmp__(PyObject *__pyx_v_lhs, PyObject
03649 *__pyx_v_rhs, int __pyx_v_op) {
03650     PyObject *__pyx_r = 0;
03651     __Pyx_RefNannyDeclarations
03652     __Pyx_RefNannySetupContext("__richcmp__ (wrapper)", 0);
03653     __pyx_r = __pyx_pf_8PyClical_9index_set_6__richcmp__(((struct __pyx_obj_8PyClical_index_set
03654 *)__pyx_v_lhs), ((PyObject *)__pyx_v_rhs), ((int)__pyx_v_op));
03655 }

```

```

03652  /* function exit code */
03653  __Pyx_RefNannyFinishContext();
03654  return __pyx_r;
03655 }
03656
03657 static PyObject *__pyx_pf_8PyClical_9index_set_6__richcmp__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op) {
03658     PyObject *__pyx_v_eq = NULL;
03659     PyObject *__pyx_v_lt = NULL;
03660     PyObject *__pyx_r = NULL;
03661     __Pyx_RefNannyDeclarations
03662     int __pyx_t_1;
03663     int __pyx_t_2;
03664     int __pyx_t_3;
03665     PyObject *__pyx_t_4 = NULL;
03666     int __pyx_lineno = 0;
03667     const char *__pyx_filename = NULL;
03668     int __pyx_clineno = 0;
03669     __Pyx_RefNannySetupContext("__richcmp__", 0);
03670
03671     /* "PyClical.pyx":143
03672     *         False
03673     *         """
03674     *         if (lhs is None) or (rhs is None):                # ««««««««
03675     *             eq = bool(lhs is rhs)
03676     *             if op == 2: # ==
03677     */
03678     __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
03679     __pyx_t_3 = (__pyx_t_2 != 0);
03680     if (!__pyx_t_3) {
03681     } else {
03682         __pyx_t_1 = __pyx_t_3;
03683         goto __pyx_L4_bool_binop_done;
03684     }
03685     __pyx_t_3 = (__pyx_v_rhs == Py_None);
03686     __pyx_t_2 = (__pyx_t_3 != 0);
03687     __pyx_t_1 = __pyx_t_2;
03688     __pyx_L4_bool_binop_done;
03689     if (__pyx_t_1) {
03690
03691         /* "PyClical.pyx":144
03692         *         """
03693         *         if (lhs is None) or (rhs is None):
03694         *             eq = bool(lhs is rhs)                # ««««««««
03695         *             if op == 2: # ==
03696         *                 return eq
03697         */
03698         __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
03699         __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 144,
__pyx_L1_error)
03700         __Pyx_GOTREF(__pyx_t_4);
03701         __pyx_v_eq = __pyx_t_4;
03702         __pyx_t_4 = 0;
03703
03704         /* "PyClical.pyx":145
03705         *         if (lhs is None) or (rhs is None):
03706         *             eq = bool(lhs is rhs)
03707         *             if op == 2: # ==                # ««««««««
03708         *                 return eq
03709         *             elif op == 3: # !=
03710         */
03711         switch (__pyx_v_op) {
03712             case 2:
03713
03714                 /* "PyClical.pyx":146
03715                 *                 eq = bool(lhs is rhs)
03716                 *                 if op == 2: # ==
03717                 *                     return eq                # ««««««««
03718                 *                 elif op == 3: # !=
03719                 *                     return not eq
03720                 */
03721                 __Pyx_XDECREF(__pyx_r);
03722                 __Pyx_INCREF(__pyx_v_eq);
03723                 __pyx_r = __pyx_v_eq;
03724                 goto __pyx_L0;
03725
03726                 /* "PyClical.pyx":145
03727                 *                 if (lhs is None) or (rhs is None):
03728                 *                     eq = bool(lhs is rhs)
03729                 *                     if op == 2: # ==                # ««««««««
03730                 *                         return eq
03731                 *                     elif op == 3: # !=
03732                 */
03733                 break;
03734             case 3:
03735
03736                 /* "PyClical.pyx":148

```

```

03737 *          return eq
03738 *          elif op == 3: # !=
03739 *          return not eq          # ««««««««
03740 *          else:
03741 *          if op == 0: # <
03742 */
03743     __Pyx_XDECREF(__pyx_r);
03744     __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 148,
__pyx_L1_error)
03745     __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 148,
__pyx_L1_error)
03746     __Pyx_GOTREF(__pyx_t_4);
03747     __pyx_r = __pyx_t_4;
03748     __pyx_t_4 = 0;
03749     goto __pyx_L0;
03750
03751     /* "PyClical.pyx":147
03752 *          if op == 2: # ==
03753 *          return eq
03754 *          elif op == 3: # !=          # ««««««««
03755 *          return not eq
03756 *          else:
03757 */
03758     break;
03759     default:
03760
03761     /* "PyClical.pyx":150
03762 *          return not eq
03763 *          else:
03764 *          if op == 0: # <          # ««««««««
03765 *          return False
03766 *          elif op == 1: # <=
03767 */
03768     switch (__pyx_v_op) {
03769     case 0:
03770
03771     /* "PyClical.pyx":151
03772 *          else:
03773 *          if op == 0: # <          # ««««««««
03774 *          return False
03775 *          elif op == 1: # <=
03776 *          return eq
03777 */
03778     __Pyx_XDECREF(__pyx_r);
03779     __Pyx_INCREF(Py_False);
03780     __pyx_r = Py_False;
03781     goto __pyx_L0;
03782
03783     /* "PyClical.pyx":150
03784 *          return not eq
03785 *          else:
03786 *          if op == 0: # <          # ««««««««
03787 *          return False
03788 *          elif op == 1: # <=
03789 */
03790     break;
03791     case 1:
03792
03793     /* "PyClical.pyx":153
03794 *          return False
03795 *          elif op == 1: # <=
03796 *          return eq          # ««««««««
03797 *          elif op == 4: # >
03798 *          return False
03799 */
03800     __Pyx_XDECREF(__pyx_r);
03801     __Pyx_INCREF(__pyx_v_eq);
03802     __pyx_r = __pyx_v_eq;
03803     goto __pyx_L0;
03804
03805     /* "PyClical.pyx":152
03806 *          if op == 0: # <
03807 *          return False
03808 *          elif op == 1: # <=          # ««««««««
03809 *          return eq
03810 *          elif op == 4: # >
03811 */
03812     break;
03813     case 4:
03814
03815     /* "PyClical.pyx":155
03816 *          return eq
03817 *          elif op == 4: # >
03818 *          return False          # ««««««««
03819 *          elif op == 5: # >=
03820 *          return eq
03821 */

```

```

03822     __Pyx_XDECREF(__pyx_r);
03823     __Pyx_INCREF(Py_False);
03824     __pyx_r = Py_False;
03825     goto __pyx_L0;
03826
03827     /* "PyClical.pyx":154
03828     *         elif op == 1: # <=
03829     *             return eq
03830     *         elif op == 4: # >           # ««««««««
03831     *             return False
03832     *         elif op == 5: # >=
03833     */
03834     break;
03835     case 5:
03836
03837     /* "PyClical.pyx":157
03838     *             return False
03839     *         elif op == 5: # >=
03840     *             return eq           # ««««««««
03841     *         else:
03842     *             return NotImplemented
03843     */
03844     __Pyx_XDECREF(__pyx_r);
03845     __Pyx_INCREF(__pyx_v_eq);
03846     __pyx_r = __pyx_v_eq;
03847     goto __pyx_L0;
03848
03849     /* "PyClical.pyx":156
03850     *         elif op == 4: # >
03851     *             return False
03852     *         elif op == 5: # >=       # ««««««««
03853     *             return eq
03854     *         else:
03855     */
03856     break;
03857     default:
03858
03859     /* "PyClical.pyx":159
03860     *             return eq
03861     *         else:
03862     *             return NotImplemented           # ««««««««
03863     *         else:
03864     *             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03865     */
03866     __Pyx_XDECREF(__pyx_r);
03867     __Pyx_INCREF(__pyx_builtin_NotImplemented);
03868     __pyx_r = __pyx_builtin_NotImplemented;
03869     goto __pyx_L0;
03870     break;
03871 }
03872 break;
03873 }
03874
03875 /* "PyClical.pyx":143
03876 *     False
03877 *     """
03878 *     if (lhs is None) or (rhs is None):           # ««««««««
03879 *         eq = bool(lhs is rhs)
03880 *         if op == 2: # ==
03881     */
03882 }
03883
03884 /* "PyClical.pyx":161
03885 *             return NotImplemented
03886 *         else:
03887 *             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )           # ««««««««
03888 *             if op == 2: # ==
03889 *                 return eq
03890     */
03891 /*else*/ {
03892     __pyx_t_1 = (__pyx_f_8PyClical_toIndexSet(((PyObject *)__pyx_v_lhs)) ==
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs));
03893     __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 161,
__pyx_L1_error)
03894     __Pyx_GOTREF(__pyx_t_4);
03895     __pyx_v_eq = __pyx_t_4;
03896     __pyx_t_4 = 0;
03897
03898     /* "PyClical.pyx":162
03899     *         else:
03900     *             eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03901     *             if op == 2: # ==           # ««««««««
03902     *                 return eq
03903     *             elif op == 3: # !=
03904     */
03905     switch (__pyx_v_op) {
03906     case 2:

```

```

03907
03908      /* "PyClical.pyx":163
03909      *          eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03910      *          if op == 2: # ==
03911      *              return eq          # ««««««««
03912      *          elif op == 3: # !=
03913      *              return not eq
03914      */
03915      __Pyx_XDECREF(__pyx_r);
03916      __Pyx_INCREF(__pyx_v_eq);
03917      __pyx_r = __pyx_v_eq;
03918      goto __pyx_L0;
03919
03920      /* "PyClical.pyx":162
03921      *          else:
03922      *              eq = bool( toIndexSet(lhs) == toIndexSet(rhs) )
03923      *              if op == 2: # ==
03924      *                  return eq          # ««««««««
03925      *              elif op == 3: # !=
03926      */
03927      break;
03928      case 3:
03929
03930      /* "PyClical.pyx":165
03931      *          return eq
03932      *          elif op == 3: # !=
03933      *              return not eq          # ««««««««
03934      *          else:
03935      *              lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03936      */
03937      __Pyx_XDECREF(__pyx_r);
03938      __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 165,
__pyx_L1_error)
03939      __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 165,
__pyx_L1_error)
03940      __Pyx_GOTREF(__pyx_t_4);
03941      __pyx_r = __pyx_t_4;
03942      __pyx_t_4 = 0;
03943      goto __pyx_L0;
03944
03945      /* "PyClical.pyx":164
03946      *          if op == 2: # ==
03947      *              return eq
03948      *          elif op == 3: # !=
03949      *              return not eq          # ««««««««
03950      *          else:
03951      */
03952      break;
03953      default:
03954
03955      /* "PyClical.pyx":167
03956      *          return not eq
03957      *          else:
03958      *              lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )          # ««««««««
03959      *              if op == 0: # <
03960      *                  return lt
03961      */
03962      __pyx_t_1 = (__pyx_f_8PyClical_toIndexSet(((PyObject *)__pyx_v_lhs)) <
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs));
03963      __pyx_t_4 = __Pyx_PyBool_FromLong(!(!__pyx_t_1)); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 167,
__pyx_L1_error)
03964      __Pyx_GOTREF(__pyx_t_4);
03965      __pyx_v_lt = __pyx_t_4;
03966      __pyx_t_4 = 0;
03967
03968      /* "PyClical.pyx":168
03969      *          else:
03970      *              lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03971      *              if op == 0: # <
03972      *                  return lt          # ««««««««
03973      *              elif op == 1: # <=
03974      */
03975      switch (__pyx_v_op) {
03976      case 0:
03977
03978      /* "PyClical.pyx":169
03979      *          lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03980      *          if op == 0: # <
03981      *              return lt          # ««««««««
03982      *          elif op == 1: # <=
03983      *              return lt or eq
03984      */
03985      __Pyx_XDECREF(__pyx_r);
03986      __Pyx_INCREF(__pyx_v_lt);
03987      __pyx_r = __pyx_v_lt;
03988      goto __pyx_L0;
03989

```

```

03990      /* "PyClical.pyx":168
03991      *          else:
03992      *              lt = bool( toIndexSet(lhs) < toIndexSet(rhs) )
03993      *              if op == 0: # <          # ««««««««
03994      *                  return lt
03995      *              elif op == 1: # <=
03996      */
03997      break;
03998      case 1:
03999
04000      /* "PyClical.pyx":171
04001      *              return lt
04002      *              elif op == 1: # <=
04003      *                  return lt or eq          # ««««««««
04004      *              elif op == 4: # >
04005      *                  return not (lt or eq)
04006      */
04007      __Pyx_XDECREF(__pyx_r);
04008      __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 171,
__pyx_L1_error)
04009      if (!__pyx_t_1) {
04010      } else {
04011      __Pyx_INCREF(__pyx_v_lt);
04012      __pyx_t_4 = __pyx_v_lt;
04013      goto __pyx_L6_bool_binop_done;
04014      }
04015      __Pyx_INCREF(__pyx_v_eq);
04016      __pyx_t_4 = __pyx_v_eq;
04017      __pyx_L6_bool_binop_done;
04018      __pyx_r = __pyx_t_4;
04019      __pyx_t_4 = 0;
04020      goto __pyx_L0;
04021
04022      /* "PyClical.pyx":170
04023      *              if op == 0: # <
04024      *                  return lt
04025      *              elif op == 1: # <=          # ««««««««
04026      *                  return lt or eq
04027      *              elif op == 4: # >
04028      */
04029      break;
04030      case 4:
04031
04032      /* "PyClical.pyx":173
04033      *              return lt or eq
04034      *              elif op == 4: # >
04035      *                  return not (lt or eq)          # ««««««««
04036      *              elif op == 5: # >=
04037      *                  return not lt
04038      */
04039      __Pyx_XDECREF(__pyx_r);
04040      __pyx_t_2 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_2 < 0)) __PYX_ERR(0, 173,
__pyx_L1_error)
04041      if (!__pyx_t_2) {
04042      } else {
04043      __pyx_t_1 = __pyx_t_2;
04044      goto __pyx_L8_bool_binop_done;
04045      }
04046      __pyx_t_2 = __Pyx_PyObject_IsTrue(__pyx_v_eq); if (unlikely(__pyx_t_2 < 0)) __PYX_ERR(0, 173,
__pyx_L1_error)
04047      __pyx_t_1 = __pyx_t_2;
04048      __pyx_L8_bool_binop_done;
04049      __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 173,
__pyx_L1_error)
04050      __Pyx_GOTREF(__pyx_t_4);
04051      __pyx_r = __pyx_t_4;
04052      __pyx_t_4 = 0;
04053      goto __pyx_L0;
04054
04055      /* "PyClical.pyx":172
04056      *              elif op == 1: # <=
04057      *                  return lt or eq
04058      *              elif op == 4: # >          # ««««««««
04059      *                  return not (lt or eq)
04060      *              elif op == 5: # >=
04061      */
04062      break;
04063      case 5:
04064
04065      /* "PyClical.pyx":175
04066      *              return not (lt or eq)
04067      *              elif op == 5: # >=
04068      *                  return not lt          # ««««««««
04069      *              else:
04070      *                  return NotImplemented
04071      */
04072      __Pyx_XDECREF(__pyx_r);

```

```

04073     __pyx_t_1 = __Pyx_PyObject_IsTrue(__pyx_v_lt); if (unlikely(__pyx_t_1 < 0)) __PYX_ERR(0, 175,
__pyx_L1_error)
04074     __pyx_t_4 = __Pyx_PyBool_FromLong(!__pyx_t_1); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 175,
__pyx_L1_error)
04075     __Pyx_GOTREF(__pyx_t_4);
04076     __pyx_r = __pyx_t_4;
04077     __pyx_t_4 = 0;
04078     goto __pyx_L0;
04079
04080     /* "PyClical.pyx":174
04081     *         elif op == 4: # >
04082     *             return not (lt or eq)
04083     *         elif op == 5: # >=          # ««««««««
04084     *             return not lt
04085     *         else:
04086     */
04087     break;
04088     default:
04089
04090     /* "PyClical.pyx":177
04091     *             return not lt
04092     *         else:
04093     *             return NotImplemented          # ««««««««
04094     *
04095     *     def __setitem__(self, idx, val):
04096     */
04097     __Pyx_XDECREF(__pyx_r);
04098     __Pyx_INCREF(__pyx_builtin_NotImplemented);
04099     __pyx_r = __pyx_builtin_NotImplemented;
04100     goto __pyx_L0;
04101     break;
04102     }
04103     break;
04104     }
04105     }
04106
04107     /* "PyClical.pyx":122
04108     *         del self.instance
04109     *
04110     *     def __richcmp__(lhs, rhs, int op):          # ««««««««
04111     *         """
04112     *         Compare two objects of class index_set.
04113     */
04114
04115     /* function exit code */
04116     __pyx_L1_error:;
04117     __Pyx_XDECREF(__pyx_t_4);
04118     __Pyx_AddTraceback("PyClical.index_set.__richcmp__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04119     __pyx_r = NULL;
04120     __pyx_L0:;
04121     __Pyx_XDECREF(__pyx_v_eq);
04122     __Pyx_XDECREF(__pyx_v_lt);
04123     __Pyx_XGIVEREF(__pyx_r);
04124     __Pyx_RefNannyFinishContext();
04125     return __pyx_r;
04126 }
04127
04128 /* "PyClical.pyx":179
04129 *             return NotImplemented
04130 *
04131 *     def __setitem__(self, idx, val):          # ««««««««
04132 *         """
04133 *         Set the value of an index_set object at index idx to value val.
04134     */
04135
04136     /* Python wrapper */
04137     static int __pyx_pw_8PyClical_9index_set_9__setitem__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx,
PyObject *__pyx_v_val); /*proto*/
04138     static char __pyx_doc_8PyClical_9index_set_8__setitem__[] = "\n        Set the value of an index_set
object at index idx to value val.\n\n        >> s=index_set({1}); s[2] = True; print(s)\n
{1,2}\n        >> s=index_set({1,2}); s[1] = False; print(s)\n        {2}\n        ";
04139     #if CYTHON_COMPILING_IN_CPYTHON
04140     struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_8__setitem__;
04141     #endif
04142     static int __pyx_pw_8PyClical_9index_set_9__setitem__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx,
PyObject *__pyx_v_val) {
04143         int __pyx_r;
04144         __Pyx_RefNannyDeclarations
04145         __Pyx_RefNannySetupContext("__setitem__ (wrapper)", 0);
04146         __pyx_r = __pyx_pf_8PyClical_9index_set_8__setitem__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_idx), ((PyObject *)__pyx_v_val));
04147
04148     /* function exit code */
04149     __Pyx_RefNannyFinishContext();
04150     return __pyx_r;
04151 }
04152

```

```

04153 static int __pyx_pf_8PyClical_9index_set_8__setitem__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx, PyObject *__pyx_v_val) {
04154     int __pyx_r;
04155     __Pyx_RefNannyDeclarations
04156     int __pyx_t_1;
04157     int __pyx_t_2;
04158     int __pyx_lineno = 0;
04159     const char *__pyx_filename = NULL;
04160     int __pyx_clineno = 0;
04161     __Pyx_RefNannySetupContext("__setitem__", 0);
04162
04163     /* "PyClical.pyx":188
04164     *         {2}
04165     *         """
04166     *         self.instance.set(idx, val)          # ««««««««
04167     *         return
04168     *
04169     */
04170     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 188, __pyx_L1_error)
04171     __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_val); if (unlikely((__pyx_t_2 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 188, __pyx_L1_error)
04172     try {
04173         __pyx_v_self->instance->set(__pyx_t_1, __pyx_t_2);
04174     } catch (...) {
04175         __Pyx_CppExn2PyErr();
04176         __PYX_ERR(0, 188, __pyx_L1_error)
04177     }
04178
04179     /* "PyClical.pyx":189
04180     *         """
04181     *         self.instance.set(idx, val)
04182     *         return          # ««««««««
04183     *
04184     *         def __getitem__(self, idx):
04185     */
04186     __pyx_r = 0;
04187     goto __pyx_L0;
04188
04189     /* "PyClical.pyx":179
04190     *         return NotImplemented
04191     *
04192     *         def __setitem__(self, idx, val):          # ««««««««
04193     *         """
04194     *         Set the value of an index_set object at index idx to value val.
04195     */
04196
04197     /* function exit code */
04198     __pyx_L1_error:;
04199     __Pyx_AddTraceback("PyClical.index_set.__setitem__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04200     __pyx_r = -1;
04201     __pyx_L0:;
04202     __Pyx_RefNannyFinishContext();
04203     return __pyx_r;
04204 }
04205
04206 /* "PyClical.pyx":191
04207 *         return
04208 *
04209 *         def __getitem__(self, idx):          # ««««««««
04210 *         """
04211 *         Get the value of an index_set object at an index.
04212     */
04213
04214     /* Python wrapper */
04215 static PyObject *__pyx_pw_8PyClical_9index_set_11__getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_idx); /*proto*/
04216 static char __pyx_doc_8PyClical_9index_set_10__getitem__[] = "\n        Get the value of an index_set
object at an index.\n\n        >> index_set({1})[1]\n                True\n        >> index_set({1})[2]\n                False\n        >> index_set({2})[-1]\n                False\n        >> index_set({2})[1]\n                False\n        >> index_set({2})[2]\n                True\n        >> index_set({2})[33]\n                False\n        ";
04217 #if CYTHON_COMPILING_IN_CPYTHON
04218 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_10__getitem__;
04219 #endif
04220 static PyObject *__pyx_pw_8PyClical_9index_set_11__getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_idx) {
04221     PyObject *__pyx_r = 0;
04222     __Pyx_RefNannyDeclarations
04223     __Pyx_RefNannySetupContext("__getitem__ (wrapper)", 0);
04224     __pyx_r = __pyx_pf_8PyClical_9index_set_10__getitem__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_idx));
04225
04226     /* function exit code */
04227     __Pyx_RefNannyFinishContext();
04228     return __pyx_r;
04229 }
04230

```



```

04231 static PyObject *__pyx_pf_8PyClical_9index_set_10__getitem__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx) {
04232     PyObject *__pyx_r = NULL;
04233     __Pyx_RefNannyDeclarations
04234     int __pyx_t_1;
04235     PyObject *__pyx_t_2 = NULL;
04236     int __pyx_lineno = 0;
04237     const char *__pyx_filename = NULL;
04238     int __pyx_clineno = 0;
04239     __Pyx_RefNannySetupContext("__getitem__", 0);
04240
04241     /* "PyClical.pyx":208
04242     *         False
04243     *         """
04244     *         return self.instance.getitem(idx) # ««««««««
04245     *
04246     *     def __contains__(self, idx):
04247     */
04248     __Pyx_XDECREF(__pyx_r);
04249     __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 208, __pyx_L1_error)
04250     __pyx_t_2 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->operator[](__pyx_t_1)); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 208, __pyx_L1_error)
04251     __Pyx_GOTREF(__pyx_t_2);
04252     __pyx_r = __pyx_t_2;
04253     __pyx_t_2 = 0;
04254     goto __pyx_L0;
04255
04256     /* "PyClical.pyx":191
04257     *         return
04258     *
04259     *     def __getitem__(self, idx): # ««««««««
04260     *         """
04261     *         Get the value of an index_set object at an index.
04262     */
04263
04264     /* function exit code */
04265     __pyx_L1_error:;
04266     __Pyx_XDECREF(__pyx_t_2);
04267     __Pyx_AddTraceback("PyClical.index_set.__getitem__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04268     __pyx_r = NULL;
04269     __pyx_L0:;
04270     __Pyx_XGIVEREF(__pyx_r);
04271     __Pyx_RefNannyFinishContext();
04272     return __pyx_r;
04273 }
04274
04275 /* "PyClical.pyx":210
04276 *         return self.instance.getitem(idx)
04277 *
04278 *     def __contains__(self, idx): # ««««««««
04279 *         """
04280 *         Check that an index_set object contains the index idx: idx in self.
04281     */
04282
04283 /* Python wrapper */
04284 static int __pyx_pw_8PyClical_9index_set_13__contains__(PyObject *__pyx_v_self, PyObject
*__pyx_v_idx); /*proto*/
04285 static char __pyx_doc_8PyClical_9index_set_12__contains__[] = "\n        Check that an index_set
object contains the index idx: idx in self.\n\n        >> 1 in index_set({1})\n        True\n
>> 2 in index_set({1})\n        False\n        >> -1 in index_set({2})\n        False\n        >> 1 in
index_set({2})\n        False\n        >> 2 in index_set({2})\n        True\n        >> 33 in
index_set({2})\n        False\n        ";
04286 #if CYTHON_COMPILING_IN_CPYTHON
04287 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_12__contains__;
04288 #endif
04289 static int __pyx_pw_8PyClical_9index_set_13__contains__(PyObject *__pyx_v_self, PyObject *__pyx_v_idx)
{
04290     int __pyx_r;
04291     __Pyx_RefNannyDeclarations
04292     __Pyx_RefNannySetupContext("__contains__ (wrapper)", 0);
04293     __pyx_r = __pyx_pf_8PyClical_9index_set_12__contains__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_idx));
04294
04295     /* function exit code */
04296     __Pyx_RefNannyFinishContext();
04297     return __pyx_r;
04298 }
04299
04300 static int __pyx_pf_8PyClical_9index_set_12__contains__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_idx) {
04301     int __pyx_r;
04302     __Pyx_RefNannyDeclarations
04303     int __pyx_t_1;
04304     int __pyx_lineno = 0;
04305     const char *__pyx_filename = NULL;
04306     int __pyx_clineno = 0;

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04307  __Pyx_RefNannySetupContext("__contains__", 0);
04308
04309  /* "PyClicl.pyx":227
04310  *      False
04311  *      """
04312  *      return self.instance.getitem(idx)          # ««««««««
04313  *
04314  *      def __iter__(self):
04315  */
04316  __pyx_t_1 = __Pyx_PyInt_As_int(__pyx_v_idx); if (unlikely((__pyx_t_1 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 227, __pyx_L1_error)
04317  __pyx_r = __pyx_v_self->instance->operator[](__pyx_t_1);
04318  goto __pyx_L0;
04319
04320  /* "PyClicl.pyx":210
04321  *      return self.instance.getitem(idx)
04322  *
04323  *      def __contains__(self, idx):          # ««««««««
04324  *      """
04325  *      Check that an index_set object contains the index idx: idx in self.
04326  */
04327
04328  /* function exit code */
04329  __pyx_L1_error:;
04330  __Pyx_AddTraceback("PyClicl.index_set.__contains__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04331  __pyx_r = -1;
04332  __pyx_L0:;
04333  __Pyx_RefNannyFinishContext();
04334  return __pyx_r;
04335 }
04336 static PyObject * __pyx_gb_8PyClicl_9index_set_16generator(__pyx_CoroutineObject * __pyx_generator,
CYTHON_UNUSED PyThreadState * __pyx_tstate, PyObject * __pyx_sent_value); /* proto */
04337
04338 /* "PyClicl.pyx":229
04339 *      return self.instance.getitem(idx)
04340 *
04341 *      def __iter__(self):          # ««««««««
04342 *      """
04343 *      Iterate over the indices of an index_set.
04344 */
04345
04346 /* Python wrapper */
04347 static PyObject * __pyx_pw_8PyClicl_9index_set_15_iter__(PyObject * __pyx_v_self); /*proto*/
04348 static char __pyx_doc_8PyClicl_9index_set_14_iter__[] = "\n          Iterate over the indices of an
index_set.\n\n          >> for i in index_set({-3,4,7}):print(i, end=\",\\n\")\n          -3,4,7,\n          ";
04349 #if CYTHON_COMPILING_IN_CPYTHON
04350 struct wrapperbase __pyx_wrapperbase_8PyClicl_9index_set_14_iter__;
04351 #endif
04352 static PyObject * __pyx_pw_8PyClicl_9index_set_15_iter__(PyObject * __pyx_v_self) {
04353     PyObject * __pyx_r = 0;
04354     __Pyx_RefNannyDeclarations
04355     __Pyx_RefNannySetupContext("__iter__ (wrapper)", 0);
04356     __pyx_r = __pyx_pf_8PyClicl_9index_set_14_iter__(((struct __pyx_obj_8PyClicl_index_set
*) __pyx_v_self));
04357
04358     /* function exit code */
04359     __Pyx_RefNannyFinishContext();
04360     return __pyx_r;
04361 }
04362
04363 static PyObject * __pyx_pf_8PyClicl_9index_set_14_iter__(struct __pyx_obj_8PyClicl_index_set
* __pyx_v_self) {
04364     struct __pyx_obj_8PyClicl__pyx_scope_struct__iter__ * __pyx_cur_scope;
04365     PyObject * __pyx_r = NULL;
04366     __Pyx_RefNannyDeclarations
04367     int __pyx_lineno = 0;
04368     const char * __pyx_filename = NULL;
04369     int __pyx_clineno = 0;
04370     __Pyx_RefNannySetupContext("__iter__", 0);
04371     __pyx_cur_scope = (struct __pyx_obj_8PyClicl__pyx_scope_struct__iter__
*) __pyx_tp_new_8PyClicl__pyx_scope_struct__iter__((__pyx_ptype_8PyClicl__pyx_scope_struct__iter__,
__pyx_empty_tuple, NULL);
04372     if (unlikely(!__pyx_cur_scope)) {
04373         __pyx_cur_scope = ((struct __pyx_obj_8PyClicl__pyx_scope_struct__iter__ *)Py_None);
04374         __Pyx_INCREF(Py_None);
04375         __PYX_ERR(0, 229, __pyx_L1_error)
04376     } else {
04377         __Pyx_GOTREF(__pyx_cur_scope);
04378     }
04379     __pyx_cur_scope->__pyx_v_self = __pyx_v_self;
04380     __Pyx_INCREF((PyObject *) __pyx_cur_scope->__pyx_v_self);
04381     __Pyx_GIVEREF((PyObject *) __pyx_cur_scope->__pyx_v_self);
04382     {
04383         __pyx_CoroutineObject *gen = __Pyx_Generator_New((__pyx_coroutine_body_t)
__pyx_gb_8PyClicl_9index_set_16generator, NULL, (PyObject *) __pyx_cur_scope, __pyx_n_s_iter,
__pyx_n_s_index_set__iter__, __pyx_n_s_PyClicl); if (unlikely(!gen)) __PYX_ERR(0, 229, __pyx_L1_error)
04384         __Pyx_DECREF(__pyx_cur_scope);

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```

04385     __Pyx_RefNannyFinishContext();
04386     return (PyObject *) gen;
04387 }
04388
04389 /* function exit code */
04390 __pyx_l1_error:;
04391 __Pyx_AddTraceback("PyClical.index_set.__iter__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04392 __pyx_r = NULL;
04393 __Pyx_DECREF(((PyObject *) __pyx_cur_scope));
04394 __Pyx_XGIVEREF(__pyx_r);
04395 __Pyx_RefNannyFinishContext();
04396 return __pyx_r;
04397 }
04398
04399 static PyObject * __pyx_gb_8PyClical_9index_set_16generator(__pyx_CoroutineObject * __pyx_generator,
CYTHON_UNUSED PyThreadState * __pyx_tstate, PyObject * __pyx_sent_value) /* generator body */
04400 {
04401     struct __pyx_obj_8PyClical__pyx_scope_struct__iter__ * __pyx_cur_scope = ((struct
__pyx_obj_8PyClical__pyx_scope_struct__iter__ *) __pyx_generator->closure);
04402     PyObject * __pyx_r = NULL;
04403     PyObject * __pyx_t_1 = NULL;
04404     PyObject * __pyx_t_2 = NULL;
04405     PyObject * __pyx_t_3 = NULL;
04406     PyObject * __pyx_t_4 = NULL;
04407     Py_ssize_t __pyx_t_5;
04408     PyObject * (*__pyx_t_6)(PyObject *);
04409     int __pyx_t_7;
04410     int __pyx_t_8;
04411     int __pyx_lineno = 0;
04412     const char * __pyx_filename = NULL;
04413     int __pyx_clineno = 0;
04414     __Pyx_RefNannyDeclarations
04415     __Pyx_RefNannySetupContext("__iter__", 0);
04416     switch (__pyx_generator->resume_label) {
04417         case 0: goto __pyx_L3_first_run;
04418         case 1: goto __pyx_L7_resume_from_yield;
04419         default: /* CPython raises the right error here */
04420             __Pyx_RefNannyFinishContext();
04421             return NULL;
04422     }
04423     __pyx_L3_first_run:;
04424     if (unlikely(!__pyx_sent_value)) __PYX_ERR(0, 229, __pyx_L1_error)
04425
04426     /* "PyClical.pyx":236
04427     *         -3,4,7,
04428     *         """
04429     *         for idx in range(self.min(), self.max()+1):
04430     *             if idx in self:
04431     *                 yield idx
04432     */
04433     __pyx_t_2 = __Pyx_PyObject_GetAttrStr(((PyObject *) __pyx_cur_scope->__pyx_v_self), __pyx_n_s_min);
04434     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04435     __Pyx_GOTREF(__pyx_t_2);
04436     __pyx_t_3 = NULL;
04437     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_2))) {
04438         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_2);
04439         if (likely(__pyx_t_3)) {
04440             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_2);
04441             __Pyx_INCREF(__pyx_t_3);
04442             __Pyx_INCREF(function);
04443             __Pyx_DECREF_SET(__pyx_t_2, function);
04444         }
04445     }
04446     __pyx_t_1 = (__pyx_t_3) ? __Pyx_PyObject_CallOneArg(__pyx_t_2, __pyx_t_3) :
__Pyx_PyObject_CallNoArg(__pyx_t_2);
04447     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
04448     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 236, __pyx_L1_error)
04449     __Pyx_GOTREF(__pyx_t_1);
04450     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04451     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(((PyObject *) __pyx_cur_scope->__pyx_v_self), __pyx_n_s_max);
04452     if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 236, __pyx_L1_error)
04453     __Pyx_GOTREF(__pyx_t_3);
04454     __pyx_t_4 = NULL;
04455     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
04456         __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
04457         if (likely(__pyx_t_4)) {
04458             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
04459             __Pyx_INCREF(__pyx_t_4);
04460             __Pyx_INCREF(function);
04461             __Pyx_DECREF_SET(__pyx_t_3, function);
04462         }
04463     }
04464     __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
04465     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
04466     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04467     __Pyx_GOTREF(__pyx_t_2);

```

```

04466     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
04467     __pyx_t_3 = __Pyx_PyInt_AddObjC(__pyx_t_2, __pyx_int_1, 1, 0, 0); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04468     __Pyx_GOTREF(__pyx_t_3);
04469     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04470     __pyx_t_2 = PyTuple_New(2); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 236, __pyx_L1_error)
04471     __Pyx_GOTREF(__pyx_t_2);
04472     __Pyx_GIVEREF(__pyx_t_1);
04473     PyTuple_SET_ITEM(__pyx_t_2, 0, __pyx_t_1);
04474     __Pyx_GIVEREF(__pyx_t_3);
04475     PyTuple_SET_ITEM(__pyx_t_2, 1, __pyx_t_3);
04476     __pyx_t_1 = 0;
04477     __pyx_t_3 = 0;
04478     __pyx_t_3 = __Pyx_PyObject_Call(__pyx_builtin_range, __pyx_t_2, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04479     __Pyx_GOTREF(__pyx_t_3);
04480     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04481     if (likely(PyList_CheckExact(__pyx_t_3)) || PyTuple_CheckExact(__pyx_t_3)) {
04482         __pyx_t_2 = __pyx_t_3; __Pyx_INCREF(__pyx_t_2); __pyx_t_5 = 0;
04483         __pyx_t_6 = NULL;
04484     } else {
04485         __pyx_t_5 = -1; __pyx_t_2 = PyObject_GetIter(__pyx_t_3); if (unlikely(!__pyx_t_2)) __PYX_ERR(0,
236, __pyx_L1_error)
04486         __Pyx_GOTREF(__pyx_t_2);
04487         __pyx_t_6 = Py_TYPE(__pyx_t_2)->tp_iternext; if (unlikely(!__pyx_t_6)) __PYX_ERR(0, 236,
__pyx_L1_error)
04488     }
04489     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
04490     for (;;) {
04491         if (likely(!__pyx_t_6)) {
04492             if (likely(PyList_CheckExact(__pyx_t_2))) {
04493                 if (__pyx_t_5 >= PyList_GET_SIZE(__pyx_t_2)) break;
04494                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
04495                 __pyx_t_3 = PyList_GET_ITEM(__pyx_t_2, __pyx_t_5); __Pyx_INCREF(__pyx_t_3); __pyx_t_5++; if
(unlikely(0 < 0)) __PYX_ERR(0, 236, __pyx_L1_error)
04496             #else
04497                 __pyx_t_3 = PySequence_ITEM(__pyx_t_2, __pyx_t_5); __pyx_t_5++; if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04498                 __Pyx_GOTREF(__pyx_t_3);
04499             #endif
04500         } else {
04501             if (__pyx_t_5 >= PyTuple_GET_SIZE(__pyx_t_2)) break;
04502             #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
04503             __pyx_t_3 = PyTuple_GET_ITEM(__pyx_t_2, __pyx_t_5); __Pyx_INCREF(__pyx_t_3); __pyx_t_5++; if
(unlikely(0 < 0)) __PYX_ERR(0, 236, __pyx_L1_error)
04504             #else
04505             __pyx_t_3 = PySequence_ITEM(__pyx_t_2, __pyx_t_5); __pyx_t_5++; if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 236, __pyx_L1_error)
04506             __Pyx_GOTREF(__pyx_t_3);
04507             #endif
04508         }
04509     } else {
04510         __pyx_t_3 = __pyx_t_6(__pyx_t_2);
04511         if (unlikely(!__pyx_t_3)) {
04512             PyObject* exc_type = PyErr_Occurred();
04513             if (exc_type) {
04514                 if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration))) PyErr_Clear();
04515                 else __PYX_ERR(0, 236, __pyx_L1_error)
04516             }
04517             break;
04518         }
04519         __Pyx_GOTREF(__pyx_t_3);
04520     }
04521     __Pyx_XGOTREF(__pyx_cur_scope->__pyx_v_idx);
04522     __Pyx_XDECREF_SET(__pyx_cur_scope->__pyx_v_idx, __pyx_t_3);
04523     __Pyx_GIVEREF(__pyx_t_3);
04524     __pyx_t_3 = 0;
04525
04526     /* "PyClical.pyx":237
04527     *
04528     *     for idx in range(self.min(), self.max()+1):
04529     *         if idx in self:
04530     *             # ««««««««
04531     *             yield idx
04532     */
04533     __pyx_t_7 = (__Pyx_PySequence_ContainsTF(__pyx_cur_scope->__pyx_v_idx, ((PyObject
*)__pyx_cur_scope->__pyx_v_self), Py_EQ)); if (unlikely(__pyx_t_7 < 0)) __PYX_ERR(0, 237,
__pyx_L1_error)
04534     __pyx_t_8 = (__pyx_t_7 != 0);
04535     if (__pyx_t_8) {
04536
04537         /* "PyClical.pyx":238
04538         *     for idx in range(self.min(), self.max()+1):
04539         *         if idx in self:
04540         *             yield idx
04541         *             # ««««««««
04542         *     def __invert__(self):

```

```

04543 */
04544     __Pyx_INCREF(__pyx_cur_scope->__pyx_v_idx);
04545     __pyx_r = __pyx_cur_scope->__pyx_v_idx;
04546     __Pyx_XGIVEREF(__pyx_t_2);
04547     __pyx_cur_scope->__pyx_t_0 = __pyx_t_2;
04548     __pyx_cur_scope->__pyx_t_1 = __pyx_t_5;
04549     __pyx_cur_scope->__pyx_t_2 = __pyx_t_6;
04550     __Pyx_XGIVEREF(__pyx_r);
04551     __Pyx_RefNannyFinishContext();
04552     __Pyx_Coroutine_ResetAndClearException(__pyx_generator);
04553     /* return from generator, yielding value */
04554     __pyx_generator->resume_label = 1;
04555     return __pyx_r;
04556     __pyx_L7_resume_from_yield:;
04557     __pyx_t_2 = __pyx_cur_scope->__pyx_t_0;
04558     __pyx_cur_scope->__pyx_t_0 = 0;
04559     __Pyx_XGOTREF(__pyx_t_2);
04560     __pyx_t_5 = __pyx_cur_scope->__pyx_t_1;
04561     __pyx_t_6 = __pyx_cur_scope->__pyx_t_2;
04562     if (unlikely(!__pyx_sent_value)) __PYX_ERR(0, 238, __pyx_L1_error)
04563
04564     /* "PyClical.pyx":237
04565     *
04566     *     for idx in range(self.min(), self.max()+1):
04567     *         if idx in self:             # ««««««««
04568     *             yield idx
04569     *
04570     */
04571 }
04572
04573 /* "PyClical.pyx":236
04574 *     -3,4,7,
04575 *     """
04576 *     for idx in range(self.min(), self.max()+1):             # ««««««««
04577 *         if idx in self:
04578 *             yield idx
04579 */
04580 }
04581 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
04582 CYTHON_MAYBE_UNUSED_VAR(__pyx_cur_scope);
04583
04584 /* "PyClical.pyx":229
04585 *     return self.instance.getitem(idx)
04586 *
04587 *     def __iter__(self):             # ««««««««
04588 *         """
04589 *         Iterate over the indices of an index_set.
04590 */
04591
04592 /* function exit code */
04593 PyErr_SetNone(PyExc_StopIteration);
04594 goto __pyx_L0;
04595 __pyx_L1_error:;
04596 __Pyx_XDECREF(__pyx_t_1);
04597 __Pyx_XDECREF(__pyx_t_2);
04598 __Pyx_XDECREF(__pyx_t_3);
04599 __Pyx_XDECREF(__pyx_t_4);
04600 __Pyx_AddTraceback("__iter__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04601 __pyx_L0:;
04602 __Pyx_XDECREF(__pyx_r); __pyx_r = 0;
04603 #if !CYTHON_USE_EXC_INFO_STACK
04604     __Pyx_Coroutine_ResetAndClearException(__pyx_generator);
04605 #endif
04606     __pyx_generator->resume_label = -1;
04607     __Pyx_Coroutine_clear((PyObject*)__pyx_generator);
04608     __Pyx_RefNannyFinishContext();
04609     return __pyx_r;
04610 }
04611
04612 /* "PyClical.pyx":240
04613 *         yield idx
04614 *
04615 *     def __invert__(self):             # ««««««««
04616 *         """
04617 *         Set complement: not.
04618 */
04619
04620 /* Python wrapper */
04621 static PyObject *__pyx_pw_8PyClical_9index_set_18__invert__(PyObject *__pyx_v_self); /*proto*/
04622 static char __pyx_doc_8PyClical_9index_set_17__invert__[] = "\n        Set complement: not.\n\n\n    print(~index_set({-16,-15,-14,-13,-12,-11,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16}))\n    {-32,-31,-30,-29,-28,-27,-26,-25,-24,-23,-22,-21,-20,-19,-18,-17,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32})\n    ";
04623 #if CYTHON_COMPILING_IN_CPYTHON
04624 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_17__invert__;
04625 #endif

```

```

04626 static PyObject *__pyx_pw_8PyClical_9index_set_18__invert__(PyObject *__pyx_v_self) {
04627     PyObject *__pyx_r = 0;
04628     __Pyx_RefNannyDeclarations
04629     __Pyx_RefNannySetupContext("__invert__ (wrapper)", 0);
04630     __pyx_r = __pyx_pf_8PyClical_9index_set_17__invert__((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
04631
04632     /* function exit code */
04633     __Pyx_RefNannyFinishContext();
04634     return __pyx_r;
04635 }
04636
04637 static PyObject *__pyx_pf_8PyClical_9index_set_17__invert__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
04638     PyObject *__pyx_r = NULL;
04639     __Pyx_RefNannyDeclarations
04640     PyObject *__pyx_t_1 = NULL;
04641     PyObject *__pyx_t_2 = NULL;
04642     int __pyx_lineno = 0;
04643     const char *__pyx_filename = NULL;
04644     int __pyx_clineno = 0;
04645     __Pyx_RefNannySetupContext("__invert__", 0);
04646
04647     /* "PyClical.pyx":247
04648     *
04649     *      """
04650     *          return index_set().wrap( self.instance.invert() )          # ««««««««
04651     *
04652     *      def __xor__(lhs, rhs):
04653     */
04654     __Pyx_XDECREF(__pyx_r);
04655     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 247, __pyx_L1_error)
04656     __Pyx_GOTREF(__pyx_t_1);
04657     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
__pyx_v_self->instance->operator~()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 247, __pyx_L1_error)
04658     __Pyx_GOTREF(__pyx_t_2);
04659     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04660     __pyx_r = __pyx_t_2;
04661     __pyx_t_2 = 0;
04662     goto __pyx_L0;
04663
04664     /* "PyClical.pyx":240
04665     *          yield idx
04666     *
04667     *      def __invert__(self):          # ««««««««
04668     *          """
04669     *          Set complement: not.
04670     */
04671
04672     /* function exit code */
04673     __pyx_L1_error:;
04674     __Pyx_XDECREF(__pyx_t_1);
04675     __Pyx_XDECREF(__pyx_t_2);
04676     __Pyx_AddTraceback("PyClical.index_set.__invert__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04677     __pyx_r = NULL;
04678     __pyx_L0:;
04679     __Pyx_XGIVEREF(__pyx_r);
04680     __Pyx_RefNannyFinishContext();
04681     return __pyx_r;
04682 }
04683
04684 /* "PyClical.pyx":249
04685     *          return index_set().wrap( self.instance.invert() )
04686     *
04687     *      def __xor__(lhs, rhs):          # ««««««««
04688     *          """
04689     *          Symmetric set difference: exclusive or.
04690     */
04691
04692 /* Python wrapper */
04693 static PyObject *__pyx_pw_8PyClical_9index_set_20__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
04694 static char __pyx_doc_8PyClical_9index_set_19__xor__[] = "\n          Symmetric set difference:
exclusive or.\n\n          >> print(index_set({1}) ^ index_set({2}))\n          {1,2}\n          >>
print(index_set({1,2}) ^ index_set({2}))\n          {1}\n          ";
04695 #if CYTHON_COMPILING_IN_CPYTHON
04696 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_19__xor__;
04697 #endif
04698 static PyObject *__pyx_pw_8PyClical_9index_set_20__xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
{
04699     PyObject *__pyx_r = 0;
04700     __Pyx_RefNannyDeclarations
04701     __Pyx_RefNannySetupContext("__xor__ (wrapper)", 0);
04702     __pyx_r = __pyx_pf_8PyClical_9index_set_19__xor__(((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));

```

```

04703
04704 /* function exit code */
04705 __Pyx_RefNannyFinishContext();
04706 return __pyx_r;
04707 }
04708
04709 static PyObject *__pyx_pf_8PyClical_9index_set_19__xor__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
04710 {
04711     PyObject *__pyx_r = NULL;
04712     __Pyx_RefNannyDeclarations
04713     PyObject *__pyx_t_1 = NULL;
04714     PyObject *__pyx_t_2 = NULL;
04715     int __pyx_lineno = 0;
04716     const char *__pyx_filename = NULL;
04717     int __pyx_clineno = 0;
04718     __Pyx_RefNannySetupContext("__xor__", 0);
04719
04720     /* "PyClical.pyx":258
04721     *         {1}
04722     *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )           # ««««««««
04723     *
04724     *         def __ixor__(self, rhs):
04725     */
04726     __Pyx_XDECREF(__pyx_r);
04727     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 258, __pyx_L1_error)
04728     __Pyx_GOTREF(__pyx_t_1);
04729     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) ^ __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 258, __pyx_L1_error)
04730     __Pyx_GOTREF(__pyx_t_2);
04731     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04732     __pyx_r = __pyx_t_2;
04733     __pyx_t_2 = 0;
04734     goto __pyx_L0;
04735
04736     /* "PyClical.pyx":249
04737     *         return index_set().wrap( self.instance.invert() )
04738     *
04739     *         def __xor__(lhs, rhs):           # ««««««««
04740     *         """
04741     *         Symmetric set difference: exclusive or.
04742     */
04743
04744     /* function exit code */
04745     __pyx_L1_error:;
04746     __Pyx_XDECREF(__pyx_t_1);
04747     __Pyx_XDECREF(__pyx_t_2);
04748     __Pyx_AddTraceback("PyClical.index_set.__xor__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04749     __pyx_r = NULL;
04750     __pyx_L0:;
04751     __Pyx_XGIVEREF(__pyx_r);
04752     __Pyx_RefNannyFinishContext();
04753     return __pyx_r;
04754 }
04755
04756 /* "PyClical.pyx":260
04757 *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
04758 *
04759 *         def __ixor__(self, rhs):           # ««««««««
04760 *         """
04761 *         Symmetric set difference: exclusive or.
04762     */
04763
04764 /* Python wrapper */
04765 static PyObject *__pyx_pw_8PyClical_9index_set_22__ixor__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
04766 static char __pyx_doc_8PyClical_9index_set_21__ixor__[] = "\n          Symmetric set difference:
exclusive or.\n\n          >> x = index_set({1}); x ^= index_set({2}); print(x)\n          {1,2}\n
>> x = index_set({1,2}); x ^= index_set({2}); print(x)\n          {1}\n          ";
04767 #if CYTHON_COMPILING_IN_CPYTHON
04768 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_21__ixor__;
04769 #endif
04770 static PyObject *__pyx_pw_8PyClical_9index_set_22__ixor__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
04771     PyObject *__pyx_r = 0;
04772     __Pyx_RefNannyDeclarations
04773     __Pyx_RefNannySetupContext("__ixor__ (wrapper)", 0);
04774     __pyx_r = __pyx_pf_8PyClical_9index_set_21__ixor__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
04775
04776     /* function exit code */
04777     __Pyx_RefNannyFinishContext();
04778     return __pyx_r;
04779 }
04780

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```

04781 static PyObject *__pyx_pf_8PyClical_9index_set_21_ixor__(struct __pyx_obj_8PyClical_index_set
    *__pyx_v_self, PyObject *__pyx_v_rhs) {
04782     PyObject *__pyx_r = NULL;
04783     __Pyx_RefNannyDeclarations
04784     PyObject *__pyx_t_1 = NULL;
04785     int __pyx_lineno = 0;
04786     const char *__pyx_filename = NULL;
04787     int __pyx_clineno = 0;
04788     __Pyx_RefNannySetupContext("__ixor__", 0);
04789
04790     /* "PyClical.pyx":269
04791     *         {1}
04792     *         """
04793     *         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )           # ««««««««
04794     *
04795     *     def __and__(lhs, rhs):
04796     */
04797     __Pyx_XDECREF(__pyx_r);
04798     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
    (__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) ^ __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 269, __pyx_L1_error)
04799     __Pyx_GOTREF(__pyx_t_1);
04800     __pyx_r = __pyx_t_1;
04801     __pyx_t_1 = 0;
04802     goto __pyx_L0;
04803
04804     /* "PyClical.pyx":260
04805     *         return index_set().wrap( toIndexSet(lhs) ^ toIndexSet(rhs) )
04806     *
04807     *     def __ixor__(self, rhs):           # ««««««««
04808     *         """
04809     *         Symmetric set difference: exclusive or.
04810     */
04811
04812     /* function exit code */
04813     __pyx_L1_error:;
04814     __Pyx_XDECREF(__pyx_t_1);
04815     __Pyx_AddTraceback("PyClical.index_set.__ixor__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04816     __pyx_r = NULL;
04817     __pyx_L0:;
04818     __Pyx_XGIVEREF(__pyx_r);
04819     __Pyx_RefNannyFinishContext();
04820     return __pyx_r;
04821 }
04822
04823 /* "PyClical.pyx":271
04824 *         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
04825 *
04826 *     def __and__(lhs, rhs):           # ««««««««
04827 *         """
04828 *         Set intersection: and.
04829     */
04830
04831 /* Python wrapper */
04832 static PyObject *__pyx_pw_8PyClical_9index_set_24_and__(PyObject *__pyx_v_lhs, PyObject
    *__pyx_v_rhs); /*proto*/
04833 static char __pyx_doc_8PyClical_9index_set_23_and__[] = "\n        Set intersection: and.\n\n
    >> print(index_set({1}) & index_set({2}))\n                {}\n                >> print(index_set({1,2}) &
    index_set({2}))\n                {2}\n                ";
04834 #if CYTHON_COMPILING_IN_CPYTHON
04835 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_23_and__;
04836 #endif
04837 static PyObject *__pyx_pw_8PyClical_9index_set_24_and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04838     PyObject *__pyx_r = 0;
04839     __Pyx_RefNannyDeclarations
04840     __Pyx_RefNannySetupContext("__and__ (wrapper)", 0);
04841     __pyx_r = __pyx_pf_8PyClical_9index_set_23_and__(((PyObject *)__pyx_v_lhs), ((PyObject
    *)__pyx_v_rhs));
04842
04843     /* function exit code */
04844     __Pyx_RefNannyFinishContext();
04845     return __pyx_r;
04846 }
04847
04848 static PyObject *__pyx_pf_8PyClical_9index_set_23_and__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
    {
04849     PyObject *__pyx_r = NULL;
04850     __Pyx_RefNannyDeclarations
04851     PyObject *__pyx_t_1 = NULL;
04852     PyObject *__pyx_t_2 = NULL;
04853     int __pyx_lineno = 0;
04854     const char *__pyx_filename = NULL;
04855     int __pyx_clineno = 0;
04856     __Pyx_RefNannySetupContext("__and__", 0);
04857
04858     /* "PyClical.pyx":280

```



```

04859 *         {2}
04860 *         """
04861 *         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )           # ««««««««
04862 *
04863 *         def __iand__(self, rhs):
04864 */
04865     __Pyx_XDECREF(__pyx_r);
04866     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 280, __pyx_L1_error)
04867     __Pyx_GOTREF(__pyx_t_1);
04868     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_t_1),
(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) & __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 280, __pyx_L1_error)
04869     __Pyx_GOTREF(__pyx_t_2);
04870     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
04871     __pyx_r = __pyx_t_2;
04872     __pyx_t_2 = 0;
04873     goto __pyx_L0;
04874
04875     /* "PyClical.pyx":271
04876     *         return self.wrap( self.unwrap() ^ toIndexSet(rhs) )
04877     *
04878     *         def __and__(lhs, rhs):           # ««««««««
04879     *         """
04880     *         Set intersection: and.
04881     */
04882
04883     /* function exit code */
04884     __pyx_L1_error:;
04885     __Pyx_XDECREF(__pyx_t_1);
04886     __Pyx_XDECREF(__pyx_t_2);
04887     __Pyx_AddTraceback("PyClical.index_set.__and__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04888     __pyx_r = NULL;
04889     __pyx_L0:;
04890     __Pyx_XGIVEREF(__pyx_r);
04891     __Pyx_RefNannyFinishContext();
04892     return __pyx_r;
04893 }
04894
04895 /* "PyClical.pyx":282
04896 *         return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
04897 *
04898 *         def __iand__(self, rhs):           # ««««««««
04899 *         """
04900 *         Set intersection: and.
04901     */
04902
04903     /* Python wrapper */
04904     static PyObject *__pyx_pw_8PyClical_9index_set_26__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
04905     static char __pyx_doc_8PyClical_9index_set_25__iand__[] = "\n        Set intersection: and.\n\n
>> x = index_set({1}); x &= index_set({2}); print(x)\n        {}\n        >> x = index_set({1,2}); x
&= index_set({2}); print(x)\n        {2}\n        ";
04906     #if CYTHON_COMPILING_IN_CPYTHON
04907     struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_25__iand__;
04908     #endif
04909     static PyObject *__pyx_pw_8PyClical_9index_set_26__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
04910     PyObject *__pyx_r = 0;
04911     __Pyx_RefNannyDeclarations
04912     __Pyx_RefNannySetupContext("__iand__ (wrapper)", 0);
04913     __pyx_r = __pyx_pf_8PyClical_9index_set_25__iand__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
04914
04915     /* function exit code */
04916     __Pyx_RefNannyFinishContext();
04917     return __pyx_r;
04918 }
04919
04920     static PyObject *__pyx_pf_8PyClical_9index_set_25__iand__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self, PyObject *__pyx_v_rhs) {
04921     PyObject *__pyx_r = NULL;
04922     __Pyx_RefNannyDeclarations
04923     PyObject *__pyx_t_1 = NULL;
04924     int __pyx_lineno = 0;
04925     const char *__pyx_filename = NULL;
04926     int __pyx_clineno = 0;
04927     __Pyx_RefNannySetupContext("__iand__", 0);
04928
04929     /* "PyClical.pyx":291
04930     *         {2}
04931     *         """
04932     *         return self.wrap( self.unwrap() & toIndexSet(rhs) )           # ««««««««
04933     *
04934     *         def __or__(lhs, rhs):
04935     */
04936     __Pyx_XDECREF(__pyx_r);

```

```

04937  __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
  (__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) & __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
  (unlikely(!__pyx_t_1)) __PYX_ERR(0, 291, __pyx_L1_error)
04938  __Pyx_GOTREF(__pyx_t_1);
04939  __pyx_r = __pyx_t_1;
04940  __pyx_t_1 = 0;
04941  goto __pyx_L0;
04942
04943  /* "PyClical.pyx":282
04944  *      return index_set().wrap( toIndexSet(lhs) & toIndexSet(rhs) )
04945  *
04946  *      def __iand__(self, rhs):          # ««««««««
04947  *          """
04948  *          Set intersection: and.
04949  */
04950
04951  /* function exit code */
04952  __pyx_L1_error:;
04953  __Pyx_XDECREF(__pyx_t_1);
04954  __Pyx_AddTraceback("PyClical.index_set.__iand__", __pyx_clineno, __pyx_lineno, __pyx_filename);
04955  __pyx_r = NULL;
04956  __pyx_L0:;
04957  __Pyx_XGIVEREF(__pyx_r);
04958  __Pyx_RefNannyFinishContext();
04959  return __pyx_r;
04960 }
04961
04962 /* "PyClical.pyx":293
04963 *      return self.wrap( self.unwrap() & toIndexSet(rhs) )
04964 *
04965 *      def __or__(lhs, rhs):          # ««««««««
04966 *          """
04967 *          Set union: or.
04968 */
04969
04970 /* Python wrapper */
04971 static PyObject *__pyx_pw_8PyClical_9index_set_28__or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs);
04972 /*proto*/
04973 static char __pyx_doc_8PyClical_9index_set_27__or__[] = "\n      Set union: or.\n\n      >>
  print(index_set({1}) | index_set({2}))\n          {1,2}\n      >> print(index_set({1,2}) |
  index_set({2}))\n          {1,2}\n      ";
04974 #if CYTHON_COMPILING_IN_CPYTHON
04975 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_27__or__;
04976 #endif
04977 static PyObject *__pyx_pw_8PyClical_9index_set_28__or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
  {
04978     PyObject *__pyx_r = 0;
04979     __Pyx_RefNannyDeclarations
04980     __Pyx_RefNannySetupContext("__or__ (wrapper)", 0);
04981     __pyx_r = __pyx_pf_8PyClical_9index_set_27__or__(((PyObject *) __pyx_v_lhs), ((PyObject
  *) __pyx_v_rhs));
04982
04983     /* function exit code */
04984     __Pyx_RefNannyFinishContext();
04985     return __pyx_r;
04986 }
04987 static PyObject *__pyx_pf_8PyClical_9index_set_27__or__(PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs)
  {
04988     PyObject *__pyx_r = NULL;
04989     __Pyx_RefNannyDeclarations
04990     PyObject *__pyx_t_1 = NULL;
04991     PyObject *__pyx_t_2 = NULL;
04992     int __pyx_lineno = 0;
04993     const char *__pyx_filename = NULL;
04994     int __pyx_clineno = 0;
04995     __Pyx_RefNannySetupContext("__or__", 0);
04996
04997     /* "PyClical.pyx":302
04998     *      {1,2}
04999     *      """
05000     *      return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )          # ««««««««
05001     *
05002     *      def __ior__(self, rhs):
05003     */
05004     __Pyx_XDECREF(__pyx_r);
05005     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_index_set)); if
  (unlikely(!__pyx_t_1)) __PYX_ERR(0, 302, __pyx_L1_error)
05006     __Pyx_GOTREF(__pyx_t_1);
05007     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set *) __pyx_t_1),
  (__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs) | __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
  (unlikely(!__pyx_t_2)) __PYX_ERR(0, 302, __pyx_L1_error)
05008     __Pyx_GOTREF(__pyx_t_2);
05009     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05010     __pyx_r = __pyx_t_2;
05011     __pyx_t_2 = 0;
05012     goto __pyx_L0;

```

```

05013
05014 /* "PyClical.pyx":293
05015 *         return self.wrap( self.unwrap() & toIndexSet(rhs) )
05016 *
05017 *         def __or__(lhs, rhs):             # ««««««««
05018 *             """
05019 *             Set union: or.
05020 */
05021
05022 /* function exit code */
05023 __pyx_L1_error:;
05024 __Pyx_XDECREF(__pyx_t_1);
05025 __Py_XDECREF(__pyx_t_2);
05026 __Pyx_AddTraceback("PyClical.index_set.__or__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05027 __pyx_r = NULL;
05028 __pyx_L0:;
05029 __Pyx_XGIVEREF(__pyx_r);
05030 __Pyx_RefNannyFinishContext();
05031 return __pyx_r;
05032 }
05033
05034 /* "PyClical.pyx":304
05035 *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
05036 *
05037 *         def __ior__(self, rhs):             # ««««««««
05038 *             """
05039 *             Set union: or.
05040 */
05041
05042 /* Python wrapper */
05043 static PyObject *__pyx_pw_8PyClical_9index_set_30__ior__(PyObject *__pyx_v_self, PyObject
05044 *__pyx_v_rhs); /*proto*/
05045 static char __pyx_doc_8PyClical_9index_set_29__ior__[] = "\n        Set union: or.\n\n        >> x =
05046 index_set({1}); x |= index_set({2}); print(x)\n        {1,2}\n        >> x = index_set({1,2}); x |=
05047 index_set({2}); print(x)\n        {1,2}\n        ";
05048 #if CYTHON_COMPILING_IN_CPYTHON
05049 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_29__ior__;
05050 #endif
05051 static PyObject *__pyx_pw_8PyClical_9index_set_30__ior__(PyObject *__pyx_v_self, PyObject
05052 *__pyx_v_rhs) {
05053     PyObject *__pyx_r = 0;
05054     __Pyx_RefNannyDeclarations
05055     __Pyx_RefNannySetupContext("__ior__ (wrapper)", 0);
05056     __pyx_r = __pyx_pf_8PyClical_9index_set_29__ior__(((struct __pyx_obj_8PyClical_index_set
05057 *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
05058
05059 /* function exit code */
05060 __Pyx_RefNannyFinishContext();
05061 return __pyx_r;
05062 }
05063
05064 static PyObject *__pyx_pf_8PyClical_9index_set_29__ior__(struct __pyx_obj_8PyClical_index_set
05065 *__pyx_v_self, PyObject *__pyx_v_rhs) {
05066     PyObject *__pyx_r = NULL;
05067     __Pyx_RefNannyDeclarations
05068     PyObject *__pyx_t_1 = NULL;
05069     int __pyx_lineno = 0;
05070     const char *__pyx_filename = NULL;
05071     int __pyx_clineno = 0;
05072     __Pyx_RefNannySetupContext("__ior__", 0);
05073
05074 /* "PyClical.pyx":313
05075 *         {1,2}
05076 *         """
05077 *         return self.wrap( self.unwrap() | toIndexSet(rhs) )             # ««««««««
05078 *
05079 *         def count(self):
05080 */
05081     __Pyx_XDECREF(__pyx_r);
05082     __pyx_t_1 = __pyx_f_8PyClical_9index_set_wrap(__pyx_v_self,
05083     (__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self) | __pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if
05084     (unlikely(!__pyx_t_1)) __PYX_ERR(0, 313, __pyx_L1_error)
05085     __Pyx_GOTREF(__pyx_t_1);
05086     __pyx_r = __pyx_t_1;
05087     __pyx_t_1 = 0;
05088     goto __pyx_L0;
05089
05090 /* "PyClical.pyx":304
05091 *         return index_set().wrap( toIndexSet(lhs) | toIndexSet(rhs) )
05092 *
05093 *         def __ior__(self, rhs):             # ««««««««
05094 *             """
05095 *             Set union: or.
05096 */
05097
05098 /* function exit code */
05099 __pyx_L1_error:;

```

```

05092     __Pyx_XDECREF(__pyx_t_1);
05093     __Pyx_AddTraceback("PyClical.index_set.__ior__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05094     __pyx_r = NULL;
05095     __pyx_L0:;
05096     __Pyx_XGIVEREF(__pyx_r);
05097     __Pyx_RefNannyFinishContext();
05098     return __pyx_r;
05099 }
05100
05101 /* "PyClical.pyx":315
05102 *         return self.wrap( self.unwrap() | toIndexSet(rhs) )
05103 *
05104 *     def count(self):
05105 *         """
05106 *         Cardinality: Number of indices included in set.
05107 */
05108
05109 /* Python wrapper */
05110 static PyObject *__pyx_pw_8PyClical_9index_set_32count(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
05111 *unused); /*proto*/
05112 static char __pyx_doc_8PyClical_9index_set_31count[] = "\n        Cardinality: Number of indices\n        included in set.\n\n        >> index_set({-1,1,2}).count()\n        3\n        ";
05113 static PyObject *__pyx_pw_8PyClical_9index_set_32count(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
05114 *unused) {
05115     PyObject *__pyx_r = 0;
05116     __Pyx_RefNannyDeclarations
05117     __Pyx_RefNannySetupContext("count (wrapper)", 0);
05118     __pyx_r = __pyx_pf_8PyClical_9index_set_31count(((struct __pyx_obj_8PyClical_index_set
05119 *)__pyx_v_self));
05120
05121     /* function exit code */
05122     __Pyx_RefNannyFinishContext();
05123     return __pyx_r;
05124 }
05125
05126 static PyObject *__pyx_pf_8PyClical_9index_set_31count(struct __pyx_obj_8PyClical_index_set
05127 *__pyx_v_self) {
05128     PyObject *__pyx_r = NULL;
05129     __Pyx_RefNannyDeclarations
05130     PyObject *__pyx_t_1 = NULL;
05131     int __pyx_lineno = 0;
05132     const char *__pyx_filename = NULL;
05133     int __pyx_clineno = 0;
05134     __Pyx_RefNannySetupContext("count", 0);
05135
05136     /* "PyClical.pyx":322
05137     *         3
05138     *         """
05139     *         return self.instance.count()
05140     *         # ««««««««
05141     *
05142     *     def count_neg(self):
05143     */
05144     __Pyx_XDECREF(__pyx_r);
05145     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count()); if (unlikely(!__pyx_t_1))
05146     __PYX_ERR(0, 322, __pyx_L1_error)
05147     __Pyx_GOTREF(__pyx_t_1);
05148     __pyx_r = __pyx_t_1;
05149     __pyx_t_1 = 0;
05150     goto __pyx_L0;
05151
05152     /* "PyClical.pyx":315
05153     *         return self.wrap( self.unwrap() | toIndexSet(rhs) )
05154     *
05155     *     def count(self):
05156     *         """
05157     *         Cardinality: Number of indices included in set.
05158     */
05159
05160     /* function exit code */
05161     __pyx_L1_error:;
05162     __Pyx_XDECREF(__pyx_t_1);
05163     __Pyx_AddTraceback("PyClical.index_set.count", __pyx_clineno, __pyx_lineno, __pyx_filename);
05164     __pyx_r = NULL;
05165     __pyx_L0:;
05166     __Pyx_XGIVEREF(__pyx_r);
05167     __Pyx_RefNannyFinishContext();
05168     return __pyx_r;
05169 }
05170
05171 /* "PyClical.pyx":324
05172 *         return self.instance.count()
05173 *
05174 *     def count_neg(self):
05175 *         """
05176 *         Number of negative indices included in set.
05177 */

```

```

05173 /* Python wrapper */
05174 static PyObject *__pyx_pw_8PyClical_9index_set_34count_neg(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05175 static char __pyx_doc_8PyClical_9index_set_33count_neg[] = "\n          Number of negative indices
included in set.\n\n          >> index_set({-1,1,2}).count_neg()\n          1\n          ";
05176 static PyObject *__pyx_pw_8PyClical_9index_set_34count_neg(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05177     PyObject *__pyx_r = 0;
05178     __Pyx_RefNannyDeclarations
05179     __Pyx_RefNannySetupContext("count_neg (wrapper)", 0);
05180     __pyx_r = __pyx_pf_8PyClical_9index_set_33count_neg(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05181
05182     /* function exit code */
05183     __Pyx_RefNannyFinishContext();
05184     return __pyx_r;
05185 }
05186
05187 static PyObject *__pyx_pf_8PyClical_9index_set_33count_neg(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05188     PyObject *__pyx_r = NULL;
05189     __Pyx_RefNannyDeclarations
05190     PyObject *__pyx_t_1 = NULL;
05191     int __pyx_lineno = 0;
05192     const char *__pyx_filename = NULL;
05193     int __pyx_clineno = 0;
05194     __Pyx_RefNannySetupContext("count_neg", 0);
05195
05196     /* "PyClical.pyx":331
05197     *         1
05198     *         """
05199     *         return self.instance.count_neg()          # ««««««««
05200     *
05201     *     def count_pos(self):
05202     */
05203     __Pyx_XDECREF(__pyx_r);
05204     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count_neg()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 331, __pyx_L1_error)
05205     __Pyx_GOTREF(__pyx_t_1);
05206     __pyx_r = __pyx_t_1;
05207     __pyx_t_1 = 0;
05208     goto __pyx_L0;
05209
05210     /* "PyClical.pyx":324
05211     *         return self.instance.count()
05212     *
05213     *     def count_neg(self):          # ««««««««
05214     *         """
05215     *         Number of negative indices included in set.
05216     */
05217
05218     /* function exit code */
05219     __pyx_L1_error:;
05220     __Pyx_XDECREF(__pyx_t_1);
05221     __Pyx_AddTraceback("PyClical.index_set.count_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
05222     __pyx_r = NULL;
05223     __pyx_L0:;
05224     __Pyx_XGIVEREF(__pyx_r);
05225     __Pyx_RefNannyFinishContext();
05226     return __pyx_r;
05227 }
05228
05229 /* "PyClical.pyx":333
05230 *         return self.instance.count_neg()
05231 *
05232 *     def count_pos(self):          # ««««««««
05233 *         """
05234 *         Number of positive indices included in set.
05235     */
05236
05237 /* Python wrapper */
05238 static PyObject *__pyx_pw_8PyClical_9index_set_36count_pos(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05239 static char __pyx_doc_8PyClical_9index_set_35count_pos[] = "\n          Number of positive indices
included in set.\n\n          >> index_set({-1,1,2}).count_pos()\n          2\n          ";
05240 static PyObject *__pyx_pw_8PyClical_9index_set_36count_pos(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05241     PyObject *__pyx_r = 0;
05242     __Pyx_RefNannyDeclarations
05243     __Pyx_RefNannySetupContext("count_pos (wrapper)", 0);
05244     __pyx_r = __pyx_pf_8PyClical_9index_set_35count_pos(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05245
05246     /* function exit code */
05247     __Pyx_RefNannyFinishContext();
05248     return __pyx_r;
05249 }

```

```

05250
05251 static PyObject *__pyx_pf_8PyClical_9index_set_35count_pos(struct __pyx_obj_8PyClical_index_set
    __pyx_v_self) {
05252     PyObject *__pyx_r = NULL;
05253     __Pyx_RefNannyDeclarations
05254     PyObject *__pyx_t_1 = NULL;
05255     int __pyx_lineno = 0;
05256     const char *__pyx_filename = NULL;
05257     int __pyx_clineno = 0;
05258     __Pyx_RefNannySetupContext("count_pos", 0);
05259
05260     /* "PyClical.pyx":340
05261     *         2
05262     *         """
05263     *         return self.instance.count_pos()          # ««««««««
05264     *
05265     *     def min(self):
05266     */
05267     __Pyx_XDECREF(__pyx_r);
05268     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->count_pos()); if (unlikely(!__pyx_t_1))
    __PYX_ERR(0, 340, __pyx_L1_error)
05269     __Pyx_GOTREF(__pyx_t_1);
05270     __pyx_r = __pyx_t_1;
05271     __pyx_t_1 = 0;
05272     goto __pyx_L0;
05273
05274     /* "PyClical.pyx":333
05275     *         return self.instance.count_neg()
05276     *
05277     *     def count_pos(self):          # ««««««««
05278     *         """
05279     *         Number of positive indices included in set.
05280     */
05281
05282     /* function exit code */
05283     __pyx_L1_error;
05284     __Pyx_XDECREF(__pyx_t_1);
05285     __Pyx_AddTraceback("PyClical.index_set.count_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
05286     __pyx_r = NULL;
05287     __pyx_L0;
05288     __Pyx_XGIVEREF(__pyx_r);
05289     __Pyx_RefNannyFinishContext();
05290     return __pyx_r;
05291 }
05292
05293 /* "PyClical.pyx":342
05294 *         return self.instance.count_pos()
05295 *
05296 *     def min(self):          # ««««««««
05297 *         """
05298 *         Minimum member.
05299 */
05300
05301 /* Python wrapper */
05302 static PyObject *__pyx_pw_8PyClical_9index_set_38min(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused); /*proto*/
05303 static char __pyx_doc_8PyClical_9index_set_37min[] = "\n        Minimum member.\n\n        >>
    index_set({-1,1,2}).min()\n        -1\n        ";
05304 static PyObject *__pyx_pw_8PyClical_9index_set_38min(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
    *unused) {
05305     PyObject *__pyx_r = 0;
05306     __Pyx_RefNannyDeclarations
05307     __Pyx_RefNannySetupContext("min (wrapper)", 0);
05308     __pyx_r = __pyx_pf_8PyClical_9index_set_37min(((struct __pyx_obj_8PyClical_index_set
    *)__pyx_v_self));
05309
05310     /* function exit code */
05311     __Pyx_RefNannyFinishContext();
05312     return __pyx_r;
05313 }
05314
05315 static PyObject *__pyx_pf_8PyClical_9index_set_37min(struct __pyx_obj_8PyClical_index_set
    __pyx_v_self) {
05316     PyObject *__pyx_r = NULL;
05317     __Pyx_RefNannyDeclarations
05318     PyObject *__pyx_t_1 = NULL;
05319     int __pyx_lineno = 0;
05320     const char *__pyx_filename = NULL;
05321     int __pyx_clineno = 0;
05322     __Pyx_RefNannySetupContext("min", 0);
05323
05324     /* "PyClical.pyx":349
05325     *         -1
05326     *         """
05327     *         return self.instance.min()          # ««««««««
05328     *
05329     *     def max(self):

```

```

05330 */
05331 __Pyx_XDECREF(__pyx_r);
05332 __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->min()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 349, __pyx_L1_error)
05333 __Pyx_GOTREF(__pyx_t_1);
05334 __pyx_r = __pyx_t_1;
05335 __pyx_t_1 = 0;
05336 goto __pyx_L0;
05337
05338 /* "PyClical.pyx":342
05339 *     return self.instance.count_pos()
05340 *
05341 *     def min(self):           # ««««««««
05342 *         """
05343 *         Minimum member.
05344 */
05345
05346 /* function exit code */
05347 __pyx_L1_error:;
05348 __Pyx_XDECREF(__pyx_t_1);
05349 __Pyx_AddTraceback("PyClical.index_set.min", __pyx_clineno, __pyx_lineno, __pyx_filename);
05350 __pyx_r = NULL;
05351 __pyx_L0:;
05352 __Pyx_XGIVEREF(__pyx_r);
05353 __Pyx_RefNannyFinishContext();
05354 return __pyx_r;
05355 }
05356
05357 /* "PyClical.pyx":351
05358 *     return self.instance.min()
05359 *
05360 *     def max(self):           # ««««««««
05361 *         """
05362 *         Maximum member.
05363 */
05364
05365 /* Python wrapper */
05366 static PyObject *__pyx_pw_8PyClical_9index_set_40max(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
__unused); /*proto*/
05367 static char __pyx_doc_8PyClical_9index_set_39max[] = "\n          Maximum member.\n\n          >>
index_set({-1,1,2}).max()\n          2\n          ";
05368 static PyObject *__pyx_pw_8PyClical_9index_set_40max(PyObject *__pyx_v_self, CYTHON_UNUSED PyObject
__unused) {
05369     PyObject *__pyx_r = 0;
05370     __Pyx_RefNannyDeclarations
05371     __Pyx_RefNannySetupContext("max (wrapper)", 0);
05372     __pyx_r = __pyx_pf_8PyClical_9index_set_39max(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05373
05374 /* function exit code */
05375 __Pyx_RefNannyFinishContext();
05376 return __pyx_r;
05377 }
05378
05379 static PyObject *__pyx_pf_8PyClical_9index_set_39max(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05380     PyObject *__pyx_r = NULL;
05381     __Pyx_RefNannyDeclarations
05382     PyObject *__pyx_t_1 = NULL;
05383     int __pyx_lineno = 0;
05384     const char *__pyx_filename = NULL;
05385     int __pyx_clineno = 0;
05386     __Pyx_RefNannySetupContext("max", 0);
05387
05388 /* "PyClical.pyx":358
05389 *         2
05390 *         """
05391 *     return self.instance.max()           # ««««««««
05392 *
05393 *     def hash_fn(self):
05394 */
05395     __Pyx_XDECREF(__pyx_r);
05396 __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->max()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 358, __pyx_L1_error)
05397 __Pyx_GOTREF(__pyx_t_1);
05398 __pyx_r = __pyx_t_1;
05399 __pyx_t_1 = 0;
05400 goto __pyx_L0;
05401
05402 /* "PyClical.pyx":351
05403 *     return self.instance.min()
05404 *
05405 *     def max(self):           # ««««««««
05406 *         """
05407 *         Maximum member.
05408 */
05409

```

```

05410  /* function exit code */
05411  __pyx_L1_error:;
05412  __Pyx_XDECREF(__pyx_t_1);
05413  __Pyx_AddTraceback("PyCliclal.index_set.max", __pyx_clineno, __pyx_lineno, __pyx_filename);
05414  __pyx_r = NULL;
05415  __pyx_L0:;
05416  __Pyx_XGIVEREF(__pyx_r);
05417  __Pyx_RefNannyFinishContext();
05418  return __pyx_r;
05419 }
05420
05421 /* "PyCliclal.pyx":360
05422 *      return self.instance.max()
05423 *
05424 *      def hash_fn(self):          # ««««««««
05425 *          """
05426 *          Hash function.
05427 */
05428
05429 /* Python wrapper */
05430 static PyObject *__pyx_pw_8PyCliclal_9index_set_42hash_fn(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05431 static char __pyx_doc_8PyCliclal_9index_set_41hash_fn[] = "\n      Hash function.\n";
05432 static PyObject *__pyx_pw_8PyCliclal_9index_set_42hash_fn(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
05433     PyObject *__pyx_r = 0;
05434     __Pyx_RefNannyDeclarations
05435     __Pyx_RefNannySetupContext("hash_fn (wrapper)", 0);
05436     __pyx_r = __pyx_pf_8PyCliclal_9index_set_41hash_fn(((struct __pyx_obj_8PyCliclal_index_set
*)__pyx_v_self));
05437
05438     /* function exit code */
05439     __Pyx_RefNannyFinishContext();
05440     return __pyx_r;
05441 }
05442
05443 static PyObject *__pyx_pf_8PyCliclal_9index_set_41hash_fn(struct __pyx_obj_8PyCliclal_index_set
*__pyx_v_self) {
05444     PyObject *__pyx_r = NULL;
05445     __Pyx_RefNannyDeclarations
05446     PyObject *__pyx_t_1 = NULL;
05447     int __pyx_lineno = 0;
05448     const char *__pyx_filename = NULL;
05449     int __pyx_clineno = 0;
05450     __Pyx_RefNannySetupContext("hash_fn", 0);
05451
05452     /* "PyCliclal.pyx":364
05453 *          Hash function.
05454 *
05455 *          return self.instance.hash_fn()          # ««««««««
05456 *
05457 *          def sign_of_mult(self, rhs):
05458 */
05459     __Pyx_XDECREF(__pyx_r);
05460     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->hash_fn()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 364, __pyx_L1_error)
05461     __Pyx_GOTREF(__pyx_t_1);
05462     __pyx_r = __pyx_t_1;
05463     __pyx_t_1 = 0;
05464     goto __pyx_L0;
05465
05466     /* "PyCliclal.pyx":360
05467 *      return self.instance.max()
05468 *
05469 *      def hash_fn(self):          # ««««««««
05470 *          """
05471 *          Hash function.
05472 */
05473
05474     /* function exit code */
05475     __pyx_L1_error:;
05476     __Pyx_XDECREF(__pyx_t_1);
05477     __Pyx_AddTraceback("PyCliclal.index_set.hash_fn", __pyx_clineno, __pyx_lineno, __pyx_filename);
05478     __pyx_r = NULL;
05479     __pyx_L0:;
05480     __Pyx_XGIVEREF(__pyx_r);
05481     __Pyx_RefNannyFinishContext();
05482     return __pyx_r;
05483 }
05484
05485 /* "PyCliclal.pyx":366
05486 *      return self.instance.hash_fn()
05487 *
05488 *      def sign_of_mult(self, rhs):          # ««««««««
05489 *          """
05490 *          Sign of geometric product of two Clifford basis elements.
05491 */

```



```

05492
05493 /* Python wrapper */
05494 static PyObject *__pyx_pw_8PyClical_9index_set_44sign_of_mult(PyObject *__pyx_v_self, PyObject
05495 *__pyx_v_rhs); /*proto*/
05496 static char __pyx_doc_8PyClical_9index_set_43sign_of_mult[] = "\n          Sign of geometric product of
05497 two Clifford basis elements.\n\n          >> s = index_set({1,2}); t=index_set({-1});
05498 s.sign_of_mult(t)\n          1\n          ";
05499 static PyObject *__pyx_pw_8PyClical_9index_set_44sign_of_mult(PyObject *__pyx_v_self, PyObject
05500 *__pyx_v_rhs) {
05501     PyObject *__pyx_r = 0;
05502     __Pyx_RefNannyDeclarations
05503     __Pyx_RefNannySetupContext("sign_of_mult (wrapper)", 0);
05504     __pyx_r = __pyx_pf_8PyClical_9index_set_43sign_of_mult(((struct __pyx_obj_8PyClical_index_set
05505 *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
05506
05507 /* function exit code */
05508 __Pyx_RefNannyFinishContext();
05509 return __pyx_r;
05510 }
05511
05512 static PyObject *__pyx_pf_8PyClical_9index_set_43sign_of_mult(struct __pyx_obj_8PyClical_index_set
05513 *__pyx_v_self, PyObject *__pyx_v_rhs) {
05514     PyObject *__pyx_r = NULL;
05515     __Pyx_RefNannyDeclarations
05516     PyObject *__pyx_t_1 = NULL;
05517     int __pyx_lineno = 0;
05518     const char *__pyx_filename = NULL;
05519     int __pyx_clineno = 0;
05520     __Pyx_RefNannySetupContext("sign_of_mult", 0);
05521
05522     /* "PyClical.pyx":373
05523     *
05524     *     """
05525     *     return self.instance.sign_of_mult(toIndexSet(rhs))
05526     *                                     # ««««««««
05527     *
05528     *     def sign_of_square(self):
05529     */
05530     __Pyx_XDECREF(__pyx_r);
05531     __pyx_t_1 =
05532     __Pyx_PyInt_From_int(__pyx_v_self->instance->sign_of_mult(__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs)));
05533     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 373, __pyx_L1_error)
05534     __Pyx_GOTREF(__pyx_t_1);
05535     __pyx_r = __pyx_t_1;
05536     __pyx_t_1 = 0;
05537     goto __pyx_L0;
05538
05539     /* "PyClical.pyx":366
05540     *
05541     *     return self.instance.hash_fn()
05542     *
05543     *     def sign_of_mult(self, rhs):
05544     *                                     # ««««««««
05545     *     """
05546     *     Sign of geometric product of two Clifford basis elements.
05547     */
05548
05549     /* function exit code */
05550     __Pyx_L1_error:;
05551     __Pyx_XDECREF(__pyx_t_1);
05552     __Pyx_AddTraceback("PyClical.index_set.sign_of_mult", __pyx_clineno, __pyx_lineno, __pyx_filename);
05553     __pyx_r = NULL;
05554     __pyx_L0:;
05555     __Pyx_XGIVEREF(__pyx_r);
05556     __Pyx_RefNannyFinishContext();
05557     return __pyx_r;
05558 }
05559
05560 /* "PyClical.pyx":375
05561 *
05562 *     return self.instance.sign_of_mult(toIndexSet(rhs))
05563 *
05564 *     def sign_of_square(self):
05565 *                                     # ««««««««
05566 *     """
05567 *     Sign of geometric square of a Clifford basis element.
05568 */
05569
05570 /* Python wrapper */
05571 static PyObject *__pyx_pw_8PyClical_9index_set_46sign_of_square(PyObject *__pyx_v_self, CYTHON_UNUSED
05572 PyObject *unused); /*proto*/
05573 static char __pyx_doc_8PyClical_9index_set_45sign_of_square[] = "\n          Sign of geometric square of
05574 a Clifford basis element.\n\n          >> s = index_set({1,2}); s.sign_of_square()\n          -1\n          ";
05575 static PyObject *__pyx_pw_8PyClical_9index_set_46sign_of_square(PyObject *__pyx_v_self, CYTHON_UNUSED
05576 PyObject *unused) {
05577     PyObject *__pyx_r = 0;
05578     __Pyx_RefNannyDeclarations
05579     __Pyx_RefNannySetupContext("sign_of_square (wrapper)", 0);
05580     __pyx_r = __pyx_pf_8PyClical_9index_set_45sign_of_square(((struct __pyx_obj_8PyClical_index_set
05581 *)__pyx_v_self));
05582
05583 }

```

```

05566  /* function exit code */
05567  __Pyx_RefNannyFinishContext();
05568  return __pyx_r;
05569 }
05570
05571 static PyObject *__pyx_pf_8PyClical_9index_set_45sign_of_square(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05572     PyObject *__pyx_r = NULL;
05573     __Pyx_RefNannyDeclarations
05574     PyObject *__pyx_t_1 = NULL;
05575     int __pyx_lineno = 0;
05576     const char *__pyx_filename = NULL;
05577     int __pyx_clineno = 0;
05578     __Pyx_RefNannySetupContext("sign_of_square", 0);
05579
05580     /* "PyClical.pyx":382
05581     *         -1
05582     *         """
05583     *         return self.instance.sign_of_square()          # ««««««««
05584     *
05585     *     def __repr__(self):
05586     */
05587     __Pyx_XDECREF(__pyx_r);
05588     __pyx_t_1 = __Pyx_PyInt_From_int(__pyx_v_self->instance->sign_of_square()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 382, __pyx_L1_error)
05589     __Pyx_GOTREF(__pyx_t_1);
05590     __pyx_r = __pyx_t_1;
05591     __pyx_t_1 = 0;
05592     goto __pyx_L0;
05593
05594     /* "PyClical.pyx":375
05595     *         return self.instance.sign_of_mult(toIndexSet(rhs))
05596     *
05597     *     def sign_of_square(self):          # ««««««««
05598     *         """
05599     *         Sign of geometric square of a Clifford basis element.
05600     */
05601
05602     /* function exit code */
05603     __pyx_L1_error:;
05604     __Pyx_XDECREF(__pyx_t_1);
05605     __Pyx_AddTraceback("PyClical.index_set.sign_of_square", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05606     __pyx_r = NULL;
05607     __pyx_L0:;
05608     __Pyx_XGIVEREF(__pyx_r);
05609     __Pyx_RefNannyFinishContext();
05610     return __pyx_r;
05611 }
05612
05613 /* "PyClical.pyx":384
05614 *         return self.instance.sign_of_square()
05615 *
05616 *     def __repr__(self):          # ««««««««
05617 *         """
05618 *         The official string representation of self.
05619     */
05620
05621 /* Python wrapper */
05622 static PyObject *__pyx_pw_8PyClical_9index_set_48__repr__(PyObject *__pyx_v_self); /*proto*/
05623 static char __pyx_doc_8PyClical_9index_set_47__repr__[] = "\n        The
        \342\200\234official\342\200\235 string representation of self.\n\n        >>
        index_set({1,2}).__repr__()\n        'index_set({1,2})'\n        >> repr(index_set({1,2}))\n
        'index_set({1,2})'\n        ";
05624 #if CYTHON_COMPILING_IN_CPYTHON
05625 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_47__repr__;
05626 #endif
05627 static PyObject *__pyx_pw_8PyClical_9index_set_48__repr__(PyObject *__pyx_v_self) {
05628     PyObject *__pyx_r = 0;
05629     __Pyx_RefNannyDeclarations
05630     __Pyx_RefNannySetupContext("__repr__ (wrapper)", 0);
05631     __pyx_r = __pyx_pf_8PyClical_9index_set_47__repr__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05632
05633     /* function exit code */
05634     __Pyx_RefNannyFinishContext();
05635     return __pyx_r;
05636 }
05637
05638 static PyObject *__pyx_pf_8PyClical_9index_set_47__repr__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05639     PyObject *__pyx_r = NULL;
05640     __Pyx_RefNannyDeclarations
05641     PyObject *__pyx_t_1 = NULL;
05642     int __pyx_lineno = 0;
05643     const char *__pyx_filename = NULL;
05644     int __pyx_clineno = 0;

```

```

05645  __Pyx_RefNannySetupContext("__repr__", 0);
05646
05647  /* "PyClical.pyx":393
05648  *      'index_set({1,2})'
05649  *      """
05650  *      return index_set_to_repr( self.unwrap() ).decode()          # ««««««««
05651  *
05652  *      def __str__(self):
05653  */
05654  __Pyx_XDECREF(__pyx_r);
05655  __pyx_t_1 =
__Pyx_decode_cpp_string(index_set_to_repr(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self)), 0,
PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 393, __pyx_L1_error)
05656  __Pyx_GOTREF(__pyx_t_1);
05657  __pyx_r = __pyx_t_1;
05658  __pyx_t_1 = 0;
05659  goto __pyx_L0;
05660
05661  /* "PyClical.pyx":384
05662  *      return self.instance.sign_of_square()
05663  *
05664  *      def __repr__(self):          # ««««««««
05665  *      """
05666  *      The official string representation of self.
05667  */
05668
05669  /* function exit code */
05670  __pyx_L1_error:;
05671  __Pyx_XDECREF(__pyx_t_1);
05672  __Pyx_AddTraceback("PyClical.index_set.__repr__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05673  __pyx_r = NULL;
05674  __pyx_L0:;
05675  __Pyx_XGIVEREF(__pyx_r);
05676  __Pyx_RefNannyFinishContext();
05677  return __pyx_r;
05678 }
05679
05680 /* "PyClical.pyx":395
05681 *      return index_set_to_repr( self.unwrap() ).decode()
05682 *
05683 *      def __str__(self):          # ««««««««
05684 *      """
05685 *      The informal string representation of self.
05686  */
05687
05688 /* Python wrapper */
05689 static PyObject *__pyx_pw_8PyClical_9index_set_50_str__(PyObject *__pyx_v_self); /*proto*/
05690 static char __pyx_doc_8PyClical_9index_set_49_str__[] = "\n      The\n      \342\200\234informal\342\200\235 string representation of self.\n\n      >>\n      index_set({1,2}).__str__()\n      ' {1,2} '\n      >> str(index_set({1,2}))\n      ' {1,2} '\n";
05691 #if CYTHON_COMPILING_IN_CPYTHON
05692 struct wrapperbase __pyx_wrapperbase_8PyClical_9index_set_49_str__;
05693 #endif
05694 static PyObject *__pyx_pw_8PyClical_9index_set_50_str__(PyObject *__pyx_v_self) {
05695     PyObject *__pyx_r = 0;
05696     __Pyx_RefNannyDeclarations
05697     __Pyx_RefNannySetupContext("__str__ (wrapper)", 0);
05698     __pyx_r = __pyx_pf_8PyClical_9index_set_49_str__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self));
05699
05700     /* function exit code */
05701     __Pyx_RefNannyFinishContext();
05702     return __pyx_r;
05703 }
05704
05705 static PyObject *__pyx_pf_8PyClical_9index_set_49_str__(struct __pyx_obj_8PyClical_index_set
*__pyx_v_self) {
05706     PyObject *__pyx_r = NULL;
05707     __Pyx_RefNannyDeclarations
05708     PyObject *__pyx_t_1 = NULL;
05709     int __pyx_lineno = 0;
05710     const char *__pyx_filename = NULL;
05711     int __pyx_clineno = 0;
05712     __Pyx_RefNannySetupContext("__str__", 0);
05713
05714     /* "PyClical.pyx":404
05715     *      ' {1,2} '
05716     *      """
05717     *      return index_set_to_str( self.unwrap() ).decode()          # ««««««««
05718     *
05719     *      def index_set_hidden_doctests():
05720     */
05721     __Pyx_XDECREF(__pyx_r);
05722     __pyx_t_1 =
__Pyx_decode_cpp_string(index_set_to_str(__pyx_f_8PyClical_9index_set_unwrap(__pyx_v_self)), 0,
PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 404, __pyx_L1_error)

```

```

05723     __Pyx_GOTREF(__pyx_t_1);
05724     __pyx_r = __pyx_t_1;
05725     __pyx_t_1 = 0;
05726     goto __pyx_L0;
05727
05728     /* "PyClical.pyx":395
05729     *         return index_set_to_repr( self.unwrap() ).decode()
05730     *
05731     *     def __str__(self):                # ««««««««
05732     *         """
05733     *         The informal string representation of self.
05734     */
05735
05736     /* function exit code */
05737     __pyx_L1_error:;
05738     __Pyx_XDECREF(__pyx_t_1);
05739     __Pyx_AddTraceback("PyClical.index_set.__str__", __pyx_clineno, __pyx_lineno, __pyx_filename);
05740     __pyx_r = NULL;
05741     __pyx_L0:;
05742     __Pyx_XGIVEREF(__pyx_r);
05743     __Pyx_RefNannyFinishContext();
05744     return __pyx_r;
05745 }
05746
05747 /* "(tree fragment)":1
05748 * def __reduce_cython__(self):                # ««««««««
05749 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05750 * def __setstate_cython__(self, __pyx_state):
05751 */
05752
05753 /* Python wrapper */
05754 static PyObject *__pyx_pw_8PyClical_9index_set_52__reduce_cython__(PyObject *__pyx_v_self,
05755 CYTHON_UNUSED PyObject *unused); /*proto*/
05756 static PyObject *__pyx_pw_8PyClical_9index_set_52__reduce_cython__(PyObject *__pyx_v_self,
05757 CYTHON_UNUSED PyObject *unused) {
05758     PyObject *__pyx_r = 0;
05759     __Pyx_RefNannyDeclarations
05760     __Pyx_RefNannySetupContext("__reduce_cython__ (wrapper)", 0);
05761     __pyx_r = __pyx_pf_8PyClical_9index_set_51__reduce_cython__(((struct __pyx_obj_8PyClical_index_set
05762 *)__pyx_v_self));
05763
05764     /* function exit code */
05765     __Pyx_RefNannyFinishContext();
05766     return __pyx_r;
05767 }
05768
05769 static PyObject *__pyx_pf_8PyClical_9index_set_51__reduce_cython__(CYTHON_UNUSED struct
05770 __pyx_obj_8PyClical_index_set *__pyx_v_self) {
05771     PyObject *__pyx_r = NULL;
05772     __Pyx_RefNannyDeclarations
05773     PyObject *__pyx_t_1 = NULL;
05774     int __pyx_lineno = 0;
05775     const char *__pyx_filename = NULL;
05776     int __pyx_clineno = 0;
05777     __Pyx_RefNannySetupContext("__reduce_cython__", 0);
05778
05779     /* "(tree fragment)":2
05780     * def __reduce_cython__(self):
05781     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05782     * def __setstate_cython__(self, __pyx_state):
05783     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05784     */
05785     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__3, NULL); if
05786     (unlikely(!__pyx_t_1)) __PYX_ERR(1, 2, __pyx_L1_error)
05787     __Pyx_GOTREF(__pyx_t_1);
05788     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
05789     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05790     __PYX_ERR(1, 2, __pyx_L1_error)
05791
05792     /* "(tree fragment)":1
05793     * def __reduce_cython__(self):                # ««««««««
05794     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05795     * def __setstate_cython__(self, __pyx_state):
05796     */
05797
05798     /* function exit code */
05799     __pyx_L1_error:;
05800     __Pyx_XDECREF(__pyx_t_1);
05801     __Pyx_AddTraceback("PyClical.index_set.__reduce_cython__", __pyx_clineno, __pyx_lineno,
05802 __pyx_filename);
05803     __pyx_r = NULL;
05804     __Pyx_XGIVEREF(__pyx_r);
05805     __Pyx_RefNannyFinishContext();
05806     return __pyx_r;
05807 }
05808
05809 /* "(tree fragment)":3

```

```

05804 * def __reduce_cython__(self):
05805 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05806 * def __setstate_cython__(self, __pyx_state): # ««««««««
05807 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05808 */
05809
05810 /* Python wrapper */
05811 static PyObject *__pyx_pw_8PyClical_9index_set_54__setstate_cython__(PyObject *__pyx_v_self, PyObject
*__pyx_v__pyx_state); /*proto*/
05812 static PyObject *__pyx_pw_8PyClical_9index_set_54__setstate_cython__(PyObject *__pyx_v_self, PyObject
*__pyx_v__pyx_state) {
05813     PyObject *__pyx_r = 0;
05814     __Pyx_RefNannyDeclarations
05815     __Pyx_RefNannySetupContext("__setstate_cython__ (wrapper)", 0);
05816     __pyx_r = __pyx_pf_8PyClical_9index_set_53__setstate_cython__(((struct __pyx_obj_8PyClical_index_set
*)__pyx_v_self), ((PyObject *)__pyx_v__pyx_state));
05817
05818     /* function exit code */
05819     __Pyx_RefNannyFinishContext();
05820     return __pyx_r;
05821 }
05822
05823 static PyObject *__pyx_pf_8PyClical_9index_set_53__setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_index_set *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state) {
05824     PyObject *__pyx_r = NULL;
05825     __Pyx_RefNannyDeclarations
05826     PyObject *__pyx_t_1 = NULL;
05827     int __pyx_lineno = 0;
05828     const char *__pyx_filename = NULL;
05829     int __pyx_clineno = 0;
05830     __Pyx_RefNannySetupContext("__setstate_cython__", 0);
05831
05832     /* (tree fragment)":4
05833     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05834     * def __setstate_cython__(self, __pyx_state): # ««««««««
05835     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05836     */
05837     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__4, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 4, __pyx_L1_error)
05838     __Pyx_GOTREF(__pyx_t_1);
05839     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
05840     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
05841     __PYX_ERR(1, 4, __pyx_L1_error)
05842
05843     /* (tree fragment)":3
05844     * def __reduce_cython__(self):
05845     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05846     * def __setstate_cython__(self, __pyx_state): # ««««««««
05847     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
05848     */
05849
05850     /* function exit code */
05851     __pyx_L1_error:;
05852     __Pyx_XDECREF(__pyx_t_1);
05853     __Pyx_AddTraceback("PyClical.index_set.__setstate_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
05854     __pyx_r = NULL;
05855     __Pyx_XGIVEREF(__pyx_r);
05856     __Pyx_RefNannyFinishContext();
05857     return __pyx_r;
05858 }
05859
05860 /* "PyClical.pyx":406
05861 *     return index_set_to_str( self.unwrap() ).decode()
05862 *
05863 * def index_set_hidden_doctests(): # ««««««««
05864 *     """
05865 *     Tests for functions that Doctest cannot see.
05866 *
05867 */
05868 /* Python wrapper */
05869 static PyObject *__pyx_pw_8PyClical_1index_set_hidden_doctests(PyObject *__pyx_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
05870 static char __pyx_doc_8PyClical_index_set_hidden_doctests[] = "\n    Tests for functions that Doctest
cannot see.\n\n    For index_set.__cinit__: Construct index_set.\n\n    >> print(index_set(1))\n
{1}\n    >> print(index_set({1,2}))\n    {1,2}\n    >> print(index_set(index_set({1,2})))\n    {1,2}\n
>> print(index_set({1,2}))\n    {1,2}\n    >> print(index_set({1,2,1}))\n    {1,2}\n    >>
print(index_set({1,2,1}))\n    {1,2}\n    >> print(index_set(\"{1}\"))\n    {} \n    >>
print(index_set(\"{\"}))\n    Traceback (most recent call last):\n    ... \n    ValueError: Cannot
initialize index_set object from invalid string '{'.\n    >> print(index_set(\"{1}\"))\n    Traceback
(most recent call last):\n    ... \n    ValueError: Cannot initialize index_set object from invalid
string '{1'.\n    >> print(index_set(\"{1,2,100}\"))\n    Traceback (most recent call last):\n
... \n    ValueError: Cannot initialize index_set object from invalid string '{1,2,100}'.\n    >>
print(index_set({1,2,100}))\n    Traceback (most recent call last):\n    ... \n    IndexError: Cannot
initialize index_set object from invalid {1, 2, 100}.\n    >> print(index_set([1,2]))\n    Traceback
(most recent call last):\n    ... \n    TypeError: Cannot initialize index_set object from <class
'list'>.\n\n    For index_set.__richcmp__: Compare two objects of class index_set.\n\n    >>

```

```

index_set(1) == index_set({1})\n    True\n    >> index_set({1}) != index_set({1})\n    False\n    >>
index_set({1}) != index_set({2})\n    True\n    >> index_set({1}) == index_set({2})\n    False\n    >>
index_set({1}) < index_set({2})\n    True\n    >> index_set({1}) <= index_set({2})\n    True\n    >>
index_set({1}) > index_set({2})\n    False\n    >> index_set({1}) >= index_set({2})\n    False\n    >>
None == index_set({1,2})\n    False\n    >> None != index_set({1,2})\n    True\n    >> None <
index_set({1,2})\n    False\n    >> None <= index_set({1,2})\n    False\n    >> None >
index_set({1,2})\n    False\n    >> None >= index_set({1,2})\n    False\n    >> "index_set({1,2}) ==
None\n    False\n    >> index_set({1,2}) != None\n    True\n    >> index_set({1,2}) < None\n
False\n    >> index_set({1,2}) <= None\n    False\n    >> index_set({1,2}) > None\n    False\n    >>
index_set({1,2}) >= None\n    False\n    ";
05871     static PyMethodDef __pyx_mdef_8PyClical_index_set_hidden_doctests =
{"index_set_hidden_doctests", (PyCFunction)__pyx_pw_8PyClical_index_set_hidden_doctests, METH_NOARGS,
__pyx_doc_8PyClical_index_set_hidden_doctests};
05872     static PyObject *__pyx_pw_8PyClical_index_set_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused) {
05873         PyObject *__pyx_r = 0;
05874         __Pyx_RefNannyDeclarations
05875         __Pyx_RefNannySetupContext("index_set_hidden_doctests (wrapper)", 0);
05876         __pyx_r = __pyx_pf_8PyClical_index_set_hidden_doctests(__pyx_self);
05877
05878         /* function exit code */
05879         __Pyx_RefNannyFinishContext();
05880         return __pyx_r;
05881     }
05882
05883     static PyObject *__pyx_pf_8PyClical_index_set_hidden_doctests(CYTHON_UNUSED PyObject
*__pyx_self) {
05884         PyObject *__pyx_r = NULL;
05885         __Pyx_RefNannyDeclarations
05886         __Pyx_RefNannySetupContext("index_set_hidden_doctests", 0);
05887
05888         /* "PyClical.pyx":490
05889         *     False
05890         *     """
05891         *     return          # ««««««««
05892         *
05893         * cpdef inline compare(lhs,rhs):
05894         */
05895         __Pyx_XDECREF(__pyx_r);
05896         __pyx_r = Py_None; __Pyx_INCREF(Py_None);
05897         goto __pyx_L0;
05898
05899         /* "PyClical.pyx":406
05900         *     return index_set_to_str( self.unwrap() ).decode()
05901         *
05902         * def index_set_hidden_doctests():          # ««««««««
05903         *     """
05904         *     Tests for functions that Doctest cannot see.
05905         */
05906
05907         /* function exit code */
05908         __pyx_L0:;
05909         __Pyx_XGIVEREF(__pyx_r);
05910         __Pyx_RefNannyFinishContext();
05911         return __pyx_r;
05912     }
05913
05914         /* "PyClical.pyx":492
05915         *     return
05916         *
05917         * cpdef inline compare(lhs,rhs):          # ««««««««
05918         *     """
05919         *     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
05920         */
05921
05922         static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
05923         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_compare(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, CYTHON_UNUSED int __pyx_skip_dispatch) {
05924             PyObject *__pyx_r = NULL;
05925             __Pyx_RefNannyDeclarations
05926             PyObject *__pyx_t_1 = NULL;
05927             int __pyx_lineno = 0;
05928             const char *__pyx_filename = NULL;
05929             int __pyx_clineno = 0;
05930             __Pyx_RefNannySetupContext("compare", 0);
05931
05932             /* "PyClical.pyx":502
05933             *     1
05934             *     """
05935             *     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )          # ««««««««
05936             *
05937             * cpdef inline min_neg(obj):
05938             */
05939             __Pyx_XDECREF(__pyx_r);
05940             __pyx_t_1 = __Pyx_PyInt_From_int(compare(__pyx_f_8PyClical_toIndexSet(__pyx_v_lhs),
__pyx_f_8PyClical_toIndexSet(__pyx_v_rhs))); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 502,

```

```

__pyx_L1_error)
05941     __Pyx_GOTREF(__pyx_t_1);
05942     __pyx_r = __pyx_t_1;
05943     __pyx_t_1 = 0;
05944     goto __pyx_L0;
05945
05946     /* "PyClical.pyx":492
05947     *     return
05948     *
05949     * cpdef inline compare(lhs,rhs):          # ««««««««
05950     *     """
05951     *     "lexicographic compare" eg. {3,4,5} is less than {3,7,8};
05952     */
05953
05954     /* function exit code */
05955     __pyx_L1_error++;
05956     __Pyx_XDECREF(__pyx_t_1);
05957     __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
05958     __pyx_r = 0;
05959     __pyx_L0++;
05960     __Pyx_XGIVEREF(__pyx_r);
05961     __Pyx_RefNannyFinishContext();
05962     return __pyx_r;
05963 }
05964
05965 /* Python wrapper */
05966 static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
05967 static char __pyx_doc_8PyClical_2compare[] = "\n    \"lexicographic compare\" eg. {3,4,5} is
less than {3,7,8};\n    -1 if a<b, +1 if a>b, 0 if a==b.\n\n    >>
compare(index_set({1,2}),index_set({-1,3}))\n    -1\n    >>
compare(index_set({-1,4}),index_set({-1,3}))\n    1\n    ";
05968 static PyObject *__pyx_pw_8PyClical_3compare(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
05969     PyObject *__pyx_v_lhs = 0;
05970     PyObject *__pyx_v_rhs = 0;
05971     int __pyx_lineno = 0;
05972     const char *__pyx_filename = NULL;
05973     int __pyx_clineno = 0;
05974     PyObject *__pyx_r = 0;
05975     __Pyx_RefNannyDeclarations
05976     __Pyx_RefNannySetupContext("compare (wrapper)", 0);
05977     {
05978         static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_lhs,&__pyx_n_s_rhs,0};
05979         PyObject* values[2] = {0,0};
05980         if (unlikely(__pyx_kwds)) {
05981             Py_ssize_t kw_args;
05982             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
05983             switch (pos_args) {
05984                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
05985                 CYTHON_FALLTHROUGH;
05986                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
05987                 CYTHON_FALLTHROUGH;
05988                 case 0: break;
05989                 default: goto __pyx_L5_argtuple_error;
05990             }
05991             kw_args = PyDict_Size(__pyx_kwds);
05992             switch (pos_args) {
05993                 case 0:
05994                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) != 0))
05995                         kw_args--;
05996                     else goto __pyx_L5_argtuple_error;
05997                     CYTHON_FALLTHROUGH;
05998                     case 1:
05999                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) != 0))
06000                             kw_args--;
06001                         else {
06002                             __Pyx_RaiseArgtupleInvalid("compare", 1, 2, 2, 1); __PYX_ERR(0, 492, __pyx_L3_error)
06003                         }
06004                     if (unlikely(kw_args > 0)) {
06005                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
pos_args, "compare") < 0)) __PYX_ERR(0, 492, __pyx_L3_error)
06006                     }
06007                     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
06008                         goto __pyx_L5_argtuple_error;
06009                     } else {
06010                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06011                         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06012                     }
06013                     __pyx_v_lhs = values[0];
06014                     __pyx_v_rhs = values[1];
06015                 }
06016                 goto __pyx_L4_argument_unpacking_done;
06017             __pyx_L5_argtuple_error:;
06018             __Pyx_RaiseArgtupleInvalid("compare", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
492, __pyx_L3_error)

```

```

06018     __pyx_L3_error;;
06019     __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
06020     __Pyx_RefNannyFinishContext();
06021     return NULL;
06022     __pyx_L4_argument_unpacking_done;;
06023     __pyx_r = __pyx_pf_8PyClical_2compare(__pyx_self, __pyx_v_lhs, __pyx_v_rhs);
06024
06025     /* function exit code */
06026     __Pyx_RefNannyFinishContext();
06027     return __pyx_r;
06028 }
06029
06030 static PyObject *__pyx_pf_8PyClical_2compare(CYTHON_UNUSED PyObject *__pyx_self, PyObject
06031 *__pyx_v_lhs, PyObject *__pyx_v_rhs) {
06032     PyObject *__pyx_r = NULL;
06033     __Pyx_RefNannyDeclarations
06034     PyObject *__pyx_t_1 = NULL;
06035     int __pyx_lineno = 0;
06036     const char *__pyx_filename = NULL;
06037     int __pyx_clineno = 0;
06038     __Pyx_RefNannySetupContext("compare", 0);
06039     __Pyx_XDECREF(__pyx_r);
06040     __pyx_t_1 = __pyx_f_8PyClical_compare(__pyx_v_lhs, __pyx_v_rhs, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 492, __pyx_L1_error)
06041     __Pyx_GOTREF(__pyx_t_1);
06042     __pyx_r = __pyx_t_1;
06043     __pyx_t_1 = 0;
06044     goto __pyx_L0;
06045
06046     /* function exit code */
06047     __Pyx_XDECREF(__pyx_t_1);
06048     __Pyx_AddTraceback("PyClical.compare", __pyx_clineno, __pyx_lineno, __pyx_filename);
06049     __pyx_r = NULL;
06050     __pyx_L0;;
06051     __Pyx_XGIVEREF(__pyx_r);
06052     __Pyx_RefNannyFinishContext();
06053     return __pyx_r;
06054 }
06055
06056     /* "PyClical.pyx":504
06057 *     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
06058 *
06059 * cpdef inline min_neg(obj):                # ««««««««
06060 *     """
06061 *     Minimum negative index, or 0 if none.
06062 */
06063
06064     static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06065 /*proto*/
06066 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_min_neg(PyObject *__pyx_v_obj, CYTHON_UNUSED
int __pyx_skip_dispatch) {
06067     PyObject *__pyx_r = NULL;
06068     __Pyx_RefNannyDeclarations
06069     PyObject *__pyx_t_1 = NULL;
06070     int __pyx_lineno = 0;
06071     const char *__pyx_filename = NULL;
06072     int __pyx_clineno = 0;
06073     __Pyx_RefNannySetupContext("min_neg", 0);
06074
06075     /* "PyClical.pyx":511
06076 *     0
06077 *     """
06078 *     return glucat.min_neg( toIndexSet(obj) )                # ««««««««
06079 * cpdef inline max_pos(obj):
06080 */
06081     __Pyx_XDECREF(__pyx_r);
06082     __pyx_t_1 = __Pyx_PyInt_From_int(min_neg(__pyx_f_8PyClical_toIndexSet(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 511, __pyx_L1_error)
06083     __Pyx_GOTREF(__pyx_t_1);
06084     __pyx_r = __pyx_t_1;
06085     __pyx_t_1 = 0;
06086     goto __pyx_L0;
06087
06088     /* "PyClical.pyx":504
06089 *     return glucat.compare( toIndexSet(lhs), toIndexSet(rhs) )
06090 *
06091 * cpdef inline min_neg(obj):                # ««««««««
06092 *     """
06093 *     Minimum negative index, or 0 if none.
06094 */
06095
06096     /* function exit code */
06097     __Pyx_L1_error;;
06098     __Pyx_XDECREF(__pyx_t_1);
06099     __Pyx_AddTraceback("PyClical.min_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);

```



```

06100         __pyx_r = 0;
06101         __pyx_L0:;
06102         __Pyx_XGIVEREF(__pyx_r);
06103         __Pyx_RefNannyFinishContext();
06104         return __pyx_r;
06105     }
06106
06107     /* Python wrapper */
06108     static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06109     /*proto*/
06110     static char __pyx_doc_8PyClical_4min_neg[] = "\n    Minimum negative index, or 0 if none.\n\n
>> min_neg(index_set({1,2}))\n    0\n    ";
06111     static PyObject *__pyx_pw_8PyClical_5min_neg(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
06112         PyObject *__pyx_r = 0;
06113         __Pyx_RefNannyDeclarations
06114         __Pyx_RefNannySetupContext("min_neg (wrapper)", 0);
06115         __pyx_r = __pyx_pf_8PyClical_4min_neg(__pyx_self, ((PyObject *)__pyx_v_obj));
06116
06117         /* function exit code */
06118         __Pyx_RefNannyFinishContext();
06119         return __pyx_r;
06120     }
06121
06122     static PyObject *__pyx_pf_8PyClical_4min_neg(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj) {
06123         PyObject *__pyx_r = NULL;
06124         __Pyx_RefNannyDeclarations
06125         PyObject *__pyx_t_1 = NULL;
06126         int __pyx_lineno = 0;
06127         const char *__pyx_filename = NULL;
06128         int __pyx_clineno = 0;
06129         __Pyx_RefNannySetupContext("min_neg", 0);
06130         __Pyx_XDECREF(__pyx_r);
06131         __pyx_t_1 = __pyx_f_8PyClical_min_neg(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 504, __pyx_L1_error)
06132         __Pyx_GOTREF(__pyx_t_1);
06133         __pyx_r = __pyx_t_1;
06134         __pyx_t_1 = 0;
06135         goto __pyx_L0;
06136
06137         /* function exit code */
06138         __pyx_L1_error:;
06139         __Pyx_XDECREF(__pyx_t_1);
06140         __Pyx_AddTraceback("PyClical.min_neg", __pyx_clineno, __pyx_lineno, __pyx_filename);
06141         __pyx_r = NULL;
06142         __pyx_L0:;
06143         __Pyx_XGIVEREF(__pyx_r);
06144         __Pyx_RefNannyFinishContext();
06145         return __pyx_r;
06146     }
06147
06148     /* "PyClical.pyx":513
*     return glucat.min_neg( toIndexSet(obj) )
06149 *
06150 * cpdef inline max_pos(obj):
*     """
06151 *     Maximum positive index, or 0 if none.
06152 *
06153 */
06154
06155     static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj);
06156     /*proto*/
06157     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_pos(PyObject *__pyx_v_obj, CYTHON_UNUSED
int __pyx_skip_dispatch) {
06158         PyObject *__pyx_r = NULL;
06159         __Pyx_RefNannyDeclarations
06160         PyObject *__pyx_t_1 = NULL;
06161         int __pyx_lineno = 0;
06162         const char *__pyx_filename = NULL;
06163         int __pyx_clineno = 0;
06164         __Pyx_RefNannySetupContext("max_pos", 0);
06165
06166         /* "PyClical.pyx":520
*     2
*     """
06167 *     return glucat.max_pos( toIndexSet(obj) )
*
06168 *
06169 * cdef inline vector[scalar_t] list_to_vector(lst):
06170 */
06171         __Pyx_XDECREF(__pyx_r);
06172         __pyx_t_1 = __Pyx_PyInt_From_int(max_pos(__pyx_f_8PyClical_toIndexSet(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 520, __pyx_L1_error)
06173         __Pyx_GOTREF(__pyx_t_1);
06174         __pyx_r = __pyx_t_1;
06175         __pyx_t_1 = 0;
06176         goto __pyx_L0;
06177
06178         /* "PyClical.pyx":513

```

```

06180 *      return glucat.min_neg( toIndexSet(obj) )
06181 *
06182 * cpdef inline max_pos(obj):          # ««««««««
06183 *      """
06184 *      Maximum positive index, or 0 if none.
06185 */
06186
06187     /* function exit code */
06188     __pyx_Ll_error++;
06189     __Pyx_XDECREF(__pyx_t_1);
06190     __Pyx_AddTraceback("PyClical.max_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
06191     __pyx_r = 0;
06192     __pyx_L0++;
06193     __Pyx_XGIVEREF(__pyx_r);
06194     __Pyx_RefNannyFinishContext();
06195     return __pyx_r;
06196 }
06197
06198 /* Python wrapper */
06199 static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj);
/*proto*/
06200 static char __pyx_doc_8PyClical_6max_pos[] = "\n      Maximum positive index, or 0 if none.\n\n
>> max_pos(index_set({1,2}))\n      2\n      ";
06201 static PyObject *__pyx_pw_8PyClical_7max_pos(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
06202     PyObject *__pyx_r = 0;
06203     __Pyx_RefNannyDeclarations
06204     __Pyx_RefNannySetupContext("max_pos (wrapper)", 0);
06205     __pyx_r = __pyx_pf_8PyClical_6max_pos(__pyx_self, ((PyObject *)__pyx_v_obj));
06206
06207     /* function exit code */
06208     __Pyx_RefNannyFinishContext();
06209     return __pyx_r;
06210 }
06211
06212 static PyObject *__pyx_pf_8PyClical_6max_pos(CYTHON_UNUSED PyObject *__pyx_self, PyObject
__pyx_v_obj) {
06213     PyObject *__pyx_r = NULL;
06214     __Pyx_RefNannyDeclarations
06215     PyObject *__pyx_t_1 = NULL;
06216     int __pyx_lineno = 0;
06217     const char *__pyx_filename = NULL;
06218     int __pyx_clineno = 0;
06219     __Pyx_RefNannySetupContext("max_pos", 0);
06220     __Pyx_XDECREF(__pyx_r);
06221     __pyx_t_1 = __pyx_f_8PyClical_max_pos(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 513, __pyx_Ll_error)
06222     __Pyx_GOTREF(__pyx_t_1);
06223     __pyx_r = __pyx_t_1;
06224     __pyx_t_1 = 0;
06225     goto __pyx_L0;
06226
06227     /* function exit code */
06228     __pyx_Ll_error++;
06229     __Pyx_XDECREF(__pyx_t_1);
06230     __Pyx_AddTraceback("PyClical.max_pos", __pyx_clineno, __pyx_lineno, __pyx_filename);
06231     __pyx_r = NULL;
06232     __pyx_L0++;
06233     __Pyx_XGIVEREF(__pyx_r);
06234     __Pyx_RefNannyFinishContext();
06235     return __pyx_r;
06236 }
06237
06238 /* "PyClical.pyx":522
06239 *      return glucat.max_pos( toIndexSet(obj) )
06240 *
06241 * cdef inline vector[scalar_t] list_to_vector(lst):          # ««««««««
06242 *      """
06243 *      Create a C++ std::vector[scalar_t] from an iterable Python object.
06244 */
06245
06246 static CYTHON_INLINE std::vector<scalar_t> __pyx_f_8PyClical_list_to_vector(PyObject
__pyx_v_lst) {
06247     std::vector<scalar_t> __pyx_v_v;
06248     PyObject *__pyx_v_s = NULL;
06249     std::vector<scalar_t> __pyx_r;
06250     __Pyx_RefNannyDeclarations
06251     PyObject *__pyx_t_1 = NULL;
06252     Py_ssize_t __pyx_t_2;
06253     PyObject *(*__pyx_t_3)(PyObject *);
06254     PyObject *__pyx_t_4 = NULL;
06255     scalar_t __pyx_t_5;
06256     int __pyx_lineno = 0;
06257     const char *__pyx_filename = NULL;
06258     int __pyx_clineno = 0;
06259     __Pyx_RefNannySetupContext("list_to_vector", 0);
06260
06261     /* "PyClical.pyx":527

```

```

06262 *      """
06263 *      cdef vector[scalar_t] v
06264 *      for s in lst:          # ««««««««
06265 *          v.push_back(<scalar_t>s)
06266 *      return v
06267 */
06268         if (likely(PyList_CheckExact(__pyx_v_lst)) || PyTuple_CheckExact(__pyx_v_lst)) {
06269             __pyx_t_1 = __pyx_v_lst; __Pyx_INCREF(__pyx_t_1); __pyx_t_2 = 0;
06270             __pyx_t_3 = NULL;
06271         } else {
06272             __pyx_t_2 = -1; __pyx_t_1 = PyObject_GetIter(__pyx_v_lst); if (unlikely(!__pyx_t_1))
06273             __PYX_ERR(0, 527, __pyx_L1_error)
06274             __Pyx_GOTREF(__pyx_t_1);
06275             __pyx_t_3 = Py_TYPE(__pyx_t_1)->tp_iternext; if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 527,
06276             __pyx_L1_error)
06277         }
06278         for (;;) {
06279             if (likely(!__pyx_t_3)) {
06280                 if (likely(PyList_CheckExact(__pyx_t_1))) {
06281                     if (__pyx_t_2 >= PyList_GET_SIZE(__pyx_t_1)) break;
06282                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
06283                     __pyx_t_4 = PyList_GET_ITEM(__pyx_t_1, __pyx_t_2); __Pyx_INCREF(__pyx_t_4);
06284                     __pyx_t_2++; if (unlikely(0 < 0)) __PYX_ERR(0, 527, __pyx_L1_error)
06285                     #else
06286                     __pyx_t_4 = PySequence_ITEM(__pyx_t_1, __pyx_t_2); __pyx_t_2++; if
06287                     (unlikely(!__pyx_t_4)) __PYX_ERR(0, 527, __pyx_L1_error)
06288                     __Pyx_GOTREF(__pyx_t_4);
06289                     #endif
06290                 } else {
06291                     if (__pyx_t_2 >= PyTuple_GET_SIZE(__pyx_t_1)) break;
06292                     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
06293                     __pyx_t_4 = PyTuple_GET_ITEM(__pyx_t_1, __pyx_t_2); __Pyx_INCREF(__pyx_t_4);
06294                     __pyx_t_2++; if (unlikely(0 < 0)) __PYX_ERR(0, 527, __pyx_L1_error)
06295                     #else
06296                     __pyx_t_4 = PySequence_ITEM(__pyx_t_1, __pyx_t_2); __pyx_t_2++; if
06297                     (unlikely(!__pyx_t_4)) __PYX_ERR(0, 527, __pyx_L1_error)
06298                     __Pyx_GOTREF(__pyx_t_4);
06299                     #endif
06300                 }
06301             } else {
06302                 __pyx_t_4 = __pyx_t_3(__pyx_t_1);
06303                 if (unlikely(!__pyx_t_4)) {
06304                     PyObject* exc_type = PyErr_Occurred();
06305                     if (exc_type) {
06306                         if (likely(__Pyx_PyErr_GivenExceptionMatches(exc_type, PyExc_StopIteration)))
06307                             PyErr_Clear();
06308                         else __PYX_ERR(0, 527, __pyx_L1_error)
06309                     }
06310                     break;
06311                 }
06312                 __Pyx_GOTREF(__pyx_t_4);
06313             }
06314             __Pyx_XDECREF_SET(__pyx_v_s, __pyx_t_4);
06315             __pyx_t_4 = 0;
06316         }
06317         /* "PyClical.pyx":528
06318         *      cdef vector[scalar_t] v
06319         *      for s in lst:
06320         *          v.push_back(<scalar_t>s)          # ««««««««
06321         *      return v
06322         */
06323         __pyx_t_5 = __pyx_PyFloat_AsDouble(__pyx_v_s); if (unlikely((__pyx_t_5 == ((scalar_t)-1))
06324         && PyErr_Occurred())) __PYX_ERR(0, 528, __pyx_L1_error)
06325         try {
06326             __pyx_v_v.push_back(((scalar_t)__pyx_t_5));
06327         } catch (...) {
06328             __Pyx_CppExn2PyErr();
06329             __PYX_ERR(0, 528, __pyx_L1_error)
06330         }
06331         /* "PyClical.pyx":527
06332         *      """
06333         *      cdef vector[scalar_t] v
06334         *      for s in lst:          # ««««««««
06335         *          v.push_back(<scalar_t>s)
06336         *      return v
06337         */
06338         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06339         /* "PyClical.pyx":529
06340         *      for s in lst:
06341         *          v.push_back(<scalar_t>s)
06342         *      return v          # ««««««««
06343         *      # Forward reference.

```

```

06341 */
06342     __pyx_r = __pyx_v_v;
06343     goto __pyx_L0;
06344
06345     /* "PyClical.pyx":522
06346 *     return glucat.max_pos( toIndexSet(obj) )
06347 *
06348 * cdef inline vector[scalar_t] list_to_vector(lst):           # ««««««««
06349 *     """
06350 *     Create a C++ std::vector[scalar_t] from an iterable Python object.
06351 */
06352
06353     /* function exit code */
06354     __pyx_L1_error:;
06355     __Pyx_XDECREF(__pyx_t_1);
06356     __Pyx_XDECREF(__pyx_t_4);
06357     __Pyx_WriteUnraisable("PyClical.list_to_vector", __pyx_clineno, __pyx_lineno,
__pyx_filename, 1, 0);
06358     __Pyx_pretend_to_initialize(&__pyx_r);
06359     __pyx_L0:;
06360     __Pyx_XDECREF(__pyx_v_s);
06361     __Pyx_RefNannyFinishContext();
06362     return __pyx_r;
06363 }
06364
06365     /* "PyClical.pyx":534
06366 * cdef class clifford
06367 *
06368 * cdef inline Clifford toClifford(obj):           # ««««««««
06369 *     return clifford(obj).instance[0]
06370 *
06371 */
06372
06373     static CYTHON_INLINE Clifford __pyx_f_8PyClical_toClifford(PyObject *__pyx_v_obj) {
06374         Clifford __pyx_r;
06375         __Pyx_RefNannyDeclarations
06376         PyObject *__pyx_t_1 = NULL;
06377         int __pyx_lineno = 0;
06378         const char *__pyx_filename = NULL;
06379         int __pyx_clineno = 0;
06380         __Pyx_RefNannySetupContext("toClifford", 0);
06381
06382     /* "PyClical.pyx":535
06383 *
06384 * cdef inline Clifford toClifford(obj):
06385 *     return clifford(obj).instance[0]           # ««««««««
06386 *
06387 * cdef class clifford:
06388 */
06389     __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 535, __pyx_L1_error)
06390     __Pyx_GOTREF(__pyx_t_1);
06391     __pyx_r = (((struct __pyx_obj_8PyClical_clifford *)__pyx_t_1)->instance[0]);
06392     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06393     goto __pyx_L0;
06394
06395     /* "PyClical.pyx":534
06396 * cdef class clifford
06397 *
06398 * cdef inline Clifford toClifford(obj):           # ««««««««
06399 *     return clifford(obj).instance[0]
06400 *
06401 */
06402
06403     /* function exit code */
06404     __pyx_L1_error:;
06405     __Pyx_XDECREF(__pyx_t_1);
06406     __Pyx_WriteUnraisable("PyClical.toClifford", __pyx_clineno, __pyx_lineno, __pyx_filename, 1,
0);
06407     __Pyx_pretend_to_initialize(&__pyx_r);
06408     __pyx_L0:;
06409     __Pyx_RefNannyFinishContext();
06410     return __pyx_r;
06411 }
06412
06413     /* "PyClical.pyx":543
06414 * cdef Clifford *instance # Wrapped instance of C++ class Clifford.
06415 *
06416 * cdef inline wrap(clifford self, Clifford other):           # ««««««««
06417 *     """
06418 *     Wrap an instance of the C++ class Clifford.
06419 */
06420
06421     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_8clifford_wrap(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, Clifford __pyx_v_other) {
06422     PyObject *__pyx_r = NULL;
06423     __Pyx_RefNannyDeclarations

```

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06424     __Pyx_RefNannySetupContext("wrap", 0);
06425
06426     /* "PyClical.pyx":547
06427     *     Wrap an instance of the C++ class Clifford.
06428     *     """
06429     *     self.instance[0] = other          # ««««««««
06430     *     return self
06431     *
06432     */
06433     (__pyx_v_self->instance[0]) = __pyx_v_other;
06434
06435     /* "PyClical.pyx":548
06436     *     """
06437     *     self.instance[0] = other
06438     *     return self          # ««««««««
06439     *
06440     * cdef inline Clifford unwrap(clifford self):
06441     */
06442     __Pyx_XDECREF(__pyx_r);
06443     __Pyx_INCREF((PyObject *)__pyx_v_self);
06444     __pyx_r = (PyObject *)__pyx_v_self;
06445     goto __pyx_L0;
06446
06447     /* "PyClical.pyx":543
06448     * cdef Clifford *instance # Wrapped instance of C++ class Clifford.
06449     *
06450     * cdef inline wrap(clifford self, Clifford other):          # ««««««««
06451     *     """
06452     *     Wrap an instance of the C++ class Clifford.
06453     */
06454
06455     /* function exit code */
06456     __pyx_L0:;
06457     __Pyx_XGIVEREF(__pyx_r);
06458     __Pyx_RefNannyFinishContext();
06459     return __pyx_r;
06460 }
06461
06462     /* "PyClical.pyx":550
06463     *     return self
06464     *
06465     * cdef inline Clifford unwrap(clifford self):          # ««««««««
06466     *     """
06467     *     Return the wrapped C++ Clifford instance.
06468     */
06469
06470     static CYTHON_INLINE Clifford __pyx_f_8PyClical_8clifford_unwrap(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
06471     Clifford __pyx_r;
06472     __Pyx_RefNannyDeclarations
06473     __Pyx_RefNannySetupContext("unwrap", 0);
06474
06475     /* "PyClical.pyx":554
06476     *     Return the wrapped C++ Clifford instance.
06477     *     """
06478     *     return self.instance[0]          # ««««««««
06479     *
06480     * cpdef copy(clifford self):
06481     */
06482     __pyx_r = (__pyx_v_self->instance[0]);
06483     goto __pyx_L0;
06484
06485     /* "PyClical.pyx":550
06486     *     return self
06487     *
06488     * cdef inline Clifford unwrap(clifford self):          # ««««««««
06489     *     """
06490     *     Return the wrapped C++ Clifford instance.
06491     */
06492
06493     /* function exit code */
06494     __pyx_L0:;
06495     __Pyx_RefNannyFinishContext();
06496     return __pyx_r;
06497 }
06498
06499     /* "PyClical.pyx":556
06500     *     return self.instance[0]
06501     *
06502     * cpdef copy(clifford self):          # ««««««««
06503     *     """
06504     *     Copy this clifford object.
06505     */
06506
06507     static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
06508     static PyObject *__pyx_f_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford

```

```

    *__pyx_v_self, int __pyx_skip_dispatch) {
06509     PyObject *__pyx_r = NULL;
06510     __Pyx_RefNannyDeclarations
06511     PyObject *__pyx_t_1 = NULL;
06512     PyObject *__pyx_t_2 = NULL;
06513     PyObject *__pyx_t_3 = NULL;
06514     PyObject *__pyx_t_4 = NULL;
06515     int __pyx_lineno = 0;
06516     const char *__pyx_filename = NULL;
06517     int __pyx_clineno = 0;
06518     __Pyx_RefNannySetupContext("copy", 0);
06519     /* Check if called by wrapper */
06520     if (unlikely(__pyx_skip_dispatch));
06521     /* Check if overridden in Python */
06522     else if (unlikely((Py_TYPE((PyObject *)__pyx_v_self)->tp_dictoffset != 0) ||
(Py_TYPE((PyObject *)__pyx_v_self)->tp_flags & (Py_TPFLAGS_IS_ABSTRACT | Py_TPFLAGS_HEAPTYPE)))) {
06523         #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06524             static PY_UINT64_T __pyx_tp_dict_version = __PYX_DICT_VERSION_INIT, __pyx_obj_dict_version
= __PYX_DICT_VERSION_INIT;
06525             if (unlikely(!__Pyx_object_dict_version_matches((PyObject *)__pyx_v_self),
__pyx_tp_dict_version, __pyx_obj_dict_version)) {
06526                 PY_UINT64_T __pyx_type_dict_guard = __Pyx_get_tp_dict_version((PyObject
*)__pyx_v_self));
06527                 #endif
06528                 __pyx_t_1 = __Pyx_PyObject_GetAttrStr((PyObject *)__pyx_v_self, __pyx_n_s_copy); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 556, __pyx_L1_error)
06529                 __Pyx_GOTREF(__pyx_t_1);
06530                 if (!PyCFunction_Check(__pyx_t_1) || (PyCFunction_GET_FUNCTION(__pyx_t_1) !=
(PyCFunction)(void*)__pyx_pw_8PyClical_8clifford_lcopy)) {
06531                     __Pyx_XDECREF(__pyx_r);
06532                     __Pyx_INCREF(__pyx_t_1);
06533                     __pyx_t_3 = __pyx_t_1; __pyx_t_4 = NULL;
06534                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_3))) {
06535                         __pyx_t_4 = PyMethod_GET_SELF(__pyx_t_3);
06536                         if (likely(__pyx_t_4)) {
06537                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
06538                             __Pyx_INCREF(__pyx_t_4);
06539                             __Pyx_INCREF(function);
06540                             __Pyx_DECREF_SET(__pyx_t_3, function);
06541                         }
06542                     }
06543                     __pyx_t_2 = (__pyx_t_4) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_4) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
06544                     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
06545                     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 556, __pyx_L1_error)
06546                     __Pyx_GOTREF(__pyx_t_2);
06547                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
06548                     __pyx_r = __pyx_t_2;
06549                     __pyx_t_2 = 0;
06550                     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06551                     goto __pyx_L0;
06552                 }
06553                 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06554                     __pyx_tp_dict_version = __Pyx_get_tp_dict_version((PyObject *)__pyx_v_self);
06555                     __pyx_obj_dict_version = __Pyx_get_object_dict_version((PyObject *)__pyx_v_self);
06556                     if (unlikely(__pyx_type_dict_guard != __pyx_tp_dict_version)) {
06557                         __pyx_tp_dict_version = __pyx_obj_dict_version = __PYX_DICT_VERSION_INIT;
06558                     }
06559                 #endif
06560                 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
06561                 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_PYTYPE_LOOKUP && CYTHON_USE_TYPE_SLOTS
06562             }
06563         #endif
06564     }
06565
06566     /* "PyClical.pyx":563
06567     *     {2}
06568     *     """
06569     *     return clifford(self) # ««««««««
06570     *
06571     *     def __cinit__(self, other = 0, ixt = None):
06572     */
06573     __Pyx_XDECREF(__pyx_r);
06574     __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
((PyObject *)__pyx_v_self); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 563, __pyx_L1_error)
06575     __Pyx_GOTREF(__pyx_t_1);
06576     __pyx_r = __pyx_t_1;
06577     __pyx_t_1 = 0;
06578     goto __pyx_L0;
06579
06580     /* "PyClical.pyx":556
06581     *     return self.instance[0]
06582     *
06583     *     cpdef copy(clifford self): # ««««««««
06584     *         """
06585     *         Copy this clifford object.
06586     */

```

```

06587
06588     /* function exit code */
06589     __pyx_L1_error++;
06590     __Pyx_XDECREF(__pyx_t_1);
06591     __Pyx_XDECREF(__pyx_t_2);
06592     __Pyx_XDECREF(__pyx_t_3);
06593     __Pyx_XDECREF(__pyx_t_4);
06594     __Pyx_AddTraceback("PyClical.clifford.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
06595     __pyx_r = 0;
06596     __pyx_L0++;
06597     __Pyx_XGIVEREF(__pyx_r);
06598     __Pyx_RefNannyFinishContext();
06599     return __pyx_r;
06600 }
06601
06602     /* Python wrapper */
06603     static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
06604     static char __pyx_doc_8PyClical_8clifford_copy[] = "\n          Copy this clifford object.\n\n
>> x=clifford(\"1{2}\"); y=x.copy(); print(y)\n          {2}\n          ";
06605     static PyObject *__pyx_pw_8PyClical_8clifford_1copy(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
06606         PyObject *__pyx_r = 0;
06607         __Pyx_RefNannyDeclarations
06608         __Pyx_RefNannySetupContext("copy (wrapper)", 0);
06609         __pyx_r = __pyx_pf_8PyClical_8clifford_copy(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
06610
06611         /* function exit code */
06612         __Pyx_RefNannyFinishContext();
06613         return __pyx_r;
06614     }
06615
06616     static PyObject *__pyx_pf_8PyClical_8clifford_copy(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
06617         PyObject *__pyx_r = NULL;
06618         __Pyx_RefNannyDeclarations
06619         PyObject *__pyx_t_1 = NULL;
06620         int __pyx_lineno = 0;
06621         const char *__pyx_filename = NULL;
06622         int __pyx_clineno = 0;
06623         __Pyx_RefNannySetupContext("copy", 0);
06624         __Pyx_XDECREF(__pyx_r);
06625         __pyx_t_1 = __pyx_f_8PyClical_8clifford_copy(__pyx_v_self, 1); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 556, __pyx_L1_error)
06626         __Pyx_GOTREF(__pyx_t_1);
06627         __pyx_r = __pyx_t_1;
06628         __pyx_t_1 = 0;
06629         goto __pyx_L0;
06630
06631         /* function exit code */
06632         __pyx_L1_error++;
06633         __Pyx_XDECREF(__pyx_t_1);
06634         __Pyx_AddTraceback("PyClical.clifford.copy", __pyx_clineno, __pyx_lineno, __pyx_filename);
06635         __pyx_r = NULL;
06636         __pyx_L0++;
06637         __Pyx_XGIVEREF(__pyx_r);
06638         __Pyx_RefNannyFinishContext();
06639         return __pyx_r;
06640     }
06641
06642     /* "PyClical.pyx":565
06643     *         return clifford(self)
06644     *
06645     * def __cinit__(self, other = 0, ixt = None):          # ««««««««
06646     *     """
06647     *     Construct an object of type clifford.
06648     */
06649
06650     /* Python wrapper */
06651     static int __pyx_pw_8PyClical_8clifford_3__cinit__(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
06652     static int __pyx_pw_8PyClical_8clifford_3__cinit__(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
06653         PyObject *__pyx_v_other = 0;
06654         PyObject *__pyx_v_ixt = 0;
06655         int __pyx_lineno = 0;
06656         const char *__pyx_filename = NULL;
06657         int __pyx_clineno = 0;
06658         int __pyx_r;
06659         __Pyx_RefNannyDeclarations
06660         __Pyx_RefNannySetupContext("__cinit__ (wrapper)", 0);
06661         {
06662             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_other,&__pyx_n_s_ixt,0};
06663             PyObject* values[2] = {0,0};
06664             values[0] = ((PyObject *)__pyx_int_0);
06665             values[1] = ((PyObject *)Py_None);

```

```

06666         if (unlikely(__pyx_kwds)) {
06667             Py_ssize_t kw_args;
06668             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
06669             switch (pos_args) {
06670                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06671                     CYTHON_FALLTHROUGH;
06672                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06673                     CYTHON_FALLTHROUGH;
06674                 case 0: break;
06675                 default: goto __pyx_L5_argtuple_error;
06676             }
06677             kw_args = PyDict_Size(__pyx_kwds);
06678             switch (pos_args) {
06679                 case 0:
06680                     if (kw_args > 0) {
06681                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_other);
06682                         if (value) { values[0] = value; kw_args--; }
06683                     }
06684                     CYTHON_FALLTHROUGH;
06685                 case 1:
06686                     if (kw_args > 0) {
06687                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_ixt);
06688                         if (value) { values[1] = value; kw_args--; }
06689                     }
06690             }
06691             if (unlikely(kw_args > 0)) {
06692                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
06693 pos_args, "__cinit__") < 0)) __PYX_ERR(0, 565, __pyx_L3_error)
06694             } else {
06695                 switch (PyTuple_GET_SIZE(__pyx_args)) {
06696                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
06697                         CYTHON_FALLTHROUGH;
06698                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
06699                         CYTHON_FALLTHROUGH;
06700                     case 0: break;
06701                     default: goto __pyx_L5_argtuple_error;
06702                 }
06703             }
06704             __pyx_v_other = values[0];
06705             __pyx_v_ixt = values[1];
06706         }
06707         goto __pyx_L4_argument_unpacking_done;
06708     __pyx_L5_argtuple_error:;
06709     __Pyx_RaiseArgtupleInvalid("__cinit__", 0, 0, 2, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
565, __pyx_L3_error)
06710     __pyx_L3_error:;
06711     __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
06712     __Pyx_RefNannyFinishContext();
06713     return -1;
06714     __pyx_L4_argument_unpacking_done:;
06715     __pyx_r = __pyx_pf_8PyClical_8clifford_2__cinit__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), __pyx_v_other, __pyx_v_ixt);
06716
06717     /* function exit code */
06718     __Pyx_RefNannyFinishContext();
06719     return __pyx_r;
06720 }
06721
06722     static int __pyx_pf_8PyClical_8clifford_2__cinit__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_other, PyObject *__pyx_v_ixt) {
06723         PyObject *__pyx_v_error_msg_prefix = NULL;
06724         PyObject *__pyx_v_bother = NULL;
06725         PyObject *__pyx_v_err = NULL;
06726         int __pyx_r;
06727         __Pyx_RefNannyDeclarations
06728         int __pyx_t_1;
06729         int __pyx_t_2;
06730         PyObject *__pyx_t_3 = NULL;
06731         PyObject *__pyx_t_4 = NULL;
06732         PyObject *__pyx_t_5 = NULL;
06733         Clifford *__pyx_t_6;
06734         PyObject *__pyx_t_7 = NULL;
06735         PyObject *__pyx_t_8 = NULL;
06736         scalar_t __pyx_t_9;
06737         PyObject *__pyx_t_10 = NULL;
06738         PyObject *__pyx_t_11 = NULL;
06739         PyObject *__pyx_t_12 = NULL;
06740         PyObject *__pyx_t_13 = NULL;
06741         char *__pyx_t_14;
06742         int __pyx_t_15;
06743         PyObject *__pyx_t_16 = NULL;
06744         PyObject *__pyx_t_17 = NULL;
06745         PyObject *__pyx_t_18 = NULL;
06746         int __pyx_t_19;
06747         char const *__pyx_t_20;

```



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06748 PyObject *__pyx_t_21 = NULL;
06749 PyObject *__pyx_t_22 = NULL;
06750 PyObject *__pyx_t_23 = NULL;
06751 int __pyx_lineno = 0;
06752 const char *__pyx_filename = NULL;
06753 int __pyx_clineno = 0;
06754 __Pyx_RefNannySetupContext("__cinit__", 0);
06755
06756 /* "PyClical.pyx":588
06757 *      2{1}+3{2}
06758 *      """
06759 *      error_msg_prefix = "Cannot initialize clifford object from"          # ««««««««
06760 *      if ixt is None:
06761 *          try:
06762 */
06763 __Pyx_INCREF(__pyx_kp_u_Cannot_initialize_clifford_objec);
06764 __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_initialize_clifford_objec;
06765
06766 /* "PyClical.pyx":589
06767 *      """
06768 *      error_msg_prefix = "Cannot initialize clifford object from"
06769 *      if ixt is None:          # ««««««««
06770 *          try:
06771 *              if isinstance(other, clifford):
06772 */
06773 __pyx_t_1 = (__pyx_v_ixt == Py_None);
06774 __pyx_t_2 = (__pyx_t_1 != 0);
06775 if (__pyx_t_2) {
06776
06777     /* "PyClical.pyx":590
06778     *      error_msg_prefix = "Cannot initialize clifford object from"
06779     *      if ixt is None:
06780     *          try:          # ««««««««
06781     *              if isinstance(other, clifford):
06782     *                  self.instance = new Clifford((<clifford>other).unwrap())
06783 */
06784     {
06785         __Pyx_PyThreadState_declare
06786         __Pyx_PyThreadState_assign
06787         __Pyx_ExceptionSave(&__pyx_t_3, &__pyx_t_4, &__pyx_t_5);
06788         __Pyx_XGOTREF(__pyx_t_3);
06789         __Pyx_XGOTREF(__pyx_t_4);
06790         __Pyx_XGOTREF(__pyx_t_5);
06791     /*try:*/ {
06792
06793         /* "PyClical.pyx":591
06794         *      if ixt is None:
06795         *          try:
06796         *              if isinstance(other, clifford):          # ««««««««
06797         *                  self.instance = new Clifford((<clifford>other).unwrap())
06798         *              elif isinstance(other, index_set):
06799         */
06800         __pyx_t_2 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_clifford);
06801         __pyx_t_1 = (__pyx_t_2 != 0);
06802         if (__pyx_t_1) {
06803
06804             /* "PyClical.pyx":592
06805             *      try:
06806             *          if isinstance(other, clifford):
06807             *              self.instance = new Clifford((<clifford>other).unwrap())          # ««««««««
06808             *          elif isinstance(other, index_set):
06809             *              self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)
06810             */
06811             try {
06812                 __pyx_t_6 = new Clifford(__pyx_f_8PyClical_8clifford_unwrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_other)));
06813             } catch (...) {
06814                 __Pyx_CppExn2PyErr();
06815                 __PYX_ERR(0, 592, __pyx_L4_error)
06816             }
06817             __pyx_v_self->instance = __pyx_t_6;
06818
06819             /* "PyClical.pyx":591
06820             *      if ixt is None:
06821             *          try:
06822             *              if isinstance(other, clifford):          # ««««««««
06823             *                  self.instance = new Clifford((<clifford>other).unwrap())
06824             *              elif isinstance(other, index_set):
06825             */
06826             goto __pyx_L10;
06827         }
06828
06829         /* "PyClical.pyx":593
06830         *      if isinstance(other, clifford):
06831         *          self.instance = new Clifford((<clifford>other).unwrap())
06832         *      elif isinstance(other, index_set):          # ««««««««
06833         *          self.instance = new Clifford((<index_set>other).unwrap(), <scalar_t>1.0)

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06834 *             elif isinstance(other, numbers.Real):
06835 */
06836         __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_other, __pyx_ptype_8PyClical_index_set);
06837         __pyx_t_2 = (__pyx_t_1 != 0);
06838         if (__pyx_t_2) {
06839
06840             /* "PyClical.pyx":594
06841 *             self.instance = new Clifford(<clifford>other).unwrap()
06842 *             elif isinstance(other, index_set):
06843 *             self.instance = new Clifford(<index_set>other).unwrap(), <scalar_t>1.0)
06844 # ««««««««
06844 *             elif isinstance(other, numbers.Real):
06845 *             self.instance = new Clifford(<scalar_t>other)
06846 */
06847             try {
06848                 __pyx_t_6 = new Clifford(__pyx_f_8PyClical_9index_set_unwrap(((struct
__pyx_obj_8PyClical_index_set *)__pyx_v_other)), ((scalar_t)1.0));
06849             } catch(...) {
06850                 __Pyx_CppExn2PyErr();
06851                 __PYX_ERR(0, 594, __pyx_L4_error)
06852             }
06853             __pyx_v_self->instance = __pyx_t_6;
06854
06855             /* "PyClical.pyx":593
06856 *             if isinstance(other, clifford):
06857 *             self.instance = new Clifford(<clifford>other).unwrap()
06858 *             elif isinstance(other, index_set):
06859 *             self.instance = new Clifford(<index_set>other).unwrap(), <scalar_t>1.0)
06860 *             elif isinstance(other, numbers.Real):
06861 */
06862             goto __pyx_L10;
06863         }
06864
06865         /* "PyClical.pyx":595
06866 *             elif isinstance(other, index_set):
06867 *             self.instance = new Clifford(<index_set>other).unwrap(), <scalar_t>1.0)
06868 *             elif isinstance(other, numbers.Real):
06869 *             self.instance = new Clifford(<scalar_t>other)
06870 *             elif isinstance(other, str):
06871 */
06872         __Pyx_GetModuleGlobalName(__pyx_t_7, __pyx_n_s_numbers); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 595, __pyx_L4_error)
06873         __Pyx_GOTREF(__pyx_t_7);
06874         __pyx_t_8 = __Pyx_PyObject_GetAttrStr(__pyx_t_7, __pyx_n_s_Real); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 595, __pyx_L4_error)
06875         __Pyx_GOTREF(__pyx_t_8);
06876         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
06877         __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_8); if (unlikely(__pyx_t_2 ==
((int)-1))) __PYX_ERR(0, 595, __pyx_L4_error)
06878         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
06879         __pyx_t_1 = (__pyx_t_2 != 0);
06880         if (__pyx_t_1) {
06881
06882             /* "PyClical.pyx":596
06883 *             self.instance = new Clifford(<index_set>other).unwrap(), <scalar_t>1.0)
06884 *             elif isinstance(other, numbers.Real):
06885 *             self.instance = new Clifford(<scalar_t>other)
06886 *             elif isinstance(other, str):
06887 *             try:
06888 */
06889         __pyx_t_9 = __pyx_PyFloat_AsDouble(__pyx_v_other); if (unlikely((__pyx_t_9 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 596, __pyx_L4_error)
06890         try {
06891             __pyx_t_6 = new Clifford(((scalar_t)__pyx_t_9));
06892         } catch(...) {
06893             __Pyx_CppExn2PyErr();
06894             __PYX_ERR(0, 596, __pyx_L4_error)
06895         }
06896         __pyx_v_self->instance = __pyx_t_6;
06897
06898         /* "PyClical.pyx":595
06899 *             elif isinstance(other, index_set):
06900 *             self.instance = new Clifford(<index_set>other).unwrap(), <scalar_t>1.0)
06901 *             elif isinstance(other, numbers.Real):
06902 *             self.instance = new Clifford(<scalar_t>other)
06903 *             elif isinstance(other, str):
06904 */
06905         goto __pyx_L10;
06906     }
06907
06908     /* "PyClical.pyx":597
06909 *             elif isinstance(other, numbers.Real):
06910 *             self.instance = new Clifford(<scalar_t>other)
06911 *             elif isinstance(other, str):
06912 *             try:
06913 *             bother = other.encode("UTF-8")
06914 */

```

```

06915         __pyx_t_1 = PyUnicode_Check(__pyx_v_other);
06916         __pyx_t_2 = (__pyx_t_1 != 0);
06917         if (likely(__pyx_t_2)) {
06918
06919             /* "PyClical.pyx":598
06920             *         self.instance = new Clifford(<scalar_t>other)
06921             *         elif isinstance(other, str):
06922             *             try:
06923             *                 # ««««««««
06924             *                 bother = other.encode("UTF-8")
06925             *                 self.instance = new Clifford(<char *>bother)
06926             */
06927             {
06928                 __Pyx_PyThreadState_declare
06929                 __Pyx_PyThreadState_assign
06930                 __Pyx_ExceptionSave(&__pyx_t_10, &__pyx_t_11, &__pyx_t_12);
06931                 __Pyx_XGOTREF(__pyx_t_10);
06932                 __Pyx_XGOTREF(__pyx_t_11);
06933                 __Pyx_XGOTREF(__pyx_t_12);
06934                 /*try:*/ {
06935
06936                     /* "PyClical.pyx":599
06937                     *         elif isinstance(other, str):
06938                     *             try:
06939                     *                 bother = other.encode("UTF-8")
06940                     *                 self.instance = new Clifford(<char *>bother)
06941                     *             except RuntimeError:
06942                     */
06943                     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_v_other, __pyx_n_s_encode); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 599, __pyx_L11_error)
06944                     __Pyx_GOTREF(__pyx_t_7);
06945                     __pyx_t_13 = NULL;
06946                     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_7))) {
06947                         __pyx_t_13 = PyMethod_GET_SELF(__pyx_t_7);
06948                         if (likely(__pyx_t_13)) {
06949                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_7);
06950                             __Pyx_INCREF(__pyx_t_13);
06951                             __Pyx_INCREF(function);
06952                             __Pyx_DECREF_SET(__pyx_t_7, function);
06953                         }
06954                     }
06955                     __pyx_t_8 = (__pyx_t_13) ? __Pyx_PyObject_Call2Args(__pyx_t_7, __pyx_t_13,
__pyx_kp_u_UTF_8) : __Pyx_PyObject_CallOneArg(__pyx_t_7, __pyx_kp_u_UTF_8);
06956                     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
06957                     if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 599, __pyx_L11_error)
06958                     __Pyx_GOTREF(__pyx_t_8);
06959                     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
06960                     __pyx_v_bother = __pyx_t_8;
06961                     __pyx_t_8 = 0;
06962
06963                     /* "PyClical.pyx":600
06964                     *             try:
06965                     *                 bother = other.encode("UTF-8")
06966                     *                 self.instance = new Clifford(<char *>bother)
06967                     *             except RuntimeError:
06968                     *                 raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
06969                     */
06970                     __pyx_t_14 = __Pyx_PyObject_AsWritableString(__pyx_v_bother); if
(unlikely((!__pyx_t_14) && PyErr_Occurred())) __PYX_ERR(0, 600, __pyx_L11_error)
06971                     try {
06972                         __pyx_t_6 = new Clifford(((char *)__pyx_t_14));
06973                     } catch (...) {
06974                         __Pyx_CppExn2PyErr();
06975                         __PYX_ERR(0, 600, __pyx_L11_error)
06976                     }
06977                     __pyx_v_self->instance = __pyx_t_6;
06978
06979                     /* "PyClical.pyx":598
06980                     *         self.instance = new Clifford(<scalar_t>other)
06981                     *         elif isinstance(other, str):
06982                     *             try:
06983                     *                 # ««««««««
06984                     *                 bother = other.encode("UTF-8")
06985                     *                 self.instance = new Clifford(<char *>bother)
06986                     */
06987                     {
06988                         __Pyx_XDECREF(__pyx_t_10); __pyx_t_10 = 0;
06989                         __Pyx_XDECREF(__pyx_t_11); __pyx_t_11 = 0;
06990                         __Pyx_XDECREF(__pyx_t_12); __pyx_t_12 = 0;
06991                         goto __pyx_L16_try_end;
06992                     }
06993                     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
06994                     __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
06995                     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
06996
06997                     /* "PyClical.pyx":601
06998                     *             bother = other.encode("UTF-8")
06999                     *             self.instance = new Clifford(<char *>bother)
07000                     *             except RuntimeError:
07001                     *                 # ««««««««

```

```

06999 *             raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07000 *         else:
07001 */
07002             __pyx_t_15 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07003             if (__pyx_t_15) {
07004                 __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07005                 if (__Pyx_GetException(&__pyx_t_8, &__pyx_t_7, &__pyx_t_13) < 0) __PYX_ERR(0,
601, __pyx_L13_except_error)
07006                 __Pyx_GOTREF(__pyx_t_8);
07007                 __Pyx_GOTREF(__pyx_t_7);
07008                 __Pyx_GOTREF(__pyx_t_13);
07009
07010                 /* "PyClical.pyx":602
07011                  self.instance = new Clifford(<char *>bother)
07012                  except RuntimeError:
07013                      raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07014                  # ««««««««
07015                  else:
07016                      raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07017                  */
07018                 __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix,
__pyx_kp_u_invalid_string); if (unlikely(!__pyx_t_16)) __PYX_ERR(0, 602, __pyx_L13_except_error)
07019                 __Pyx_GOTREF(__pyx_t_16);
07020                 __pyx_t_17 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_17))
__PYX_ERR(0, 602, __pyx_L13_except_error)
07021                 __Pyx_GOTREF(__pyx_t_17);
07022                 __pyx_t_18 = PyNumber_Add(__pyx_t_16, __pyx_t_17); if (unlikely(!__pyx_t_18))
__PYX_ERR(0, 602, __pyx_L13_except_error)
07023                 __Pyx_GOTREF(__pyx_t_18);
07024                 __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07025                 __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07026                 __pyx_t_17 = PyNumber_Add(__pyx_t_18, __pyx_kp_u); if (unlikely(!__pyx_t_17))
__PYX_ERR(0, 602, __pyx_L13_except_error)
07027                 __Pyx_GOTREF(__pyx_t_17);
07028                 __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07029                 __pyx_t_18 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_17); if
(unlikely(!__pyx_t_18)) __PYX_ERR(0, 602, __pyx_L13_except_error)
07030                 __Pyx_GOTREF(__pyx_t_18);
07031                 __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07032                 __Pyx_Raise(__pyx_t_18, 0, 0, 0);
07033                 __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07034                 __PYX_ERR(0, 602, __pyx_L13_except_error)
07035             }
07036             goto __pyx_L13_except_error;
07037             __pyx_L13_except_error;;
07038
07039             /* "PyClical.pyx":598
07040              self.instance = new Clifford(<scalar_t>other)
07041              elif isinstance(other, str):
07042                  try:
07043                      # ««««««««
07044                      bother = other.encode("UTF-8")
07045                      self.instance = new Clifford(<char *>bother)
07046                  */
07047                 __Pyx_XGIVEREF(__pyx_t_10);
07048                 __Pyx_XGIVEREF(__pyx_t_11);
07049                 __Pyx_XGIVEREF(__pyx_t_12);
07050                 __Pyx_ExceptionReset(__pyx_t_10, __pyx_t_11, __pyx_t_12);
07051                 goto __pyx_L4_error;
07052                 __pyx_L16_try_end;;
07053             }
07054
07055             /* "PyClical.pyx":597
07056              elif isinstance(other, numbers.Real):
07057                  self.instance = new Clifford(<scalar_t>other)
07058              elif isinstance(other, str):
07059                  # ««««««««
07060                  try:
07061                      bother = other.encode("UTF-8")
07062                  */
07063                 goto __pyx_L10;
07064             }
07065
07066             /* "PyClical.pyx":604
07067              raise ValueError(error_msg_prefix + " invalid string " + repr(other) + ".")
07068              else:
07069                  raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07070              #
07071              ««««««««
07072              except RuntimeError as err:
07073                  raise ValueError(error_msg_prefix + " " + str(type(other))
07074                  */
07075
07076              /*else*/ {
07077                 __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 604, __pyx_L4_error)
07078                 __Pyx_GOTREF(__pyx_t_13);
07079                 __pyx_t_7 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 604, __pyx_L4_error)
07080                 __Pyx_GOTREF(__pyx_t_7);

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07075     __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_t_7); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 604, __pyx_L4_error)
07076     __Pyx_GOTREF(__pyx_t_8);
07077     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07078     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07079     __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_kp_u_); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 604, __pyx_L4_error)
07080     __Pyx_GOTREF(__pyx_t_7);
07081     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07082     __pyx_t_8 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_7); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 604, __pyx_L4_error)
07083     __Pyx_GOTREF(__pyx_t_8);
07084     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07085     __Pyx_Raise(__pyx_t_8, 0, 0, 0);
07086     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07087     __PYX_ERR(0, 604, __pyx_L4_error)
07088 }
07089 __pyx_L10::
07090
07091     /* "PyClical.pyx":590
07092     *     error_msg_prefix = "Cannot initialize clifford object from"
07093     *     if ixt is None:
07094     *         try:
07095     *             # ««««««««
07096     *             if isinstance(other, clifford):
07097     *                 self.instance = new Clifford(<clifford>other).unwrap()
07098     */
07099     }
07100     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
07101     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
07102     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
07103     goto __pyx_L9_try_end;
07104     __pyx_L4_error:;
07105     __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
07106     __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
07107     __Pyx_XDECREF(__pyx_t_17); __pyx_t_17 = 0;
07108     __Pyx_XDECREF(__pyx_t_18); __pyx_t_18 = 0;
07109     __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
07110     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
07111
07112     /* "PyClical.pyx":605
07113     *     else:
07114     *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07115     *     except RuntimeError as err:
07116     *         # ««««««««
07117     *         raise ValueError(error_msg_prefix + " " + str(type(other))
07118     *             + " value " + repr(other) + ":",
07119     *             + "\n\t" + str(err))
07120     */
07121     __pyx_t_15 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07122     if (__pyx_t_15) {
07123         __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07124         if (__Pyx_GetException(&__pyx_t_8, &__pyx_t_7, &__pyx_t_13) < 0) __PYX_ERR(0, 605,
__pyx_L6_except_error)
07125         __Pyx_GOTREF(__pyx_t_8);
07126         __Pyx_GOTREF(__pyx_t_7);
07127         __Pyx_GOTREF(__pyx_t_13);
07128         __Pyx_INCREF(__pyx_t_7);
07129         __pyx_v_err = __pyx_t_7;
07130         /*try:*/ {
07131
07132             /* "PyClical.pyx":606
07133             *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07134             *     except RuntimeError as err:
07135             *         raise ValueError(error_msg_prefix + " " + str(type(other))
07136             *             + " value " + repr(other) + ":",
07137             *             + "\n\t" + str(err))
07138             *         # ««««««««
07139             *         + "\n\t" + str(err))
07140             */
07141             __pyx_t_18 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_2); if
(unlikely(!__pyx_t_18)) __PYX_ERR(0, 606, __pyx_L24_error)
07142             __Pyx_GOTREF(__pyx_t_18);
07143             __pyx_t_17 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_17)) __PYX_ERR(0, 606, __pyx_L24_error)
07144             __Pyx_GOTREF(__pyx_t_17);
07145             __pyx_t_16 = __Pyx_PyUnicode_Concat(__pyx_t_18, __pyx_t_17); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 606, __pyx_L24_error)
07146             __Pyx_GOTREF(__pyx_t_16);
07147             __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07148             __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07149
07150             /* "PyClical.pyx":607
07151             *     except RuntimeError as err:
07152             *         raise ValueError(error_msg_prefix + " " + str(type(other))
07153             *             + " value " + repr(other) + ":",
07154             *             + "\n\t" + str(err))
07155             *         elif isinstance(ixt, index_set):
07156             *
07157             */
07158             __pyx_t_17 = __Pyx_PyUnicode_Concat(__pyx_t_16, __pyx_kp_u_value); if

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        (unlikely(!__pyx_t_17)) __PYX_ERR(0, 607, __pyx_L24_error)
07153     __Pyx_GOTREF(__pyx_t_17);
07154     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07155     __pyx_t_16 = PyObject_Repr(__pyx_v_other); if (unlikely(!__pyx_t_16)) __PYX_ERR(0,
        607, __pyx_L24_error)
07156     __Pyx_GOTREF(__pyx_t_16);
07157     __pyx_t_18 = PyNumber_Add(__pyx_t_17, __pyx_t_16); if (unlikely(!__pyx_t_18))
__PYX_ERR(0, 607, __pyx_L24_error)
07158     __Pyx_GOTREF(__pyx_t_18);
07159     __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07160     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07161     __pyx_t_16 = PyNumber_Add(__pyx_t_18, __pyx_kp_u_5); if (unlikely(!__pyx_t_16))
__PYX_ERR(0, 607, __pyx_L24_error)
07162     __Pyx_GOTREF(__pyx_t_16);
07163     __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07164
07165     /* "PyClical.pyx":608
07166     *         raise ValueError(error_msg_prefix + " " + str(type(other))
07167     *                         + " value " + repr(other) + ":"
07168     *                         + "\n\t" + str(err)) # ««««««««
07169     *     elif isinstance(ixt, index_set):
07170     *         if isinstance(other, numbers.Real):
07171     */
07172     __pyx_t_18 = PyNumber_Add(__pyx_t_16, __pyx_kp_u_6); if (unlikely(!__pyx_t_18))
__PYX_ERR(0, 608, __pyx_L24_error)
07173     __Pyx_GOTREF(__pyx_t_18);
07174     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07175     __pyx_t_16 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_err); if (unlikely(!__pyx_t_16)) __PYX_ERR(0, 608, __pyx_L24_error)
07176     __Pyx_GOTREF(__pyx_t_16);
07177     __pyx_t_17 = PyNumber_Add(__pyx_t_18, __pyx_t_16); if (unlikely(!__pyx_t_17))
__PYX_ERR(0, 608, __pyx_L24_error)
07178     __Pyx_GOTREF(__pyx_t_17);
07179     __Pyx_DECREF(__pyx_t_18); __pyx_t_18 = 0;
07180     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07181
07182     /* "PyClical.pyx":606
07183     *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07184     *     except RuntimeError as err:
07185     *         raise ValueError(error_msg_prefix + " " + str(type(other)) # ««««««««
07186     *                         + " value " + repr(other) + ":"
07187     *                         + "\n\t" + str(err))
07188     */
07189     __pyx_t_16 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_17); if
(unlikely(!__pyx_t_16)) __PYX_ERR(0, 606, __pyx_L24_error)
07190     __Pyx_GOTREF(__pyx_t_16);
07191     __Pyx_DECREF(__pyx_t_17); __pyx_t_17 = 0;
07192     __Pyx_Raise(__pyx_t_16, 0, 0, 0);
07193     __Pyx_DECREF(__pyx_t_16); __pyx_t_16 = 0;
07194     __PYX_ERR(0, 606, __pyx_L24_error)
07195 }
07196
07197     /* "PyClical.pyx":605
07198     *     else:
07199     *         raise TypeError(error_msg_prefix + " " + str(type(other)) + ".")
07200     *     except RuntimeError as err: # ««««««««
07201     *         raise ValueError(error_msg_prefix + " " + str(type(other))
07202     *                         + " value " + repr(other) + ":"
07203     */
07204     /*finally:*/ {
07205     __pyx_L24_error:;
07206     /*exception exit:*/{
07207     __Pyx_PyThreadState_declare
07208     __Pyx_PyThreadState_assign
07209     __pyx_t_12 = 0; __pyx_t_11 = 0; __pyx_t_10 = 0; __pyx_t_21 = 0; __pyx_t_22 = 0;
__pyx_t_23 = 0;
07210     __Pyx_XDECREF(__pyx_t_16); __pyx_t_16 = 0;
07211     __Pyx_XDECREF(__pyx_t_17); __pyx_t_17 = 0;
07212     __Pyx_XDECREF(__pyx_t_18); __pyx_t_18 = 0;
07213     if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_21, &__pyx_t_22,
&__pyx_t_23);
07214     if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_12,
&__pyx_t_11, &__pyx_t_10) < 0)) __Pyx_ErrFetch(&__pyx_t_12, &__pyx_t_11, &__pyx_t_10);
07215     __Pyx_XGOTREF(__pyx_t_12);
07216     __Pyx_XGOTREF(__pyx_t_11);
07217     __Pyx_XGOTREF(__pyx_t_10);
07218     __Pyx_XGOTREF(__pyx_t_21);
07219     __Pyx_XGOTREF(__pyx_t_22);
07220     __Pyx_XGOTREF(__pyx_t_23);
07221     __pyx_t_15 = __pyx_lineno; __pyx_t_19 = __pyx_clineno; __pyx_t_20 =
__pyx_filename;
07222     {
07223     __Pyx_DECREF(__pyx_v_err);
07224     __pyx_v_err = NULL;
07225     }
07226     if (PY_MAJOR_VERSION >= 3) {
07227     __Pyx_XGIVEREF(__pyx_t_21);

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07228         __Pyx_XGIVEREF(__pyx_t_22);
07229         __Pyx_XGIVEREF(__pyx_t_23);
07230         __Pyx_ExceptionReset(__pyx_t_21, __pyx_t_22, __pyx_t_23);
07231     }
07232     __Pyx_XGIVEREF(__pyx_t_12);
07233     __Pyx_XGIVEREF(__pyx_t_11);
07234     __Pyx_XGIVEREF(__pyx_t_10);
07235     __Pyx_ErrRestore(__pyx_t_12, __pyx_t_11, __pyx_t_10);
07236     __pyx_t_12 = 0; __pyx_t_11 = 0; __pyx_t_10 = 0; __pyx_t_21 = 0; __pyx_t_22 = 0;
__pyx_t_23 = 0;
07237     __pyx_lineno = __pyx_t_15; __pyx_clineno = __pyx_t_19; __pyx_filename =
__pyx_t_20;
07238         goto __pyx_L6_except_error;
07239     }
07240 }
07241 }
07242 goto __pyx_L6_except_error;
07243 __pyx_L6_except_error;;
07244
07245     /* "PyClical.pyx":590
07246     * error_msg_prefix = "Cannot initialize clifford object from"
07247     * if ixt is None:
07248     *     try:
07249     *         # ««««««««
07250     *         if isinstance(other, clifford):
07251     *             self.instance = new Clifford((<clifford>other).unwrap())
07252     */
07252     __Pyx_XGIVEREF(__pyx_t_3);
07253     __Pyx_XGIVEREF(__pyx_t_4);
07254     __Pyx_XGIVEREF(__pyx_t_5);
07255     __Pyx_ExceptionReset(__pyx_t_3, __pyx_t_4, __pyx_t_5);
07256     goto __pyx_L1_error;
07257     __pyx_L9_try_end;;
07258 }
07259
07260     /* "PyClical.pyx":589
07261     * """
07262     * error_msg_prefix = "Cannot initialize clifford object from"
07263     * if ixt is None:
07264     *     try:
07265     *         # ««««««««
07266     *         if isinstance(other, clifford):
07267     */
07267     goto __pyx_L3;
07268 }
07269
07270     /* "PyClical.pyx":609
07271     *
07272     * + " value " + repr(other) + ":"
07273     * + "\n\t" + str(err))
07274     * # ««««««««
07275     * elif isinstance(ixt, index_set):
07276     *     if isinstance(other, numbers.Real):
07277     *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07278     */
07277 __pyx_t_2 = __Pyx_TypeCheck(__pyx_v_ixt, __pyx_ptype_8PyClical_index_set);
07278 __pyx_t_1 = (__pyx_t_2 != 0);
07279 if (likely(__pyx_t_1)) {
07280
07281     /* "PyClical.pyx":610
07282     *
07283     * + "\n\t" + str(err))
07284     * elif isinstance(ixt, index_set):
07285     *     if isinstance(other, numbers.Real):
07286     *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07287     *     elif isinstance(other, collections.abc.Sequence):
07288     */
07288     __Pyx_GetModuleGlobalName(__pyx_t_13, __pyx_n_s_numbers); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 610, __pyx_L1_error)
07289     __Pyx_GOTREF(__pyx_t_13);
07290     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_t_13, __pyx_n_s_Real); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 610, __pyx_L1_error)
07291     __Pyx_GOTREF(__pyx_t_7);
07292     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07293     __pyx_t_1 = PyObject_IsInstance(__pyx_v_other, __pyx_t_7); if (unlikely(__pyx_t_1 ==
((int)-1))) __PYX_ERR(0, 610, __pyx_L1_error)
07294     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07295     __pyx_t_2 = (__pyx_t_1 != 0);
07296     if (__pyx_t_2) {
07297
07298     /* "PyClical.pyx":611
07299     * elif isinstance(ixt, index_set):
07300     *     if isinstance(other, numbers.Real):
07301     *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
07302     * # ««««««««
07303     * elif isinstance(other, collections.abc.Sequence):
07304     *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07305     */
07305     __pyx_t_9 = __pyx_PyFloat_AsDouble(__pyx_v_other); if (unlikely((__pyx_t_9 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 611, __pyx_L1_error)
07306     try {
07307         __pyx_t_6 = new Clifford(__pyx_f_8PyClical_9index_set_unwrap((struct

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__pyx_obj_8PyClical_index_set *)__pyx_v_ixt)), ((scalar_t)__pyx_t_9));
77308         } catch(...) {
77309             __Pyx_CppExn2PyErr();
77310             __PYX_ERR(0, 611, __pyx_L1_error)
77311         }
77312         __pyx_v_self->instance = __pyx_t_6;
77313
77314         /* "PyClical.pyx":610
77315          *
77316          * elif isinstance(ixt, index_set):
77317          *     if isinstance(other, numbers.Real):
77318          *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
77319          *     elif isinstance(other, collections.abc.Sequence):
77320          */
77321         goto __pyx_L30;
77322     }
77323
77324     /* "PyClical.pyx":612
77325     *     if isinstance(other, numbers.Real):
77326     *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
77327     *     elif isinstance(other, collections.abc.Sequence):
77328     *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
77329     *     else:
77330     */
77331     __Pyx_GetModuleGlobalName(__pyx_t_7, __pyx_n_s_collections); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 612, __pyx_L1_error)
77332     __Pyx_GOTREF(__pyx_t_7);
77333     __pyx_t_13 = __Pyx_PyObject_GetAttrStr(__pyx_t_7, __pyx_n_s_abc); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 612, __pyx_L1_error)
77334     __Pyx_GOTREF(__pyx_t_13);
77335     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
77336     __pyx_t_7 = __Pyx_PyObject_GetAttrStr(__pyx_t_13, __pyx_n_s_Sequence); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 612, __pyx_L1_error)
77337     __Pyx_GOTREF(__pyx_t_7);
77338     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
77339     __pyx_t_2 = PyObject_IsInstance(__pyx_v_other, __pyx_t_7); if (unlikely(__pyx_t_2 ==
((int)-1))) __PYX_ERR(0, 612, __pyx_L1_error)
77340     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
77341     __pyx_t_1 = (__pyx_t_2 != 0);
77342     if (likely(__pyx_t_1)) {
77343
77344         /* "PyClical.pyx":613
77345          *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
77346          *     elif isinstance(other, collections.abc.Sequence):
77347          *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
77348          *     else:
77349          *         raise TypeError(error_msg_prefix + " (" + str(type(other))
77350          */
77351         try {
77352             __pyx_t_6 = new Clifford(__pyx_f_8PyClical_list_to_vector(__pyx_v_other),
__pyx_f_8PyClical_9index_set_unwrap(((struct __pyx_obj_8PyClical_index_set *)__pyx_v_ixt)));
77353         } catch(...) {
77354             __Pyx_CppExn2PyErr();
77355             __PYX_ERR(0, 613, __pyx_L1_error)
77356         }
77357         __pyx_v_self->instance = __pyx_t_6;
77358
77359         /* "PyClical.pyx":612
77360          *     if isinstance(other, numbers.Real):
77361          *         self.instance = new Clifford((<index_set>ixt).unwrap(), <scalar_t>other)
77362          *     elif isinstance(other, collections.abc.Sequence):
77363          *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
77364          *     else:
77365          */
77366         goto __pyx_L30;
77367     }
77368
77369     /* "PyClical.pyx":615
77370     *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
77371     *     else:
77372     *         raise TypeError(error_msg_prefix + " (" + str(type(other))
77373     *             + ", " + repr(ixt) + ").")
77374     *     else:
77375     */
77376     /*else*/ {
77377         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_7); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 615, __pyx_L1_error)
77378         __Pyx_GOTREF(__pyx_t_7);
77379         __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 615, __pyx_L1_error)
77380         __Pyx_GOTREF(__pyx_t_13);
77381         __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_t_13); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 615, __pyx_L1_error)
77382         __Pyx_GOTREF(__pyx_t_8);
77383         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
77384         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;

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07385
07386         /* "PyClical.pyx":616
07387         *         else:
07388         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07389         *                 + ", " + repr(ixt) + ").") # ««««««««
07390         *         else:
07391         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07392         */
07393         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_kp_u_8); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 616, __pyx_L1_error)
07394         __Pyx_GOTREF(__pyx_t_13);
07395         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07396         __pyx_t_8 = PyObject_Repr(__pyx_v_ixt); if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 616,
__pyx_L1_error)
07397         __Pyx_GOTREF(__pyx_t_8);
07398         __pyx_t_7 = PyNumber_Add(__pyx_t_13, __pyx_t_8); if (unlikely(!__pyx_t_7)) __PYX_ERR(0,
616, __pyx_L1_error)
07399         __Pyx_GOTREF(__pyx_t_7);
07400         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07401         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07402         __pyx_t_8 = PyNumber_Add(__pyx_t_7, __pyx_kp_u_9); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 616, __pyx_L1_error)
07403         __Pyx_GOTREF(__pyx_t_8);
07404         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07405
07406         /* "PyClical.pyx":615
07407         *         self.instance = new Clifford(list_to_vector(other), (<index_set>ixt).unwrap())
07408         *         else:
07409         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07410         *                 + ", " + repr(ixt) + ").") # ««««««««
07411         *         else:
07412         */
07413         __pyx_t_7 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_8); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 615, __pyx_L1_error)
07414         __Pyx_GOTREF(__pyx_t_7);
07415         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07416         __Pyx_Raise(__pyx_t_7, 0, 0, 0);
07417         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07418         __PYX_ERR(0, 615, __pyx_L1_error)
07419     }
07420     __pyx_L30:;
07421
07422     /* "PyClical.pyx":609
07423     *         + " value " + repr(other) + ":"
07424     *         + "\n\t" + str(err))
07425     *         elif isinstance(ixt, index_set): # ««««««««
07426     *             if isinstance(other, numbers.Real):
07427     *                 self.instance = new Clifford(<index_set>ixt).unwrap(), <scalar_t>other)
07428     */
07429     goto __pyx_L3;
07430 }
07431
07432     /* "PyClical.pyx":618
07433     *         + ", " + repr(ixt) + ").")
07434     *         else:
07435     *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07436     *                 + ", " + str(type(ixt)) + ").") # ««««««««
07437     *
07438     */
07439     /*else*/ {
07440         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_7); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 618, __pyx_L1_error)
07441         __Pyx_GOTREF(__pyx_t_7);
07442         __pyx_t_8 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_other))); if (unlikely(!__pyx_t_8)) __PYX_ERR(0, 618, __pyx_L1_error)
07443         __Pyx_GOTREF(__pyx_t_8);
07444         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_t_8); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 618, __pyx_L1_error)
07445         __Pyx_GOTREF(__pyx_t_13);
07446         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07447         __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07448
07449         /* "PyClical.pyx":619
07450         *         else:
07451         *             raise TypeError(error_msg_prefix + " (" + str(type(other))
07452         *                 + ", " + str(type(ixt)) + ").") # ««««««««
07453         *
07454         *         def __dealloc__(self):
07455         */
07456         __pyx_t_8 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_8); if (unlikely(!__pyx_t_8))
__PYX_ERR(0, 619, __pyx_L1_error)
07457         __Pyx_GOTREF(__pyx_t_8);
07458         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07459         __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_ixt))); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 619, __pyx_L1_error)
07460         __Pyx_GOTREF(__pyx_t_13);
07461         __pyx_t_7 = __Pyx_PyUnicode_Concat(__pyx_t_8, __pyx_t_13); if (unlikely(!__pyx_t_7))

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__PYX_ERR(0, 619, __pyx_L1_error)
07462     __Pyx_GOTREF(__pyx_t_7);
07463     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
07464     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07465     __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_7, __pyx_kp_u_9); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 619, __pyx_L1_error)
07466     __Pyx_GOTREF(__pyx_t_13);
07467     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07468
07469     /* "PyClical.pyx":618
07470     *                                     + ", " + repr(ixt) + ").")
07471     *     else:
07472     *         raise TypeError(error_msg_prefix + " (" + str(type(other))          # ««««««««
07473     *         + ", " + str(type(ixt)) + ").")
07474     *
07475     */
07476     __pyx_t_7 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_13); if
(unlikely(!__pyx_t_7)) __PYX_ERR(0, 618, __pyx_L1_error)
07477     __Pyx_GOTREF(__pyx_t_7);
07478     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07479     __Pyx_Raise(__pyx_t_7, 0, 0, 0);
07480     __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
07481     __PYX_ERR(0, 618, __pyx_L1_error)
07482 }
07483 __pyx_L3:;
07484
07485     /* "PyClical.pyx":565
07486     *     return clifford(self)
07487     *
07488     * def __cinit__(self, other = 0, ixt = None):          # ««««««««
07489     *     """
07490     *     Construct an object of type clifford.
07491     */
07492
07493     /* function exit code */
07494     __pyx_r = 0;
07495     goto __pyx_L0;
07496     __pyx_L1_error:;
07497     __Pyx_XDECREF(__pyx_t_7);
07498     __Pyx_XDECREF(__pyx_t_8);
07499     __Pyx_XDECREF(__pyx_t_13);
07500     __Pyx_XDECREF(__pyx_t_16);
07501     __Pyx_XDECREF(__pyx_t_17);
07502     __Pyx_XDECREF(__pyx_t_18);
07503     __Pyx_AddTraceback("PyClical.clifford.__cinit__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07504     __pyx_r = -1;
07505     __pyx_L0:;
07506     __Pyx_XDECREF(__pyx_v_error_msg_prefix);
07507     __Pyx_XDECREF(__pyx_v_bother);
07508     __Pyx_XDECREF(__pyx_v_err);
07509     __Pyx_RefNannyFinishContext();
07510     return __pyx_r;
07511 }
07512
07513     /* "PyClical.pyx":621
07514     *                                     + ", " + str(type(ixt)) + ").")
07515     *
07516     * def __dealloc__(self):          # ««««««««
07517     *     """
07518     *     Clean up by deallocating the instance of C++ class Clifford.
07519     */
07520
07521     /* Python wrapper */
07522     static void __pyx_pw_8PyClical_8clifford_5__dealloc__(PyObject *__pyx_v_self); /*proto*/
07523     static void __pyx_pw_8PyClical_8clifford_5__dealloc__(PyObject *__pyx_v_self) {
07524         __Pyx_RefNannyDeclarations
07525         __Pyx_RefNannySetupContext("__dealloc__ (wrapper)", 0);
07526         __pyx_pf_8PyClical_8clifford_4__dealloc__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
07527
07528     /* function exit code */
07529     __Pyx_RefNannyFinishContext();
07530 }
07531
07532     static void __pyx_pf_8PyClical_8clifford_4__dealloc__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
07533         __Pyx_RefNannyDeclarations
07534         __Pyx_RefNannySetupContext("__dealloc__", 0);
07535
07536     /* "PyClical.pyx":625
07537     *     Clean up by deallocating the instance of C++ class Clifford.
07538     *     """
07539     *     del self.instance          # ««««««««
07540     *
07541     * def __contains__(self, x):
07542     */

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07543         delete __pyx_v_self->instance;
07544
07545         /* "PyClical.pyx":621
07546         *                                     + ", " + str(type(ixt)) + ").")
07547         *
07548         *     def __dealloc__(self):
07549         *         """
07550         *         Clean up by deallocating the instance of C++ class Clifford.
07551         */
07552
07553         /* function exit code */
07554         __Pyx_RefNannyFinishContext();
07555     }
07556
07557     /* "PyClical.pyx":627
07558     *     del self.instance
07559     *
07560     *     def __contains__(self, x):
07561     *         """
07562     *         Not applicable.
07563     */
07564
07565     /* Python wrapper */
07566     static int __pyx_pw_8PyClical_8clifford_7__contains__(PyObject *__pyx_v_self, PyObject
07567     *__pyx_v_x); /*proto*/
07567     static char __pyx_doc_8PyClical_8clifford_6__contains__[] = "\n        Not applicable.\n\n
    >> x=clifford(index_set({-3,4,7})); -3 in x\n        Traceback (most recent call last):\n
    ... \n        TypeError: Not applicable.\n        ";
07568     #if CYTHON_COMPILING_IN_CPYTHON
07569     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_6__contains__;
07570     #endif
07571     static int __pyx_pw_8PyClical_8clifford_7__contains__(PyObject *__pyx_v_self, PyObject
07572     *__pyx_v_x) {
07572         int __pyx_r;
07573         __Pyx_RefNannyDeclarations
07574         __Pyx_RefNannySetupContext("__contains__ (wrapper)", 0);
07575         __pyx_r = __pyx_pf_8PyClical_8clifford_6__contains__(((struct __pyx_obj_8PyClical_clifford
07576     *)__pyx_v_self), ((PyObject *)__pyx_v_x));
07577
07577         /* function exit code */
07578         __Pyx_RefNannyFinishContext();
07579         return __pyx_r;
07580     }
07581
07582     static int __pyx_pf_8PyClical_8clifford_6__contains__(CYTHON_UNUSED struct
07583     __pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v_x) {
07583         int __pyx_r;
07584         __Pyx_RefNannyDeclarations
07585         PyObject *__pyx_t_1 = NULL;
07586         int __pyx_lineno = 0;
07587         const char *__pyx_filename = NULL;
07588         int __pyx_clineno = 0;
07589         __Pyx_RefNannySetupContext("__contains__", 0);
07590
07591         /* "PyClical.pyx":636
07592         *         TypeError: Not applicable.
07593         *         """
07594         *         raise TypeError("Not applicable.")
07595         *         # ««««««««
07596         *
07597         *     def __iter__(self):
07598         */
07598         __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__10, NULL); if
07599     (unlikely(!__pyx_t_1)) __PYX_ERR(0, 636, __pyx_L1_error)
07599         __Pyx_GOTREF(__pyx_t_1);
07600         __Pyx_Raise(__pyx_t_1, 0, 0, 0);
07601         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
07602         __PYX_ERR(0, 636, __pyx_L1_error)
07603
07604         /* "PyClical.pyx":627
07605         *     del self.instance
07606         *
07607         *     def __contains__(self, x):
07608         *         """
07609         *         Not applicable.
07610         */
07611
07612         /* function exit code */
07613         __pyx_L1_error:;
07614         __Pyx_XDECREF(__pyx_t_1);
07615         __Pyx_AddTraceback("PyClical.clifford.__contains__", __pyx_clineno, __pyx_lineno,
07616     __pyx_filename);
07616         __pyx_r = -1;
07617         __Pyx_RefNannyFinishContext();
07618         return __pyx_r;
07619     }
07620
07621     /* "PyClical.pyx":638

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07622 *         raise TypeError("Not applicable.")
07623 *
07624 *     def __iter__(self):                 # ««««««««
07625 *         """
07626 *         Not applicable.
07627 */
07628
07629 /* Python wrapper */
07630 static PyObject *__pyx_pw_8PyClical_8clifford_9__iter__(PyObject *__pyx_v_self); /*proto*/
07631 static char __pyx_doc_8PyClical_8clifford_8__iter__[] = "\n        Not applicable.\n\n
>> for a in clifford(index_set({-3,4,7})):print(a, end=',')\n        Traceback (most recent call
last):\n        ... \n        TypeError: Not applicable.\n        ";
07632 #if CYTHON_COMPILING_IN_CPYTHON
07633 struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_8__iter__;
07634 #endif
07635 static PyObject *__pyx_pw_8PyClical_8clifford_9__iter__(PyObject *__pyx_v_self) {
07636     PyObject *__pyx_r = 0;
07637     __Pyx_RefNannyDeclarations
07638     __Pyx_RefNannySetupContext("__iter__ (wrapper)", 0);
07639     __pyx_r = __pyx_pf_8PyClical_8clifford_8__iter__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
07640
07641 /* function exit code */
07642 __Pyx_RefNannyFinishContext();
07643 return __pyx_r;
07644 }
07645
07646 static PyObject *__pyx_pf_8PyClical_8clifford_8__iter__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
07647     PyObject *__pyx_r = NULL;
07648     __Pyx_RefNannyDeclarations
07649     PyObject *__pyx_t_1 = NULL;
07650     int __pyx_lineno = 0;
07651     const char *__pyx_filename = NULL;
07652     int __pyx_clineno = 0;
07653     __Pyx_RefNannySetupContext("__iter__", 0);
07654
07655 /* "PyClical.pyx":647
07656 *         raise TypeError("Not applicable.
07657 *         """
07658 *         raise TypeError("Not applicable.")           # ««««««««
07659 *
07660 *     def reframe(self, ixt):
07661 */
07662     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__10, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 647, __pyx_L1_error)
07663     __Pyx_GOTREF(__pyx_t_1);
07664     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
07665     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
07666     __PYX_ERR(0, 647, __pyx_L1_error)
07667
07668 /* "PyClical.pyx":638
07669 *         raise TypeError("Not applicable.")
07670 *
07671 *     def __iter__(self):                 # ««««««««
07672 *         """
07673 *         Not applicable.
07674 */
07675
07676 /* function exit code */
07677 __pyx_L1_error:;
07678 __Pyx_XDECREF(__pyx_t_1);
07679 __Pyx_AddTraceback("PyClical.clifford.__iter__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07680     __pyx_r = NULL;
07681     __Pyx_XGIVEREF(__pyx_r);
07682     __Pyx_RefNannyFinishContext();
07683     return __pyx_r;
07684 }
07685
07686 /* "PyClical.pyx":649
07687 *         raise TypeError("Not applicable.")
07688 *
07689 *     def reframe(self, ixt):                 # ««««««««
07690 *         """
07691 *         Put self into a larger frame, containing the union of self.frame() and index set ixt.
07692 */
07693
07694 /* Python wrapper */
07695 static PyObject *__pyx_pw_8PyClical_8clifford_11reframe(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt); /*proto*/
07696 static char __pyx_doc_8PyClical_8clifford_10reframe[] = "\n        Put self into a larger
frame, containing the union of self.frame() and index set ixt.\n        This can be used to make
multiplication faster, by multiplying within a common frame.\n\n        >>
clifford(\"2+3{1}\").reframe(index_set({1,2,3}))\n        clifford(\"2+3{1}\")\n        >>
s=index_set({1,2,3});t=index_set({-3,-2,-1});x=random_clifford(s); x.reframe(t).frame() == (s|t);\n
True\n        ";

```

```

07697     static PyObject *__pyx_pw_8PyClical_8clifford_1lreframe(PyObject *__pyx_v_self, PyObject
07698     *__pyx_v_ixt) {
07698         PyObject *__pyx_r = 0;
07699         __Pyx_RefNannyDeclarations
07700         __Pyx_RefNannySetupContext("reframe (wrapper)", 0);
07701         __pyx_r = __pyx_pf_8PyClical_8clifford_1lreframe(((struct __pyx_obj_8PyClical_clifford
07702     *)__pyx_v_self), ((PyObject *)__pyx_v_ixt));
07703
07703         /* function exit code */
07704         __Pyx_RefNannyFinishContext();
07705         return __pyx_r;
07706     }
07707
07708     static PyObject *__pyx_pf_8PyClical_8clifford_1lreframe(struct __pyx_obj_8PyClical_clifford
07709     *__pyx_v_self, PyObject *__pyx_v_ixt) {
07709         PyObject *__pyx_v_error_msg_prefix = NULL;
07710         struct __pyx_obj_8PyClical_clifford *__pyx_v_result = NULL;
07711         PyObject *__pyx_v_err = NULL;
07712         PyObject *__pyx_r = NULL;
07713         __Pyx_RefNannyDeclarations
07714         int __pyx_t_1;
07715         int __pyx_t_2;
07716         PyObject *__pyx_t_3 = NULL;
07717         PyObject *__pyx_t_4 = NULL;
07718         PyObject *__pyx_t_5 = NULL;
07719         PyObject *__pyx_t_6 = NULL;
07720         Clifford *__pyx_t_7;
07721         int __pyx_t_8;
07722         PyObject *__pyx_t_9 = NULL;
07723         PyObject *__pyx_t_10 = NULL;
07724         PyObject *__pyx_t_11 = NULL;
07725         PyObject *__pyx_t_12 = NULL;
07726         PyObject *__pyx_t_13 = NULL;
07727         int __pyx_t_14;
07728         char const *__pyx_t_15;
07729         PyObject *__pyx_t_16 = NULL;
07730         PyObject *__pyx_t_17 = NULL;
07731         PyObject *__pyx_t_18 = NULL;
07732         PyObject *__pyx_t_19 = NULL;
07733         PyObject *__pyx_t_20 = NULL;
07734         PyObject *__pyx_t_21 = NULL;
07735         int __pyx_lineno = 0;
07736         const char *__pyx_filename = NULL;
07737         int __pyx_clineno = 0;
07738         __Pyx_RefNannySetupContext("reframe", 0);
07739
07740         /* "PyClical.pyx":659
07741         *     True
07742         *     """
07743         *     error_msg_prefix = "Cannot reframe" # ««««««««
07744         *     if isinstance(ixt, index_set):
07745         *         try:
07746         */
07747         __Pyx_INCREF(__pyx_kp_u_Cannot_reframe);
07748         __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_reframe;
07749
07750         /* "PyClical.pyx":660
07751         *     """
07752         *     error_msg_prefix = "Cannot reframe"
07753         *     if isinstance(ixt, index_set): # ««««««««
07754         *         try:
07755         *             result = clifford()
07756         */
07757         __pyx_t_1 = __Pyx_TypeCheck(__pyx_v_ixt, __pyx_ptype_8PyClical_index_set);
07758         __pyx_t_2 = (__pyx_t_1 != 0);
07759         if (likely(__pyx_t_2)) {
07760
07761             /* "PyClical.pyx":661
07762             *     error_msg_prefix = "Cannot reframe"
07763             *     if isinstance(ixt, index_set):
07764             *         try: # ««««««««
07765             *             result = clifford()
07766             *             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07767             */
07768             {
07769                 __Pyx_PyThreadState_declare
07770                 __Pyx_PyThreadState_assign
07771                 __Pyx_ExceptionSave(&__pyx_t_3, &__pyx_t_4, &__pyx_t_5);
07772                 __Pyx_XGOTREF(__pyx_t_3);
07773                 __Pyx_XGOTREF(__pyx_t_4);
07774                 __Pyx_XGOTREF(__pyx_t_5);
07775                 /*try:*/ {
07776
07777                     /* "PyClical.pyx":662
07778                     *     if isinstance(ixt, index_set):
07779                     *         try:
07780                     *             result = clifford() # ««««««««

```

```

07781 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07782 *     except RuntimeError as err:
07783 */
07784     __pyx_t_6 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 662, __pyx_L4_error)
07785     __Pyx_GOTREF(__pyx_t_6);
07786     __pyx_v_result = ((struct __pyx_obj_8PyClical_clifford *) __pyx_t_6);
07787     __pyx_t_6 = 0;
07788
07789     /* "PyClical.pyx":663
07790 *     try:
07791 *         result = clifford()
07792 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07793 # ««««««««
07793 *     except RuntimeError as err:
07794 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07795 */
07796     try {
07797         __pyx_t_7 = new Clifford(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self),
__pyx_f_8PyClical_9index_set_unwrap(((struct __pyx_obj_8PyClical_index_set *) __pyx_v_ixt)));
07798     } catch (...) {
07799         __Pyx_CppExn2PyErr();
07800         __PYX_ERR(0, 663, __pyx_L4_error)
07801     }
07802     __pyx_v_result->instance = __pyx_t_7;
07803
07804     /* "PyClical.pyx":661
07805 *     error_msg_prefix = "Cannot reframe"
07806 *     if isinstance(ixt, index_set):
07807 *         try:
07808 *             # ««««««««
07808 *             result = clifford()
07809 *             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07810 */
07811     }
07812     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
07813     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
07814     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
07815     goto __pyx_L9_try_end;
07816     __pyx_L4_error:;
07817     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
07818
07819     /* "PyClical.pyx":664
07820 *         result = clifford()
07821 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07822 *     except RuntimeError as err:
07823 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07824 *             + str(ixt) + ":"
07825 */
07826     __pyx_t_8 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);
07827     if (__pyx_t_8) {
07828         __Pyx_AddTraceback("PyClical.clifford.reframe", __pyx_clineno, __pyx_lineno,
__pyx_filename);
07829     if (__Pyx_GetException(&__pyx_t_6, &__pyx_t_9, &__pyx_t_10) < 0) __PYX_ERR(0, 664,
__pyx_L6_except_error)
07830     __Pyx_GOTREF(__pyx_t_6);
07831     __Pyx_GOTREF(__pyx_t_9);
07832     __Pyx_GOTREF(__pyx_t_10);
07833     __Pyx_INCREF(__pyx_t_9);
07834     __pyx_v_err = __pyx_t_9;
07835     /*try:*/ {
07836
07837         /* "PyClical.pyx":665
07838 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07839 *     except RuntimeError as err:
07840 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07841 # ««««««««
07841 *             + str(ixt) + ":"
07842 *             + "\n\t" + str(err))
07843 */
07844     __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_from); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 665, __pyx_L15_error)
07845     __Pyx_GOTREF(__pyx_t_11);
07846     __pyx_t_12 = __Pyx_PyObject_CallOneArg(((PyObject *) (&PyUnicode_Type)), ((PyObject
*) __pyx_v_self)); if (unlikely(!__pyx_t_12)) __PYX_ERR(0, 665, __pyx_L15_error)
07847     __Pyx_GOTREF(__pyx_t_12);
07848     __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_t_12); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 665, __pyx_L15_error)
07849     __Pyx_GOTREF(__pyx_t_13);
07850     __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07851     __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07852     __pyx_t_12 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_to_frame); if
(unlikely(!__pyx_t_12)) __PYX_ERR(0, 665, __pyx_L15_error)
07853     __Pyx_GOTREF(__pyx_t_12);
07854     __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07855
07856     /* "PyClical.pyx":666
07857 *     except RuntimeError as err:

```

```

07858 *             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07859 *                               + str(ixt) + ":", # ««««««««
07860 *                               + "\n\t" + str(err))
07861 *         else:
07862 */
07863         __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_ixt); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 666, __pyx_L15_error)
07864         __Pyx_GOTREF(__pyx_t_13);
07865         __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_t_12, __pyx_t_13); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 666, __pyx_L15_error)
07866         __Pyx_GOTREF(__pyx_t_11);
07867         __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07868         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07869         __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_kp_u_5); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 666, __pyx_L15_error)
07870         __Pyx_GOTREF(__pyx_t_13);
07871         __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07872
07873         /* "PyClical.pyx":667
07874 *             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07875 *                               + str(ixt) + ":",
07876 *                               + "\n\t" + str(err)) #
07877 *         else:
07878 *             raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
07879 */
07880         __pyx_t_11 = __Pyx_PyUnicode_Concat(__pyx_t_13, __pyx_kp_u_6); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 667, __pyx_L15_error)
07881         __Pyx_GOTREF(__pyx_t_11);
07882         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07883         __pyx_t_13 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)),
__pyx_v_err); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 667, __pyx_L15_error)
07884         __Pyx_GOTREF(__pyx_t_13);
07885         __pyx_t_12 = __Pyx_PyUnicode_Concat(__pyx_t_11, __pyx_t_13); if
(unlikely(!__pyx_t_12)) __PYX_ERR(0, 667, __pyx_L15_error)
07886         __Pyx_GOTREF(__pyx_t_12);
07887         __Pyx_DECREF(__pyx_t_11); __pyx_t_11 = 0;
07888         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07889
07890         /* "PyClical.pyx":665
07891 *             result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07892 *         except RuntimeError as err:
07893 *             raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07894 *                               + str(ixt) + ":",
07895 *                               + "\n\t" + str(err))
07896 */
07897         __pyx_t_13 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_12); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 665, __pyx_L15_error)
07898         __Pyx_GOTREF(__pyx_t_13);
07899         __Pyx_DECREF(__pyx_t_12); __pyx_t_12 = 0;
07900         __Pyx_Raise(__pyx_t_13, 0, 0, 0);
07901         __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
07902         __PYX_ERR(0, 665, __pyx_L15_error)
07903     }
07904
07905     /* "PyClical.pyx":664
07906 *         result = clifford()
07907 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07908 *     except RuntimeError as err: # ««««««««
07909 *         raise ValueError(error_msg_prefix + " from " + str(self) + " to frame "
07910 *                               + str(ixt) + ":",
07911 */
07912     /*finally:*/ {
07913         __pyx_L15_error;;
07914         /*exception exit:*/{
07915             __Pyx_PyThreadState_declare
07916             __Pyx_PyThreadState_assign
07917             __pyx_t_16 = 0; __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0;
__pyx_t_21 = 0;
07918             __Pyx_XDECREF(__pyx_t_11); __pyx_t_11 = 0;
07919             __Pyx_XDECREF(__pyx_t_12); __pyx_t_12 = 0;
07920             __Pyx_XDECREF(__pyx_t_13); __pyx_t_13 = 0;
07921             if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_19, &__pyx_t_20,
&__pyx_t_21);
07922             if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_16,
&__pyx_t_17, &__pyx_t_18) < 0)) __Pyx_ErrFetch(&__pyx_t_16, &__pyx_t_17, &__pyx_t_18);
07923             __Pyx_XGOTREF(__pyx_t_16);
07924             __Pyx_XGOTREF(__pyx_t_17);
07925             __Pyx_XGOTREF(__pyx_t_18);
07926             __Pyx_XGOTREF(__pyx_t_19);
07927             __Pyx_XGOTREF(__pyx_t_20);
07928             __Pyx_XGOTREF(__pyx_t_21);
07929             __pyx_t_8 = __pyx_lineno; __pyx_t_14 = __pyx_clineno; __pyx_t_15 = __pyx_filename;
07930             {
07931                 __Pyx_DECREF(__pyx_v_err);
07932                 __pyx_v_err = NULL;

```

```

07933         }
07934         if (PY_MAJOR_VERSION >= 3) {
07935             __Pyx_XGIVEREF(__pyx_t_19);
07936             __Pyx_XGIVEREF(__pyx_t_20);
07937             __Pyx_XGIVEREF(__pyx_t_21);
07938             __Pyx_ExceptionReset(__pyx_t_19, __pyx_t_20, __pyx_t_21);
07939         }
07940         __Pyx_XGIVEREF(__pyx_t_16);
07941         __Pyx_XGIVEREF(__pyx_t_17);
07942         __Pyx_XGIVEREF(__pyx_t_18);
07943         __Pyx_ErrRestore(__pyx_t_16, __pyx_t_17, __pyx_t_18);
07944         __pyx_t_16 = 0; __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0;
__pyx_t_21 = 0;
07945         __pyx_lineno = __pyx_t_8; __pyx_clineno = __pyx_t_14; __pyx_filename = __pyx_t_15;
07946         goto __pyx_L6_except_error;
07947     }
07948 }
07949 }
07950 goto __pyx_L6_except_error;
07951 __pyx_L6_except_error;
07952
07953 /* "PyClical.pyx":661
07954 * error_msg_prefix = "Cannot reframe"
07955 * if isinstance(ixt, index_set):
07956 *     try:
07957 *         # ««««««««
07958 *         result = clifford()
07959 *         result.instance = new Clifford(self.unwrap(), (<index_set>ixt).unwrap())
07960 */
07961         __Pyx_XGIVEREF(__pyx_t_3);
07962         __Pyx_XGIVEREF(__pyx_t_4);
07963         __Pyx_XGIVEREF(__pyx_t_5);
07964         __Pyx_ExceptionReset(__pyx_t_3, __pyx_t_4, __pyx_t_5);
07965         goto __pyx_L1_error;
07966         __pyx_L9_try_end;
07967     }
07968
07969 /* "PyClical.pyx":660
07970 * """
07971 * error_msg_prefix = "Cannot reframe"
07972 * if isinstance(ixt, index_set):
07973 *     try:
07974 *         # ««««««««
07975 *         result = clifford()
07976 */
07977         goto __pyx_L3;
07978 }
07979
07980 /* "PyClical.pyx":669
07981 *
07982 *                                     + "\n\t" + str(err))
07983 *
07984 *     else:
07985 *         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
07986 *                                     #
07987 *                                     ««««««««
07988 *
07989 *     return result
07990 *
07991 */
07992         /*else*/ {
07993             __pyx_t_10 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_kp_u_using); if
(unlikely(!__pyx_t_10)) __PYX_ERR(0, 669, __pyx_L1_error)
07994             __Pyx_GOTREF(__pyx_t_10);
07995             __pyx_t_9 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type)), ((PyObject
*)Py_TYPE(__pyx_v_ixt)); if (unlikely(!__pyx_t_9)) __PYX_ERR(0, 669, __pyx_L1_error)
07996             __Pyx_GOTREF(__pyx_t_9);
07997             __pyx_t_6 = __Pyx_PyUnicode_Concat(__pyx_t_10, __pyx_t_9); if (unlikely(!__pyx_t_6))
__PYX_ERR(0, 669, __pyx_L1_error)
07998             __Pyx_GOTREF(__pyx_t_6);
07999             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
08000             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
08001             __pyx_t_9 = __Pyx_PyUnicode_Concat(__pyx_t_6, __pyx_kp_u_9); if (unlikely(!__pyx_t_9))
__PYX_ERR(0, 669, __pyx_L1_error)
08002             __Pyx_GOTREF(__pyx_t_9);
08003             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08004             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
08005             __Pyx_Raise(__pyx_t_6, 0, 0, 0);
08006             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08007             __PYX_ERR(0, 669, __pyx_L1_error)
08008         }
08009         __pyx_L3;
08010
08011 /* "PyClical.pyx":670
08012 *
08013 *     else:
08014 *         raise TypeError(error_msg_prefix + " using (" + str(type(ixt)) + ").")
08015 *
08016 *     return result
08017 *                                     # ««««««««
08018 *
08019 * def __richcmp__(lhs, rhs, int op):
08020 */

```



```

08013     __Pyx_XDECREF(__pyx_r);
08014     __Pyx_INCREF((PyObject *)__pyx_v_result);
08015     __pyx_r = ((PyObject *)__pyx_v_result);
08016     goto __pyx_L0;
08017
08018     /* "PyClical.pyx":649
08019  *     raise TypeError("Not applicable.")
08020  *
08021  * def reframe(self, ixt):          # ««««««««
08022  *     """
08023  *     Put self into a larger frame, containing the union of self.frame() and index set ixt.
08024  */
08025
08026     /* function exit code */
08027     __pyx_L1_error:;
08028     __Pyx_XDECREF(__pyx_t_6);
08029     __Pyx_XDECREF(__pyx_t_9);
08030     __Pyx_XDECREF(__pyx_t_10);
08031     __Pyx_XDECREF(__pyx_t_11);
08032     __Pyx_XDECREF(__pyx_t_12);
08033     __Pyx_XDECREF(__pyx_t_13);
08034     __Pyx_AddTraceback("PyClical.clifford.reframe", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08035     __pyx_r = NULL;
08036     __pyx_L0:;
08037     __Pyx_XDECREF(__pyx_v_error_msg_prefix);
08038     __Pyx_XDECREF((PyObject *)__pyx_v_result);
08039     __Pyx_XDECREF(__pyx_v_err);
08040     __Pyx_XGIVEREF(__pyx_r);
08041     __Pyx_RefNannyFinishContext();
08042     return __pyx_r;
08043 }
08044
08045     /* "PyClical.pyx":672
08046  *     return result
08047  *
08048  * def __richcmp__(lhs, rhs, int op):          # ««««««««
08049  *     """
08050  *     Compare objects of type clifford.
08051  */
08052
08053     /* Python wrapper */
08054     static PyObject *__pyx_pw_8PyClical_8clifford_13__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op); /*proto*/
08055     static PyObject *__pyx_pw_8PyClical_8clifford_13__richcmp__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs, int __pyx_v_op) {
08056         PyObject *__pyx_r = 0;
08057         __Pyx_RefNannyDeclarations
08058         __Pyx_RefNannySetupContext("__richcmp__ (wrapper)", 0);
08059         __pyx_r = __pyx_pf_8PyClical_8clifford_12__richcmp__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_lhs), ((PyObject *)__pyx_v_rhs), ((int)__pyx_v_op));
08060
08061         /* function exit code */
08062         __Pyx_RefNannyFinishContext();
08063         return __pyx_r;
08064     }
08065
08066     static PyObject *__pyx_pf_8PyClical_8clifford_12__richcmp__(struct
__pyx_obj_8PyClical_clifford *__pyx_v_lhs, PyObject *__pyx_v_rhs, int __pyx_v_op) {
08067         PyObject *__pyx_r = NULL;
08068         __Pyx_RefNannyDeclarations
08069         int __pyx_t_1;
08070         int __pyx_t_2;
08071         int __pyx_t_3;
08072         PyObject *__pyx_t_4 = NULL;
08073         PyObject *__pyx_t_5 = NULL;
08074         PyObject *__pyx_t_6 = NULL;
08075         int __pyx_lineno = 0;
08076         const char *__pyx_filename = NULL;
08077         int __pyx_clineno = 0;
08078         __Pyx_RefNannySetupContext("__richcmp__", 0);
08079
08080         /* "PyClical.pyx":691
08081  *     True
08082  *     """
08083  *     if op == 2: # ==          # ««««««««
08084  *         if (lhs is None) or (rhs is None):
08085  *             return bool(lhs is rhs)
08086  */
08087         __pyx_t_1 = ((__pyx_v_op == 2) != 0);
08088         if (__pyx_t_1) {
08089
08090             /* "PyClical.pyx":692
08091  *     """
08092  *     if op == 2: # ==
08093  *         if (lhs is None) or (rhs is None):          # ««««««««
08094  *             return bool(lhs is rhs)

```

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08095 *         else:
08096 */
08097     __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
08098     __pyx_t_3 = (__pyx_t_2 != 0);
08099     if (!__pyx_t_3) {
08100     } else {
08101         __pyx_t_1 = __pyx_t_3;
08102         goto __pyx_L5_bool_binop_done;
08103     }
08104     __pyx_t_3 = (__pyx_v_rhs == Py_None);
08105     __pyx_t_2 = (__pyx_t_3 != 0);
08106     __pyx_t_1 = __pyx_t_2;
08107     __pyx_L5_bool_binop_done;
08108     if (__pyx_t_1) {
08109
08110         /* "PyClicl.pyx":693
08111 *         if op == 2: # ==
08112 *             if (lhs is None) or (rhs is None):
08113 *                 return bool(lhs is rhs) # ««««««««
08114 *             else:
08115 *                 return bool( toClifford(lhs) == toClifford(rhs) )
08116 */
08117         __Pyx_XDECREF(__pyx_r);
08118         __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
08119         __pyx_t_4 = __Pyx_PyBool_FromLong((!(!__pyx_t_1))); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 693, __pyx_L1_error)
08120         __Pyx_GOTREF(__pyx_t_4);
08121         __pyx_r = __pyx_t_4;
08122         __pyx_t_4 = 0;
08123         goto __pyx_L0;
08124
08125         /* "PyClicl.pyx":692
08126 *         """
08127 *         if op == 2: # ==
08128 *             if (lhs is None) or (rhs is None): # ««««««««
08129 *                 return bool(lhs is rhs)
08130 *             else:
08131 */
08132     }
08133
08134     /* "PyClicl.pyx":695
08135 *         return bool(lhs is rhs)
08136 *     else:
08137 *         return bool( toClifford(lhs) == toClifford(rhs) ) # ««««««««
08138 *     elif op == 3: # !=
08139 *         if (lhs is None) or (rhs is None):
08140 */
08141     /*else*/ {
08142         __Pyx_XDECREF(__pyx_r);
08143         __pyx_t_1 = (__pyx_f_8PyClicl_toClifford(((PyObject *)__pyx_v_lhs)) ==
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs));
08144         __pyx_t_4 = __Pyx_PyBool_FromLong((!(!__pyx_t_1))); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 695, __pyx_L1_error)
08145         __Pyx_GOTREF(__pyx_t_4);
08146         __pyx_r = __pyx_t_4;
08147         __pyx_t_4 = 0;
08148         goto __pyx_L0;
08149     }
08150
08151     /* "PyClicl.pyx":691
08152 *         True
08153 *         """
08154 *         if op == 2: # == # ««««««««
08155 *             if (lhs is None) or (rhs is None):
08156 *                 return bool(lhs is rhs)
08157 */
08158     }
08159
08160     /* "PyClicl.pyx":696
08161 *         else:
08162 *             return bool( toClifford(lhs) == toClifford(rhs) )
08163 *         elif op == 3: # != # ««««««««
08164 *             if (lhs is None) or (rhs is None):
08165 *                 return not bool(lhs is rhs)
08166 */
08167     __pyx_t_1 = ((__pyx_v_op == 3) != 0);
08168     if (__pyx_t_1) {
08169
08170         /* "PyClicl.pyx":697
08171 *             return bool( toClifford(lhs) == toClifford(rhs) )
08172 *         elif op == 3: # !=
08173 *             if (lhs is None) or (rhs is None): # ««««««««
08174 *                 return not bool(lhs is rhs)
08175 *             else:
08176 */
08177         __pyx_t_2 = (((PyObject *)__pyx_v_lhs) == Py_None);
08178         __pyx_t_3 = (__pyx_t_2 != 0);

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08179         if (!__pyx_t_3) {
08180         } else {
08181             __pyx_t_1 = __pyx_t_3;
08182             goto __pyx_L8_bool_binop_done;
08183         }
08184         __pyx_t_3 = (__pyx_v_rhs == Py_None);
08185         __pyx_t_2 = (__pyx_t_3 != 0);
08186         __pyx_t_1 = __pyx_t_2;
08187         __pyx_L8_bool_binop_done;
08188         if (__pyx_t_1) {
08189
08190             /* "PyClical.pyx":698
08191             *
08192             *     if (lhs is None) or (rhs is None):
08193             *         return not bool(lhs is rhs)                # ««««««««
08194             *
08195             *         else:
08196             *             return bool( toClifford(lhs) != toClifford(rhs) )
08197             */
08198             __Pyx_XDECREF(__pyx_r);
08199             __pyx_t_1 = (((PyObject *)__pyx_v_lhs) == __pyx_v_rhs);
08200             __pyx_t_4 = __Pyx_PyBool_FromLong((((!(!__pyx_t_1)) != 0))); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 698, __pyx_L1_error)
08201             __Pyx_GOTREF(__pyx_t_4);
08202             __pyx_r = __pyx_t_4;
08203             __pyx_t_4 = 0;
08204             goto __pyx_L0;
08205
08206             /* "PyClical.pyx":697
08207             *
08208             *         return bool( toClifford(lhs) == toClifford(rhs) )
08209             *
08210             *     elif op == 3: # !=
08211             *         if (lhs is None) or (rhs is None):                # ««««««««
08212             *             return not bool(lhs is rhs)
08213             *
08214             *         else:
08215             */
08216             }
08217
08218             /* "PyClical.pyx":700
08219             *
08220             *         return not bool(lhs is rhs)
08221             *
08222             *         else:
08223             *             return bool( toClifford(lhs) != toClifford(rhs) )                # ««««««««
08224             *
08225             *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08226             *             raise TypeError("This comparison operator is not implemented for "
08227             */
08228             /*else*/ {
08229                 __Pyx_XDECREF(__pyx_r);
08230                 __pyx_t_1 = (__pyx_f_8PyClical_toClifford(((PyObject *)__pyx_v_lhs)) !=
__pyx_f_8PyClical_toClifford(__pyx_v_rhs));
08231                 __pyx_t_4 = __Pyx_PyBool_FromLong((((!(!__pyx_t_1)) != 0))); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 700, __pyx_L1_error)
08232                 __Pyx_GOTREF(__pyx_t_4);
08233                 __pyx_r = __pyx_t_4;
08234                 __pyx_t_4 = 0;
08235                 goto __pyx_L0;
08236             }
08237
08238             /* "PyClical.pyx":696
08239             *
08240             *         else:
08241             *             return bool( toClifford(lhs) == toClifford(rhs) )
08242             *
08243             *     elif op == 3: # !=
08244             *         if (lhs is None) or (rhs is None):                # ««««««««
08245             *             return not bool(lhs is rhs)
08246             */
08247             }
08248
08249             /* "PyClical.pyx":701
08250             *
08251             *         else:
08252             *             return bool( toClifford(lhs) != toClifford(rhs) )
08253             *
08254             *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08255             *             raise TypeError("This comparison operator is not implemented for "
08256             *                 + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08257             */
08258             __pyx_t_2 = __Pyx_TypeCheck(((PyObject *)__pyx_v_lhs), __pyx_ptype_8PyClical_clifford);
08259             __pyx_t_3 = (__pyx_t_2 != 0);
08260             if (!__pyx_t_3) {
08261             } else {
08262                 __pyx_t_1 = __pyx_t_3;
08263                 goto __pyx_L10_bool_binop_done;
08264             }
08265             __pyx_t_3 = __Pyx_TypeCheck(__pyx_v_rhs, __pyx_ptype_8PyClical_clifford);
08266             __pyx_t_2 = (__pyx_t_3 != 0);
08267             __pyx_t_1 = __pyx_t_2;
08268             __pyx_L10_bool_binop_done;
08269             if (unlikely(__pyx_t_1)) {
08270
08271                 /* "PyClical.pyx":703
08272                 *
08273                 *         elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08274                 *             raise TypeError("This comparison operator is not implemented for "

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08263 *                                     + str(type(lhs)) + ", " + str(type(rhs)) + ".") # ««««««««
08264 *     else:
08265 *         return NotImplemented
08266 */
08267     __pyx_t_4 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), ((PyObject
*)Py_TYPE(((PyObject *)__pyx_v_lhs))))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 703, __pyx_L1_error)
08268     __Pyx_GOTREF(__pyx_t_4);
08269     __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_kp_u_This_comparison_operator_is_not, __pyx_t_4);
08270     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 703, __pyx_L1_error)
08271     __Pyx_GOTREF(__pyx_t_5);
08272     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
08273     __pyx_t_4 = __Pyx_PyUnicode_Concat(__pyx_t_5, __pyx_kp_u_8); if (unlikely(!__pyx_t_4))
__PYX_ERR(0, 703, __pyx_L1_error)
08274     __Pyx_GOTREF(__pyx_t_4);
08275     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08276     __pyx_t_5 = __Pyx_PyObject_CallOneArg((PyObject *)(&PyUnicode_Type), ((PyObject
*)Py_TYPE(__pyx_v_rhs))); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 703, __pyx_L1_error)
08277     __Pyx_GOTREF(__pyx_t_5);
08278     __pyx_t_6 = __Pyx_PyUnicode_Concat(__pyx_t_4, __pyx_t_5); if (unlikely(!__pyx_t_6))
__PYX_ERR(0, 703, __pyx_L1_error)
08279     __Pyx_GOTREF(__pyx_t_6);
08280     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
08281     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08282     __pyx_t_5 = __Pyx_PyUnicode_Concat(__pyx_t_6, __pyx_kp_u_); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 703, __pyx_L1_error)
08283     __Pyx_GOTREF(__pyx_t_5);
08284     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08285     /* "PyClicl.pyx":702
08286     *         return bool( toClifford(lhs) != toClifford(rhs) )
08287     *     elif isinstance(lhs, clifford) or isinstance(rhs, clifford):
08288     *         raise TypeError("This comparison operator is not implemented for " #
««««««««
08289     *                                     + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08290     *     else:
08291     */
08292     __pyx_t_6 = __Pyx_PyObject_CallOneArg(__pyx_builtin_TypeError, __pyx_t_5); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 702, __pyx_L1_error)
08293     __Pyx_GOTREF(__pyx_t_6);
08294     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
08295     __Pyx_Raise(__pyx_t_6, 0, 0, 0);
08296     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
08297     __PYX_ERR(0, 702, __pyx_L1_error)
08298     /* "PyClicl.pyx":701
08299     *     else:
08300     *         return bool( toClifford(lhs) != toClifford(rhs) )
08301     *     elif isinstance(lhs, clifford) or isinstance(rhs, clifford): # ««««««««
08302     *         raise TypeError("This comparison operator is not implemented for "
08303     *                                     + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08304     *
08305     */
08306     }
08307
08308     /* "PyClicl.pyx":705
08309     *                                     + str(type(lhs)) + ", " + str(type(rhs)) + ".")
08310     *     else:
08311     *         return NotImplemented # ««««««««
08312     *
08313     def __getitem__(self, ixt):
08314     */
08315     /*else*/ {
08316         __Pyx_XDECREF(__pyx_r);
08317         __Pyx_INCREF(__pyx_builtin_NotImplemented);
08318         __pyx_r = __pyx_builtin_NotImplemented;
08319         goto __pyx_L0;
08320     }
08321
08322     /* "PyClicl.pyx":672
08323     *     return result
08324     *
08325     def __richcmp__(lhs, rhs, int op): # ««««««««
08326     *         """
08327     *         Compare objects of type clifford.
08328     */
08329
08330     /* function exit code */
08331     __pyx_L1_error:;
08332     __Pyx_XDECREF(__pyx_t_4);
08333     __Pyx_XDECREF(__pyx_t_5);
08334     __Pyx_XDECREF(__pyx_t_6);
08335     __Pyx_AddTraceback("PyClicl.clifford.__richcmp__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08336     __pyx_r = NULL;
08337     __pyx_L0:;
08338     __Pyx_XGIVEREF(__pyx_r);
08339     __Pyx_RefNannyFinishContext();
08340     return __pyx_r;

```

```

08341     }
08342
08343     /* "PyClical.pyx":707
08344     *         return NotImplemented
08345     *
08346     *     def __getitem__(self, ixt):          # ««««««««
08347     *         """
08348     *         Subscripting: map from index set to scalar coordinate.
08349     */
08350
08351     /* Python wrapper */
08352     static PyObject *__pyx_pw_8PyClical_8clifford_15__getitem__(PyObject *__pyx_v_self, PyObject
08353 *__pyx_v_ixt); /*proto*/
08354     static char __pyx_doc_8PyClical_8clifford_14__getitem__[] = "\n        Subscripting: map from
index set to scalar coordinate.\n\n        >> clifford(\"{1}\") [index_set(1)]\n                1.0\nclifford(\"{1}\") [index_set({1})]\n                1.0\n        >> clifford(\"{1}\") [index_set({1,2})]\n        0.0\n        >> clifford(\"2{1,2}\") [index_set({1,2})]\n                2.0\n        ";
08354     #if CYTHON_COMPILING_IN_CPYTHON
08355     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_14__getitem__;
08356     #endif
08357     static PyObject *__pyx_pw_8PyClical_8clifford_15__getitem__(PyObject *__pyx_v_self, PyObject
*__pyx_v_ixt) {
08358         PyObject *__pyx_r = 0;
08359         __Pyx_RefNannyDeclarations
08360         __Pyx_RefNannySetupContext("__getitem__ (wrapper)", 0);
08361         __pyx_r = __pyx_pf_8PyClical_8clifford_14__getitem__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_ixt));
08362
08363         /* function exit code */
08364         __Pyx_RefNannyFinishContext();
08365         return __pyx_r;
08366     }
08367
08368     static PyObject *__pyx_pf_8PyClical_8clifford_14__getitem__(struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, PyObject *__pyx_v_ixt) {
08369         PyObject *__pyx_r = NULL;
08370         __Pyx_RefNannyDeclarations
08371         PyObject *__pyx_t_1 = NULL;
08372         int __pyx_lineno = 0;
08373         const char *__pyx_filename = NULL;
08374         int __pyx_clineno = 0;
08375         __Pyx_RefNannySetupContext("__getitem__", 0);
08376
08377         /* "PyClical.pyx":720
08378         *         2.0
08379         *         """
08380         *         return self.instance.getitem(toIndexSet(ixt))          # ««««««««
08381         *
08382         *     def __neg__(self):
08383         */
08384         __Pyx_XDECREF(__pyx_r);
08385         __pyx_t_1 =
PyFloat_FromDouble(__pyx_v_self->instance->operator[](__pyx_f_8PyClical_toIndexSet(__pyx_v_ixt))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 720, __pyx_L1_error)
08386         __Pyx_GOTREF(__pyx_t_1);
08387         __pyx_r = __pyx_t_1;
08388         __pyx_t_1 = 0;
08389         goto __pyx_L0;
08390
08391         /* "PyClical.pyx":707
08392         *         return NotImplemented
08393         *
08394         *     def __getitem__(self, ixt):          # ««««««««
08395         *         """
08396         *         Subscripting: map from index set to scalar coordinate.
08397         */
08398
08399         /* function exit code */
08400         __pyx_L1_error;
08401         __Pyx_XDECREF(__pyx_t_1);
08402         __Pyx_AddTraceback("PyClical.clifford.__getitem__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08403         __pyx_r = NULL;
08404         __pyx_L0;
08405         __Pyx_XGIVEREF(__pyx_r);
08406         __Pyx_RefNannyFinishContext();
08407         return __pyx_r;
08408     }
08409
08410     /* "PyClical.pyx":722
08411     *         return self.instance.getitem(toIndexSet(ixt))
08412     *
08413     *     def __neg__(self):          # ««««««««
08414     *         """
08415     *         Unary -.
08416     */
08417

```

```

08418         /* Python wrapper */
08419         static PyObject *__pyx_pw_8PyClical_8clifford_17__neg__(PyObject *__pyx_v_self); /*proto*/
08420         static char __pyx_doc_8PyClical_8clifford_16__neg__[] = "\n        Unary -. \n\n        »>
print(-clifford("{1}\n"))\n        -{1}\n        ";
08421         #if CYTHON_COMPILING_IN_CPYTHON
08422         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_16__neg__;
08423         #endif
08424         static PyObject *__pyx_pw_8PyClical_8clifford_17__neg__(PyObject *__pyx_v_self) {
08425             PyObject *__pyx_r = 0;
08426             __Pyx_RefNannyDeclarations
08427             __Pyx_RefNannySetupContext("__neg__ (wrapper)", 0);
08428             __pyx_r = __pyx_pf_8PyClical_8clifford_16__neg__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
08429
08430             /* function exit code */
08431             __Pyx_RefNannyFinishContext();
08432             return __pyx_r;
08433         }
08434
08435         static PyObject *__pyx_pf_8PyClical_8clifford_16__neg__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
08436             PyObject *__pyx_r = NULL;
08437             __Pyx_RefNannyDeclarations
08438             PyObject *__pyx_t_1 = NULL;
08439             PyObject *__pyx_t_2 = NULL;
08440             int __pyx_lineno = 0;
08441             const char *__pyx_filename = NULL;
08442             int __pyx_clineno = 0;
08443             __Pyx_RefNannySetupContext("__neg__", 0);
08444
08445             /* "PyClical.pyx":729
08446             *         -{1}
08447             *         """
08448             *         return clifford().wrap( self.instance.neg() )           # ««««««««
08449             *
08450             *     def __pos__(self):
08451             */
08452             __Pyx_XDECREF(__pyx_r);
08453             __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 729, __pyx_L1_error)
08454             __Pyx_GOTREF(__pyx_t_1);
08455             __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->operator-()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 729,
__pyx_L1_error)
08456             __Pyx_GOTREF(__pyx_t_2);
08457             __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08458             __pyx_r = __pyx_t_2;
08459             __pyx_t_2 = 0;
08460             goto __pyx_L0;
08461
08462             /* "PyClical.pyx":722
08463             *         return self.instance.getitem(toIndexSet(ixt))
08464             *
08465             *     def __neg__(self):           # ««««««««
08466             *         """
08467             *         Unary -.
08468             */
08469
08470             /* function exit code */
08471             __pyx_L1_error:;
08472             __Pyx_XDECREF(__pyx_t_1);
08473             __Pyx_XDECREF(__pyx_t_2);
08474             __Pyx_AddTraceback("PyClical.clifford.__neg__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08475             __pyx_r = NULL;
08476             __pyx_L0:;
08477             __Pyx_XGIVEREF(__pyx_r);
08478             __Pyx_RefNannyFinishContext();
08479             return __pyx_r;
08480         }
08481
08482             /* "PyClical.pyx":731
08483             *         return clifford().wrap( self.instance.neg() )
08484             *
08485             *     def __pos__(self):           # ««««««««
08486             *         """
08487             *         Unary +.
08488             */
08489
08490             /* Python wrapper */
08491             static PyObject *__pyx_pw_8PyClical_8clifford_19__pos__(PyObject *__pyx_v_self); /*proto*/
08492             static char __pyx_doc_8PyClical_8clifford_18__pos__[] = "\n        Unary +. \n\n        »>
print(+clifford("{1}\n"))\n        {1}\n        ";
08493             #if CYTHON_COMPILING_IN_CPYTHON
08494             struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_18__pos__;
08495             #endif
08496             static PyObject *__pyx_pw_8PyClical_8clifford_19__pos__(PyObject *__pyx_v_self) {

```

```

08497         PyObject *__pyx_r = 0;
08498         __Pyx_RefNannyDeclarations
08499         __Pyx_RefNannySetupContext("__pos__ (wrapper)", 0);
08500         __pyx_r = __pyx_pf_8PyClical_8clifford_18__pos__((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
08501
08502         /* function exit code */
08503         __Pyx_RefNannyFinishContext();
08504         return __pyx_r;
08505     }
08506
08507     static PyObject *__pyx_pf_8PyClical_8clifford_18__pos__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
08508         PyObject *__pyx_r = NULL;
08509         __Pyx_RefNannyDeclarations
08510         PyObject *__pyx_t_1 = NULL;
08511         int __pyx_lineno = 0;
08512         const char *__pyx_filename = NULL;
08513         int __pyx_clineno = 0;
08514         __Pyx_RefNannySetupContext("__pos__", 0);
08515
08516         /* "PyClical.pyx":738
08517         *     {1}
08518         *     """
08519         *     return clifford(self)                # ««««««««
08520         *
08521         *     def __add__(lhs, rhs):
08522         */
08523         __Pyx_XDECREF(__pyx_r);
08524         __pyx_t_1 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
((PyObject *)__pyx_v_self)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 738, __pyx_L1_error)
08525         __Pyx_GOTREF(__pyx_t_1);
08526         __pyx_r = __pyx_t_1;
08527         __pyx_t_1 = 0;
08528         goto __pyx_L0;
08529
08530         /* "PyClical.pyx":731
08531         *     return clifford().wrap( self.instance.neg() )
08532         *
08533         *     def __pos__(self):                # ««««««««
08534         *     """
08535         *     Unary +.
08536         */
08537
08538         /* function exit code */
08539         __pyx_L1_error:;
08540         __Pyx_XDECREF(__pyx_t_1);
08541         __Pyx_AddTraceback("PyClical.clifford.__pos__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08542         __pyx_r = NULL;
08543         __pyx_L0:;
08544         __Pyx_XGIVEREF(__pyx_r);
08545         __Pyx_RefNannyFinishContext();
08546         return __pyx_r;
08547     }
08548
08549     /* "PyClical.pyx":740
08550     *     return clifford(self)
08551     *
08552     *     def __add__(lhs, rhs):                # ««««««««
08553     *     """
08554     *     Geometric sum.
08555     */
08556
08557     /* Python wrapper */
08558     static PyObject *__pyx_pw_8PyClical_8clifford_21__add__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
08559     static char __pyx_doc_8PyClical_8clifford_20__add__[] = "\n          Geometric sum.\n\n
>> print(clifford(1) + clifford(\"{2}\"))\n          1+{2}\n          >> print(clifford(\"{1}\") +
clifford(\"{2}\"))\n          {1}+{2}\n          ";
08560     #if CYTHON_COMPILING_IN_CPYTHON
08561     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_20__add__;
08562     #endif
08563     static PyObject *__pyx_pw_8PyClical_8clifford_21__add__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08564         PyObject *__pyx_r = 0;
08565         __Pyx_RefNannyDeclarations
08566         __Pyx_RefNannySetupContext("__add__ (wrapper)", 0);
08567         __pyx_r = __pyx_pf_8PyClical_8clifford_20__add__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
08568
08569         /* function exit code */
08570         __Pyx_RefNannyFinishContext();
08571         return __pyx_r;
08572     }
08573
08574     static PyObject *__pyx_pf_8PyClical_8clifford_20__add__(PyObject *__pyx_v_lhs, PyObject

```

```

__pyx_v_rhs) {
08575     PyObject *__pyx_r = NULL;
08576     __Pyx_RefNannyDeclarations
08577     PyObject *__pyx_t_1 = NULL;
08578     PyObject *__pyx_t_2 = NULL;
08579     int __pyx_lineno = 0;
08580     const char *__pyx_filename = NULL;
08581     int __pyx_clineno = 0;
08582     __Pyx_RefNannySetupContext("__add__", 0);
08583
08584     /* "PyClicl.pyx":749
08585     *
08586     *
08587     *     return clifford().wrap( toClifford(lhs) + toClifford(rhs) )           # ««««««««
08588     *
08589     *     def __iadd__(self, rhs):
08590     */
08591     __Pyx_XDECREF(__pyx_r);
08592     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 749, __pyx_L1_error)
08593     __Pyx_GOTREF(__pyx_t_1);
08594     __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), (__pyx_f_8PyClicl_toClifford(__pyx_v_lhs) +
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 749,
__pyx_L1_error)
08595     __Pyx_GOTREF(__pyx_t_2);
08596     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08597     __pyx_r = __pyx_t_2;
08598     __pyx_t_2 = 0;
08599     goto __pyx_L0;
08600
08601     /* "PyClicl.pyx":740
08602     *
08603     *
08604     *     def __add__(lhs, rhs):           # ««««««««
08605     *
08606     *         Geometric sum.
08607     */
08608
08609     /* function exit code */
08610     __pyx_L1_error:;
08611     __Pyx_XDECREF(__pyx_t_1);
08612     __Pyx_XDECREF(__pyx_t_2);
08613     __Pyx_AddTraceback("PyClicl.clifford.__add__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08614     __pyx_r = NULL;
08615     __pyx_L0:;
08616     __Pyx_XGIVEREF(__pyx_r);
08617     __Pyx_RefNannyFinishContext();
08618     return __pyx_r;
08619 }
08620
08621     /* "PyClicl.pyx":751
08622     *
08623     *
08624     *     def __iadd__(self, rhs):           # ««««««««
08625     *
08626     *         Geometric sum.
08627     */
08628
08629     /* Python wrapper */
08630     static PyObject *__pyx_pw_8PyClicl_8clifford_23__iadd__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08631     static char __pyx_doc_8PyClicl_8clifford_22__iadd__[] = "\n         Geometric sum.\n\n
>> x = clifford(1); x += clifford('{2}'); print(x)\n         1+{2}\n         ";
08632     #if CYTHON_COMPILING_IN_CPYTHON
08633     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_22__iadd__;
08634     #endif
08635     static PyObject *__pyx_pw_8PyClicl_8clifford_23__iadd__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08636     PyObject *__pyx_r = 0;
08637     __Pyx_RefNannyDeclarations
08638     __Pyx_RefNannySetupContext("__iadd__ (wrapper)", 0);
08639     __pyx_r = __pyx_pf_8PyClicl_8clifford_22__iadd__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08640
08641     /* function exit code */
08642     __Pyx_RefNannyFinishContext();
08643     return __pyx_r;
08644 }
08645
08646     static PyObject *__pyx_pf_8PyClicl_8clifford_22__iadd__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
08647     PyObject *__pyx_r = NULL;
08648     __Pyx_RefNannyDeclarations
08649     PyObject *__pyx_t_1 = NULL;
08650     int __pyx_lineno = 0;

```



```

08651         const char *__pyx_filename = NULL;
08652         int __pyx_clineno = 0;
08653         __Pyx_RefNannySetupContext("__iadd__", 0);
08654
08655         /* "PyClical.pyx":758
08656         *         1+{2}
08657         *         """
08658         *         return self.wrap( self.unwrap() + toClifford(rhs) ) # ««««««««
08659         *
08660         *     def __sub__(lhs, rhs):
08661         */
08662         __Pyx_XDECREF(__pyx_r);
08663         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
08664         (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) + __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
08665         (unlikely(!__pyx_t_1)) __PYX_ERR(0, 758, __pyx_L1_error)
08666         __Pyx_GOTREF(__pyx_t_1);
08667         __pyx_r = __pyx_t_1;
08668         __pyx_t_1 = 0;
08669         goto __pyx_L0;
08670
08671         /* "PyClical.pyx":751
08672         *         return clifford().wrap( toClifford(lhs) + toClifford(rhs) )
08673         *
08674         *     def __iadd__(self, rhs): # ««««««««
08675         *         """
08676         *         Geometric sum.
08677         */
08678         /* function exit code */
08679         __pyx_L1_error++;
08680         __Pyx_XDECREF(__pyx_t_1);
08681         __Pyx_AddTraceback("PyClical.clifford.__iadd__", __pyx_clineno, __pyx_lineno,
08682         __pyx_filename);
08683         __pyx_r = NULL;
08684         __pyx_L0++;
08685         __Pyx_XGIVEREF(__pyx_r);
08686         __Pyx_RefNannyFinishContext();
08687         return __pyx_r;
08688     }
08689
08690         /* "PyClical.pyx":760
08691         *         return self.wrap( self.unwrap() + toClifford(rhs) )
08692         *
08693         *     def __sub__(lhs, rhs): # ««««««««
08694         *         """
08695         *         Geometric difference.
08696         */
08697         /* Python wrapper */
08698         static PyObject *__pyx_pw_8PyClical_8clifford_25__sub__(PyObject *__pyx_v_lhs, PyObject
08699         *__pyx_v_rhs); /*proto*/
08700         static char __pyx_doc_8PyClical_8clifford_24__sub__[] = "\n         Geometric difference.\n\n
08701         >> print(clifford(1) - clifford(\{2\})\n\n         1-2}\n         >> print(clifford(\{1\}) -
08702         clifford(\{2\})\n\n         {1}-{2}\n         ";
08703         #if CYTHON_COMPILING_IN_CPYTHON
08704         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_24__sub__;
08705         #endif
08706         static PyObject *__pyx_pw_8PyClical_8clifford_25__sub__(PyObject *__pyx_v_lhs, PyObject
08707         *__pyx_v_rhs) {
08708             PyObject *__pyx_r = 0;
08709             __Pyx_RefNannyDeclarations
08710             __Pyx_RefNannySetupContext("__sub__ (wrapper)", 0);
08711             __pyx_r = __pyx_pf_8PyClical_8clifford_24__sub__(((PyObject *)__pyx_v_lhs), ((PyObject
08712             *)__pyx_v_rhs));
08713
08714             /* function exit code */
08715             __Pyx_RefNannyFinishContext();
08716             return __pyx_r;
08717         }
08718
08719         static PyObject *__pyx_pf_8PyClical_8clifford_24__sub__(PyObject *__pyx_v_lhs, PyObject
08720         *__pyx_v_rhs) {
08721             PyObject *__pyx_r = NULL;
08722             __Pyx_RefNannyDeclarations
08723             PyObject *__pyx_t_1 = NULL;
08724             PyObject *__pyx_t_2 = NULL;
08725             int __pyx_lineno = 0;
08726             const char *__pyx_filename = NULL;
08727             int __pyx_clineno = 0;
08728             __Pyx_RefNannySetupContext("__sub__", 0);
08729
08730         /* "PyClical.pyx":769
08731         *         {1}-{2}
08732         *         """
08733         *         return clifford().wrap( toClifford(lhs) - toClifford(rhs) ) # ««««««««
08734         *
08735         *     def __isub__(self, rhs):

```

```

08729  */
08730      __Pyx_XDECREF(__pyx_r);
08731      __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 769, __pyx_L1_error)
08732      __Pyx_GOTREF(__pyx_t_1);
08733      __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) -
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 769,
__pyx_L1_error)
08734      __Pyx_GOTREF(__pyx_t_2);
08735      __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08736      __pyx_r = __pyx_t_2;
08737      __pyx_t_2 = 0;
08738      goto __pyx_L0;
08739
08740      /* "PyClical.pyx":760
08741      *      return self.wrap( self.unwrap() + toClifford(rhs) )
08742      *
08743      *      def __sub__(lhs, rhs):          # ««««««««
08744      *          """
08745      *          Geometric difference.
08746      */
08747
08748      /* function exit code */
08749      __pyx_L1_error:;
08750      __Pyx_XDECREF(__pyx_t_1);
08751      __Pyx_XDECREF(__pyx_t_2);
08752      __Pyx_AddTraceback("PyClical.clifford.__sub__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08753      __pyx_r = NULL;
08754      __pyx_L0:;
08755      __Pyx_XGIVEREF(__pyx_r);
08756      __Pyx_RefNannyFinishContext();
08757      return __pyx_r;
08758  }
08759
08760      /* "PyClical.pyx":771
08761      *      return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
08762      *
08763      *      def __isub__(self, rhs):          # ««««««««
08764      *          """
08765      *          Geometric difference.
08766      */
08767
08768      /* Python wrapper */
08769      static PyObject *__pyx_pw_8PyClical_8clifford_27__isub__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08770      static char __pyx_doc_8PyClical_8clifford_26__isub__[] = "\n          Geometric difference.\n\n
>> x = clifford(1); x -= clifford('{2}'); print(x)\n          1-{2}\n          ";
08771      #if CYTHON_COMPILING_IN_CPYTHON
08772      struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_26__isub__;
08773      #endif
08774      static PyObject *__pyx_pw_8PyClical_8clifford_27__isub__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08775      PyObject *__pyx_r = 0;
08776      __Pyx_RefNannyDeclarations
08777      __Pyx_RefNannySetupContext("__isub__ (wrapper)", 0);
08778      __pyx_r = __pyx_pf_8PyClical_8clifford_26__isub__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08779
08780      /* function exit code */
08781      __Pyx_RefNannyFinishContext();
08782      return __pyx_r;
08783  }
08784
08785      static PyObject *__pyx_pf_8PyClical_8clifford_26__isub__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
08786      PyObject *__pyx_r = NULL;
08787      __Pyx_RefNannyDeclarations
08788      PyObject *__pyx_t_1 = NULL;
08789      int __pyx_lineno = 0;
08790      const char *__pyx_filename = NULL;
08791      int __pyx_clineno = 0;
08792      __Pyx_RefNannySetupContext("__isub__", 0);
08793
08794      /* "PyClical.pyx":778
08795      *      1-{2}
08796      *      """
08797      *      return self.wrap( self.unwrap() - toClifford(rhs) )          # ««««««««
08798      *
08799      *      def __mul__(lhs, rhs):
08800      */
08801      __Pyx_XDECREF(__pyx_r);
08802      __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) - __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 778, __pyx_L1_error)
08803      __Pyx_GOTREF(__pyx_t_1);

```

```

08804         __pyx_r = __pyx_t_1;
08805         __pyx_t_1 = 0;
08806         goto __pyx_L0;
08807
08808         /* "PyClical.pyx":771
08809  *         return clifford().wrap( toClifford(lhs) - toClifford(rhs) )
08810  *
08811  *         def __isub__(self, rhs):           # ««««««««
08812  *             """
08813  *             Geometric difference.
08814  */
08815
08816         /* function exit code */
08817         __pyx_L1_error++;
08818         __Pyx_XDECREF(__pyx_t_1);
08819         __Pyx_AddTraceback("PyClical.clifford.__isub__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08820         __pyx_r = NULL;
08821         __pyx_L0;;
08822         __Pyx_XGIVEREF(__pyx_r);
08823         __Pyx_RefNannyFinishContext();
08824         return __pyx_r;
08825     }
08826
08827     /* "PyClical.pyx":780
08828  *         return self.wrap( self.unwrap() - toClifford(rhs) )
08829  *
08830  *         def __mul__(lhs, rhs):           # ««««««««
08831  *             """
08832  *             Geometric product.
08833  */
08834
08835     /* Python wrapper */
08836     static PyObject *__pyx_pw_8PyClical_8clifford_29__mul__(PyObject *__pyx_v_lhs, PyObject
__pyx_v_rhs); /*proto*/
08837     static char __pyx_doc_8PyClical_8clifford_28__mul__[] = "\n        Geometric product.\n\n
>> print(clifford(\"{1}\") * clifford(\"{2}\"))\n        {1,2}\n        >> print(clifford(2) *
clifford(\"{2}\"))\n        2{2}\n        >> print(clifford(\"{1}\") * clifford(\"{1,2}\"))\n
{2}\n        ";
08838     #if CYTHON_COMPILING_IN_CPYTHON
08839     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_28__mul__;
08840     #endif
08841     static PyObject *__pyx_pw_8PyClical_8clifford_29__mul__(PyObject *__pyx_v_lhs, PyObject
__pyx_v_rhs) {
08842         PyObject *__pyx_r = 0;
08843         __Pyx_RefNannyDeclarations
08844         __Pyx_RefNannySetupContext("__mul__ (wrapper)", 0);
08845         __pyx_r = __pyx_pf_8PyClical_8clifford_28__mul__(((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
08846
08847         /* function exit code */
08848         __Pyx_RefNannyFinishContext();
08849         return __pyx_r;
08850     }
08851
08852     static PyObject *__pyx_pf_8PyClical_8clifford_28__mul__(PyObject *__pyx_v_lhs, PyObject
__pyx_v_rhs) {
08853         PyObject *__pyx_r = NULL;
08854         __Pyx_RefNannyDeclarations
08855         PyObject *__pyx_t_1 = NULL;
08856         PyObject *__pyx_t_2 = NULL;
08857         int __pyx_lineno = 0;
08858         const char *__pyx_filename = NULL;
08859         int __pyx_clineno = 0;
08860         __Pyx_RefNannySetupContext("__mul__", 0);
08861
08862         /* "PyClical.pyx":791
08863  *         {2}
08864  *         """
08865  *         return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08866  *
08867  *         def __imul__(self, rhs):
08868  */
08869         __Pyx_XDECREF(__pyx_r);
08870         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 791, __pyx_L1_error)
08871         __Pyx_GOTREF(__pyx_t_1);
08872         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) *
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 791,
__pyx_L1_error)
08873         __Pyx_GOTREF(__pyx_t_2);
08874         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
08875         __pyx_r = __pyx_t_2;
08876         __pyx_t_2 = 0;
08877         goto __pyx_L0;
08878

```

```

08879      /* "PyClicl.pyx":780
08880      *      return self.wrap( self.unwrap() - toClifford(rhs) )
08881      *
08882      *      def __mul__(lhs, rhs):          # ««««««««
08883      *          """
08884      *          Geometric product.
08885      */
08886
08887      /* function exit code */
08888      __pyx_L1_error:;
08889      __Pyx_XDECREF(__pyx_t_1);
08890      __Pyx_XDECREF(__pyx_t_2);
08891      __Pyx_AddTraceback("PyClicl.clifford.__mul__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08892      __pyx_r = NULL;
08893      __pyx_L0:;
08894      __Pyx_XGIVEREF(__pyx_r);
08895      __Pyx_RefNannyFinishContext();
08896      return __pyx_r;
08897  }
08898
08899      /* "PyClicl.pyx":793
08900      *      return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08901      *
08902      *      def __imul__(self, rhs):          # ««««««««
08903      *          """
08904      *          Geometric product.
08905      */
08906
08907      /* Python wrapper */
08908      static PyObject *__pyx_pw_8PyClicl_8clifford_31__imul__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
08909      static char __pyx_doc_8PyClicl_8clifford_30__imul__[] = "\n          Geometric product.\n\n
>> x = clifford(2); x *= clifford('{2}'); print(x)\n          2{2}\n          >> x = clifford('{1}');
x *= clifford('{2}'); print(x)\n          {1,2}\n          >> x = clifford('{1}'); x *=
clifford('{1,2}'); print(x)\n          {2}\n          ";
08910      #if CYTHON_COMPILING_IN_CPYTHON
08911      struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_30__imul__;
08912      #endif
08913      static PyObject *__pyx_pw_8PyClicl_8clifford_31__imul__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
08914          PyObject *__pyx_r = 0;
08915          __Pyx_RefNannyDeclarations
08916          __Pyx_RefNannySetupContext("__imul__ (wrapper)", 0);
08917          __pyx_r = __pyx_pf_8PyClicl_8clifford_30__imul__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
08918
08919      /* function exit code */
08920      __Pyx_RefNannyFinishContext();
08921      return __pyx_r;
08922  }
08923
08924      static PyObject *__pyx_pf_8PyClicl_8clifford_30__imul__(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
08925          PyObject *__pyx_r = NULL;
08926          __Pyx_RefNannyDeclarations
08927          PyObject *__pyx_t_1 = NULL;
08928          int __pyx_lineno = 0;
08929          const char *__pyx_filename = NULL;
08930          int __pyx_clineno = 0;
08931          __Pyx_RefNannySetupContext("__imul__", 0);
08932
08933      /* "PyClicl.pyx":804
08934      *          {2}
08935      *          """
08936      *          return self.wrap( self.unwrap() * toClifford(rhs) )          # ««««««««
08937      *
08938      *      def __mod__(lhs, rhs):
08939      */
08940      __Pyx_XDECREF(__pyx_r);
08941      __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) * __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 804, __pyx_L1_error)
08942      __Pyx_GOTREF(__pyx_t_1);
08943      __pyx_r = __pyx_t_1;
08944      __pyx_t_1 = 0;
08945      goto __pyx_L0;
08946
08947      /* "PyClicl.pyx":793
08948      *      return clifford().wrap( toClifford(lhs) * toClifford(rhs) )
08949      *
08950      *      def __imul__(self, rhs):          # ««««««««
08951      *          """
08952      *          Geometric product.
08953      */
08954
08955      /* function exit code */

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```

08956         __pyx_L1_error;;
08957         __Pyx_XDECREF(__pyx_t_1);
08958         __Pyx_AddTraceback("PyClicl.clifford.__imul__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
08959         __pyx_r = NULL;
08960         __pyx_L0;;
08961         __Pyx_XGIVEREF(__pyx_r);
08962         __Pyx_RefNannyFinishContext();
08963         return __pyx_r;
08964     }
08965
08966     /* "PyClicl.pyx":806
08967     *         return self.wrap( self.unwrap() * toClifford(rhs) )
08968     *
08969     *     def __mod__(lhs, rhs):                # ««««««««
08970     *         """
08971     *         Contraction.
08972     */
08973
08974     /* Python wrapper */
08975     static PyObject *__pyx_pw_8PyClicl_8clifford_33__mod__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
08976     static char __pyx_doc_8PyClicl_8clifford_32__mod__[] = "\n        Contraction.\n\n    >>
print(clifford(\"{1}\") % clifford(\"{2}\"))\n        0\n    >> print(clifford(2) %
clifford(\"{2}\"))\n        2{2}\n    >> print(clifford(\"{1}\") % clifford(\"{1}\"))\n    1\n
>> print(clifford(\"{1}\") % clifford(\"{1,2}\"))\n        {2}\n        ";
08977     #if CYTHON_COMPILING_IN_CPYTHON
08978     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_32__mod__;
08979     #endif
08980     static PyObject *__pyx_pw_8PyClicl_8clifford_33__mod__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08981         PyObject *__pyx_r = 0;
08982         __Pyx_RefNannyDeclarations
08983         __Pyx_RefNannySetupContext("__mod__ (wrapper)", 0);
08984         __pyx_r = __pyx_pf_8PyClicl_8clifford_32__mod__(((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
08985
08986         /* function exit code */
08987         __Pyx_RefNannyFinishContext();
08988         return __pyx_r;
08989     }
08990
08991     static PyObject *__pyx_pf_8PyClicl_8clifford_32__mod__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
08992         PyObject *__pyx_r = NULL;
08993         __Pyx_RefNannyDeclarations
08994         PyObject *__pyx_t_1 = NULL;
08995         PyObject *__pyx_t_2 = NULL;
08996         int __pyx_lineno = 0;
08997         const char *__pyx_filename = NULL;
08998         int __pyx_clineno = 0;
08999         __Pyx_RefNannySetupContext("__mod__", 0);
09000
09001         /* "PyClicl.pyx":819
09002     *         {2}
09003     *         """
09004     *         return clifford().wrap( toClifford(lhs) % toClifford(rhs) )                # ««««««««
09005     *
09006     *     def __imod__(self, rhs):
09007     */
09008         __Pyx_XDECREF(__pyx_r);
09009         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 819, __pyx_L1_error)
09010         __Pyx_GOTREF(__pyx_t_1);
09011         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), (__pyx_f_8PyClicl_toClifford(__pyx_v_lhs) %
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 819,
__pyx_L1_error)
09012         __Pyx_GOTREF(__pyx_t_2);
09013         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09014         __pyx_r = __pyx_t_2;
09015         __pyx_t_2 = 0;
09016         goto __pyx_L0;
09017
09018         /* "PyClicl.pyx":806
09019     *         return self.wrap( self.unwrap() * toClifford(rhs) )
09020     *
09021     *     def __mod__(lhs, rhs):                # ««««««««
09022     *         """
09023     *         Contraction.
09024     */
09025
09026         /* function exit code */
09027         __pyx_L1_error;;
09028         __Pyx_XDECREF(__pyx_t_1);
09029         __Pyx_XDECREF(__pyx_t_2);
09030         __Pyx_AddTraceback("PyClicl.clifford.__mod__", __pyx_clineno, __pyx_lineno,

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```

__pyx_filename);
09031     __pyx_r = NULL;
09032     __pyx_L0:;
09033     __Pyx_XGIVEREF(__pyx_r);
09034     __Pyx_RefNannyFinishContext();
09035     return __pyx_r;
09036 }
09037
09038 /* "PyClicl.pyx":821
09039 *     return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
09040 *
09041 * def __imod__(self, rhs):          # ««««««««
09042 *     """
09043 *     Contraction.
09044 */
09045
09046     /* Python wrapper */
09047     static PyObject *__pyx_pw_8PyClicl_8clifford_35__imod__(PyObject *__pyx_v_self, PyObject
__pyx_v_rhs); /*proto*/
09048     static char __pyx_doc_8PyClicl_8clifford_34__imod__[] = "\n        Contraction.\n\n        >>
x = clifford(\"{1}\"); x %= clifford(\"{2}\"); print(x)\n        0\n        >> x = clifford(2); x
%= clifford(\"{2}\"); print(x)\n        2{2}\n        >> x = clifford(\"{1}\"); x %= clifford(\"{1}\");
print(x)\n        1\n        >> x = clifford(\"{1}\"); x %= clifford(\"{1,2}\"); print(x)\n
{2}\n        ";
09049     #if CYTHON_COMPILING_IN_CPYTHON
09050     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_34__imod__;
09051     #endif
09052     static PyObject *__pyx_pw_8PyClicl_8clifford_35__imod__(PyObject *__pyx_v_self, PyObject
__pyx_v_rhs) {
09053         PyObject *__pyx_r = 0;
09054         __Pyx_RefNannyDeclarations
09055         __Pyx_RefNannySetupContext("__imod__ (wrapper)", 0);
09056         __pyx_r = __pyx_pf_8PyClicl_8clifford_34__imod__(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09057
09058         /* function exit code */
09059         __Pyx_RefNannyFinishContext();
09060         return __pyx_r;
09061     }
09062
09063     static PyObject *__pyx_pf_8PyClicl_8clifford_34__imod__(struct __pyx_obj_8PyClicl_clifford
__pyx_v_self, PyObject *__pyx_v_rhs) {
09064         PyObject *__pyx_r = NULL;
09065         __Pyx_RefNannyDeclarations
09066         PyObject *__pyx_t_1 = NULL;
09067         int __pyx_lineno = 0;
09068         const char *__pyx_filename = NULL;
09069         int __pyx_clineno = 0;
09070         __Pyx_RefNannySetupContext("__imod__", 0);
09071
09072         /* "PyClicl.pyx":834
09073 *     {2}
09074 *     """
09075 *     return self.wrap( self.unwrap() % toClifford(rhs) )          # ««««««««
09076 *
09077 * def __and__(lhs, rhs):
09078 */
09079         __Pyx_XDECREF(__pyx_r);
09080         __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) % __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 834, __pyx_L1_error)
09081         __Pyx_GOTREF(__pyx_t_1);
09082         __pyx_r = __pyx_t_1;
09083         __pyx_t_1 = 0;
09084         goto __pyx_L0;
09085
09086         /* "PyClicl.pyx":821
09087 *     return clifford().wrap( toClifford(lhs) % toClifford(rhs) )
09088 *
09089 * def __imod__(self, rhs):          # ««««««««
09090 *     """
09091 *     Contraction.
09092 */
09093
09094         /* function exit code */
09095         __pyx_L1_error:;
09096         __Pyx_XDECREF(__pyx_t_1);
09097         __Pyx_AddTraceback("PyClicl.clifford.__imod__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09098         __pyx_r = NULL;
09099         __pyx_L0:;
09100         __Pyx_XGIVEREF(__pyx_r);
09101         __Pyx_RefNannyFinishContext();
09102         return __pyx_r;
09103     }
09104
09105     /* "PyClicl.pyx":836

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09181 *          """
09182 *          Inner product.
09183 */
09184
09185 /* Python wrapper */
09186 static PyObject *__pyx_pw_8PyClical_8clifford_39__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09187 static char __pyx_doc_8PyClical_8clifford_38__iand__[] = "\n          Inner product.\n\n
>> x = clifford(\"{1}\"); x &= clifford(\"{2}\"); print(x)\n          0\n          >> x = clifford(2); x
&= clifford(\"{2}\"); print(x)\n          0\n          >> x = clifford(\"{1}\"); x &= clifford(\"{1}\");
print(x)\n          1\n          >> x = clifford(\"{1}\"); x &= clifford(\"{1,2}\"); print(x)\n
{2}\n          ";
09188 #if CYTHON_COMPILING_IN_CPYTHON
09189 struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_38__iand__;
09190 #endif
09191 static PyObject *__pyx_pw_8PyClical_8clifford_39__iand__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09192     PyObject *__pyx_r = 0;
09193     __Pyx_RefNannyDeclarations
09194     __Pyx_RefNannySetupContext("__iand__ (wrapper)", 0);
09195     __pyx_r = __pyx_pf_8PyClical_8clifford_38__iand__((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09196
09197     /* function exit code */
09198     __Pyx_RefNannyFinishContext();
09199     return __pyx_r;
09200 }
09201
09202 static PyObject *__pyx_pf_8PyClical_8clifford_38__iand__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
09203     PyObject *__pyx_r = NULL;
09204     __Pyx_RefNannyDeclarations
09205     PyObject *__pyx_t_1 = NULL;
09206     int __pyx_lineno = 0;
09207     const char *__pyx_filename = NULL;
09208     int __pyx_clineno = 0;
09209     __Pyx_RefNannySetupContext("__iand__", 0);
09210
09211     /* "PyClical.pyx":864
09212 *          {2}
09213 *          """
09214 *          return self.wrap( self.unwrap() & toClifford(rhs) )          # ««««««««
09215 *
09216 *          def __xor__(lhs, rhs):
09217 */
09218     __Pyx_XDECREF(__pyx_r);
09219     __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) & __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 864, __pyx_L1_error)
09220     __Pyx_GOTREF(__pyx_t_1);
09221     __pyx_r = __pyx_t_1;
09222     __pyx_t_1 = 0;
09223     goto __pyx_L0;
09224
09225     /* "PyClical.pyx":851
09226 *          return clifford().wrap( toClifford(lhs) & toClifford(rhs) )
09227 *
09228 *          def __iand__(self, rhs):          # ««««««««
09229 *          """
09230 *          Inner product.
09231 */
09232
09233     /* function exit code */
09234     __pyx_L1_error:;
09235     __Pyx_XDECREF(__pyx_t_1);
09236     __Pyx_AddTraceback("PyClical.clifford.__iand__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09237     __pyx_r = NULL;
09238     __pyx_L0:;
09239     __Pyx_XGIVEREF(__pyx_r);
09240     __Pyx_RefNannyFinishContext();
09241     return __pyx_r;
09242 }
09243
09244 /* "PyClical.pyx":866
09245 *          return self.wrap( self.unwrap() & toClifford(rhs) )
09246 *
09247 *          def __xor__(lhs, rhs):          # ««««««««
09248 *          """
09249 *          Outer product.
09250 */
09251
09252 /* Python wrapper */
09253 static PyObject *__pyx_pw_8PyClical_8clifford_41__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
09254 static char __pyx_doc_8PyClical_8clifford_40__xor__[] = "\n          Outer product.\n\n
>> print(clifford(\"{1}\") ^ clifford(\"{2}\"))\n          {1,2}\n          >> print(clifford(2) ^

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        clifford("\{2}\")\n        2{2}\n        >> print(clifford("\{1}\") ^ clifford("\{1}\"))\n        0\n
>> print(clifford("\{1}\") ^ clifford("\{1,2}\"))\n        0\n        ";
09255     #if CYTHON_COMPILING_IN_CPYTHON
09256     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_40__xor__;
09257     #endif
09258     static PyObject *__pyx_pw_8PyClicl_8clifford_41__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09259         PyObject *__pyx_r = 0;
09260         __Pyx_RefNannyDeclarations
09261         __Pyx_RefNannySetupContext("__xor__ (wrapper)", 0);
09262         __pyx_r = __pyx_pf_8PyClicl_8clifford_40__xor__((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09263
09264         /* function exit code */
09265         __Pyx_RefNannyFinishContext();
09266         return __pyx_r;
09267     }
09268
09269     static PyObject *__pyx_pf_8PyClicl_8clifford_40__xor__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09270         PyObject *__pyx_r = NULL;
09271         __Pyx_RefNannyDeclarations
09272         PyObject *__pyx_t_1 = NULL;
09273         PyObject *__pyx_t_2 = NULL;
09274         int __pyx_lineno = 0;
09275         const char *__pyx_filename = NULL;
09276         int __pyx_clineno = 0;
09277         __Pyx_RefNannySetupContext("__xor__", 0);
09278
09279         /* "PyClicl.pyx":879
09280         *
09281         *
09282         * return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09283         *
09284         * def __ixor__(self, rhs):
09285         */
09286         __Pyx_XDECREF(__pyx_r);
09287         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 879, __pyx_L1_error)
09288         __Pyx_GOTREF(__pyx_t_1);
09289         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), (__pyx_f_8PyClicl_toClifford(__pyx_v_lhs) ^
__pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 879,
__pyx_L1_error)
09290         __Pyx_GOTREF(__pyx_t_2);
09291         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09292         __pyx_r = __pyx_t_2;
09293         __pyx_t_2 = 0;
09294         goto __pyx_L0;
09295
09296         /* "PyClicl.pyx":866
09297         * return self.wrap( self.unwrap() & toClifford(rhs) )
09298         *
09299         * def __xor__(lhs, rhs):
09300         *
09301         * Outer product.
09302         */
09303
09304         /* function exit code */
09305         __pyx_L1_error;
09306         __Pyx_XDECREF(__pyx_t_1);
09307         __Pyx_XDECREF(__pyx_t_2);
09308         __Pyx_AddTraceback("PyClicl.clifford.__xor__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09309         __pyx_r = NULL;
09310         __pyx_L0;
09311         __Pyx_XGIVEREF(__pyx_r);
09312         __Pyx_RefNannyFinishContext();
09313         return __pyx_r;
09314     }
09315
09316     /* "PyClicl.pyx":881
09317     * return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09318     *
09319     * def __ixor__(self, rhs):
09320     *
09321     * Outer product.
09322     */
09323
09324     /* Python wrapper */
09325     static PyObject *__pyx_pw_8PyClicl_8clifford_43__ixor__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09326     static char __pyx_doc_8PyClicl_8clifford_42__ixor__[] = "\n        Outer product.\n\n
>> x = clifford("\{1}\"); x ^= clifford("\{2}\"); print(x)\n        {1,2}\n        >> x = clifford(2);
x ^= clifford("\{2}\"); print(x)\n        2{2}\n        >> x = clifford("\{1}\"); x ^=
clifford("\{1}\"); print(x)\n        0\n        >> x = clifford("\{1}\"); x ^= clifford("\{1,2}\");
print(x)\n        0\n        ";

```

```

09327         #if CYTHON_COMPILING_IN_CPYTHON
09328         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_42__ixor__;
09329         #endif
09330         static PyObject *__pyx_pw_8PyClical_8clifford_43__ixor__(PyObject *__pyx_v_self, PyObject
09331 *__pyx_v_rhs) {
09332         PyObject *__pyx_r = 0;
09333         __Pyx_RefNannyDeclarations
09334         __Pyx_RefNannySetupContext("__ixor__ (wrapper)", 0);
09335         __pyx_r = __pyx_pf_8PyClical_8clifford_42__ixor__(((struct __pyx_obj_8PyClical_clifford
09336 *)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09337
09338         /* function exit code */
09339         __Pyx_RefNannyFinishContext();
09340         return __pyx_r;
09341     }
09342
09343     static PyObject *__pyx_pf_8PyClical_8clifford_42__ixor__(struct __pyx_obj_8PyClical_clifford
09344 *__pyx_v_self, PyObject *__pyx_v_rhs) {
09345     PyObject *__pyx_r = NULL;
09346     __Pyx_RefNannyDeclarations
09347     PyObject *__pyx_t_1 = NULL;
09348     int __pyx_lineno = 0;
09349     const char *__pyx_filename = NULL;
09350     int __pyx_clineno = 0;
09351     __Pyx_RefNannySetupContext("__ixor__", 0);
09352
09353     /* "PyClical.pyx":894
09354 *
09355 *     return self.wrap( self.unwrap() ^ toClifford(rhs) )           # ««««««««
09356 *
09357 *     def __truediv__(lhs, rhs):
09358 */
09359     __Pyx_XDECREF(__pyx_r);
09360     __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
09361 (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) ^ __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
09362 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 894, __pyx_L1_error)
09363     __Pyx_GOTREF(__pyx_t_1);
09364     __pyx_r = __pyx_t_1;
09365     __pyx_t_1 = 0;
09366     goto __pyx_L0;
09367
09368     /* "PyClical.pyx":881
09369 *     return clifford().wrap( toClifford(lhs) ^ toClifford(rhs) )
09370 *
09371 *     def __ixor__(self, rhs):           # ««««««««
09372 *     """
09373 *     Outer product.
09374 */
09375     /* function exit code */
09376     __pyx_L1_error:;
09377     __Pyx_XDECREF(__pyx_t_1);
09378     __Pyx_AddTraceback("PyClical.clifford.__ixor__", __pyx_clineno, __pyx_lineno,
09379 __pyx_filename);
09380     __pyx_r = NULL;
09381     __pyx_L0:;
09382     __Pyx_XGIVEREF(__pyx_r);
09383     __Pyx_RefNannyFinishContext();
09384     return __pyx_r;
09385 }
09386
09387     /* "PyClical.pyx":896
09388 *     return self.wrap( self.unwrap() ^ toClifford(rhs) )
09389 *
09390 *     def __truediv__(lhs, rhs):           # ««««««««
09391 *     """
09392 *     Geometric quotient.
09393 */
09394     /* Python wrapper */
09395     static PyObject *__pyx_pw_8PyClical_8clifford_45__truediv__(PyObject *__pyx_v_lhs, PyObject
09396 *__pyx_v_rhs); /*proto*/
09397     static char __pyx_doc_8PyClical_8clifford_44__truediv__[] = "\n          Geometric quotient.\n\n
09398 >> print(clifford(\"{1}\") / clifford(\"{2}\"))\n          {1,2}\n          >> print(clifford(2) /
09399 clifford(\"{2}\"))\n          2{2}\n          >> print(clifford(\"{1}\") / clifford(\"{1}\"))\n          1\n
09400 >> print(clifford(\"{1}\") / clifford(\"{1,2}\"))\n          -{2}\n          ";
09401
09402     #if CYTHON_COMPILING_IN_CPYTHON
09403     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_44__truediv__;
09404     #endif
09405     static PyObject *__pyx_pw_8PyClical_8clifford_45__truediv__(PyObject *__pyx_v_lhs, PyObject
09406 *__pyx_v_rhs) {
09407     PyObject *__pyx_r = 0;
09408     __Pyx_RefNannyDeclarations
09409     __Pyx_RefNannySetupContext("__truediv__ (wrapper)", 0);
09410     __pyx_r = __pyx_pf_8PyClical_8clifford_44__truediv__(((PyObject *)__pyx_v_lhs), ((PyObject
09411 *)__pyx_v_rhs));

```

```

09402
09403     /* function exit code */
09404     __Pyx_RefNannyFinishContext();
09405     return __pyx_r;
09406 }
09407
09408 static PyObject *__pyx_pf_8PyClical_8clifford_44__truediv__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09409     PyObject *__pyx_r = NULL;
09410     __Pyx_RefNannyDeclarations
09411     PyObject *__pyx_t_1 = NULL;
09412     PyObject *__pyx_t_2 = NULL;
09413     int __pyx_lineno = 0;
09414     const char *__pyx_filename = NULL;
09415     int __pyx_clineno = 0;
09416     __Pyx_RefNannySetupContext("__truediv__", 0);
09417
09418     /* "PyClical.pyx":909
09419     *
09420     *
09421     *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09422     *
09423     *     def __idiv__(self, rhs):
09424     */
09425     __Pyx_XDECREF(__pyx_r);
09426     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 909, __pyx_L1_error)
09427     __Pyx_GOTREF(__pyx_t_1);
09428     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) /
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 909,
__pyx_L1_error)
09429     __Pyx_GOTREF(__pyx_t_2);
09430     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09431     __pyx_r = __pyx_t_2;
09432     __pyx_t_2 = 0;
09433     goto __pyx_L0;
09434
09435     /* "PyClical.pyx":896
09436     *
09437     *     return self.wrap( self.unwrap() ^ toClifford(rhs) )
09438     *
09439     *     def __truediv__(lhs, rhs):
09440     *
09441     *         Geometric quotient.
09442     */
09443     /* function exit code */
09444     __pyx_L1_error:;
09445     __Pyx_XDECREF(__pyx_t_1);
09446     __Pyx_XDECREF(__pyx_t_2);
09447     __Pyx_AddTraceback("PyClical.clifford.__truediv__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09448     __pyx_r = NULL;
09449     __pyx_L0:;
09450     __Pyx_XGIVEREF(__pyx_r);
09451     __Pyx_RefNannyFinishContext();
09452     return __pyx_r;
09453 }
09454
09455     /* "PyClical.pyx":911
09456     *
09457     *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09458     *
09459     *     def __idiv__(self, rhs):
09460     *
09461     *         Geometric quotient.
09462     */
09463     /* Python wrapper */
09464     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
09465     static PyObject *__pyx_pw_8PyClical_8clifford_47__idiv__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09466     static char __pyx_doc_8PyClical_8clifford_46__idiv__[] = "\n        Geometric quotient.\n\n
>> x = clifford(\"{1}\"); x /= clifford(\"{2}\"); print(x)\n        {1,2}\n        >> x = clifford(2);
x /= clifford(\"{2}\"); print(x)\n        2{2}\n        >> x = clifford(\"{1}\"); x /=
clifford(\"{1}\"); print(x)\n        1\n        >> x = clifford(\"{1}\"); x /= clifford(\"{1,2}\");
print(x)\n        -{2}\n        ";
09467     #if CYTHON_COMPILING_IN_CPYTHON
09468     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_46__idiv__;
09469     #endif
09470     static PyObject *__pyx_pw_8PyClical_8clifford_47__idiv__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09471     PyObject *__pyx_r = 0;
09472     __Pyx_RefNannyDeclarations
09473     __Pyx_RefNannySetupContext("__idiv__ (wrapper)", 0);
09474     __pyx_r = __pyx_pf_8PyClical_8clifford_46__idiv__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09475

```

```

09476         /* function exit code */
09477         __Pyx_RefNannyFinishContext();
09478         return __pyx_r;
09479     }
09480     #endif
09481     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
09482     static PyObject *__pyx_pf_8PyClical_8clifford_46_idiv__(struct __pyx_obj_8PyClical_clifford
09483 *__pyx_v_self, PyObject *__pyx_v_rhs) {
09484         PyObject *__pyx_r = NULL;
09485         __Pyx_RefNannyDeclarations
09486         PyObject *__pyx_t_1 = NULL;
09487         int __pyx_lineno = 0;
09488         const char *__pyx_filename = NULL;
09489         int __pyx_clineno = 0;
09490         __Pyx_RefNannySetupContext("__idiv__", 0);
09491
09492         /* "PyClical.pyx":924
09493          *
09494          *
09495          *     return self.wrap( self.unwrap() / toClifford(rhs) )
09496          *
09497          *     def inv(self):
09498          */
09499         __Pyx_XDECREF(__pyx_r);
09500         __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(__pyx_v_self,
09501         (__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self) / __pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if
09502         (unlikely(!__pyx_t_1)) __PYX_ERR(0, 924, __pyx_L1_error)
09503         __Pyx_GOTREF(__pyx_t_1);
09504         __pyx_r = __pyx_t_1;
09505         __pyx_t_1 = 0;
09506         goto __pyx_L0;
09507
09508         /* "PyClical.pyx":911
09509          *
09510          *     return clifford().wrap( toClifford(lhs) / toClifford(rhs) )
09511          *
09512          *     def __idiv__(self, rhs):
09513          *
09514          *         """
09515          *         Geometric quotient.
09516          *
09517          */
09518         /* function exit code */
09519         __pyx_L1_error:;
09520         __Pyx_XDECREF(__pyx_t_1);
09521         __Pyx_AddTraceback("PyClical.clifford.__idiv__", __pyx_clineno, __pyx_lineno,
09522         __pyx_filename);
09523         __pyx_r = NULL;
09524         __pyx_L0:;
09525         __Pyx_XGIVEREF(__pyx_r);
09526         __Pyx_RefNannyFinishContext();
09527         return __pyx_r;
09528     }
09529     #endif
09530     /* "PyClical.pyx":926
09531          *
09532          *     return self.wrap( self.unwrap() / toClifford(rhs) )
09533          *
09534          *     def inv(self):
09535          *
09536          *         """
09537          *         Geometric multiplicative inverse.
09538          *
09539          */
09540         /* Python wrapper */
09541         static PyObject *__pyx_pw_8PyClical_8clifford_49inv(PyObject *__pyx_v_self, CYTHON_UNUSED
09542         PyObject *unused); /*proto*/
09543         static char __pyx_doc_8PyClical_8clifford_48inv[] = "\n          Geometric multiplicative
09544         inverse.\n\n          >> x = clifford('{1}'); print(x.inv())\n          {1}\n          >> x = clifford(2);
09545         print(x.inv())\n          0.5\n          >> x = clifford('{1,2}'); print(x.inv())\n          -{1,2}\n
09546         ";
09547         static PyObject *__pyx_pw_8PyClical_8clifford_49inv(PyObject *__pyx_v_self, CYTHON_UNUSED
09548         PyObject *unused) {
09549             PyObject *__pyx_r = 0;
09550             __Pyx_RefNannyDeclarations
09551             __Pyx_RefNannySetupContext("inv (wrapper)", 0);
09552             __pyx_r = __pyx_pf_8PyClical_8clifford_48inv(((struct __pyx_obj_8PyClical_clifford
09553 *)__pyx_v_self));
09554
09555         /* function exit code */
09556         __Pyx_RefNannyFinishContext();
09557         return __pyx_r;
09558     }
09559
09560     static PyObject *__pyx_pf_8PyClical_8clifford_48inv(struct __pyx_obj_8PyClical_clifford
09561 *__pyx_v_self) {
09562         PyObject *__pyx_r = NULL;
09563         __Pyx_RefNannyDeclarations
09564         PyObject *__pyx_t_1 = NULL;
09565         PyObject *__pyx_t_2 = NULL;
09566         int __pyx_lineno = 0;

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09554     const char *__pyx_filename = NULL;
09555     int __pyx_clineno = 0;
09556     __Pyx_RefNannySetupContext("inv", 0);
09557
09558     /* "PyClical.pyx":937
09559     *     -{1,2}
09560     *     """
09561     *     return clifford().wrap( self.instance.inv() )           # ««««««««
09562     *
09563     *     def __or__(lhs, rhs):
09564     */
09565         __Pyx_XDECREF(__pyx_r);
09566         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 937, __pyx_L1_error)
09567         __Pyx_GOTREF(__pyx_t_1);
09568         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->inv()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 937,
__pyx_L1_error)
09569         __Pyx_GOTREF(__pyx_t_2);
09570         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09571         __pyx_r = __pyx_t_2;
09572         __pyx_t_2 = 0;
09573         goto __pyx_L0;
09574
09575     /* "PyClical.pyx":926
09576     *     return self.wrap( self.unwrap() / toClifford(rhs) )
09577     *
09578     *     def inv(self):           # ««««««««
09579     *     """
09580     *     Geometric multiplicative inverse.
09581     */
09582
09583     /* function exit code */
09584     __pyx_L1_error:;
09585     __Pyx_XDECREF(__pyx_t_1);
09586     __Pyx_XDECREF(__pyx_t_2);
09587     __Pyx_AddTraceback("PyClical.clifford.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
09588     __pyx_r = NULL;
09589     __pyx_L0:;
09590     __Pyx_XGIVEREF(__pyx_r);
09591     __Pyx_RefNannyFinishContext();
09592     return __pyx_r;
09593 }
09594
09595     /* "PyClical.pyx":939
09596     *     return clifford().wrap( self.instance.inv() )
09597     *
09598     *     def __or__(lhs, rhs):           # ««««««««
09599     *     """
09600     *     Transform left hand side, using right hand side as a transformation.
09601     */
09602
09603     /* Python wrapper */
09604     static PyObject *__pyx_pw_8PyClical_8clifford_51_or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs); /*proto*/
09605     static char __pyx_doc_8PyClical_8clifford_50_or__[] = "\n        Transform left hand side,
using right hand side as a transformation.\n\n        >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); print(y|x)\n        -{1}\n        >> x=clifford(\"{1,2}\") * pi/2;
y=clifford(\"{1}\"); print(y|exp(x))\n        -{1}\n        ";
09606     #if CYTHON_COMPILING_IN_CPYTHON
09607     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_50_or__;
09608     #endif
09609     static PyObject *__pyx_pw_8PyClical_8clifford_51_or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09610         PyObject *__pyx_r = 0;
09611         __Pyx_RefNannyDeclarations
09612         __Pyx_RefNannySetupContext("__or__ (wrapper)", 0);
09613         __pyx_r = __pyx_pf_8PyClical_8clifford_50_or__(((PyObject *)__pyx_v_lhs), ((PyObject
*)__pyx_v_rhs));
09614
09615     /* function exit code */
09616     __Pyx_RefNannyFinishContext();
09617     return __pyx_r;
09618 }
09619
09620     static PyObject *__pyx_pf_8PyClical_8clifford_50_or__(PyObject *__pyx_v_lhs, PyObject
*__pyx_v_rhs) {
09621         PyObject *__pyx_r = NULL;
09622         __Pyx_RefNannyDeclarations
09623         PyObject *__pyx_t_1 = NULL;
09624         PyObject *__pyx_t_2 = NULL;
09625         int __pyx_lineno = 0;
09626         const char *__pyx_filename = NULL;
09627         int __pyx_clineno = 0;
09628         __Pyx_RefNannySetupContext("__or__", 0);
09629
09630         /* "PyClical.pyx":948

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```

09631 *         -{1}
09632 *         """
09633 *         return clifford().wrap( toClifford(lhs) | toClifford(rhs) ) # ««««««««
09634 *
09635 *     def __ior__(self, rhs):
09636 */
09637         __Pyx_XDECREF(__pyx_r);
09638         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 948, __pyx_L1_error)
09639         __Pyx_GOTREF(__pyx_t_1);
09640         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), (__pyx_f_8PyClical_toClifford(__pyx_v_lhs) |
__pyx_f_8PyClical_toClifford(__pyx_v_rhs))); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 948,
__pyx_L1_error)
09641         __Pyx_GOTREF(__pyx_t_2);
09642         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09643         __pyx_r = __pyx_t_2;
09644         __pyx_t_2 = 0;
09645         goto __pyx_L0;
09646
09647         /* "PyClical.pyx":939
09648 *         return clifford().wrap( self.instance.inv() )
09649 *
09650 *     def __or__(lhs, rhs): # ««««««««
09651 *         """
09652 *         Transform left hand side, using right hand side as a transformation.
09653 */
09654
09655         /* function exit code */
09656         __pyx_L1_error;;
09657         __Pyx_XDECREF(__pyx_t_1);
09658         __Pyx_XDECREF(__pyx_t_2);
09659         __Pyx_AddTraceback("PyClical.clifford.__or__", __pyx_clineno, __pyx_lineno, __pyx_filename);
09660         __pyx_r = NULL;
09661         __pyx_L0;;
09662         __Pyx_XGIVEREF(__pyx_r);
09663         __Pyx_RefNannyFinishContext();
09664         return __pyx_r;
09665     }
09666
09667         /* "PyClical.pyx":950
09668 *         return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09669 *
09670 *     def __ior__(self, rhs): # ««««««««
09671 *         """
09672 *         Transform left hand side, using right hand side as a transformation.
09673 */
09674
09675         /* Python wrapper */
09676         static PyObject *__pyx_pw_8PyClical_8clifford_53__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs); /*proto*/
09677         static char __pyx_doc_8PyClical_8clifford_52__ior__[] = "\n        Transform left hand side,
using right hand side as a transformation.\n\n        >>> x=clifford('{1,2}\n') * pi/2;
y=clifford('{1}\n'); y|=x; print(y)\n        -{1}\n        >>> x=clifford('{1,2}\n') * pi/2;
y=clifford('{1}\n'); y|=exp(x); print(y)\n        -{1}\n        ";
09678         #if CYTHON_COMPILING_IN_CPYTHON
09679         struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_52__ior__;
09680         #endif
09681         static PyObject *__pyx_pw_8PyClical_8clifford_53__ior__(PyObject *__pyx_v_self, PyObject
*__pyx_v_rhs) {
09682             PyObject *__pyx_r = 0;
09683             __Pyx_RefNannyDeclarations
09684             __Pyx_RefNannySetupContext("__ior__ (wrapper)", 0);
09685             __pyx_r = __pyx_pf_8PyClical_8clifford_52__ior__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_rhs));
09686
09687             /* function exit code */
09688             __Pyx_RefNannyFinishContext();
09689             return __pyx_r;
09690         }
09691
09692         static PyObject *__pyx_pf_8PyClical_8clifford_52__ior__(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_rhs) {
09693             PyObject *__pyx_r = NULL;
09694             __Pyx_RefNannyDeclarations
09695             PyObject *__pyx_t_1 = NULL;
09696             int __pyx_lineno = 0;
09697             const char *__pyx_filename = NULL;
09698             int __pyx_clineno = 0;
09699             __Pyx_RefNannySetupContext("__ior__", 0);
09700
09701             /* "PyClical.pyx":959
09702 *         -{1}
09703 *         """
09704 *         return self.wrap( self.unwrap() | toClifford(rhs) ) # ««««««««
09705 *
09706 *     def __pow__(self, m, dummy):

```

```

09707 */
09708     __Pyx_XDECREF(__pyx_r);
09709     __pyx_t_1 = __pyx_f_8PyClicl_8clifford_wrap(__pyx_v_self,
    (__pyx_f_8PyClicl_8clifford_unwrap(__pyx_v_self) | __pyx_f_8PyClicl_toClifford(__pyx_v_rhs))); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 959, __pyx_L1_error)
09710     __Pyx_GOTREF(__pyx_t_1);
09711     __pyx_r = __pyx_t_1;
09712     __pyx_t_1 = 0;
09713     goto __pyx_L0;
09714
09715     /* "PyClicl.pyx":950
09716     *     return clifford().wrap( toClifford(lhs) | toClifford(rhs) )
09717     *
09718     *     def __ior__(self, rhs):          # ««««««««
09719     *         """
09720     *         Transform left hand side, using right hand side as a transformation.
09721     */
09722
09723     /* function exit code */
09724     __pyx_L1_error:;
09725     __Pyx_XDECREF(__pyx_t_1);
09726     __Pyx_AddTraceback("PyClicl.clifford.__ior__", __pyx_clineno, __pyx_lineno,
    __pyx_filename);
09727     __pyx_r = NULL;
09728     __pyx_L0:;
09729     __Pyx_XGIVEREF(__pyx_r);
09730     __Pyx_RefNannyFinishContext();
09731     return __pyx_r;
09732 }
09733
09734     /* "PyClicl.pyx":961
09735     *     return self.wrap( self.unwrap() | toClifford(rhs) )
09736     *
09737     *     def __pow__(self, m, dummy):          # ««««««««
09738     *         """
09739     *         Power: self to the m.
09740     */
09741
09742     /* Python wrapper */
09743     static PyObject *__pyx_pw_8PyClicl_8clifford_55__pow__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_m, PyObject *__pyx_v_dummy); /*proto*/
09744     static char __pyx_doc_8PyClicl_8clifford_54__pow__[] = "\n        Power: self to the m.\n\n
    >> x=clifford(\"{1}\"); print(x ** 2)\n        1\n        >> x=clifford(\"2\"); print(x ** 2)\n
    4\n        >> x=clifford(\"2+{1}\"); print(x ** 0)\n        1\n        >> x=clifford(\"2+{1}\");
    print(x ** 1)\n        2+{1}\n        >> x=clifford(\"2+{1}\"); print(x ** 2)\n        5+4{1}\n
    >> i=clifford(\"{1,2}\"); print(exp(pi/2) * (i ** i))\n        1\n        ";
09745     #if CYTHON_COMPILING_IN_CPYTHON
09746     struct wrapperbase __pyx_wrapperbase_8PyClicl_8clifford_54__pow__;
09747     #endif
09748     static PyObject *__pyx_pw_8PyClicl_8clifford_55__pow__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_m, PyObject *__pyx_v_dummy) {
09749         PyObject *__pyx_r = 0;
09750         __Pyx_RefNannyDeclarations
09751         __Pyx_RefNannySetupContext("__pow__ (wrapper)", 0);
09752         __pyx_r = __pyx_pf_8PyClicl_8clifford_54__pow__((PyObject *)__pyx_v_self), ((PyObject
    *)__pyx_v_m), ((PyObject *)__pyx_v_dummy));
09753
09754     /* function exit code */
09755     __Pyx_RefNannyFinishContext();
09756     return __pyx_r;
09757 }
09758
09759     static PyObject *__pyx_pf_8PyClicl_8clifford_54__pow__(PyObject *__pyx_v_self, PyObject
    *__pyx_v_m, CYTHON_UNUSED PyObject *__pyx_v_dummy) {
09760         PyObject *__pyx_r = NULL;
09761         __Pyx_RefNannyDeclarations
09762         PyObject *__pyx_t_1 = NULL;
09763         int __pyx_lineno = 0;
09764         const char *__pyx_filename = NULL;
09765         int __pyx_clineno = 0;
09766         __Pyx_RefNannySetupContext("__pow__", 0);
09767
09768     /* "PyClicl.pyx":978
09769     *     1
09770     *     """
09771     *     return pow(self, m)          # ««««««««
09772     *
09773     *     def pow(self, m):
09774     */
09775     __Pyx_XDECREF(__pyx_r);
09776     __pyx_t_1 = __pyx_f_8PyClicl_pow(__pyx_v_self, __pyx_v_m, 0); if (unlikely(!__pyx_t_1))
    __PYX_ERR(0, 978, __pyx_L1_error)
09777     __Pyx_GOTREF(__pyx_t_1);
09778     __pyx_r = __pyx_t_1;
09779     __pyx_t_1 = 0;
09780     goto __pyx_L0;
09781

```

```

09782         /* "PyClical.pyx":961
09783         *         return self.wrap( self.unwrap() | toClifford(rhs) )
09784         *
09785         *     def __pow__(self, m, dummy):          # ««««««««
09786         *         """
09787         *         Power: self to the m.
09788         */
09789
09790         /* function exit code */
09791         __pyx_L1_error++;
09792         __Pyx_XDECREF(__pyx_t_1);
09793         __Pyx_AddTraceback("PyClical.clifford.__pow__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09794         __pyx_r = NULL;
09795         __pyx_L0++;
09796         __Pyx_XGIVEREF(__pyx_r);
09797         __Pyx_RefNannyFinishContext();
09798         return __pyx_r;
09799     }
09800
09801     /* "PyClical.pyx":980
09802     *         return pow(self, m)
09803     *
09804     *     def pow(self, m):          # ««««««««
09805     *         """
09806     *         Power: self to the m.
09807     */
09808
09809     /* Python wrapper */
09810     static PyObject *__pyx_pw_8PyClical_8clifford_57pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m); /*proto*/
09811     static char __pyx_doc_8PyClical_8clifford_56pow[] = "\n          Power: self to the m.\n\n
>> x=clifford(\"{1}\"); print(x.pow(2))\n          1\n          >> x=clifford(\"2\"); print(x.pow(2))\n
4\n          >> x=clifford(\"2+{1}\"); print(x.pow(0))\n          1\n          >> x=clifford(\"2+{1}\");
print(x.pow(1))\n          2+{1}\n          >> x=clifford(\"2+{1}\"); print(x.pow(2))\n          5+4{1}\n
>> print(clifford(\"1+{1}+{1,2}\").pow(3))\n          1+3{1}+3{1,2}\n          >> i=clifford(\"{1,2}\");
print(exp(pi/2) * i.pow(i))\n          1\n          ";
09812     static PyObject *__pyx_pw_8PyClical_8clifford_57pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m) {
09813         PyObject *__pyx_r = 0;
09814         __Pyx_RefNannyDeclarations
09815         __Pyx_RefNannySetupContext("pow (wrapper)", 0);
09816         __pyx_r = __pyx_pf_8PyClical_8clifford_56pow(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_m));
09817
09818         /* function exit code */
09819         __Pyx_RefNannyFinishContext();
09820         return __pyx_r;
09821     }
09822
09823     static PyObject *__pyx_pf_8PyClical_8clifford_56pow(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_m) {
09824         PyObject *__pyx_r = NULL;
09825         __Pyx_RefNannyDeclarations
09826         PyObject *__pyx_t_1 = NULL;
09827         PyObject *__pyx_t_2 = NULL;
09828         int __pyx_t_3;
09829         int __pyx_t_4;
09830         int __pyx_t_5;
09831         int __pyx_lineno = 0;
09832         const char *__pyx_filename = NULL;
09833         int __pyx_clineno = 0;
09834         __Pyx_RefNannySetupContext("pow", 0);
09835
09836         /* "PyClical.pyx":999
09837         *         1
09838         *         """
09839         *         if isinstance(m, numbers.Integral):          # ««««««««
09840         *             return clifford().wrap( self.instance.pow(m) )
09841         *         else:
09842         */
09843         __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_numbers); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 999, __pyx_L1_error)
09844         __Pyx_GOTREF(__pyx_t_1);
09845         __pyx_t_2 = __Pyx_PyObject_GetAttrStr(__pyx_t_1, __pyx_n_s_Integral); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 999, __pyx_L1_error)
09846         __Pyx_GOTREF(__pyx_t_2);
09847         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09848         __pyx_t_3 = PyObject_IsInstance(__pyx_v_m, __pyx_t_2); if (unlikely(__pyx_t_3 == ((int)-1)))
__PYX_ERR(0, 999, __pyx_L1_error)
09849         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09850         __pyx_t_4 = (__pyx_t_3 != 0);
09851         if (__pyx_t_4) {
09852
09853             /* "PyClical.pyx":1000
09854             *         """
09855             *         if isinstance(m, numbers.Integral):

```



```

09856 *         return clifford().wrap( self.instance.pow(m) )           # ««««««««
09857 *     else:
09858 *         return exp(m * log(self))
09859 */
09860     __Pyx_XDECREF(__pyx_r);
09861     __pyx_t_2 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1000, __pyx_L1_error)
09862     __Pyx_GOTREF(__pyx_t_2);
09863     __pyx_t_5 = __Pyx_PyInt_As_int(__pyx_v_m); if (unlikely((__pyx_t_5 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1000, __pyx_L1_error)
09864     __pyx_t_1 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_2), __pyx_v_self->instance->pow(__pyx_t_5)); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1000,
__pyx_L1_error)
09865     __Pyx_GOTREF(__pyx_t_1);
09866     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09867     __pyx_r = __pyx_t_1;
09868     __pyx_t_1 = 0;
09869     goto __pyx_L0;
09870
09871     /* "PyClical.pyx":999
09872 *     1
09873 *     """
09874 *     if isinstance(m, numbers.Integral):           # ««««««««
09875 *         return clifford().wrap( self.instance.pow(m) )
09876 *     else:
09877 */
09878 }
09879
09880 /* "PyClical.pyx":1002
09881 *         return clifford().wrap( self.instance.pow(m) )
09882 *     else:
09883 *         return exp(m * log(self))           # ««««««««
09884 *
09885 *     def outer_pow(self, m):
09886 */
09887     /*else*/ {
09888     __Pyx_XDECREF(__pyx_r);
09889     __pyx_t_1 = __pyx_f_8PyClical_log(((PyObject *)__pyx_v_self), 0, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1002, __pyx_L1_error)
09890     __Pyx_GOTREF(__pyx_t_1);
09891     __pyx_t_2 = PyNumber_Multiply(__pyx_v_m, __pyx_t_1); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1002, __pyx_L1_error)
09892     __Pyx_GOTREF(__pyx_t_2);
09893     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09894     __pyx_t_1 = __pyx_f_8PyClical_exp(__pyx_t_2, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1002, __pyx_L1_error)
09895     __Pyx_GOTREF(__pyx_t_1);
09896     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
09897     __pyx_r = __pyx_t_1;
09898     __pyx_t_1 = 0;
09899     goto __pyx_L0;
09900 }
09901
09902 /* "PyClical.pyx":980
09903 *     return pow(self, m)
09904 *
09905 *     def pow(self, m):           # ««««««««
09906 *         """
09907 *         Power: self to the m.
09908 */
09909
09910 /* function exit code */
09911 __pyx_L1_error:;
09912 __Pyx_XDECREF(__pyx_t_1);
09913 __Pyx_XDECREF(__pyx_t_2);
09914 __Pyx_AddTraceback("PyClical.clifford.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
09915 __pyx_r = NULL;
09916 __pyx_L0:;
09917 __Pyx_XGIVEREF(__pyx_r);
09918 __Pyx_RefNannyFinishContext();
09919 return __pyx_r;
09920 }
09921
09922 /* "PyClical.pyx":1004
09923 *         return exp(m * log(self))
09924 *
09925 *     def outer_pow(self, m):           # ««««««««
09926 *         """
09927 *         Outer product power.
09928 */
09929
09930 /* Python wrapper */
09931 static PyObject *__pyx_pw_8PyClical_8clifford_59outer_pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m); /*proto*/
09932 static char __pyx_doc_8PyClical_8clifford_58outer_pow[] = "\n        Outer product power.\n\n
>> x=clifford(\"2+{1}\")\n                1\n                >> x=clifford(\"2+{1}\");
print(x.outer_pow(1))\n                2+{1}\n                >> x=clifford(\"2+{1}\"); print(x.outer_pow(2))\n

```

```

4+4{1}\n        >>> print(clifford("\n1+{1}+{1,2}\n").outer_pow(3))\n        1+3{1}+3{1,2}\n\n";
09933     static PyObject *__pyx_pw_8PyClical_8clifford_59outer_pow(PyObject *__pyx_v_self, PyObject
*__pyx_v_m) {
09934         PyObject *__pyx_r = 0;
09935         __Pyx_RefNannyDeclarations
09936         __Pyx_RefNannySetupContext("outer_pow (wrapper)", 0);
09937         __pyx_r = __pyx_pf_8PyClical_8clifford_58outer_pow(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_m));
09938
09939         /* function exit code */
09940         __Pyx_RefNannyFinishContext();
09941         return __pyx_r;
09942     }
09943
09944     static PyObject *__pyx_pf_8PyClical_8clifford_58outer_pow(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self, PyObject *__pyx_v_m) {
09945         PyObject *__pyx_r = NULL;
09946         __Pyx_RefNannyDeclarations
09947         PyObject *__pyx_t_1 = NULL;
09948         int __pyx_t_2;
09949         PyObject *__pyx_t_3 = NULL;
09950         int __pyx_lineno = 0;
09951         const char *__pyx_filename = NULL;
09952         int __pyx_clineno = 0;
09953         __Pyx_RefNannySetupContext("outer_pow", 0);
09954
09955         /* "PyClical.pyx":1018
09956         *
09957         *      """
09958         *      return clifford().wrap( self.instance.outer_pow(m) )
09959         *
09960         *      def __call__(self, grade):
09961         */
09962         __Pyx_XDECREF(__pyx_r);
09963         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1018, __pyx_L1_error)
09964         __Pyx_GOTREF(__pyx_t_1);
09965         __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_m); if (unlikely((__pyx_t_2 == (int)-1) &&
PyErr_Occurred())) __PYX_ERR(0, 1018, __pyx_L1_error)
09966         __pyx_t_3 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->outer_pow(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
1018, __pyx_L1_error)
09967         __Pyx_GOTREF(__pyx_t_3);
09968         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
09969         __pyx_r = __pyx_t_3;
09970         __pyx_t_3 = 0;
09971         goto __pyx_L0;
09972
09973         /* "PyClical.pyx":1004
09974         *      return exp(m * log(self))
09975         *
09976         *      def outer_pow(self, m):
09977         *          """
09978         *          Outer product power.
09979         */
09980
09981         /* function exit code */
09982         __pyx_L1_error:;
09983         __Pyx_XDECREF(__pyx_t_1);
09984         __Pyx_XDECREF(__pyx_t_3);
09985         __Pyx_AddTraceback("PyClical.clifford.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
09986         __pyx_r = NULL;
09987         __pyx_L0:;
09988         __Pyx_XGIVEREF(__pyx_r);
09989         __Pyx_RefNannyFinishContext();
09990         return __pyx_r;
09991     }
09992
09993     /* "PyClical.pyx":1020
09994     *      return clifford().wrap( self.instance.outer_pow(m) )
09995     *
09996     *      def __call__(self, grade):
09997     *          """
09998     *          Pure grade-vector part.
09999     */
10000
10001     /* Python wrapper */
10002     static PyObject *__pyx_pw_8PyClical_8clifford_61__call__(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
10003     static char __pyx_doc_8PyClical_8clifford_60__call__[] = "\n        Pure grade-vector
part.\n\n        >>> print(clifford("\n{1}\n")(1))\n        {1}\n        >>> print(clifford("\n{1}\n")(0))\n
0\n        >>> print(clifford("\n1+{1}+{1,2}\n")(0))\n        1\n        >>>
print(clifford("\n1+{1}+{1,2}\n")(1))\n        {1}\n        >>> print(clifford("\n1+{1}+{1,2}\n")(2))\n
{1,2}\n        >>> print(clifford("\n1+{1}+{1,2}\n")(3))\n        0\n        ";
10004     #if CYTHON_COMPILING_IN_CPYTHON
10005     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_60__call__;

```

```

10006         #endif
10007         static PyObject *__pyx_pw_8PyClical_8clifford_61__call__(PyObject *__pyx_v_self, PyObject
10008         *__pyx_args, PyObject *__pyx_kwds) {
10009             PyObject *__pyx_v_grade = 0;
10010             int __pyx_lineno = 0;
10011             const char *__pyx_filename = NULL;
10012             int __pyx_clineno = 0;
10013             PyObject *__pyx_r = 0;
10014             __Pyx_RefNannyDeclarations
10015             __Pyx_RefNannySetupContext("__call__ (wrapper)", 0);
10016             {
10017                 static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_grade, 0};
10018                 PyObject* values[1] = {0};
10019                 if (unlikely(__pyx_kwds)) {
10020                     Py_ssize_t kw_args;
10021                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
10022                     switch (pos_args) {
10023                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10024                         CYTHON_FALLTHROUGH;
10025                         case 0: break;
10026                         default: goto __pyx_L5_argtuple_error;
10027                     }
10028                     kw_args = PyDict_Size(__pyx_kwds);
10029                     switch (pos_args) {
10030                         case 0:
10031                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_grade)) != 0))
10032                             kw_args--;
10033                         else goto __pyx_L5_argtuple_error;
10034                     }
10035                     if (unlikely(kw_args > 0)) {
10036                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
10037                         pos_args, "__call__") < 0)) __PYX_ERR(0, 1020, __pyx_L3_error)
10038                     }
10039                     } else if (PyTuple_GET_SIZE(__pyx_args) != 1) {
10040                         goto __pyx_L5_argtuple_error;
10041                     } else {
10042                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10043                     }
10044                     __pyx_v_grade = values[0];
10045                 }
10046                 goto __pyx_L4_argument_unpacking_done;
10047                 __pyx_L5_argtuple_error:;
10048                 __Pyx_RaiseArgtupleInvalid("__call__", 1, 1, 1, PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0,
10049                 1020, __pyx_L3_error)
10050                 __pyx_L3_error:;
10051                 __Pyx_AddTraceback("PyClical.clifford.__call__", __pyx_clineno, __pyx_lineno,
10052                 __pyx_filename);
10053                 __Pyx_RefNannyFinishContext();
10054                 return NULL;
10055                 __pyx_L4_argument_unpacking_done:;
10056                 __pyx_r = __pyx_pf_8PyClical_8clifford_60__call__(((struct __pyx_obj_8PyClical_clifford
10057                 *)__pyx_v_self), __pyx_v_grade);
10058                 /* function exit code */
10059                 __Pyx_RefNannyFinishContext();
10060                 return __pyx_r;
10061             }
10062             static PyObject *__pyx_pf_8PyClical_8clifford_60__call__(struct __pyx_obj_8PyClical_clifford
10063             *__pyx_v_self, PyObject *__pyx_v_grade) {
10064                 PyObject *__pyx_r = NULL;
10065                 __Pyx_RefNannyDeclarations
10066                 PyObject *__pyx_t_1 = NULL;
10067                 int __pyx_t_2;
10068                 PyObject *__pyx_t_3 = NULL;
10069                 int __pyx_lineno = 0;
10070                 const char *__pyx_filename = NULL;
10071                 int __pyx_clineno = 0;
10072                 __Pyx_RefNannySetupContext("__call__", 0);
10073                 /* "PyClical.pyx":1037
10074                 *
10075                 *     0
10076                 *     """
10077                 *     return clifford().wrap( self.instance.call(grade) )
10078                 *                                     # ««««««««
10079                 *
10080                 *     def scalar(self):
10081                 */
10082                 __Pyx_XDECREF(__pyx_r);
10083                 __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
10084                 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1037, __pyx_L1_error)
10085                 __Pyx_GOTREF(__pyx_t_1);
10086                 __pyx_t_2 = __Pyx_PyInt_As_int(__pyx_v_grade); if (unlikely((__pyx_t_2 == (int)-1) &&
10087                 PyErr_Occurred())) __PYX_ERR(0, 1037, __pyx_L1_error)
10088                 __pyx_t_3 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
10089                 *)__pyx_t_1), __pyx_v_self->instance->operator()(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
10090                 1037, __pyx_L1_error)
10091                 __Pyx_GOTREF(__pyx_t_3);

```

```

10082     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10083     __pyx_r = __pyx_t_3;
10084     __pyx_t_3 = 0;
10085     goto __pyx_L0;
10086
10087     /* "PyClical.pyx":1020
10088  *     return clifford().wrap( self.instance.outer_pow(m) )
10089  *
10090  * def __call__(self, grade):          # ««««««««
10091  *     """
10092  *     Pure grade-vector part.
10093  */
10094
10095     /* function exit code */
10096     __pyx_L1_error++;
10097     __Pyx_XDECREF(__pyx_t_1);
10098     __Pyx_XDECREF(__pyx_t_3);
10099     __Pyx_AddTraceback("PyClical.clifford.__call__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10100     __pyx_r = NULL;
10101     __pyx_L0++;
10102     __Pyx_XGIVEREF(__pyx_r);
10103     __Pyx_RefNannyFinishContext();
10104     return __pyx_r;
10105 }
10106
10107     /* "PyClical.pyx":1039
10108  *     return clifford().wrap( self.instance.call(grade) )
10109  *
10110  * def scalar(self):                  # ««««««««
10111  *     """
10112  *     Scalar part.
10113  */
10114
10115     /* Python wrapper */
10116     static PyObject *__pyx_pw_8PyClical_8clifford_63scalar(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10117     static char __pyx_doc_8PyClical_8clifford_62scalar[] = "\n          Scalar part.\n\n          >> clifford(\n\"1+{1}+{1},2)\n\".scalar()\n          1.0\n          >> clifford(\n\"{1},2)\n\".scalar()\n          0.0\n          ";
10118     static PyObject *__pyx_pw_8PyClical_8clifford_63scalar(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10119         PyObject *__pyx_r = 0;
10120         __Pyx_RefNannyDeclarations
10121         __Pyx_RefNannySetupContext("scalar (wrapper)", 0);
10122         __pyx_r = __pyx_pf_8PyClical_8clifford_62scalar(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10123
10124         /* function exit code */
10125         __Pyx_RefNannyFinishContext();
10126         return __pyx_r;
10127     }
10128
10129     static PyObject *__pyx_pf_8PyClical_8clifford_62scalar(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10130         PyObject *__pyx_r = NULL;
10131         __Pyx_RefNannyDeclarations
10132         PyObject *__pyx_t_1 = NULL;
10133         int __pyx_lineno = 0;
10134         const char *__pyx_filename = NULL;
10135         int __pyx_clineno = 0;
10136         __Pyx_RefNannySetupContext("scalar", 0);
10137
10138         /* "PyClical.pyx":1048
10139  *     0.0
10140  *     """
10141  *     return self.instance.scalar()          # ««««««««
10142  *
10143  * def pure(self):
10144  */
10145         __Pyx_XDECREF(__pyx_r);
10146         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->scalar()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1048, __pyx_L1_error)
10147         __Pyx_GOTREF(__pyx_t_1);
10148         __pyx_r = __pyx_t_1;
10149         __pyx_t_1 = 0;
10150         goto __pyx_L0;
10151
10152         /* "PyClical.pyx":1039
10153  *     return clifford().wrap( self.instance.call(grade) )
10154  *
10155  * def scalar(self):                  # ««««««««
10156  *     """
10157  *     Scalar part.
10158  */
10159
10160         /* function exit code */

```

```

10161         __pyx_L1_error;;
10162         __Pyx_XDECREF(__pyx_t_1);
10163         __Pyx_AddTraceback("PyClicl.clifford.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
10164         __pyx_r = NULL;
10165         __pyx_L0;
10166         __Pyx_XGIVEREF(__pyx_r);
10167         __Pyx_RefNannyFinishContext();
10168         return __pyx_r;
10169     }
10170
10171     /* "PyClicl.pyx":1050
10172     *     return self.instance.scalar()
10173     *
10174     *     def pure(self):
10175     *         """
10176     *         Pure part.
10177     */
10178
10179     /* Python wrapper */
10180     static PyObject *__pyx_pw_8PyClicl_8clifford_65pure(PyObject *__pyx_v_self, CYTHON_UNUSED
10181 PyObject *unused); /*proto*/
10182     static char __pyx_doc_8PyClicl_8clifford_64pure[] = "\n        Pure part.\n\n    >>
10181 print(clifford('{1}+{1}+{1,2}') .pure())\n                {1}+{1,2}\n    >>
10182 print(clifford('{1,2}') .pure())\n                {1,2}\n    ";
10182     static PyObject *__pyx_pw_8PyClicl_8clifford_65pure(PyObject *__pyx_v_self, CYTHON_UNUSED
10183 PyObject *unused) {
10183         PyObject *__pyx_r = 0;
10184         __Pyx_RefNannyDeclarations
10185         __Pyx_RefNannySetupContext("pure (wrapper)", 0);
10186         __pyx_r = __pyx_pf_8PyClicl_8clifford_64pure(((struct __pyx_obj_8PyClicl_clifford
10187 *)__pyx_v_self));
10188
10189         /* function exit code */
10189         __Pyx_RefNannyFinishContext();
10190         return __pyx_r;
10191     }
10192
10193     static PyObject *__pyx_pf_8PyClicl_8clifford_64pure(struct __pyx_obj_8PyClicl_clifford
10194 *__pyx_v_self) {
10194         PyObject *__pyx_r = NULL;
10195         __Pyx_RefNannyDeclarations
10196         PyObject *__pyx_t_1 = NULL;
10197         PyObject *__pyx_t_2 = NULL;
10198         int __pyx_lineno = 0;
10199         const char *__pyx_filename = NULL;
10200         int __pyx_clineno = 0;
10201         __Pyx_RefNannySetupContext("pure", 0);
10202
10203         /* "PyClicl.pyx":1059
10204     *     {1,2}
10205     *     """
10206     *     return clifford().wrap( self.instance.pure() )
10207     *
10208     *     def even(self):
10209     */
10210         __Pyx_XDECREF(__pyx_r);
10211         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
10212 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1059, __pyx_L1_error)
10212         __Pyx_GOTREF(__pyx_t_1);
10213         __pyx_t_2 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
10214 *)__pyx_t_1), __pyx_v_self->instance->pure()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1059,
10215 __pyx_L1_error)
10214         __Pyx_GOTREF(__pyx_t_2);
10215         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10216         __pyx_r = __pyx_t_2;
10217         __pyx_t_2 = 0;
10218         goto __pyx_L0;
10219
10220         /* "PyClicl.pyx":1050
10221     *     return self.instance.scalar()
10222     *
10223     *     def pure(self):
10224     *         """
10225     *         Pure part.
10226     */
10227
10228         /* function exit code */
10229         __pyx_L1_error;;
10230         __Pyx_XDECREF(__pyx_t_1);
10231         __Pyx_XDECREF(__pyx_t_2);
10232         __Pyx_AddTraceback("PyClicl.clifford.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
10233         __pyx_r = NULL;
10234         __pyx_L0;
10235         __Pyx_XGIVEREF(__pyx_r);
10236         __Pyx_RefNannyFinishContext();
10237         return __pyx_r;
10238     }

```

```

10239
10240     /* "PyCliclal.pyx":1061
10241     *         return clifford().wrap( self.instance.pure() )
10242     *
10243     *     def even(self):             # ««««««««
10244     *         """
10245     *         Even part of multivector, sum of even grade terms.
10246     */
10247
10248     /* Python wrapper */
10249     static PyObject *__pyx_pw_8PyCliclal_8clifford_67even(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10250     static char __pyx_doc_8PyCliclal_8clifford_66even[] = "\n        Even part of multivector, sum
of even grade terms.\n\n        >> print(clifford(\"1+{1}+{1,2}\").even())\n        1+{1,2}\n
";
10251     static PyObject *__pyx_pw_8PyCliclal_8clifford_67even(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10252         PyObject *__pyx_r = 0;
10253         __Pyx_RefNannyDeclarations
10254         __Pyx_RefNannySetupContext("even (wrapper)", 0);
10255         __pyx_r = __pyx_pf_8PyCliclal_8clifford_66even(((struct __pyx_obj_8PyCliclal_clifford
*)__pyx_v_self));
10256
10257         /* function exit code */
10258         __Pyx_RefNannyFinishContext();
10259         return __pyx_r;
10260     }
10261
10262     static PyObject *__pyx_pf_8PyCliclal_8clifford_66even(struct __pyx_obj_8PyCliclal_clifford
*__pyx_v_self) {
10263         PyObject *__pyx_r = NULL;
10264         __Pyx_RefNannyDeclarations
10265         PyObject *__pyx_t_1 = NULL;
10266         PyObject *__pyx_t_2 = NULL;
10267         int __pyx_lineno = 0;
10268         const char *__pyx_filename = NULL;
10269         int __pyx_clineno = 0;
10270         __Pyx_RefNannySetupContext("even", 0);
10271
10272         /* "PyCliclal.pyx":1068
10273         *         1+{1,2}
10274         *         """
10275         *         return clifford().wrap( self.instance.even() )             # ««««««««
10276         *
10277         *     def odd(self):
10278         */
10279         __Pyx_XDECREF(__pyx_r);
10280         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyCliclal_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1068, __pyx_L1_error)
10281         __Pyx_GOTREF(__pyx_t_1);
10282         __pyx_t_2 = __pyx_f_8PyCliclal_8clifford_wrap(((struct __pyx_obj_8PyCliclal_clifford
*)__pyx_t_1), __pyx_v_self->instance->even()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1068,
__pyx_L1_error)
10283         __Pyx_GOTREF(__pyx_t_2);
10284         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10285         __pyx_r = __pyx_t_2;
10286         __pyx_t_2 = 0;
10287         goto __pyx_L0;
10288
10289         /* "PyCliclal.pyx":1061
10290         *         return clifford().wrap( self.instance.pure() )
10291         *
10292         *     def even(self):             # ««««««««
10293         *         """
10294         *         Even part of multivector, sum of even grade terms.
10295         */
10296
10297         /* function exit code */
10298         __pyx_L1_error;
10299         __Pyx_XDECREF(__pyx_t_1);
10300         __Pyx_XDECREF(__pyx_t_2);
10301         __Pyx_AddTraceback("PyCliclal.clifford.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
10302         __pyx_r = NULL;
10303         __pyx_L0;
10304         __Pyx_XGIVEREF(__pyx_r);
10305         __Pyx_RefNannyFinishContext();
10306         return __pyx_r;
10307     }
10308
10309     /* "PyCliclal.pyx":1070
10310     *         return clifford().wrap( self.instance.even() )
10311     *
10312     *     def odd(self):             # ««««««««
10313     *         """
10314     *         Odd part of multivector, sum of odd grade terms.
10315     */
10316

```

```

10317     /* Python wrapper */
10318     static PyObject *__pyx_pw_8PyClical_8clifford_69odd(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10319     static char __pyx_doc_8PyClical_8clifford_68odd[] = "\n          Odd part of multivector, sum of
odd grade terms.\n\n          >> print(clifford(\"1+{1}+{1,2}\").odd())\n          {1}\n          ";
10320     static PyObject *__pyx_pw_8PyClical_8clifford_69odd(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10321         PyObject *__pyx_r = 0;
10322         __Pyx_RefNannyDeclarations
10323         __Pyx_RefNannySetupContext("odd (wrapper)", 0);
10324         __pyx_r = __pyx_pf_8PyClical_8clifford_68odd(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10325
10326         /* function exit code */
10327         __Pyx_RefNannyFinishContext();
10328         return __pyx_r;
10329     }
10330
10331     static PyObject *__pyx_pf_8PyClical_8clifford_68odd(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10332         PyObject *__pyx_r = NULL;
10333         __Pyx_RefNannyDeclarations
10334         PyObject *__pyx_t_1 = NULL;
10335         PyObject *__pyx_t_2 = NULL;
10336         int __pyx_lineno = 0;
10337         const char *__pyx_filename = NULL;
10338         int __pyx_clineno = 0;
10339         __Pyx_RefNannySetupContext("odd", 0);
10340
10341         /* "PyClical.pyx":1077
10342         *
10343         *
10344         *     return clifford().wrap( self.instance.odd() )
10345         *
10346         *
10347         */
10348         __Pyx_XDECREF(__pyx_r);
10349         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1077, __pyx_L1_error)
10350         __Pyx_GOTREF(__pyx_t_1);
10351         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->odd()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1077,
__pyx_L1_error)
10352         __Pyx_GOTREF(__pyx_t_2);
10353         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10354         __pyx_r = __pyx_t_2;
10355         __pyx_t_2 = 0;
10356         goto __pyx_L0;
10357
10358         /* "PyClical.pyx":1070
10359         *     return clifford().wrap( self.instance.even() )
10360         *
10361         *
10362         *
10363         *     Odd part of multivector, sum of odd grade terms.
10364         */
10365
10366         /* function exit code */
10367         __pyx_L1_error:;
10368         __Pyx_XDECREF(__pyx_t_1);
10369         __Pyx_XDECREF(__pyx_t_2);
10370         __Pyx_AddTraceback("PyClical.clifford.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
10371         __pyx_r = NULL;
10372         __pyx_L0:;
10373         __Pyx_XGIVEREF(__pyx_r);
10374         __Pyx_RefNannyFinishContext();
10375         return __pyx_r;
10376     }
10377
10378     /* "PyClical.pyx":1079
10379     *     return clifford().wrap( self.instance.odd() )
10380     *
10381     *
10382     *
10383     *     Vector part of multivector, as a Python list, with respect to frm.
10384     */
10385
10386     /* Python wrapper */
10387     static PyObject *__pyx_pw_8PyClical_8clifford_71vector_part(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
10388     static char __pyx_doc_8PyClical_8clifford_70vector_part[] = "\n          Vector part of
multivector, as a Python list, with respect to frm.\n\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\").vector_part())\n          [2.0, 3.0]\n          >>
print(clifford(\"1+2{1}+3{2}+4{1,2}\").vector_part(index_set((-1,1,2))))\n          [0.0, 2.0, 3.0]\n
          ";
10389     static PyObject *__pyx_pw_8PyClical_8clifford_71vector_part(PyObject *__pyx_v_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {

```

```

10390     PyObject * __pyx_v_frm = 0;
10391     int __pyx_lineno = 0;
10392     const char * __pyx_filename = NULL;
10393     int __pyx_clineno = 0;
10394     PyObject * __pyx_r = 0;
10395     __Pyx_RefNannyDeclarations
10396     __Pyx_RefNannySetupContext("vector_part (wrapper)", 0);
10397     {
10398         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_frm, 0};
10399         PyObject* values[1] = {0};
10400         values[0] = ((PyObject *)Py_None);
10401         if (unlikely(__pyx_kwds)) {
10402             Py_ssize_t kw_args;
10403             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
10404             switch (pos_args) {
10405                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10406                     CYTHON_FALLTHROUGH;
10407                 case 0: break;
10408                 default: goto __pyx_L5_argtuple_error;
10409             }
10410             kw_args = PyDict_Size(__pyx_kwds);
10411             switch (pos_args) {
10412                 case 0:
10413                     if (kw_args > 0) {
10414                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_frm);
10415                         if (value) { values[0] = value; kw_args--; }
10416                     }
10417             }
10418             if (unlikely(kw_args > 0)) {
10419                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0, values,
10420 pos_args, "vector_part") < 0)) __PYX_ERR(0, 1079, __pyx_L3_error)
10421             }
10422             else {
10423                 switch (PyTuple_GET_SIZE(__pyx_args)) {
10424                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
10425                         CYTHON_FALLTHROUGH;
10426                     case 0: break;
10427                     default: goto __pyx_L5_argtuple_error;
10428                 }
10429             }
10430             __pyx_v_frm = values[0];
10431         }
10432         goto __pyx_L4_argument_unpacking_done;
10433         __pyx_L5_argtuple_error:;
10434         __Pyx_RaiseArgtupleInvalid("vector_part", 0, 0, 1, PyTuple_GET_SIZE(__pyx_args));
10435         __PYX_ERR(0, 1079, __pyx_L3_error)
10436         __pyx_L3_error:;
10437         __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
10438 __pyx_filename);
10439         __Pyx_RefNannyFinishContext();
10440         return NULL;
10441         __pyx_L4_argument_unpacking_done:;
10442         __pyx_r = __pyx_pf_8PyClical_8clifford_70vector_part(((struct __pyx_obj_8PyClical_clifford
10443 *)__pyx_v_self), __pyx_v_frm);
10444
10445         /* function exit code */
10446         __Pyx_RefNannyFinishContext();
10447         return __pyx_r;
10448     }
10449
10450     static PyObject * __pyx_pf_8PyClical_8clifford_70vector_part(struct
10451 __pyx_obj_8PyClical_clifford * __pyx_v_self, PyObject * __pyx_v_frm) {
10452     PyObject * __pyx_v_error_msg_prefix = NULL;
10453     std::vector<scalar_t> __pyx_v_vec;
10454     int __pyx_v_n;
10455     int __pyx_v_i;
10456     PyObject * __pyx_v_lst = NULL;
10457     PyObject * __pyx_v_err = NULL;
10458     PyObject * __pyx_r = NULL;
10459     __Pyx_RefNannyDeclarations
10460     PyObject * __pyx_t_1 = NULL;
10461     PyObject * __pyx_t_2 = NULL;
10462     PyObject * __pyx_t_3 = NULL;
10463     int __pyx_t_4;
10464     int __pyx_t_5;
10465     std::vector<scalar_t> __pyx_t_6;
10466     PyObject * __pyx_t_7 = NULL;
10467     int __pyx_t_8;
10468     int __pyx_t_9;
10469     int __pyx_t_10;
10470     PyObject * __pyx_t_11 = NULL;
10471     PyObject * __pyx_t_12 = NULL;
10472     PyObject * __pyx_t_13 = NULL;
10473     PyObject * __pyx_t_14 = NULL;
10474     PyObject * __pyx_t_15 = NULL;
10475     char const * __pyx_t_16;
10476     PyObject * __pyx_t_17 = NULL;

```



```

10472 PyObject *__pyx_t_18 = NULL;
10473 PyObject *__pyx_t_19 = NULL;
10474 PyObject *__pyx_t_20 = NULL;
10475 PyObject *__pyx_t_21 = NULL;
10476 PyObject *__pyx_t_22 = NULL;
10477 int __pyx_lineno = 0;
10478 const char *__pyx_filename = NULL;
10479 int __pyx_clineno = 0;
10480 __Pyx_RefNannySetupContext("vector_part", 0);
10481
10482 /* "PyClical.pyx":1088
10483 * [0.0, 2.0, 3.0]
10484 * """
10485 * error_msg_prefix = "Cannot take vector part of " # ««««««««
10486 * cdef vector[scalar_t] vec
10487 * cdef int n
10488 */
10489 __Pyx_INCREF(__pyx_kp_u_Cannot_take_vector_part_of);
10490 __pyx_v_error_msg_prefix = __pyx_kp_u_Cannot_take_vector_part_of;
10491
10492 /* "PyClical.pyx":1092
10493 * cdef int n
10494 * cdef int i
10495 * try: # ««««««««
10496 * if frm is None:
10497 *     vec = self.instance.vector_part()
10498 */
10499 {
10500     __Pyx_PyThreadState_declare
10501     __Pyx_PyThreadState_assign
10502     __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
10503     __Pyx_XGOTREF(__pyx_t_1);
10504     __Pyx_XGOTREF(__pyx_t_2);
10505     __Pyx_XGOTREF(__pyx_t_3);
10506     /*try:*/ {
10507
10508         /* "PyClical.pyx":1093
10509         * cdef int i
10510         * try:
10511         *     if frm is None: # ««««««««
10512         *         vec = self.instance.vector_part()
10513         *     else:
10514         */
10515         __pyx_t_4 = (__pyx_v_frm == Py_None);
10516         __pyx_t_5 = (__pyx_t_4 != 0);
10517         if (__pyx_t_5) {
10518
10519             /* "PyClical.pyx":1094
10520             * try:
10521             *     if frm is None:
10522             *         vec = self.instance.vector_part() # ««««««««
10523             *     else:
10524             *         vec = self.instance.vector_part((<index_set>frm).unwrap())
10525             */
10526             __pyx_t_6 = __pyx_v_self->instance->vector_part();
10527             __pyx_v_vec = __pyx_t_6;
10528
10529             /* "PyClical.pyx":1093
10530             * cdef int i
10531             * try:
10532             *     if frm is None: # ««««««««
10533             *         vec = self.instance.vector_part()
10534             *     else:
10535             */
10536             goto __pyx_L9;
10537         }
10538
10539         /* "PyClical.pyx":1096
10540         *     vec = self.instance.vector_part()
10541         *     else:
10542         *         vec = self.instance.vector_part((<index_set>frm).unwrap()) # ««««««««
10543         *     n = vec.size()
10544         *     lst = [0.0]*n
10545         */
10546         /*else*/ {
10547             try {
10548                 __pyx_t_6 =
10549                 __pyx_v_self->instance->vector_part(__pyx_f_8PyClical_9index_set_unwrap(((struct
10550                 __pyx_obj_8PyClical_index_set *)__pyx_v_frm)));
10551                 } catch (...) {
10552                     __Pyx_CppExn2PyErr();
10553                     __PYX_ERR(0, 1096, __pyx_L3_error)
10554                 }
10555                 __pyx_v_vec = __pyx_t_6;
10556                 __pyx_L9:;

```

```

10557         /* "PyClical.pyx":1097
10558         *         else:
10559         *             vec = self.instance.vector_part((<index_set>frm).unwrap())
10560         *             n = vec.size() # ««««««««
10561         *             lst = [0.0]*n
10562         *             for i in xrange(n):
10563         */
10564         __pyx_v_n = __pyx_v_vec.size();
10565
10566         /* "PyClical.pyx":1098
10567         *             vec = self.instance.vector_part((<index_set>frm).unwrap())
10568         *             n = vec.size()
10569         *             lst = [0.0]*n # ««««««««
10570         *             for i in xrange(n):
10571         *                 lst[i] = vec[i]
10572         */
10573         __pyx_t_7 = PyList_New(1 * ((__pyx_v_n<0) ? 0:__pyx_v_n)); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 1098, __pyx_L3_error)
10574         __Pyx_GOTREF(__pyx_t_7);
10575         { Py_ssize_t __pyx_temp;
10576         for (__pyx_temp=0; __pyx_temp < __pyx_v_n; __pyx_temp++) {
10577             __Pyx_INCREF(__pyx_float_0_0);
10578             __Pyx_GIVEREF(__pyx_float_0_0);
10579             PyList_SET_ITEM(__pyx_t_7, __pyx_temp, __pyx_float_0_0);
10580         }
10581     }
10582     __pyx_v_lst = ((PyObject*)__pyx_t_7);
10583     __pyx_t_7 = 0;
10584
10585     /* "PyClical.pyx":1099
10586     *         n = vec.size()
10587     *         lst = [0.0]*n
10588     *         for i in xrange(n): # ««««««««
10589     *             lst[i] = vec[i]
10590     *         return lst
10591     */
10592     __pyx_t_8 = __pyx_v_n;
10593     __pyx_t_9 = __pyx_t_8;
10594     for (__pyx_t_10 = 0; __pyx_t_10 < __pyx_t_9; __pyx_t_10+=1) {
10595         __pyx_v_i = __pyx_t_10;
10596
10597         /* "PyClical.pyx":1100
10598         *         lst = [0.0]*n
10599         *         for i in xrange(n):
10600         *             lst[i] = vec[i] # ««««««««
10601         *         return lst
10602         *     except RuntimeError as err:
10603         */
10604         __pyx_t_7 = PyFloat_FromDouble((__pyx_v_vec[__pyx_v_i])); if (unlikely(!__pyx_t_7))
__PYX_ERR(0, 1100, __pyx_L3_error)
10605         __Pyx_GOTREF(__pyx_t_7);
10606         if (unlikely(__Pyx_SetItemInt(__pyx_v_lst, __pyx_v_i, __pyx_t_7, int, 1,
__Pyx_PyInt_From_int, 1, 1, 1) < 0)) __PYX_ERR(0, 1100, __pyx_L3_error)
10607         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
10608     }
10609
10610     /* "PyClical.pyx":1101
10611     *         for i in xrange(n):
10612     *             lst[i] = vec[i]
10613     *         return lst # ««««««««
10614     *     except RuntimeError as err:
10615     *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10616     */
10617     __Pyx_XDECREF(__pyx_r);
10618     __Pyx_INCREF(__pyx_v_lst);
10619     __pyx_r = __pyx_v_lst;
10620     goto __pyx_L7_try_return;
10621
10622     /* "PyClical.pyx":1092
10623     *     cdef int n
10624     *     cdef int i
10625     *     try: # ««««««««
10626     *         if frm is None:
10627     *             vec = self.instance.vector_part()
10628     */
10629     }
10630     __pyx_L3_error:;
10631     __Pyx_XDECREF(__pyx_t_7); __pyx_t_7 = 0;
10632
10633     /* "PyClical.pyx":1102
10634     *         lst[i] = vec[i]
10635     *         return lst
10636     *     except RuntimeError as err: # ««««««««
10637     *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10638     *             + repr(frm) + " as frame:\n\t"
10639     */
10640     __pyx_t_8 = __Pyx_PyErr_ExceptionMatches(__pyx_builtin_RuntimeError);

```

```

10641         if (__pyx_t_8) {
10642             __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10643             if (__Pyx_GetException(&__pyx_t_7, &__pyx_t_11, &__pyx_t_12) < 0) __PYX_ERR(0, 1102,
__pyx_L5_except_error)
10644                 __Pyx_GOTREF(__pyx_t_7);
10645                 __Pyx_GOTREF(__pyx_t_11);
10646                 __Pyx_GOTREF(__pyx_t_12);
10647                 __Pyx_INCREF(__pyx_t_11);
10648                 __pyx_v_err = __pyx_t_11;
10649                 /*try:*/ {
10650
10651                     /* "PyClical.pyx":1103
10652                     *         return lst
10653                     *     except RuntimeError as err:
10654                     *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10655                     *             + repr(frm) + " as frame:\n\t"
10656                     *             + str(err))
10657                     */
10658                 __pyx_t_13 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), ((PyObject
*)__pyx_v_self)); if (unlikely(!__pyx_t_13)) __PYX_ERR(0, 1103, __pyx_L17_error)
10659                 __Pyx_GOTREF(__pyx_t_13);
10660                 __pyx_t_14 = __Pyx_PyUnicode_Concat(__pyx_v_error_msg_prefix, __pyx_t_13); if
(unlikely(!__pyx_t_14)) __PYX_ERR(0, 1103, __pyx_L17_error)
10661                 __Pyx_GOTREF(__pyx_t_14);
10662                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10663                 __pyx_t_13 = __Pyx_PyUnicode_Concat(__pyx_t_14, __pyx_kp_u_using_invalid); if
(unlikely(!__pyx_t_13)) __PYX_ERR(0, 1103, __pyx_L17_error)
10664                 __Pyx_GOTREF(__pyx_t_13);
10665                 __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10666
10667                 /* "PyClical.pyx":1104
10668                 *     except RuntimeError as err:
10669                 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10670                 *             + repr(frm) + " as frame:\n\t"
10671                 *             + str(err))
10672                 */
10673                 __pyx_t_14 = PyObject_Repr(__pyx_v_frm); if (unlikely(!__pyx_t_14)) __PYX_ERR(0, 1104,
__pyx_L17_error)
10674                 __Pyx_GOTREF(__pyx_t_14);
10675                 __pyx_t_15 = PyNumber_Add(__pyx_t_13, __pyx_t_14); if (unlikely(!__pyx_t_15))
__PYX_ERR(0, 1104, __pyx_L17_error)
10676                 __Pyx_GOTREF(__pyx_t_15);
10677                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10678                 __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10679                 __pyx_t_14 = PyNumber_Add(__pyx_t_15, __pyx_kp_u_as_frame); if (unlikely(!__pyx_t_14))
__PYX_ERR(0, 1104, __pyx_L17_error)
10680                 __Pyx_GOTREF(__pyx_t_14);
10681                 __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10682
10683                 /* "PyClical.pyx":1105
10684                 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10685                 *             + repr(frm) + " as frame:\n\t"
10686                 *             + str(err))
10687                 *         # ««««««««
10688                 *
10689                 *     def involute(self):
10690                 */
10691                 __pyx_t_15 = __Pyx_PyObject_CallOneArg(((PyObject *)(&PyUnicode_Type)), __pyx_v_err);
if (unlikely(!__pyx_t_15)) __PYX_ERR(0, 1105, __pyx_L17_error)
10692                 __Pyx_GOTREF(__pyx_t_15);
10693                 __pyx_t_13 = PyNumber_Add(__pyx_t_14, __pyx_t_15); if (unlikely(!__pyx_t_13))
__PYX_ERR(0, 1105, __pyx_L17_error)
10694                 __Pyx_GOTREF(__pyx_t_13);
10695                 __Pyx_DECREF(__pyx_t_14); __pyx_t_14 = 0;
10696                 __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10697
10698                 /* "PyClical.pyx":1103
10699                 *         return lst
10700                 *     except RuntimeError as err:
10701                 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10702                 *             + repr(frm) + " as frame:\n\t"
10703                 *             + str(err))
10704                 */
10705                 __pyx_t_15 = __Pyx_PyObject_CallOneArg(__pyx_builtin_ValueError, __pyx_t_13); if
(unlikely(!__pyx_t_15)) __PYX_ERR(0, 1103, __pyx_L17_error)
10706                 __Pyx_GOTREF(__pyx_t_15);
10707                 __Pyx_DECREF(__pyx_t_13); __pyx_t_13 = 0;
10708                 __Pyx_Raise(__pyx_t_15, 0, 0, 0);
10709                 __Pyx_DECREF(__pyx_t_15); __pyx_t_15 = 0;
10710                 __PYX_ERR(0, 1103, __pyx_L17_error)
10711             }
10712
10713             /* "PyClical.pyx":1102
10714             *         lst[i] = vec[i]
10715             *     return lst
10716             *     except RuntimeError as err:
10717             *         # ««««««««

```

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10717 *         raise ValueError(error_msg_prefix + str(self) + " using invalid "
10718 *                           + repr(frm) + " as frame:\n\t"
10719 */
10720         /*finally:*/ {
10721             __pyx_L17_error;;
10722             /*exception exit:*/{
10723                 __Pyx_PyThreadState_declare
10724                 __Pyx_PyThreadState_assign
10725                 __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0; __pyx_t_21 = 0;
__pyx_t_22 = 0;
10726                 __Pyx_XDECREFF(__pyx_t_13); __pyx_t_13 = 0;
10727                 __Pyx_XDECREFF(__pyx_t_14); __pyx_t_14 = 0;
10728                 __Pyx_XDECREFF(__pyx_t_15); __pyx_t_15 = 0;
10729                 if (PY_MAJOR_VERSION >= 3) __Pyx_ExceptionSwap(&__pyx_t_20, &__pyx_t_21,
&__pyx_t_22);
10730                 if ((PY_MAJOR_VERSION < 3) || unlikely(__Pyx_GetException(&__pyx_t_17, &__pyx_t_18,
&__pyx_t_19) < 0)) __Pyx_ErrFetch(&__pyx_t_17, &__pyx_t_18, &__pyx_t_19);
10731                 __Pyx_XGOTREF(__pyx_t_17);
10732                 __Pyx_XGOTREF(__pyx_t_18);
10733                 __Pyx_XGOTREF(__pyx_t_19);
10734                 __Pyx_XGOTREF(__pyx_t_20);
10735                 __Pyx_XGOTREF(__pyx_t_21);
10736                 __Pyx_XGOTREF(__pyx_t_22);
10737                 __pyx_t_8 = __pyx_lineno; __pyx_t_9 = __pyx_clineno; __pyx_t_16 = __pyx_filename;
10738                 {
10739                     __Pyx_DECREF(__pyx_v_err);
10740                     __pyx_v_err = NULL;
10741                 }
10742                 if (PY_MAJOR_VERSION >= 3) {
10743                     __Pyx_XGIVEREF(__pyx_t_20);
10744                     __Pyx_XGIVEREF(__pyx_t_21);
10745                     __Pyx_XGIVEREF(__pyx_t_22);
10746                     __Pyx_ExceptionReset(__pyx_t_20, __pyx_t_21, __pyx_t_22);
10747                 }
10748                 __Pyx_XGIVEREF(__pyx_t_17);
10749                 __Pyx_XGIVEREF(__pyx_t_18);
10750                 __Pyx_XGIVEREF(__pyx_t_19);
10751                 __Pyx_ErrRestore(__pyx_t_17, __pyx_t_18, __pyx_t_19);
10752                 __pyx_t_17 = 0; __pyx_t_18 = 0; __pyx_t_19 = 0; __pyx_t_20 = 0; __pyx_t_21 = 0;
__pyx_t_22 = 0;
10753                 __pyx_lineno = __pyx_t_8; __pyx_clineno = __pyx_t_9; __pyx_filename = __pyx_t_16;
10754                 goto __pyx_L5_except_error;
10755             }
10756         }
10757     }
10758     goto __pyx_L5_except_error;
10759     __pyx_L5_except_error:;
10760
10761     /* "PyClical.pyx":1092
10762 *     cdef int n
10763 *     cdef int i
10764 *     try:
10765 *         if frm is None:
10766 *             vec = self.instance.vector_part()
10767 */
10768         __Pyx_XGIVEREF(__pyx_t_1);
10769         __Pyx_XGIVEREF(__pyx_t_2);
10770         __Pyx_XGIVEREF(__pyx_t_3);
10771         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
10772         goto __pyx_L1_error;
10773         __pyx_L7_try_return:;
10774         __Pyx_XGIVEREF(__pyx_t_1);
10775         __Pyx_XGIVEREF(__pyx_t_2);
10776         __Pyx_XGIVEREF(__pyx_t_3);
10777         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
10778         goto __pyx_L0;
10779     }
10780
10781     /* "PyClical.pyx":1079
10782 *     return clifford().wrap( self.instance.odd() )
10783 *
10784 *     def vector_part(self, frm = None):
10785 *         """
10786 *         Vector part of multivector, as a Python list, with respect to frm.
10787 */
10788
10789     /* function exit code */
10790     __pyx_L1_error:;
10791     __Pyx_XDECREFF(__pyx_t_7);
10792     __Pyx_XDECREFF(__pyx_t_11);
10793     __Pyx_XDECREFF(__pyx_t_12);
10794     __Pyx_XDECREFF(__pyx_t_13);
10795     __Pyx_XDECREFF(__pyx_t_14);
10796     __Pyx_XDECREFF(__pyx_t_15);
10797     __Pyx_AddTraceback("PyClical.clifford.vector_part", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10798     __pyx_r = NULL;

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10799     __pyx_L0;;
10800     __Pyx_XDECREF(__pyx_v_error_msg_prefix);
10801     __Pyx_XDECREF(__pyx_v_lst);
10802     __Pyx_XDECREF(__pyx_v_err);
10803     __Pyx_XGIVEREF(__pyx_r);
10804     __Pyx_RefNannyFinishContext();
10805     return __pyx_r;
10806 }
10807
10808     /* "PyClical.pyx":1107
10809     *
10810     *
10811     * def involute(self):
10812     *     """
10813     *     Main involution, each {i} is replaced by -{i} in each term,
10814     */
10815
10816     /* Python wrapper */
10817     static PyObject *__pyx_pw_8PyClical_8clifford_73involute(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10818     static char __pyx_doc_8PyClical_8clifford_72involute[] = "\n        Main involution, each {i}
is replaced by -{i} in each term,\n        eg. clifford(\"{1}\") -> -clifford(\"{1}\").\n\n    >>
print(clifford(\"{1}\").involute())\n        -{1}\n    >> print((clifford(\"{2}\") *
clifford(\"{1}\")).involute())\n        -{1,2}\n    >> print((clifford(\"{1}\") *
clifford(\"{2}\")).involute())\n        {1,2}\n    >>
print(clifford(\"1+{1}+{1,2}\").involute())\n        1-{1}+{1,2}\n    ";
10819     static PyObject *__pyx_pw_8PyClical_8clifford_73involute(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10820         PyObject *__pyx_r = 0;
10821         __Pyx_RefNannyDeclarations
10822         __Pyx_RefNannySetupContext("involute (wrapper)", 0);
10823         __pyx_r = __pyx_pf_8PyClical_8clifford_72involute(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10824
10825         /* function exit code */
10826         __Pyx_RefNannyFinishContext();
10827         return __pyx_r;
10828     }
10829
10830     static PyObject *__pyx_pf_8PyClical_8clifford_72involute(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10831         PyObject *__pyx_r = NULL;
10832         __Pyx_RefNannyDeclarations
10833         PyObject *__pyx_t_1 = NULL;
10834         PyObject *__pyx_t_2 = NULL;
10835         int __pyx_lineno = 0;
10836         const char *__pyx_filename = NULL;
10837         int __pyx_clineno = 0;
10838         __Pyx_RefNannySetupContext("involute", 0);
10839
10840         /* "PyClical.pyx":1121
10841         *     1-{1}+{1,2}
10842         *     """
10843         *     return clifford().wrap( self.instance.involute() )
10844         *
10845         * def reverse(self):
10846         */
10847         __Pyx_XDECREF(__pyx_r);
10848         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1121, __pyx_L1_error)
10849         __Pyx_GOTREF(__pyx_t_1);
10850         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->involute()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1121,
__pyx_L1_error)
10851         __Pyx_GOTREF(__pyx_t_2);
10852         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10853         __pyx_r = __pyx_t_2;
10854         __pyx_t_2 = 0;
10855         goto __pyx_L0;
10856
10857         /* "PyClical.pyx":1107
10858         *
10859         *
10860         * def involute(self):
10861         *     """
10862         *     Main involution, each {i} is replaced by -{i} in each term,
10863         */
10864
10865         /* function exit code */
10866         __pyx_L1_error:;
10867         __Pyx_XDECREF(__pyx_t_1);
10868         __Pyx_XDECREF(__pyx_t_2);
10869         __Pyx_AddTraceback("PyClical.clifford.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10870         __pyx_r = NULL;
10871         __pyx_L0:;
10872         __Pyx_XGIVEREF(__pyx_r);

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10873     __Pyx_RefNannyFinishContext();
10874     return __pyx_r;
10875 }
10876
10877 /* "PyClical.pyx":1123
10878 *     return clifford().wrap( self.instance.involute() )
10879 *
10880 *     def reverse(self):
10881 *         """
10882 *         Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
10883 */
10884
10885 /* Python wrapper */
10886 static PyObject *__pyx_pw_8PyClical_8clifford_75reverse(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10887 static char __pyx_doc_8PyClical_8clifford_74reverse[] = "\n        Reversion, eg.
clifford(\"{1}\")*clifford(\"{2}\") -> clifford(\"{2}\")*clifford(\"{1}\").\n\n        >>
print(clifford(\"{1}\").reverse())\n        {1}\n        >> print((clifford(\"{2}\") *
clifford(\"{1}\")).reverse())\n        {1,2}\n        >> print((clifford(\"{1}\") *
clifford(\"{2}\")).reverse())\n        -{1,2}\n        >> print(clifford(\"1+{1}+{1,2}\").reverse())\n
1+{1}-{1,2}\n        ";
10888 static PyObject *__pyx_pw_8PyClical_8clifford_75reverse(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10889     PyObject *__pyx_r = 0;
10890     __Pyx_RefNannyDeclarations
10891     __Pyx_RefNannySetupContext("reverse (wrapper)", 0);
10892     __pyx_r = __pyx_pf_8PyClical_8clifford_74reverse(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10893
10894     /* function exit code */
10895     __Pyx_RefNannyFinishContext();
10896     return __pyx_r;
10897 }
10898
10899 static PyObject *__pyx_pf_8PyClical_8clifford_74reverse(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10900     PyObject *__pyx_r = NULL;
10901     __Pyx_RefNannyDeclarations
10902     PyObject *__pyx_t_1 = NULL;
10903     PyObject *__pyx_t_2 = NULL;
10904     int __pyx_lineno = 0;
10905     const char *__pyx_filename = NULL;
10906     int __pyx_clineno = 0;
10907     __Pyx_RefNannySetupContext("reverse", 0);
10908
10909     /* "PyClical.pyx":1136
10910 *     1+{1}-{1,2}
10911 *     """
10912 *     return clifford().wrap( self.instance.reverse() )
10913 *
10914 *     def conj(self):
10915 */
10916     __Pyx_XDECREF(__pyx_r);
10917     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1136, __pyx_L1_error)
10918     __Pyx_GOTREF(__pyx_t_1);
10919     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->reverse()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1136,
__pyx_L1_error)
10920     __Pyx_GOTREF(__pyx_t_2);
10921     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10922     __pyx_r = __pyx_t_2;
10923     __pyx_t_2 = 0;
10924     goto __pyx_L0;
10925
10926     /* "PyClical.pyx":1123
10927 *     return clifford().wrap( self.instance.involute() )
10928 *
10929 *     def reverse(self):
10930 *         """
10931 *         Reversion, eg. clifford("{1}")*clifford("{2}") -> clifford("{2}")*clifford("{1}").
10932 */
10933
10934     /* function exit code */
10935     __pyx_L1_error:;
10936     __Pyx_XDECREF(__pyx_t_1);
10937     __Pyx_XDECREF(__pyx_t_2);
10938     __Pyx_AddTraceback("PyClical.clifford.reverse", __pyx_clineno, __pyx_lineno,
__pyx_filename);
10939     __pyx_r = NULL;
10940     __pyx_L0:;
10941     __Pyx_XGIVEREF(__pyx_r);
10942     __Pyx_RefNannyFinishContext();
10943     return __pyx_r;
10944 }
10945
10946 /* "PyClical.pyx":1138

```

```

10947 *         return clifford().wrap( self.instance.reverse() )
10948 *
10949 *     def conj(self):                 # ««««««««
10950 *         """
10951 *         Conjugation, reverse o involute == involute o reverse.
10952 */
10953
10954     /* Python wrapper */
10955     static PyObject *__pyx_pw_8PyClical_8clifford_77conj(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
10956     static char __pyx_doc_8PyClical_8clifford_76conj[] = "\n          Conjugation, reverse o
involute == involute o reverse.\n\n          >> print((clifford(\"{1}\")).conj())\n          -{1}\n
>> print((clifford(\"{2}\") * clifford(\"{1}\")).conj())\n          {1,2}\n          >>
print((clifford(\"{1}\") * clifford(\"{2}\")).conj())\n          -{1,2}\n          >>
print(clifford(\"1+{1}+{1,2}\").conj())\n          1-{1}-{1,2}\n          ";
10957     static PyObject *__pyx_pw_8PyClical_8clifford_77conj(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
10958         PyObject *__pyx_r = 0;
10959         __Pyx_RefNannyDeclarations
10960         __Pyx_RefNannySetupContext("conj (wrapper)", 0);
10961         __pyx_r = __pyx_pf_8PyClical_8clifford_76conj(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
10962
10963         /* function exit code */
10964         __Pyx_RefNannyFinishContext();
10965         return __pyx_r;
10966     }
10967
10968     static PyObject *__pyx_pf_8PyClical_8clifford_76conj(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
10969         PyObject *__pyx_r = NULL;
10970         __Pyx_RefNannyDeclarations
10971         PyObject *__pyx_t_1 = NULL;
10972         PyObject *__pyx_t_2 = NULL;
10973         int __pyx_lineno = 0;
10974         const char *__pyx_filename = NULL;
10975         int __pyx_clineno = 0;
10976         __Pyx_RefNannySetupContext("conj", 0);
10977
10978         /* "PyClical.pyx":1151
10979 *         1-{1}-{1,2}
10980 *         """
10981 *         return clifford().wrap( self.instance.conj() )                 # ««««««««
10982 *
10983 *     def quad(self):
10984 */
10985         __Pyx_XDECREF(__pyx_r);
10986         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1151, __pyx_L1_error)
10987         __Pyx_GOTREF(__pyx_t_1);
10988         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), __pyx_v_self->instance->conj()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1151,
__pyx_L1_error)
10989         __Pyx_GOTREF(__pyx_t_2);
10990         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
10991         __pyx_r = __pyx_t_2;
10992         __pyx_t_2 = 0;
10993         goto __pyx_L0;
10994
10995         /* "PyClical.pyx":1138
10996 *         return clifford().wrap( self.instance.reverse() )
10997 *
10998 *     def conj(self):                 # ««««««««
10999 *         """
11000 *         Conjugation, reverse o involute == involute o reverse.
11001 */
11002
11003         /* function exit code */
11004         __pyx_L1_error:;
11005         __Pyx_XDECREF(__pyx_t_1);
11006         __Pyx_XDECREF(__pyx_t_2);
11007         __Pyx_AddTraceback("PyClical.clifford.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
11008         __pyx_r = NULL;
11009         __pyx_L0:;
11010         __Pyx_XGIVEREF(__pyx_r);
11011         __Pyx_RefNannyFinishContext();
11012         return __pyx_r;
11013     }
11014
11015     /* "PyClical.pyx":1153
11016 *         return clifford().wrap( self.instance.conj() )
11017 *
11018 *     def quad(self):                 # ««««««««
11019 *         """
11020 *         Quadratic form == (rev(x)*x)(0).
11021 */
11022

```

```

11023     /* Python wrapper */
11024     static PyObject *__pyx_pw_8PyClical_8clifford_79quad(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11025     static char __pyx_doc_8PyClical_8clifford_78quad[] = "\n          Quadratic form ==
(rev(x)*x)(0).\n\n          >> print(clifford(\"1+{1}+{1,2}\").quad())\n          3.0\n          >>
print(clifford(\"1+{-1}+{1,2}+{1,2,3}\").quad())\n          2.0\n          ";
11026     static PyObject *__pyx_pw_8PyClical_8clifford_79quad(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11027         PyObject *__pyx_r = 0;
11028         __Pyx_RefNannyDeclarations
11029         __Pyx_RefNannySetupContext("quad (wrapper)", 0);
11030         __pyx_r = __pyx_pf_8PyClical_8clifford_78quad(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11031
11032         /* function exit code */
11033         __Pyx_RefNannyFinishContext();
11034         return __pyx_r;
11035     }
11036
11037     static PyObject *__pyx_pf_8PyClical_8clifford_78quad(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11038         PyObject *__pyx_r = NULL;
11039         __Pyx_RefNannyDeclarations
11040         PyObject *__pyx_t_1 = NULL;
11041         int __pyx_lineno = 0;
11042         const char *__pyx_filename = NULL;
11043         int __pyx_clineno = 0;
11044         __Pyx_RefNannySetupContext("quad", 0);
11045
11046         /* "PyClical.pyx":1162
11047  *         2.0
11048  *         """
11049  *         return self.instance.quad()
11050  *         # <<<<<<<<<
11051  *
11052  */
11053         __Pyx_XDECREF(__pyx_r);
11054         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->quad()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1162, __pyx_L1_error)
11055         __Pyx_GOTREF(__pyx_t_1);
11056         __pyx_r = __pyx_t_1;
11057         __pyx_t_1 = 0;
11058         goto __pyx_L0;
11059
11060         /* "PyClical.pyx":1153
11061  *         return clifford().wrap( self.instance.conj() )
11062  *
11063  *
11064  *         def quad(self):
11065  *             # <<<<<<<<<
11066  *             Quadratic form == (rev(x)*x)(0).
11067  */
11068         /* function exit code */
11069         __pyx_L1_error;
11070         __Pyx_XDECREF(__pyx_t_1);
11071         __Pyx_AddTraceback("PyClical.clifford.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
11072         __pyx_r = NULL;
11073         __pyx_L0;
11074         __Pyx_XGIVEREF(__pyx_r);
11075         __Pyx_RefNannyFinishContext();
11076         return __pyx_r;
11077     }
11078
11079     /* "PyClical.pyx":1164
11080  *         return self.instance.quad()
11081  *
11082  *
11083  *         def norm(self):
11084  *             # <<<<<<<<<
11085  *             """
11086  *             Norm == sum of squares of coordinates.
11087  */
11088         /* Python wrapper */
11089         static PyObject *__pyx_pw_8PyClical_8clifford_81norm(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11089         static char __pyx_doc_8PyClical_8clifford_80norm[] = "\n          Norm == sum of squares of
coordinates.\n\n          >> clifford(\"1+{1}+{1,2}\").norm()\n          3.0\n          >>
clifford(\"1+{-1}+{1,2}+{1,2,3}\").norm()\n          4.0\n          ";
11090         static PyObject *__pyx_pw_8PyClical_8clifford_81norm(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11091         PyObject *__pyx_r = 0;
11092         __Pyx_RefNannyDeclarations
11093         __Pyx_RefNannySetupContext("norm (wrapper)", 0);
11094         __pyx_r = __pyx_pf_8PyClical_8clifford_80norm(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11095
11096         /* function exit code */
11097         __Pyx_RefNannyFinishContext();

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11098         return __pyx_r;
11099     }
11100
11101     static PyObject *__pyx_pf_8PyClical_8clifford_80norm(struct __pyx_obj_8PyClical_clifford
11102 *__pyx_v_self) {
11103         PyObject *__pyx_r = NULL;
11104         __Pyx_RefNannyDeclarations
11105         PyObject *__pyx_t_1 = NULL;
11106         int __pyx_lineno = 0;
11107         const char *__pyx_filename = NULL;
11108         int __pyx_clineno = 0;
11109         __Pyx_RefNannySetupContext("norm", 0);
11110
11111         /* "PyClical.pyx":1173
11112         *
11113         *     return self.instance.norm()
11114         *
11115         *     def abs(self):
11116         */
11117         __Pyx_XDECREF(__pyx_r);
11118         __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->norm()); if (unlikely(!__pyx_t_1))
11119         __PYX_ERR(0, 1173, __pyx_l1_error)
11120         __Pyx_GOTREF(__pyx_t_1);
11121         __pyx_r = __pyx_t_1;
11122         __pyx_t_1 = 0;
11123         goto __pyx_L0;
11124
11125         /* "PyClical.pyx":1164
11126         *     return self.instance.quad()
11127         *
11128         *     def norm(self):
11129         *         """
11130         *         Norm == sum of squares of coordinates.
11131         */
11132         /* function exit code */
11133         __pyx_l1_error;
11134         __Pyx_XDECREF(__pyx_t_1);
11135         __Pyx_AddTraceback("PyClical.clifford.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
11136         __pyx_r = NULL;
11137         __pyx_L0;
11138         __Pyx_XGIVEREF(__pyx_r);
11139         __Pyx_RefNannyFinishContext();
11140         return __pyx_r;
11141     }
11142
11143     /* "PyClical.pyx":1175
11144     *     return self.instance.norm()
11145     *
11146     *     def abs(self):
11147     *         """
11148     *         Absolute value: square root of norm.
11149     */
11150
11151     /* Python wrapper */
11152     static PyObject *__pyx_pw_8PyClical_8clifford_83abs(PyObject *__pyx_v_self, CYTHON_UNUSED
11153 PyObject *unused); /*proto*/
11154     static char __pyx_doc_8PyClical_8clifford_82abs[] = "\n        Absolute value: square root of
11155 norm.\n\n        >> clifford(\"1+{-1}+{1,2}+{1,2,3}\").abs()\n        2.0\n        ";
11156     static PyObject *__pyx_pw_8PyClical_8clifford_83abs(PyObject *__pyx_v_self, CYTHON_UNUSED
11157 PyObject *unused) {
11158         PyObject *__pyx_r = 0;
11159         __Pyx_RefNannyDeclarations
11160         __Pyx_RefNannySetupContext("abs (wrapper)", 0);
11161         __pyx_r = __pyx_pf_8PyClical_8clifford_82abs(((struct __pyx_obj_8PyClical_clifford
11162 *)__pyx_v_self));
11163
11164         /* function exit code */
11165         __Pyx_RefNannyFinishContext();
11166         return __pyx_r;
11167     }
11168
11169     static PyObject *__pyx_pf_8PyClical_8clifford_82abs(struct __pyx_obj_8PyClical_clifford
11170 *__pyx_v_self) {
11171         PyObject *__pyx_r = NULL;
11172         __Pyx_RefNannyDeclarations
11173         PyObject *__pyx_t_1 = NULL;
11174         int __pyx_lineno = 0;
11175         const char *__pyx_filename = NULL;
11176         int __pyx_clineno = 0;
11177         __Pyx_RefNannySetupContext("abs", 0);
11178
11179         /* "PyClical.pyx":1182
11180         *     2.0
11181         *
11182         *     return glucat.abs( self.unwrap() )
11183         *
11184         *
11185         *
11186         */
11187         return glucat.abs( self.unwrap() )

```

```

11178 *
11179 *     def max_abs(self):
11180 */
11181     __Pyx_XDECREF(__pyx_r);
11182     __pyx_t_1 = PyFloat_FromDouble(abs(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1182, __pyx_L1_error)
11183     __Pyx_GOTREF(__pyx_t_1);
11184     __pyx_r = __pyx_t_1;
11185     __pyx_t_1 = 0;
11186     goto __pyx_L0;
11187
11188     /* "PyClical.pyx":1175
11189 *     return self.instance.norm()
11190 *
11191 *     def abs(self):
11192 *         """
11193 *         Absolute value: square root of norm.
11194 */
11195
11196     /* function exit code */
11197     __pyx_L1_error:;
11198     __Pyx_XDECREF(__pyx_t_1);
11199     __Pyx_AddTraceback("PyClical.clifford.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
11200     __pyx_r = NULL;
11201     __pyx_L0:;
11202     __Pyx_XGIVEREF(__pyx_r);
11203     __Pyx_RefNannyFinishContext();
11204     return __pyx_r;
11205 }
11206
11207     /* "PyClical.pyx":1184
11208 *     return glucat.abs( self.unwrap() )
11209 *
11210 *     def max_abs(self):
11211 *         """
11212 *         Maximum of absolute values of components of multivector: multivector infinity norm.
11213 */
11214
11215     /* Python wrapper */
11216     static PyObject *__pyx_pw_8PyClical_8clifford_85max_abs(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11217     static char __pyx_doc_8PyClical_8clifford_84max_abs[] = "\n        Maximum of absolute values
of components of multivector: multivector infinity norm.\n\n        >>
clifford(\\"1+{-1}+{1,2}+{1,2,3}\\").max_abs()\n        1.0\n        >>
clifford(\\"3+2{1}+{1,2}\\").max_abs()\n        3.0\n        ";
11218     static PyObject *__pyx_pw_8PyClical_8clifford_85max_abs(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11219         PyObject *__pyx_r = 0;
11220         __Pyx_RefNannyDeclarations
11221         __Pyx_RefNannySetupContext("max_abs (wrapper)", 0);
11222         __pyx_r = __pyx_pf_8PyClical_8clifford_84max_abs(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11223
11224     /* function exit code */
11225     __Pyx_RefNannyFinishContext();
11226     return __pyx_r;
11227 }
11228
11229     static PyObject *__pyx_pf_8PyClical_8clifford_84max_abs(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11230         PyObject *__pyx_r = NULL;
11231         __Pyx_RefNannyDeclarations
11232         PyObject *__pyx_t_1 = NULL;
11233         int __pyx_lineno = 0;
11234         const char *__pyx_filename = NULL;
11235         int __pyx_clineno = 0;
11236         __Pyx_RefNannySetupContext("max_abs", 0);
11237
11238     /* "PyClical.pyx":1193
11239 *     3.0
11240 *     """
11241 *     return self.instance.max_abs()
11242 *
11243 *     def truncated(self, limit):
11244 */
11245     __Pyx_XDECREF(__pyx_r);
11246     __pyx_t_1 = PyFloat_FromDouble(__pyx_v_self->instance->max_abs()); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1193, __pyx_L1_error)
11247     __Pyx_GOTREF(__pyx_t_1);
11248     __pyx_r = __pyx_t_1;
11249     __pyx_t_1 = 0;
11250     goto __pyx_L0;
11251
11252     /* "PyClical.pyx":1184
11253 *     return glucat.abs( self.unwrap() )
11254 *
11255 *     def max_abs(self):

```

```

11256 *          """
11257 *          Maximum of absolute values of components of multivector: multivector infinity norm.
11258 */
11259
11260 /* function exit code */
11261 __pyx_L1_error++;
11262 __Pyx_XDECREF(__pyx_t_1);
11263 __Pyx_AddTraceback("PyClicl.clifford.max_abs", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11264 __pyx_r = NULL;
11265 __pyx_L0;
11266 __Pyx_XGIVEREF(__pyx_r);
11267 __Pyx_RefNannyFinishContext();
11268 return __pyx_r;
11269 }
11270
11271 /* "PyClicl.pyx":1195
11272 *      return self.instance.max_abs()
11273 *
11274 *      def truncated(self, limit):                                # ««««««««
11275 *          """
11276 *          Remove all terms of self with relative size smaller than limit.
11277 */
11278
11279 /* Python wrapper */
11280 static PyObject *__pyx_pw_8PyClicl_8clifford_86truncated(PyObject *__pyx_v_self, PyObject
*__pyx_v_limit); /*proto*/
11281 static char __pyx_doc_8PyClicl_8clifford_86truncated[] = "\n      Remove all terms of self
with relative size smaller than limit.\n\n      >>
clifford(\"1e8+{1}+1e-8{1,2}\").truncated(1.0e-6)\n      clifford(\"100000000\")\n      >>
clifford(\"1e4+{1}+1e-4{1,2}\").truncated(1.0e-6)\n      clifford(\"10000+{1}\")\n      ";
11282 static PyObject *__pyx_pw_8PyClicl_8clifford_86truncated(PyObject *__pyx_v_self, PyObject
*__pyx_v_limit) {
11283     PyObject *__pyx_r = 0;
11284     __Pyx_RefNannyDeclarations
11285     __Pyx_RefNannySetupContext("truncated (wrapper)", 0);
11286     __pyx_r = __pyx_pf_8PyClicl_8clifford_86truncated(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self), ((PyObject *)__pyx_v_limit));
11287
11288 /* function exit code */
11289 __Pyx_RefNannyFinishContext();
11290 return __pyx_r;
11291 }
11292
11293 static PyObject *__pyx_pf_8PyClicl_8clifford_86truncated(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self, PyObject *__pyx_v_limit) {
11294     PyObject *__pyx_r = NULL;
11295     __Pyx_RefNannyDeclarations
11296     PyObject *__pyx_t_1 = NULL;
11297     scalar_t __pyx_t_2;
11298     PyObject *__pyx_t_3 = NULL;
11299     int __pyx_lineno = 0;
11300     const char *__pyx_filename = NULL;
11301     int __pyx_clineno = 0;
11302     __Pyx_RefNannySetupContext("truncated", 0);
11303
11304 /* "PyClicl.pyx":1204
11305 *      clifford("10000+{1}")
11306 *          """
11307 *      return clifford().wrap( self.instance.truncated(limit) )          # ««««««««
11308 *
11309 *      def isinf(self):
11310 */
11311     __Pyx_XDECREF(__pyx_r);
11312     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1204, __pyx_L1_error)
11313     __Pyx_GOTREF(__pyx_t_1);
11314     __pyx_t_2 = __pyx_PyFloat_AsDouble(__pyx_v_limit); if (unlikely((__pyx_t_2 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1204, __pyx_L1_error)
11315     __pyx_t_3 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_1), __pyx_v_self->instance->truncated(__pyx_t_2)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0,
1204, __pyx_L1_error)
11316     __Pyx_GOTREF(__pyx_t_3);
11317     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11318     __pyx_r = __pyx_t_3;
11319     __pyx_t_3 = 0;
11320     goto __pyx_L0;
11321
11322 /* "PyClicl.pyx":1195
11323 *      return self.instance.max_abs()
11324 *
11325 *      def truncated(self, limit):                                # ««««««««
11326 *          """
11327 *          Remove all terms of self with relative size smaller than limit.
11328 */
11329
11330 /* function exit code */

```

```

11331         __pyx_L1_error;;
11332         __Pyx_XDECREF(__pyx_t_1);
11333         __Pyx_XDECREF(__pyx_t_3);
11334         __Pyx_AddTraceback("PyClicl.clifford.truncated", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11335         __pyx_r = NULL;
11336         __pyx_L0;;
11337         __Pyx_XGIVEREF(__pyx_r);
11338         __Pyx_RefNannyFinishContext();
11339         return __pyx_r;
11340     }
11341
11342     /* "PyClicl.pyx":1206
11343     *         return clifford().wrap( self.instance.truncated(limit) )
11344     *
11345     *     def isinf(self):
11346     *         """
11347     *         Check if a multivector contains any infinite values.
11348     */
11349
11350     /* Python wrapper */
11351     static PyObject *__pyx_pw_8PyClicl_8clifford_89isinf(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11352     static char __pyx_doc_8PyClicl_8clifford_88isinf[] = "\n        Check if a multivector
contains any infinite values.\n\n        >> clifford().isinf()\n        False\n        ";
11353     static PyObject *__pyx_pw_8PyClicl_8clifford_89isinf(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11354         PyObject *__pyx_r = 0;
11355         __Pyx_RefNannyDeclarations
11356         __Pyx_RefNannySetupContext("isinf (wrapper)", 0);
11357         __pyx_r = __pyx_pf_8PyClicl_8clifford_88isinf(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_v_self));
11358
11359         /* function exit code */
11360         __Pyx_RefNannyFinishContext();
11361         return __pyx_r;
11362     }
11363
11364     static PyObject *__pyx_pf_8PyClicl_8clifford_88isinf(struct __pyx_obj_8PyClicl_clifford
*__pyx_v_self) {
11365         PyObject *__pyx_r = NULL;
11366         __Pyx_RefNannyDeclarations
11367         PyObject *__pyx_t_1 = NULL;
11368         int __pyx_lineno = 0;
11369         const char *__pyx_filename = NULL;
11370         int __pyx_clineno = 0;
11371         __Pyx_RefNannySetupContext("isinf", 0);
11372
11373         /* "PyClicl.pyx":1213
11374     *         False
11375     *         """
11376     *         return self.instance.isnan()
11377     *         # ««««««««
11378     *     def isnan(self):
11379     */
11380         __Pyx_XDECREF(__pyx_r);
11381         __pyx_t_1 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->isnan()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1213, __pyx_L1_error)
11382         __Pyx_GOTREF(__pyx_t_1);
11383         __pyx_r = __pyx_t_1;
11384         __pyx_t_1 = 0;
11385         goto __pyx_L0;
11386
11387         /* "PyClicl.pyx":1206
11388     *         return clifford().wrap( self.instance.truncated(limit) )
11389     *
11390     *     def isinf(self):
11391     *         """
11392     *         Check if a multivector contains any infinite values.
11393     */
11394
11395         /* function exit code */
11396         __pyx_L1_error;;
11397         __Pyx_XDECREF(__pyx_t_1);
11398         __Pyx_AddTraceback("PyClicl.clifford.isinf", __pyx_clineno, __pyx_lineno, __pyx_filename);
11399         __pyx_r = NULL;
11400         __pyx_L0;;
11401         __Pyx_XGIVEREF(__pyx_r);
11402         __Pyx_RefNannyFinishContext();
11403         return __pyx_r;
11404     }
11405
11406     /* "PyClicl.pyx":1215
11407     *         return self.instance.isnan()
11408     *
11409     *     def isnan(self):
11410     *         """

```

```

11411 *          Check if a multivector contains any IEEE NaN values.
11412 */
11413
11414 /* Python wrapper */
11415 static PyObject *__pyx_pw_8PyClical_8clifford_91isnan(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11416 static char __pyx_doc_8PyClical_8clifford_90isnan[] = "\n          Check if a multivector
contains any IEEE NaN values.\n\n          >> clifford().isnan()\n          False\n          ";
11417 static PyObject *__pyx_pf_8PyClical_8clifford_91isnan(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11418     PyObject *__pyx_r = 0;
11419     __Pyx_RefNannyDeclarations
11420     __Pyx_RefNannySetupContext("isnan (wrapper)", 0);
11421     __pyx_r = __pyx_pf_8PyClical_8clifford_90isnan(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11422
11423     /* function exit code */
11424     __Pyx_RefNannyFinishContext();
11425     return __pyx_r;
11426 }
11427
11428 static PyObject *__pyx_pf_8PyClical_8clifford_90isnan(struct __pyx_obj_8PyClical_clifford
*__pyx_v_self) {
11429     PyObject *__pyx_r = NULL;
11430     __Pyx_RefNannyDeclarations
11431     PyObject *__pyx_t_1 = NULL;
11432     int __pyx_lineno = 0;
11433     const char *__pyx_filename = NULL;
11434     int __pyx_clineno = 0;
11435     __Pyx_RefNannySetupContext("isnan", 0);
11436
11437     /* "PyClical.pyx":1222
11438     *     False
11439     *     """
11440     *     return self.instance.isnan()          # ««««««««
11441     *
11442     *     def frame(self):
11443     */
11444     __Pyx_XDECREF(__pyx_r);
11445     __pyx_t_1 = __Pyx_PyBool_FromLong(__pyx_v_self->instance->isnan()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1222, __pyx_L1_error)
11446     __Pyx_GOTREF(__pyx_t_1);
11447     __pyx_r = __pyx_t_1;
11448     __pyx_t_1 = 0;
11449     goto __pyx_L0;
11450
11451     /* "PyClical.pyx":1215
11452     *     return self.instance.isnan()
11453     *
11454     *     def isnan(self):          # ««««««««
11455     *     """
11456     *     Check if a multivector contains any IEEE NaN values.
11457     */
11458
11459     /* function exit code */
11460     __pyx_L1_error:;
11461     __Pyx_XDECREF(__pyx_t_1);
11462     __Pyx_AddTraceback("PyClical.clifford.isnan", __pyx_clineno, __pyx_lineno, __pyx_filename);
11463     __pyx_r = NULL;
11464     __pyx_L0:;
11465     __Pyx_XGIVEREF(__pyx_r);
11466     __Pyx_RefNannyFinishContext();
11467     return __pyx_r;
11468 }
11469
11470 /* "PyClical.pyx":1224
11471 *     return self.instance.isnan()
11472 *
11473 *     def frame(self):          # ««««««««
11474 *     """
11475 *     Subalgebra generated by all generators of terms of given multivector.
11476     */
11477
11478 /* Python wrapper */
11479 static PyObject *__pyx_pw_8PyClical_8clifford_93frame(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
11480 static char __pyx_doc_8PyClical_8clifford_92frame[] = "\n          Subalgebra generated by all
generators of terms of given multivector.\n\n          >>
print(clifford(\"1+3{-1}+2{1,2}+4{-2,7}\").frame())\n          {-2,-1,1,2,7}\n          >>
s=clifford(\"1+3{-1}+2{1,2}+4{-2,7}\").frame(); type(s)\n          <class 'PyClical.index_set'>\n
          ";
11481 static PyObject *__pyx_pf_8PyClical_8clifford_93frame(PyObject *__pyx_v_self, CYTHON_UNUSED
PyObject *unused) {
11482     PyObject *__pyx_r = 0;
11483     __Pyx_RefNannyDeclarations
11484     __Pyx_RefNannySetupContext("frame (wrapper)", 0);
11485     __pyx_r = __pyx_pf_8PyClical_8clifford_92frame(((struct __pyx_obj_8PyClical_clifford

```

```

    *)__pyx_v_self));
11486
11487     /* function exit code */
11488     __Pyx_RefNannyFinishContext();
11489     return __pyx_r;
11490 }
11491
11492 static PyObject *__pyx_pf_8PyClical_8clifford_92frame(struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self) {
11493     PyObject *__pyx_r = NULL;
11494     __Pyx_RefNannyDeclarations
11495     PyObject *__pyx_t_1 = NULL;
11496     PyObject *__pyx_t_2 = NULL;
11497     int __pyx_lineno = 0;
11498     const char *__pyx_filename = NULL;
11499     int __pyx_clineno = 0;
11500     __Pyx_RefNannySetupContext("frame", 0);
11501
11502     /* "PyClical.pyx":1233
11503     * <class 'PyClical.index_set'>
11504     * """
11505     *     return index_set().wrap( self.instance.frame() )           # ««««««««
11506     *
11507     *     def __repr__(self):
11508     */
11509     __Pyx_XDECREF(__pyx_r);
11510     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_index_set)); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1233, __pyx_L1_error)
11511     __Pyx_GOTREF(__pyx_t_1);
11512     __pyx_t_2 = __pyx_f_8PyClical_9index_set_wrap(((struct __pyx_obj_8PyClical_index_set
*)__pyx_t_1), __pyx_v_self->instance->frame()); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1233,
__pyx_L1_error)
11513     __Pyx_GOTREF(__pyx_t_2);
11514     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11515     __pyx_r = __pyx_t_2;
11516     __pyx_t_2 = 0;
11517     goto __pyx_L0;
11518
11519     /* "PyClical.pyx":1224
11520     *     return self.instance.isnan()
11521     *
11522     *     def frame(self):           # ««««««««
11523     *     """
11524     *         Subalgebra generated by all generators of terms of given multivector.
11525     */
11526
11527     /* function exit code */
11528     __pyx_L1_error:;
11529     __Pyx_XDECREF(__pyx_t_1);
11530     __Pyx_XDECREF(__pyx_t_2);
11531     __Pyx_AddTraceback("PyClical.clifford.frame", __pyx_clineno, __pyx_lineno, __pyx_filename);
11532     __pyx_r = NULL;
11533     __pyx_L0:;
11534     __Pyx_XGIVEREF(__pyx_r);
11535     __Pyx_RefNannyFinishContext();
11536     return __pyx_r;
11537 }
11538
11539     /* "PyClical.pyx":1235
11540     *     return index_set().wrap( self.instance.frame() )
11541     *
11542     *     def __repr__(self):           # ««««««««
11543     *     """
11544     *         The official string representation of self.
11545     */
11546
11547     /* Python wrapper */
11548     static PyObject *__pyx_pw_8PyClical_8clifford_95__repr__(PyObject *__pyx_v_self); /*proto*/
11549     static char __pyx_doc_8PyClical_8clifford_94__repr__[] = "\n        The
\342\200\234official\342\200\235 string representation of self.\n\n        >>
clifford(\\"1+3{-1}+2{1,2}+4{-2,7}\\").__repr__()\n        'clifford(\\"1+3{-1}+2{1,2}+4{-2,7}\\")'\n
";
11550     #if CYTHON_COMPILING_IN_CPYTHON
11551     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_94__repr__;
11552     #endif
11553     static PyObject *__pyx_pw_8PyClical_8clifford_95__repr__(PyObject *__pyx_v_self) {
11554         PyObject *__pyx_r = 0;
11555         __Pyx_RefNannyDeclarations
11556         __Pyx_RefNannySetupContext("__repr__ (wrapper)", 0);
11557         __pyx_r = __pyx_pf_8PyClical_8clifford_94__repr__(((struct __pyx_obj_8PyClical_clifford
*)__pyx_v_self));
11558
11559     /* function exit code */
11560     __Pyx_RefNannyFinishContext();
11561     return __pyx_r;
11562 }
11563

```

```

11564     static PyObject *__pyx_pf_8PyClical_8clifford_94__repr__(struct __pyx_obj_8PyClical_clifford
11565 *__pyx_v_self) {
11566         PyObject *__pyx_r = NULL;
11567         __Pyx_RefNannyDeclarations
11568         PyObject *__pyx_t_1 = NULL;
11569         int __pyx_lineno = 0;
11570         const char *__pyx_filename = NULL;
11571         int __pyx_clineno = 0;
11572         __Pyx_RefNannySetupContext("__repr__", 0);
11573
11574         /* "PyClical.pyx":1242
11575          *
11576          *     return clifford_to_repr( self.unwrap() ).decode()           # ««««««««
11577          *
11578          *     def __str__(self):
11579          */
11580         __Pyx_XDECREF(__pyx_r);
11581         __pyx_t_1 =
11582         __Pyx_decode_cpp_string(clifford_to_repr(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self)), 0,
11583 PY_SSIZE_T_MAX, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1242, __pyx_L1_error)
11584         __Pyx_GOTREF(__pyx_t_1);
11585         __pyx_r = __pyx_t_1;
11586         __pyx_t_1 = 0;
11587         goto __pyx_L0;
11588
11589         /* "PyClical.pyx":1235
11590          *     return index_set().wrap( self.instance.frame() )
11591          *
11592          *     def __repr__(self):           # ««««««««
11593          *         """
11594          *         The official string representation of self.
11595          */
11596         /* function exit code */
11597         __pyx_L1_error:;
11598         __Pyx_XDECREF(__pyx_t_1);
11599         __Pyx_AddTraceback("PyClical.clifford.__repr__", __pyx_clineno, __pyx_lineno,
11600 __pyx_filename);
11601         __pyx_r = NULL;
11602         __pyx_L0:;
11603         __Pyx_XGIVEREF(__pyx_r);
11604         __Pyx_RefNannyFinishContext();
11605         return __pyx_r;
11606     }
11607
11608     /* "PyClical.pyx":1244
11609     *     return clifford_to_repr( self.unwrap() ).decode()
11610     *
11611     *     def __str__(self):           # ««««««««
11612     *         """
11613     *         The informal string representation of self.
11614     */
11615
11616     /* Python wrapper */
11617     static PyObject *__pyx_pw_8PyClical_8clifford_97__str__(PyObject *__pyx_v_self); /*proto*/
11618     static char __pyx_doc_8PyClical_8clifford_96__str__[] = "\n        The
11619     \342\200\234informal\342\200\235 string representation of self.\n\n        >>
11620     clifford(\342\200\234\342\200\235\342\200\235 string representation of self.\n\n        '1+3{-1}+2{1,2}+4{-2,7}'\n        ";
11621     #if CYTHON_COMPILING_IN_CPYTHON
11622     struct wrapperbase __pyx_wrapperbase_8PyClical_8clifford_96__str__;
11623     #endif
11624     static PyObject *__pyx_pw_8PyClical_8clifford_97__str__(PyObject *__pyx_v_self) {
11625         PyObject *__pyx_r = 0;
11626         __Pyx_RefNannyDeclarations
11627         __Pyx_RefNannySetupContext("__str__ (wrapper)", 0);
11628         __pyx_r = __pyx_pf_8PyClical_8clifford_96__str__(((struct __pyx_obj_8PyClical_clifford
11629 *)__pyx_v_self));
11630
11631         /* function exit code */
11632         __Pyx_RefNannyFinishContext();
11633         return __pyx_r;
11634     }
11635
11636     static PyObject *__pyx_pf_8PyClical_8clifford_96__str__(struct __pyx_obj_8PyClical_clifford
11637 *__pyx_v_self) {
11638         PyObject *__pyx_r = NULL;
11639         __Pyx_RefNannyDeclarations
11640         PyObject *__pyx_t_1 = NULL;
11641         int __pyx_lineno = 0;
11642         const char *__pyx_filename = NULL;
11643         int __pyx_clineno = 0;
11644         __Pyx_RefNannySetupContext("__str__", 0);
11645
11646         /* "PyClical.pyx":1251
11647          *
11648          *     '1+3{-1}+2{1,2}+4{-2,7}'
11649          *
11650          *     """

```

```

11643 *         return clifford_to_str( self.unwrap() ).decode()           # ««««««««
11644 *
11645 * def clifford_hidden_doctests():
11646 */
11647     __Pyx_XDECREF(__pyx_r);
11648     __pyx_t_1 =
__Pyx_decode_cpp_string(clifford_to_str(__pyx_f_8PyClical_8clifford_unwrap(__pyx_v_self)), 0,
PY_SSIZE_T_MAX, NULL, NULL, NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1251, __pyx_L1_error)
11649     __Pyx_GOTREF(__pyx_t_1);
11650     __pyx_r = __pyx_t_1;
11651     __pyx_t_1 = 0;
11652     goto __pyx_L0;
11653
11654     /* "PyClical.pyx":1244
11655 *         return clifford_to_repr( self.unwrap() ).decode()
11656 *
11657 *     def __str__(self):           # ««««««««
11658 *         """
11659 *         The informal string representation of self.
11660 */
11661
11662     /* function exit code */
11663     __pyx_L1_error:;
11664     __Pyx_XDECREF(__pyx_t_1);
11665     __Pyx_AddTraceback("PyClical.clifford.__str__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11666     __pyx_r = NULL;
11667     __pyx_L0:;
11668     __Pyx_XGIVEREF(__pyx_r);
11669     __Pyx_RefNannyFinishContext();
11670     return __pyx_r;
11671 }
11672
11673     /* "(tree fragment)":1
11674 * def __reduce_cython__(self):           # ««««««««
11675 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11676 * def __setstate_cython__(self, __pyx_state):
11677 */
11678
11679     /* Python wrapper */
11680     static PyObject *__pyx_pw_8PyClical_8clifford_99__reduce_cython__(PyObject *__pyx_v_self,
CYTHON_UNUSED PyObject *unused); /*proto*/
11681     static PyObject *__pyx_pw_8PyClical_8clifford_99__reduce_cython__(PyObject *__pyx_v_self,
CYTHON_UNUSED PyObject *unused) {
11682         PyObject *__pyx_r = 0;
11683         __Pyx_RefNannyDeclarations
11684         __Pyx_RefNannySetupContext("__reduce_cython__ (wrapper)", 0);
11685         __pyx_r = __pyx_pf_8PyClical_8clifford_98__reduce_cython__(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_self));
11686
11687     /* function exit code */
11688     __Pyx_RefNannyFinishContext();
11689     return __pyx_r;
11690 }
11691
11692     static PyObject *__pyx_pf_8PyClical_8clifford_98__reduce_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self) {
11693         PyObject *__pyx_r = NULL;
11694         __Pyx_RefNannyDeclarations
11695         PyObject *__pyx_t_1 = NULL;
11696         int __pyx_lineno = 0;
11697         const char *__pyx_filename = NULL;
11698         int __pyx_clineno = 0;
11699         __Pyx_RefNannySetupContext("__reduce_cython__", 0);
11700
11701     /* "(tree fragment)":2
11702 * def __reduce_cython__(self):
11703 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")           # ««««««««
11704 * def __setstate_cython__(self, __pyx_state):
11705 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11706 */
11707     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__11, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 2, __pyx_L1_error)
11708     __Pyx_GOTREF(__pyx_t_1);
11709     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
11710     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11711     __PYX_ERR(1, 2, __pyx_L1_error)
11712
11713     /* "(tree fragment)":1
11714 * def __reduce_cython__(self):           # ««««««««
11715 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11716 * def __setstate_cython__(self, __pyx_state):
11717 */
11718
11719     /* function exit code */
11720     __pyx_L1_error:;
11721     __Pyx_XDECREF(__pyx_t_1);

```



```

11722     __Pyx_AddTraceback("PyClical.clifford.__reduce_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11723     __pyx_r = NULL;
11724     __Pyx_XGIVEREF(__pyx_r);
11725     __Pyx_RefNannyFinishContext();
11726     return __pyx_r;
11727 }
11728
11729 /* "(tree fragment)":3
11730 * def __reduce_cython__(self):
11731 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11732 * def __setstate_cython__(self, __pyx_state): # ««««««««
11733 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11734 */
11735
11736 /* Python wrapper */
11737 static PyObject *__pyx_pw_8PyClical_8clifford_101__setstate_cython__(PyObject *__pyx_v_self,
PyObject *__pyx_v__pyx_state); /*proto*/
11738 static PyObject *__pyx_pw_8PyClical_8clifford_101__setstate_cython__(PyObject *__pyx_v_self,
PyObject *__pyx_v__pyx_state) {
11739     PyObject *__pyx_r = 0;
11740     __Pyx_RefNannyDeclarations
11741     __Pyx_RefNannySetupContext("__setstate_cython__ (wrapper)", 0);
11742     __pyx_r = __pyx_pf_8PyClical_8clifford_100__setstate_cython__(((struct
__pyx_obj_8PyClical_clifford *)__pyx_v_self), ((PyObject *)__pyx_v__pyx_state));
11743
11744     /* function exit code */
11745     __Pyx_RefNannyFinishContext();
11746     return __pyx_r;
11747 }
11748
11749 static PyObject *__pyx_pf_8PyClical_8clifford_100__setstate_cython__(CYTHON_UNUSED struct
__pyx_obj_8PyClical_clifford *__pyx_v_self, CYTHON_UNUSED PyObject *__pyx_v__pyx_state) {
11750     PyObject *__pyx_r = NULL;
11751     __Pyx_RefNannyDeclarations
11752     PyObject *__pyx_t_1 = NULL;
11753     int __pyx_lineno = 0;
11754     const char *__pyx_filename = NULL;
11755     int __pyx_clineno = 0;
11756     __Pyx_RefNannySetupContext("__setstate_cython__", 0);
11757
11758     /* "(tree fragment)":4
11759 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11760 * def __setstate_cython__(self, __pyx_state): # ««««««««
11761 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11762 */
11763     __pyx_t_1 = __Pyx_PyObject_Call(__pyx_builtin_TypeError, __pyx_tuple__12, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 4, __pyx_L1_error)
11764     __Pyx_GOTREF(__pyx_t_1);
11765     __Pyx_Raise(__pyx_t_1, 0, 0, 0);
11766     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
11767     __PYX_ERR(1, 4, __pyx_L1_error)
11768
11769     /* "(tree fragment)":3
11770 * def __reduce_cython__(self):
11771 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11772 * def __setstate_cython__(self, __pyx_state): # ««««««««
11773 *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
11774 */
11775
11776     /* function exit code */
11777     __pyx_L1_error:;
11778     __Pyx_XDECREF(__pyx_t_1);
11779     __Pyx_AddTraceback("PyClical.clifford.__setstate_cython__", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11780     __pyx_r = NULL;
11781     __Pyx_XGIVEREF(__pyx_r);
11782     __Pyx_RefNannyFinishContext();
11783     return __pyx_r;
11784 }
11785
11786 /* "PyClical.pyx":1253
11787 *     return clifford_to_str( self.unwrap() ).decode()
11788 *
11789 * def clifford_hidden_doctests(): # ««««««««
11790 *     """
11791 *     Tests for functions that Doctest cannot see.
11792 */
11793
11794 /* Python wrapper */
11795 static PyObject *__pyx_pw_8PyClical_9clifford_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused); /*proto*/
11796 static char __pyx_doc_8PyClical_8clifford_hidden_doctests[] = "\n    Tests for functions that
Doctest cannot see.\n\n    For clifford.__cinit__: Construct an object of type clifford.\n\n    >>
print(clifford(2))\n    2\n    >> print(clifford(2.0))\n    2\n    >> print(clifford(1.0e-1))\n
0.1\n    >> print(clifford(\"2\")\n    2\n    >> print(clifford(\"{1,2,3}\")\n    2{1,2,3}\n
print(clifford(clifford(\"2{1,2,3}\")\n    2{1,2,3}\n    >> print(clifford(\"-{1}\")\n    -{1}\n

```

```

>> print(clifford(2,index_set({1,2})))\n      2{1,2}\n      >> print(clifford([2,3],index_set({1,2})))\n
2{1}+3{2}\n      >> print(clifford([1,2]))\n      Traceback (most recent call last):\n      ...
TypeError: Cannot initialize clifford object from <class 'list'>.\n      >> print(clifford(None))\n
Traceback (most recent call last):\n      ...
TypeError: Cannot initialize clifford object from
<class 'NoneType'>.\n      >> print(clifford(None,[1,2]))\n      Traceback (most recent call last):\n
...
TypeError: Cannot initialize clifford object from (<class 'NoneType'>, <class 'list'>).\n
>> print(clifford([1,2],[1,2]))\n      Traceback (most recent call last):\n      ...
TypeError:
Cannot initialize clifford object from (<class 'list'>, <class 'list'>).\n      >>
print(clifford("\n"))\n      Traceback (most recent call last):\n      ...
ValueError: Cannot
initialize clifford object from invalid string ".\n      >> print(clifford("\{")\n      Traceback (most
recent call last):\n      ...
ValueError: Cannot initialize clifford object from invalid string
'\{'.\n      >> print(clifford("\{1")\n      Traceback (most recent call last):\n      ...
ValueError: Cannot initialize clifford object from invalid string '\{1'.\n      >>
print(clifford("\{+")\n      Traceback (most recent call last):\n      ...
ValueError: Cannot
initialize clifford object from invalid string '+'.\n      >> print(clifford("\{-")\n      Traceback
(most recent call last):\n      ...
ValueError: Cannot initialize clifford object fro"m invalid
string '-'.\n      >> print(clifford("\{1}+"))\n      Traceback (most recent call last):\n      ...
ValueError: Cannot initialize clifford object from invalid string '{1}+'.\n      For
clifford.__richcmp__: Compare objects of type clifford.\n      >> clifford("\{1}\n") ==
clifford("\{1{1}\n")\n      True\n      >> clifford("\{1}\n") != clifford("\{1.0{1}\n")\n      False\n      >>
clifford("\{1}\n") != clifford("\{1.0}\n")\n      True\n      >> clifford("\{1,2}\n") == None\n      False\n
>> clifford("\{1,2}\n") != None\n      True\n      >> None == clifford("\{1,2}\n")\n      False\n      >> None
!= clifford("\{1,2}\n")\n      True\n      ";

11797     static PyMethodDef __pyx_mdef_8PyClical_9clifford_hidden_doctests =
{"clifford_hidden_doctests", (PyCFunction)__pyx_pw_8PyClical_9clifford_hidden_doctests, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_hidden_doctests};
11798     static PyObject *__pyx_pw_8PyClical_9clifford_hidden_doctests(PyObject *__pyx_self,
CYTHON_UNUSED PyObject *unused) {
11799         PyObject *__pyx_r = 0;
11800         __Pyx_RefNannyDeclarations
11801         __Pyx_RefNannySetupContext("clifford_hidden_doctests (wrapper)", 0);
11802         __pyx_r = __pyx_pf_8PyClical_8clifford_hidden_doctests(__pyx_self);
11803
11804         /* function exit code */
11805         __Pyx_RefNannyFinishContext();
11806         return __pyx_r;
11807     }
11808
11809     static PyObject *__pyx_pf_8PyClical_8clifford_hidden_doctests(CYTHON_UNUSED PyObject
*__pyx_self) {
11810         PyObject *__pyx_r = NULL;
11811         __Pyx_RefNannyDeclarations
11812         __Pyx_RefNannySetupContext("clifford_hidden_doctests", 0);
11813
11814         /* "PyClical.pyx":1335
11815         *     True
11816         *     """
11817         *     return          # ««««««««
11818         *
11819         * cpdef inline error_squared_tol(obj):
11820         */
11821         __Pyx_XDECREF(__pyx_r);
11822         __pyx_r = Py_None; __Pyx_INCREF(Py_None);
11823         goto __pyx_L0;
11824
11825         /* "PyClical.pyx":1253
11826         *     return clifford_to_str( self.unwrap() ).decode()
11827         *
11828         * def clifford_hidden_doctests():          # ««««««««
11829         *     """
11830         *     Tests for functions that Doctest cannot see.
11831         */
11832
11833         /* function exit code */
11834         __pyx_L0:;
11835         __Pyx_XGIVEREF(__pyx_r);
11836         __Pyx_RefNannyFinishContext();
11837         return __pyx_r;
11838     }
11839
11840     /* "PyClical.pyx":1337
11841     *     return
11842     *
11843     * cpdef inline error_squared_tol(obj):          # ««««««««
11844     *     """
11845     *     Quadratic norm error tolerance relative to a specific multivector.
11846     */
11847
11848     static PyObject *__pyx_pw_8PyClical_11error_squared_tol(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
11849     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_error_squared_tol(PyObject
*__pyx_v_obj, CYTHON_UNUSED int __pyx_skip_dispatch) {
11850         PyObject *__pyx_r = NULL;
11851         __Pyx_RefNannyDeclarations
11852         PyObject *__pyx_t_1 = NULL;
11853         int __pyx_lineno = 0;
11854         const char *__pyx_filename = NULL;

```

```

11855         int __pyx_clineno = 0;
11856         __Pyx_RefNannySetupContext("error_squared_tol", 0);
11857
11858         /* "PyClical.pyx":1344
11859         *      0.0
11860         *      """
11861         *      return glucat.error_squared_tol(toClifford(obj))          # ««««««««
11862         *
11863         * cpdef inline error_squared(lhs, rhs, threshold):
11864         */
11865         __Pyx_XDECREF(__pyx_r);
11866         __pyx_t_1 =
PyFloat_FromDouble(error_squared_tol(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1344, __pyx_L1_error)
11867         __Pyx_GOTREF(__pyx_t_1);
11868         __pyx_r = __pyx_t_1;
11869         __pyx_t_1 = 0;
11870         goto __pyx_L0;
11871
11872         /* "PyClical.pyx":1337
11873         *      return
11874         *
11875         * cpdef inline error_squared_tol(obj):          # ««««««««
11876         *      """
11877         *      Quadratic norm error tolerance relative to a specific multivector.
11878         */
11879
11880         /* function exit code */
11881         __pyx_L1_error;
11882         __Pyx_XDECREF(__pyx_t_1);
11883         __Pyx_AddTraceback("PyClical.error_squared_tol", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11884         __pyx_r = 0;
11885         __pyx_L0;
11886         __Pyx_XGIVEREF(__pyx_r);
11887         __Pyx_RefNannyFinishContext();
11888         return __pyx_r;
11889     }
11890
11891     /* Python wrapper */
11892     static PyObject * __pyx_pw_8PyClical_11error_squared_tol(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
11893     static char __pyx_doc_8PyClical_10error_squared_tol[] = "\n    Quadratic norm error
tolerance relative to a specific multivector.\n\n    >> print(error_squared_tol(clifford(\"{1}\") *
3.0 - error_squared_tol(clifford(\"{1}-2{2}+3{3}\")\n    0.0\n    ";
11894     static PyObject * __pyx_pf_8PyClical_11error_squared_tol(PyObject * __pyx_self, PyObject
* __pyx_v_obj) {
11895         PyObject * __pyx_r = 0;
11896         __Pyx_RefNannyDeclarations
11897         __Pyx_RefNannySetupContext("error_squared_tol (wrapper)", 0);
11898         __pyx_r = __pyx_pf_8PyClical_10error_squared_tol(__pyx_self, ((PyObject
*) __pyx_v_obj));
11899
11900         /* function exit code */
11901         __Pyx_RefNannyFinishContext();
11902         return __pyx_r;
11903     }
11904
11905     static PyObject * __pyx_pf_8PyClical_10error_squared_tol(CYTHON_UNUSED PyObject
* __pyx_self, PyObject * __pyx_v_obj) {
11906         PyObject * __pyx_r = NULL;
11907         __Pyx_RefNannyDeclarations
11908         PyObject * __pyx_t_1 = NULL;
11909         int __pyx_lineno = 0;
11910         const char * __pyx_filename = NULL;
11911         int __pyx_clineno = 0;
11912         __Pyx_RefNannySetupContext("error_squared_tol", 0);
11913         __Pyx_XDECREF(__pyx_r);
11914         __pyx_t_1 = __pyx_f_8PyClical_error_squared_tol(__pyx_v_obj, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1337, __pyx_L1_error)
11915         __Pyx_GOTREF(__pyx_t_1);
11916         __pyx_r = __pyx_t_1;
11917         __pyx_t_1 = 0;
11918         goto __pyx_L0;
11919
11920         /* function exit code */
11921         __pyx_L1_error;
11922         __Pyx_XDECREF(__pyx_t_1);
11923         __Pyx_AddTraceback("PyClical.error_squared_tol", __pyx_clineno, __pyx_lineno,
__pyx_filename);
11924         __pyx_r = NULL;
11925         __pyx_L0;
11926         __Pyx_XGIVEREF(__pyx_r);
11927         __Pyx_RefNannyFinishContext();
11928         return __pyx_r;
11929     }
11930

```

```

11931         /* "PyClicl.pyx":1346
11932  *      return glucat.error_squared_tol(toClifford(obj))
11933  *
11934  * cpdef inline error_squared(lhs, rhs, threshold):          # ««««««««
11935  *      """
11936  *      Relative or absolute error using the quadratic norm.
11937  */
11938
11939         static PyObject *__pyx_pw_8PyClicl_13error_squared(PyObject *__pyx_self, PyObject
11940 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
11941         static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_error_squared(PyObject *__pyx_v_lhs,
11942 PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, CYTHON_UNUSED int __pyx_skip_dispatch) {
11943     PyObject *__pyx_r = NULL;
11944     __Pyx_RefNannyDeclarations
11945     scalar_t __pyx_t_1;
11946     PyObject *__pyx_t_2 = NULL;
11947     int __pyx_lineno = 0;
11948     const char *__pyx_filename = NULL;
11949     int __pyx_clineno = 0;
11950     __Pyx_RefNannySetupContext("error_squared", 0);
11951
11952     /* "PyClicl.pyx":1357
11953  *      25.0
11954  *      """
11955  *      return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
11956  *      # ««««««««
11957  *
11958  * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):
11959  */
11960     __Pyx_XDECREF(__pyx_r);
11961     __pyx_t_1 = __pyx_PyFloat_AsDouble(__pyx_v_threshold); if (unlikely((__pyx_t_1 ==
11962 ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1357, __pyx_L1_error)
11963     __pyx_t_2 =
11964     PyFloat_FromDouble(error_squared(__pyx_f_8PyClicl_toClifford(__pyx_v_lhs),
11965 __pyx_f_8PyClicl_toClifford(__pyx_v_rhs), ((scalar_t)__pyx_t_1)); if (unlikely(!__pyx_t_2))
11966     __PYX_ERR(0, 1357, __pyx_L1_error)
11967     __Pyx_GOTREF(__pyx_t_2);
11968     __pyx_r = __pyx_t_2;
11969     __pyx_t_2 = 0;
11970     goto __pyx_L0;
11971
11972     /* "PyClicl.pyx":1346
11973  *      return glucat.error_squared_tol(toClifford(obj))
11974  *
11975  * cpdef inline error_squared(lhs, rhs, threshold):          # ««««««««
11976  *      """
11977  *      Relative or absolute error using the quadratic norm.
11978  */
11979
11980     /* function exit code */
11981     __pyx_L1_error;
11982     __Pyx_XDECREF(__pyx_t_2);
11983     __Pyx_AddTraceback("PyClicl.error_squared", __pyx_clineno, __pyx_lineno,
11984 __pyx_filename);
11985     __pyx_r = 0;
11986     __pyx_L0;
11987     __Pyx_XGIVEREF(__pyx_r);
11988     __Pyx_RefNannyFinishContext();
11989     return __pyx_r;
11990 }
11991
11992     /* Python wrapper */
11993     static PyObject *__pyx_pw_8PyClicl_13error_squared(PyObject *__pyx_self, PyObject
11994 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
11995     static char __pyx_doc_8PyClicl_12error_squared[] = "\n      Relative or absolute error
11996 using the quadratic norm.\n\n      >> err2=scalar_epsilon*scalar_epsilon\n\n      >>
11997 print(error_squared(clifford(\"{1}\"), clifford(\"{1}\"), err2))\n      0.0\n      >>
11998 print(error_squared(clifford(\"{1}{2}-3{2}+4{3}\"), clifford(\"{1}\"), err2))\n      25.0\n      ";
11999     static PyObject *__pyx_pw_8PyClicl_13error_squared(PyObject *__pyx_self, PyObject
12000 *__pyx_args, PyObject *__pyx_kwds) {
12001     PyObject *__pyx_v_lhs = 0;
12002     PyObject *__pyx_v_rhs = 0;
12003     PyObject *__pyx_v_threshold = 0;
12004     int __pyx_lineno = 0;
12005     const char *__pyx_filename = NULL;
12006     int __pyx_clineno = 0;
12007     PyObject *__pyx_r = 0;
12008     __Pyx_RefNannyDeclarations
12009     __Pyx_RefNannySetupContext("error_squared (wrapper)", 0);
12010     {
12011         static PyObject *__pyx_pyargnames[] =
12012 {&__pyx_n_s_lhs,&__pyx_n_s_rhs,&__pyx_n_s_threshold,0};
12013         PyObject* values[3] = {0,0,0};
12014         if (unlikely(__pyx_kwds)) {
12015             Py_ssize_t kw_args;
12016             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
12017             switch (pos_args) {

```

```

12004         case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12005         CYTHON_FALLTHROUGH;
12006         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12007         CYTHON_FALLTHROUGH;
12008         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12009         CYTHON_FALLTHROUGH;
12010         case 0: break;
12011         default: goto __pyx_L5_argtuple_error;
12012     }
12013     kw_args = PyDict_Size(__pyx_kwds);
12014     switch (pos_args) {
12015     case 0:
12016         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) !=
12017 0)) kw_args--;
12018         else goto __pyx_L5_argtuple_error;
12019         CYTHON_FALLTHROUGH;
12020         case 1:
12021             if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) !=
12022 0)) kw_args--;
12023             else {
12024                 __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, 1); __PYX_ERR(0, 1346,
12025 __pyx_L3_error)
12026             }
12027             CYTHON_FALLTHROUGH;
12028             case 2:
12029                 if (likely((values[2] = __Pyx_PyDict_GetItemStr(__pyx_kwds,
12030 __pyx_n_s_threshold)) != 0)) kw_args--;
12031                 else {
12032                     __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, 2); __PYX_ERR(0, 1346,
12033 __pyx_L3_error)
12034                 }
12035             }
12036             if (unlikely(kw_args > 0)) {
12037                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
12038 values, pos_args, "error_squared") < 0)) __PYX_ERR(0, 1346, __pyx_L3_error)
12039             }
12040             } else if (PyTuple_GET_SIZE(__pyx_args) != 3) {
12041                 goto __pyx_L5_argtuple_error;
12042             } else {
12043                 values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12044                 values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12045                 values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12046             }
12047             __pyx_v_lhs = values[0];
12048             __pyx_v_rhs = values[1];
12049             __pyx_v_threshold = values[2];
12050         }
12051         goto __pyx_L4_argument_unpacking_done;
12052         __pyx_L5_argtuple_error:;
12053         __Pyx_RaiseArgtupleInvalid("error_squared", 1, 3, 3, PyTuple_GET_SIZE(__pyx_args));
12054         __PYX_ERR(0, 1346, __pyx_L3_error)
12055         __pyx_L3_error:;
12056         __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
12057 __pyx_filename);
12058         __Pyx_RefNannyFinishContext();
12059         return NULL;
12060         __pyx_L4_argument_unpacking_done:;
12061         __pyx_r = __pyx_pf_8PyClical_12error_squared(__pyx_self, __pyx_v_lhs, __pyx_v_rhs,
12062 __pyx_v_threshold);
12063         /* function exit code */
12064         __Pyx_RefNannyFinishContext();
12065         return __pyx_r;
12066     }
12067
12068     static PyObject * __pyx_pf_8PyClical_12error_squared(CYTHON_UNUSED PyObject
12069 * __pyx_self, PyObject * __pyx_v_lhs, PyObject * __pyx_v_rhs, PyObject * __pyx_v_threshold) {
12070         PyObject * __pyx_r = NULL;
12071         __Pyx_RefNannyDeclarations
12072         PyObject * __pyx_t_1 = NULL;
12073         int __pyx_lineno = 0;
12074         const char * __pyx_filename = NULL;
12075         int __pyx_clineno = 0;
12076         __Pyx_RefNannySetupContext("error_squared", 0);
12077         __Pyx_XDECREF(__pyx_r);
12078         __pyx_t_1 = __pyx_f_8PyClical_error_squared(__pyx_v_lhs, __pyx_v_rhs,
12079 __pyx_v_threshold, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1346, __pyx_L1_error)
12080         __Pyx_GOTREF(__pyx_t_1);
12081         __pyx_r = __pyx_t_1;
12082         __pyx_t_1 = 0;
12083         goto __pyx_L0;
12084
12085         /* function exit code */
12086         __pyx_L1_error:;
12087         __Pyx_XDECREF(__pyx_t_1);
12088         __Pyx_AddTraceback("PyClical.error_squared", __pyx_clineno, __pyx_lineno,
12089 __pyx_filename);

```

```

12079         __pyx_r = NULL;
12080         __pyx_L0:;
12081         __Pyx_XGIVEREF(__pyx_r);
12082         __Pyx_RefNannyFinishContext();
12083         return __pyx_r;
12084     }
12085
12086     /* "PyClicl.pyx":1359
12087     *     return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
12088     *
12089     * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None):           # ««««««««
12090     *     """
12091     *     Test for approximate equality of multivectors.
12092     */
12093
12094     static PyObject *__pyx_pw_8PyClicl_15approx_equal(PyObject *__pyx_self, PyObject
12095 *__pyx_args, PyObject *__pyx_kws); /*proto*/
12096     static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_approx_equal(PyObject *__pyx_v_lhs,
12097 PyObject *__pyx_v_rhs, CYTHON_UNUSED int __pyx_skip_dispatch, struct
12098 __pyx_opt_args_8PyClicl_approx_equal *__pyx_optional_args) {
12096         PyObject *__pyx_v_threshold = ((PyObject *)Py_None);
12097         PyObject *__pyx_v_tol = ((PyObject *)Py_None);
12098         PyObject *__pyx_r = NULL;
12099         __Pyx_RefNannyDeclarations
12100         PyObject *__pyx_t_1 = NULL;
12101         int __pyx_t_2;
12102         PyObject *__pyx_t_3 = NULL;
12103         scalar_t __pyx_t_4;
12104         scalar_t __pyx_t_5;
12105         int __pyx_lineno = 0;
12106         const char *__pyx_filename = NULL;
12107         int __pyx_clineno = 0;
12108         __Pyx_RefNannySetupContext("approx_equal", 0);
12109         if (__pyx_optional_args) {
12110             if (__pyx_optional_args->__pyx_n > 0) {
12111                 __pyx_v_threshold = __pyx_optional_args->threshold;
12112                 if (__pyx_optional_args->__pyx_n > 1) {
12113                     __pyx_v_tol = __pyx_optional_args->tol;
12114                 }
12115             }
12116         }
12117         __Pyx_INCREF(__pyx_v_threshold);
12118         __Pyx_INCREF(__pyx_v_tol);
12119
12120         /* "PyClicl.pyx":1374
12121         *     True
12122         *     """
12123         *     threshold = error_squared_tol(rhs) if threshold is None else threshold           # ««««««««
12124         *     tol = error_squared_tol(rhs) if tol is None else tol
12125         *     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
12126 <scalar_t>tol)
12127         */
12127         __pyx_t_2 = (__pyx_v_threshold == Py_None);
12128         if ((__pyx_t_2 != 0)) {
12129             __pyx_t_3 = __pyx_f_8PyClicl_error_squared_tol(__pyx_v_rhs, 0); if
12130 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1374, __pyx_L1_error)
12131             __Pyx_GOTREF(__pyx_t_3);
12132             __pyx_t_1 = __pyx_t_3;
12133             __pyx_t_3 = 0;
12134         } else {
12135             __Pyx_INCREF(__pyx_v_threshold);
12136             __pyx_t_1 = __pyx_v_threshold;
12137         }
12137         __Pyx_DECREF_SET(__pyx_v_threshold, __pyx_t_1);
12138         __pyx_t_1 = 0;
12139
12140         /* "PyClicl.pyx":1375
12141         *     """
12142         *     threshold = error_squared_tol(rhs) if threshold is None else threshold
12143         *     tol = error_squared_tol(rhs) if tol is None else tol           # ««««««««
12144         *     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
12145 <scalar_t>tol)
12146         */
12147         __pyx_t_2 = (__pyx_v_tol == Py_None);
12148         if ((__pyx_t_2 != 0)) {
12149             __pyx_t_3 = __pyx_f_8PyClicl_error_squared_tol(__pyx_v_rhs, 0); if
12150 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1375, __pyx_L1_error)
12151             __Pyx_GOTREF(__pyx_t_3);
12152             __pyx_t_1 = __pyx_t_3;
12153             __pyx_t_3 = 0;
12154         } else {
12155             __Pyx_INCREF(__pyx_v_tol);
12156             __pyx_t_1 = __pyx_v_tol;
12157         }
12157         __Pyx_DECREF_SET(__pyx_v_tol, __pyx_t_1);
12158         __pyx_t_1 = 0;

```

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12159
12160          /* "PyClical.pyx":1376
12161 *      threshold = error_squared_tol(rhs) if threshold is None else threshold
12162 *      tol      = error_squared_tol(rhs) if tol      is None else tol
12163 *      return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol) # ««««««««
12164 *
12165 * cpdef inline inv(obj):
12166 */
12167         __Pyx_XDECREF(__pyx_r);
12168         __pyx_t_4 = __pyx_PyFloat_AsDouble(__pyx_v_threshold); if (unlikely((__pyx_t_4 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1376, __pyx_L1_error)
12169         __pyx_t_5 = __pyx_PyFloat_AsDouble(__pyx_v_tol); if (unlikely((__pyx_t_5 ==
((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1376, __pyx_L1_error)
12170         __pyx_t_1 =
__Pyx_PyBool_FromLong(approx_equal(__pyx_f_8PyClical_toClifford(__pyx_v_lhs),
__pyx_f_8PyClical_toClifford(__pyx_v_rhs), ((scalar_t)__pyx_t_4), ((scalar_t)__pyx_t_5))); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1376, __pyx_L1_error)
12171         __Pyx_GOTREF(__pyx_t_1);
12172         __pyx_r = __pyx_t_1;
12173         __pyx_t_1 = 0;
12174         goto __pyx_L0;
12175
12176          /* "PyClical.pyx":1359
12177 *      return glucat.error_squared(toClifford(lhs), toClifford(rhs), <scalar_t>threshold)
12178 *
12179 * cpdef inline approx_equal(lhs, rhs, threshold=None, tol=None): # ««««««««
12180 *      """
12181 *      Test for approximate equality of multivectors.
12182 */
12183
12184         /* function exit code */
12185         __pyx_L1_error:;
12186         __Pyx_XDECREF(__pyx_t_1);
12187         __Pyx_XDECREF(__pyx_t_3);
12188         __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12189         __pyx_r = 0;
12190         __pyx_L0:;
12191         __Pyx_XDECREF(__pyx_v_threshold);
12192         __Pyx_XDECREF(__pyx_v_tol);
12193         __Pyx_XGIVEREF(__pyx_r);
12194         __Pyx_RefNannyFinishContext();
12195         return __pyx_r;
12196     }
12197
12198     /* Python wrapper */
12199     static PyObject * __pyx_pw_8PyClical_15approx_equal(PyObject * __pyx_self, PyObject
* __pyx_args, PyObject * __pyx_kwds); /*proto*/
12200     static char __pyx_doc_8PyClical_14approx_equal[] = "\n    Test for approximate
equality of multivectors.\n\n    >> err2=scalar_epsilon*scalar_epsilon\n\n    >>
print(approx_equal(clifford(\"1{1}\", clifford(\"1{1}\")))\n    True\n    >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}\", clifford(\"1{1}\")))\n    False\n    >>
print(approx_equal(clifford(\"1{1}-3{2}+4{3}+0.001\", clifford(\"1{1}-3{2}+4{3}\", err2, err2))\n
False\n    >> print(approx_equal(clifford(\"1{1}-3{2}+4{3}+1.0e-30\", clifford(\"1{1}-3{2}+4{3}\",
err2, err2))\n    True\n    ";
12201     static PyObject * __pyx_pw_8PyClical_15approx_equal(PyObject * __pyx_self, PyObject
* __pyx_args, PyObject * __pyx_kwds) {
12202         PyObject * __pyx_v_lhs = 0;
12203         PyObject * __pyx_v_rhs = 0;
12204         PyObject * __pyx_v_threshold = 0;
12205         PyObject * __pyx_v_tol = 0;
12206         int __pyx_lineno = 0;
12207         const char * __pyx_filename = NULL;
12208         int __pyx_clineno = 0;
12209         PyObject * __pyx_r = 0;
12210         __Pyx_RefNannyDeclarations
12211         __Pyx_RefNannySetupContext("approx_equal (wrapper)", 0);
12212         {
12213             static PyObject * __pyx_pyargnames[] =
12214             {&__pyx_n_s_lhs,&__pyx_n_s_rhs,&__pyx_n_s_threshold,&__pyx_n_s_tol,0};
12215             PyObject* values[4] = {0,0,0,0};
12216             values[2] = ((PyObject *)Py_None);
12217             values[3] = ((PyObject *)Py_None);
12218             if (unlikely(__pyx_kwds)) {
12219                 Py_ssize_t kw_args;
12220                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
12221                 switch (pos_args) {
12222                     case 4: values[3] = PyTuple_GET_ITEM(__pyx_args, 3);
CYTHON_FALLTHROUGH;
12223                     case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
CYTHON_FALLTHROUGH;
12224                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
12225                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
12226                     case 0: break;

```

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12230         default: goto __pyx_L5_argtuple_error;
12231     }
12232     kw_args = PyDict_Size(__pyx_kwds);
12233     switch (pos_args) {
12234         case 0:
12235             if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_lhs)) !=
0)) kw_args--;
12236             else goto __pyx_L5_argtuple_error;
12237             CYTHON_FALLTHROUGH;
12238         case 1:
12239             if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_rhs)) !=
0)) kw_args--;
12240             else {
12241                 __Pyx_RaiseArgtupleInvalid("approx_equal", 0, 2, 4, 1); __PYX_ERR(0, 1359,
__pyx_L3_error)
12242             }
12243             CYTHON_FALLTHROUGH;
12244         case 2:
12245             if (kw_args > 0) {
12246                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_threshold);
12247                 if (value) { values[2] = value; kw_args--; }
12248             }
12249             CYTHON_FALLTHROUGH;
12250         case 3:
12251             if (kw_args > 0) {
12252                 PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_tol);
12253                 if (value) { values[3] = value; kw_args--; }
12254             }
12255             if (unlikely(kw_args > 0)) {
12256                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "approx_equal") < 0)) __PYX_ERR(0, 1359, __pyx_L3_error)
12257             }
12258             } else {
12259                 switch (PyTuple_GET_SIZE(__pyx_args)) {
12260                     case 4: values[3] = PyTuple_GET_ITEM(__pyx_args, 3);
12261                     CYTHON_FALLTHROUGH;
12262                     case 3: values[2] = PyTuple_GET_ITEM(__pyx_args, 2);
12263                     CYTHON_FALLTHROUGH;
12264                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
12265                     values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
12266                     break;
12267                     default: goto __pyx_L5_argtuple_error;
12268                 }
12269             }
12270             __pyx_v_lhs = values[0];
12271             __pyx_v_rhs = values[1];
12272             __pyx_v_threshold = values[2];
12273             __pyx_v_tol = values[3];
12274         }
12275         goto __pyx_L4_argument_unpacking_done;
12276         __pyx_L5_argtuple_error:;
12277         __Pyx_RaiseArgtupleInvalid("approx_equal", 0, 2, 4, PyTuple_GET_SIZE(__pyx_args));
12278         __PYX_ERR(0, 1359, __pyx_L3_error)
12279         __pyx_L3_error:;
12280         __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12281         __Pyx_RefNannyFinishContext();
12282         return NULL;
12283         __pyx_L4_argument_unpacking_done:;
12284         __pyx_r = __pyx_pf_8PyClical_14approx_equal(__pyx_self, __pyx_v_lhs, __pyx_v_rhs,
__pyx_v_threshold, __pyx_v_tol);
12285
12286         /* function exit code */
12287         __Pyx_RefNannyFinishContext();
12288         return __pyx_r;
12289     }
12290
12291     static PyObject * __pyx_pf_8PyClical_14approx_equal(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_lhs, PyObject *__pyx_v_rhs, PyObject *__pyx_v_threshold, PyObject *__pyx_v_tol) {
12292         PyObject *__pyx_r = NULL;
12293         __Pyx_RefNannyDeclarations
12294         PyObject *__pyx_t_1 = NULL;
12295         struct __pyx_opt_args_8PyClical_approx_equal __pyx_t_2;
12296         int __pyx_lineno = 0;
12297         const char *__pyx_filename = NULL;
12298         int __pyx_clineno = 0;
12299         __Pyx_RefNannySetupContext("approx_equal", 0);
12300         __Pyx_XDECREF(__pyx_r);
12301         __pyx_t_2.__pyx_n = 2;
12302         __pyx_t_2.threshold = __pyx_v_threshold;
12303         __pyx_t_2.tol = __pyx_v_tol;
12304         __pyx_t_1 = __pyx_f_8PyClical_approx_equal(__pyx_v_lhs, __pyx_v_rhs, 0, &__pyx_t_2);
12305         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1359, __pyx_L1_error)
12306         __Pyx_GOTREF(__pyx_t_1);
12307         __pyx_r = __pyx_t_1;
12308         __pyx_t_1 = 0;

```



```

12308         goto __pyx_L0;
12309
12310         /* function exit code */
12311         __pyx_L1_error++;
12312         __Pyx_XDECREF(__pyx_t_1);
12313         __Pyx_AddTraceback("PyClical.approx_equal", __pyx_clineno, __pyx_lineno,
__pyx_filename);
12314         __pyx_r = NULL;
12315         __pyx_L0;
12316         __Pyx_XGIVEREF(__pyx_r);
12317         __Pyx_RefNannyFinishContext();
12318         return __pyx_r;
12319     }
12320
12321     /* "PyClical.pyx":1378
12322     *     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12323     *
12324     * cpdef inline inv(obj):          # ««««««««
12325     *     """
12326     *     Geometric multiplicative inverse.
12327     */
12328
12329     static PyObject * __pyx_pw_8PyClical_17inv(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
12330     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_inv(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12331         PyObject * __pyx_r = NULL;
12332         __Pyx_RefNannyDeclarations
12333         PyObject * __pyx_t_1 = NULL;
12334         PyObject * __pyx_t_2 = NULL;
12335         PyObject * __pyx_t_3 = NULL;
12336         int __pyx_lineno = 0;
12337         const char * __pyx_filename = NULL;
12338         int __pyx_clineno = 0;
12339         __Pyx_RefNannySetupContext("inv", 0);
12340
12341         /* "PyClical.pyx":1391
12342     *     nan
12343     *     """
12344     *     return clifford(obj).inv()          # ««««««««
12345     *
12346     * cpdef inline scalar(obj):
12347     */
12348         __Pyx_XDECREF(__pyx_r);
12349         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1391, __pyx_L1_error)
12350         __Pyx_GOTREF(__pyx_t_2);
12351         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_inv); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1391, __pyx_L1_error)
12352         __Pyx_GOTREF(__pyx_t_3);
12353         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12354         __pyx_t_2 = NULL;
12355         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12356             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12357             if (likely(__pyx_t_2)) {
12358                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12359                 __Pyx_INCREF(__pyx_t_2);
12360                 __Pyx_INCREF(function);
12361                 __Pyx_DECREF_SET(__pyx_t_3, function);
12362             }
12363         }
12364         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12365         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12366         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1391, __pyx_L1_error)
12367         __Pyx_GOTREF(__pyx_t_1);
12368         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12369         __pyx_r = __pyx_t_1;
12370         __pyx_t_1 = 0;
12371         goto __pyx_L0;
12372
12373     /* "PyClical.pyx":1378
12374     *     return glucat.approx_equal(toClifford(lhs), toClifford(rhs), <scalar_t>threshold,
<scalar_t>tol)
12375     *
12376     * cpdef inline inv(obj):          # ««««««««
12377     *     """
12378     *     Geometric multiplicative inverse.
12379     */
12380
12381     /* function exit code */
12382     __pyx_L1_error++;
12383     __Pyx_XDECREF(__pyx_t_1);
12384     __Pyx_XDECREF(__pyx_t_2);
12385     __Pyx_XDECREF(__pyx_t_3);
12386     __Pyx_AddTraceback("PyClical.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);

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```

12387         __pyx_r = 0;
12388         __pyx_L0:;
12389         __Pyx_XGIVEREF(__pyx_r);
12390         __Pyx_RefNannyFinishContext();
12391         return __pyx_r;
12392     }
12393
12394     /* Python wrapper */
12395     static PyObject *__pyx_pw_8PyClical_17inv(PyObject *__pyx_self, PyObject
12396 *__pyx_v_obj); /*proto*/
12397     static char __pyx_doc_8PyClical_16inv[] = "\n    Geometric multiplicative inverse.\n\n
12398 >> print(inv(clifford(\"{1}\")))\n    {1}\n    >> print(inv(clifford(\"{-1}\")))\n    {-1}\n    >>
12399 print(inv(clifford(\"{-2,-1}\")))\n    {-2,-1}\n    >> print(inv(clifford(\"{-1}+{1}\")))\n    nan\n
12400 ";
12401     static PyObject *__pyx_pw_8PyClical_17inv(PyObject *__pyx_self, PyObject *__pyx_v_obj)
12402 {
12403     PyObject *__pyx_r = 0;
12404     __Pyx_RefNannyDeclarations
12405     __Pyx_RefNannySetupContext("inv (wrapper)", 0);
12406     __pyx_r = __pyx_pf_8PyClical_16inv(__pyx_self, ((PyObject *)__pyx_v_obj));
12407
12408     /* function exit code */
12409     __Pyx_RefNannyFinishContext();
12410     return __pyx_r;
12411 }
12412
12413 static PyObject *__pyx_pf_8PyClical_16inv(CYTHON_UNUSED PyObject *__pyx_self, PyObject
12414 *__pyx_v_obj) {
12415     PyObject *__pyx_r = NULL;
12416     __Pyx_RefNannyDeclarations
12417     PyObject *__pyx_t_1 = NULL;
12418     int __pyx_lineno = 0;
12419     const char *__pyx_filename = NULL;
12420     int __pyx_clineno = 0;
12421     __Pyx_RefNannySetupContext("inv", 0);
12422     __Pyx_XDECREF(__pyx_r);
12423     __pyx_t_1 = __pyx_f_8PyClical_inv(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12424     __PYX_ERR(0, 1378, __pyx_L1_error)
12425     __Pyx_GOTREF(__pyx_t_1);
12426     __pyx_r = __pyx_t_1;
12427     __pyx_t_1 = 0;
12428     goto __pyx_L0;
12429
12430     /* function exit code */
12431     __pyx_L1_error:;
12432     __Pyx_XDECREF(__pyx_t_1);
12433     __Pyx_AddTraceback("PyClical.inv", __pyx_clineno, __pyx_lineno, __pyx_filename);
12434     __pyx_r = NULL;
12435     __pyx_L0:;
12436     __Pyx_XGIVEREF(__pyx_r);
12437     __Pyx_RefNannyFinishContext();
12438     return __pyx_r;
12439 }
12440
12441 /* "PyClical.pyx":1393
12442 *     return clifford(obj).inv()
12443 *
12444 * cpdef inline scalar(obj): # ««««««««
12445 *     """
12446 *     Scalar part.
12447 */
12448
12449 static PyObject *__pyx_pw_8PyClical_19scalar(PyObject *__pyx_self, PyObject
12450 *__pyx_v_obj); /*proto*/
12451 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_scalar(PyObject *__pyx_v_obj,
12452 CYTHON_UNUSED int __pyx_skip_dispatch) {
12453     PyObject *__pyx_r = NULL;
12454     __Pyx_RefNannyDeclarations
12455     PyObject *__pyx_t_1 = NULL;
12456     PyObject *__pyx_t_2 = NULL;
12457     PyObject *__pyx_t_3 = NULL;
12458     int __pyx_lineno = 0;
12459     const char *__pyx_filename = NULL;
12460     int __pyx_clineno = 0;
12461     __Pyx_RefNannySetupContext("scalar", 0);
12462
12463     /* "PyClical.pyx":1402
12464 *     0.0
12465 *     """
12466 *     return clifford(obj).scalar() # ««««««««
12467 * cpdef inline real(obj):
12468 */
12469     __Pyx_XDECREF(__pyx_r);
12470     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
12471 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1402, __pyx_L1_error)
12472     __Pyx_GOTREF(__pyx_t_2);

```

```

12464         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_scalar); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1402, __pyx_L1_error)
12465         __Pyx_GOTREF(__pyx_t_3);
12466         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12467         __pyx_t_2 = NULL;
12468         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12469             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12470             if (likely(__pyx_t_2)) {
12471                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12472                 __Pyx_INCREF(__pyx_t_2);
12473                 __Pyx_INCREF(function);
12474                 __Pyx_DECREF_SET(__pyx_t_3, function);
12475             }
12476         }
12477         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12478         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12479         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1402, __pyx_L1_error)
12480         __Pyx_GOTREF(__pyx_t_1);
12481         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12482         __pyx_r = __pyx_t_1;
12483         __pyx_t_1 = 0;
12484         goto __pyx_L0;
12485
12486         /* "PyClical.pyx":1393
12487         *     return clifford(obj).inv()
12488         *
12489         * cpdef inline scalar(obj):          # ««««««««
12490         *     """
12491         *     Scalar part.
12492         */
12493
12494         /* function exit code */
12495         __pyx_L1_error;;
12496         __Pyx_XDECREF(__pyx_t_1);
12497         __Pyx_XDECREF(__pyx_t_2);
12498         __Pyx_XDECREF(__pyx_t_3);
12499         __Pyx_AddTraceback("PyClical.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
12500         __pyx_r = 0;
12501         __pyx_L0;;
12502         __Pyx_XGIVEREF(__pyx_r);
12503         __Pyx_RefNannyFinishContext();
12504         return __pyx_r;
12505     }
12506
12507     /* Python wrapper */
12508     static PyObject * __pyx_pw_8PyClical_19scalar(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12509     static char __pyx_doc_8PyClical_18scalar[] = "\n    Scalar part.\n\n    >>
scalar(clifford(\"1+{1}+{1,2}\"))\n        1.0\n    >> scalar(clifford(\"{1,2}\"))\n        0.0\n    ";
12510     static PyObject * __pyx_pw_8PyClical_19scalar(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
12511         PyObject * __pyx_r = 0;
12512         __Pyx_RefNannyDeclarations
12513         __Pyx_RefNannySetupContext("scalar (wrapper)", 0);
12514         __pyx_r = __pyx_pf_8PyClical_18scalar(__pyx_self, ((PyObject *) __pyx_v_obj));
12515
12516         /* function exit code */
12517         __Pyx_RefNannyFinishContext();
12518         return __pyx_r;
12519     }
12520
12521     static PyObject * __pyx_pf_8PyClical_18scalar(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12522         PyObject * __pyx_r = NULL;
12523         __Pyx_RefNannyDeclarations
12524         PyObject * __pyx_t_1 = NULL;
12525         int __pyx_lineno = 0;
12526         const char * __pyx_filename = NULL;
12527         int __pyx_clineno = 0;
12528         __Pyx_RefNannySetupContext("scalar", 0);
12529         __Pyx_XDECREF(__pyx_r);
12530         __pyx_t_1 = __pyx_f_8PyClical_scalar(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1393, __pyx_L1_error)
12531         __Pyx_GOTREF(__pyx_t_1);
12532         __pyx_r = __pyx_t_1;
12533         __pyx_t_1 = 0;
12534         goto __pyx_L0;
12535
12536         /* function exit code */
12537         __pyx_L1_error;;
12538         __Pyx_XDECREF(__pyx_t_1);
12539         __Pyx_AddTraceback("PyClical.scalar", __pyx_clineno, __pyx_lineno, __pyx_filename);
12540         __pyx_r = NULL;
12541         __pyx_L0;;
12542         __Pyx_XGIVEREF(__pyx_r);
12543         __Pyx_RefNannyFinishContext();

```

```

12544         return __pyx_r;
12545     }
12546
12547     /* "PyClicl.pyx":1404
12548     *     return clifford(obj).scalar()
12549     *
12550     * cpdef inline real(obj):
12551     *     """
12552     *     Real part: synonym for scalar part.
12553     */
12554
12555     static PyObject * __pyx_pw_8PyClicl_21real(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12556     static CYTHON_INLINE PyObject * __pyx_f_8PyClicl_real(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12557         PyObject * __pyx_r = NULL;
12558         __Pyx_RefNannyDeclarations
12559         PyObject * __pyx_t_1 = NULL;
12560         PyObject * __pyx_t_2 = NULL;
12561         PyObject * __pyx_t_3 = NULL;
12562         int __pyx_lineno = 0;
12563         const char * __pyx_filename = NULL;
12564         int __pyx_clineno = 0;
12565         __Pyx_RefNannySetupContext("real", 0);
12566
12567         /* "PyClicl.pyx":1413
12568     *     0.0
12569     *     """
12570     *     return clifford(obj).scalar()
12571     *
12572     * cpdef inline imag(obj):
12573     */
12574         __Pyx_XDECREF(__pyx_r);
12575         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *) __pyx_ptype_8PyClicl_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1413, __pyx_L1_error)
12576         __Pyx_GOTREF(__pyx_t_2);
12577         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_scalar); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1413, __pyx_L1_error)
12578         __Pyx_GOTREF(__pyx_t_3);
12579         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12580         __pyx_t_2 = NULL;
12581         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12582             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12583             if (likely(__pyx_t_2)) {
12584                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12585                 __Pyx_INCREF(__pyx_t_2);
12586                 __Pyx_INCREF(function);
12587                 __Pyx_DECREF_SET(__pyx_t_3, function);
12588             }
12589         }
12590         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12591         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12592         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1413, __pyx_L1_error)
12593         __Pyx_GOTREF(__pyx_t_1);
12594         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12595         __pyx_r = __pyx_t_1;
12596         __pyx_t_1 = 0;
12597         goto __pyx_L0;
12598
12599         /* "PyClicl.pyx":1404
12600     *     return clifford(obj).scalar()
12601     *
12602     * cpdef inline real(obj):
12603     *     """
12604     *     Real part: synonym for scalar part.
12605     */
12606
12607         /* function exit code */
12608         __pyx_L1_error:;
12609         __Pyx_XDECREF(__pyx_t_1);
12610         __Pyx_XDECREF(__pyx_t_2);
12611         __Pyx_XDECREF(__pyx_t_3);
12612         __Pyx_AddTraceback("PyClicl.real", __pyx_clineno, __pyx_lineno, __pyx_filename);
12613         __pyx_r = 0;
12614         __pyx_L0:;
12615         __Pyx_XGIVEREF(__pyx_r);
12616         __Pyx_RefNannyFinishContext();
12617         return __pyx_r;
12618     }
12619
12620     /* Python wrapper */
12621     static PyObject * __pyx_pw_8PyClicl_21real(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12622     static char __pyx_doc_8PyClicl_20real[] = "\n    Real part: synonym for scalar
part.\n\n    >> real(clifford(\"1+{1}+{1,2}\"))\n    1.0\n    >> real(clifford(\"{1,2}\"))\n    0.0\n";

```

```

12623     static PyObject *__pyx_pw_8PyClical_21real(PyObject *__pyx_self, PyObject
12624     *__pyx_v_obj) {
12625         PyObject *__pyx_r = 0;
12626         __Pyx_RefNannyDeclarations
12627         __Pyx_RefNannySetupContext("real (wrapper)", 0);
12628         __pyx_r = __pyx_pf_8PyClical_20real(__pyx_self, (PyObject *)__pyx_v_obj));
12629
12630         /* function exit code */
12631         __Pyx_RefNannyFinishContext();
12632         return __pyx_r;
12633     }
12634
12635     static PyObject *__pyx_pf_8PyClical_20real(CYTHON_UNUSED PyObject *__pyx_self,
12636     PyObject *__pyx_v_obj) {
12637         PyObject *__pyx_r = NULL;
12638         __Pyx_RefNannyDeclarations
12639         PyObject *__pyx_t_1 = NULL;
12640         int __pyx_lineno = 0;
12641         const char *__pyx_filename = NULL;
12642         int __pyx_clineno = 0;
12643         __Pyx_RefNannySetupContext("real", 0);
12644         __Pyx_XDECREF(__pyx_r);
12645         __pyx_t_1 = __pyx_f_8PyClical_real(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12646         __PYX_ERR(0, 1404, __pyx_L1_error)
12647         __Pyx_GOTREF(__pyx_t_1);
12648         __pyx_r = __pyx_t_1;
12649         __pyx_t_1 = 0;
12650         goto __pyx_L0;
12651
12652         /* function exit code */
12653         __pyx_L1_error:;
12654         __Pyx_XDECREF(__pyx_t_1);
12655         __Pyx_AddTraceback("PyClical.real", __pyx_clineno, __pyx_lineno, __pyx_filename);
12656         __pyx_r = NULL;
12657         __pyx_L0:;
12658         __Pyx_XGIVEREF(__pyx_r);
12659         __Pyx_RefNannyFinishContext();
12660         return __pyx_r;
12661     }
12662
12663     /* "PyClical.pyx":1415
12664     *     return clifford(obj).scalar()
12665     *
12666     * cpdef inline imag(obj):
12667     *     """
12668     *     Imaginary part: deprecated (always 0).
12669     */
12670
12671     static PyObject *__pyx_pw_8PyClical_23imag(PyObject *__pyx_self, PyObject
12672     *__pyx_v_obj); /*proto*/
12673     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_imag(CYTHON_UNUSED PyObject
12674     *__pyx_v_obj, CYTHON_UNUSED int __pyx_skip_dispatch) {
12675         PyObject *__pyx_r = NULL;
12676         __Pyx_RefNannyDeclarations
12677         __Pyx_RefNannySetupContext("imag", 0);
12678
12679         /* "PyClical.pyx":1424
12680         *     0.0
12681         *     """
12682         *     return 0.0
12683         *     # ««««««««
12684         *
12685         * cpdef inline pure(obj):
12686         */
12687         __Pyx_XDECREF(__pyx_r);
12688         __Pyx_INCREF(__pyx_float_0_0);
12689         __pyx_r = __pyx_float_0_0;
12690         goto __pyx_L0;
12691
12692         /* "PyClical.pyx":1415
12693         *     return clifford(obj).scalar()
12694         *
12695         * cpdef inline imag(obj):
12696         *     """
12697         *     Imaginary part: deprecated (always 0).
12698         */
12699
12700         /* function exit code */
12701         __pyx_L0:;
12702         __Pyx_XGIVEREF(__pyx_r);
12703         __Pyx_RefNannyFinishContext();
12704         return __pyx_r;
12705     }
12706
12707     /* Python wrapper */
12708     static PyObject *__pyx_pw_8PyClical_23imag(PyObject *__pyx_self, PyObject
12709     *__pyx_v_obj); /*proto*/
12710     static char __pyx_doc_8PyClical_22imag[] = "\n    Imaginary part: deprecated (always

```

```

0).\n\n    >> imag(clifford("\1+\1+\1,2\"))\n    0.0\n    >> imag(clifford("\{1,2\}"))\n    0.0\n";
12704     static PyObject *__pyx_pw_8PyClical_23imag(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
12705         PyObject *__pyx_r = 0;
12706         __Pyx_RefNannyDeclarations
12707         __Pyx_RefNannySetupContext("imag (wrapper)", 0);
12708         __pyx_r = __pyx_pf_8PyClical_22imag(__pyx_self, ((PyObject *)__pyx_v_obj));
12709
12710         /* function exit code */
12711         __Pyx_RefNannyFinishContext();
12712         return __pyx_r;
12713     }
12714
12715     static PyObject *__pyx_pf_8PyClical_22imag(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
12716         PyObject *__pyx_r = NULL;
12717         __Pyx_RefNannyDeclarations
12718         PyObject *__pyx_t_1 = NULL;
12719         int __pyx_lineno = 0;
12720         const char *__pyx_filename = NULL;
12721         int __pyx_clineno = 0;
12722         __Pyx_RefNannySetupContext("imag", 0);
12723         __Pyx_XDECREF(__pyx_r);
12724         __pyx_t_1 = __pyx_f_8PyClical_imag(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1415, __pyx_L1_error)
12725         __Pyx_GOTREF(__pyx_t_1);
12726         __pyx_r = __pyx_t_1;
12727         __pyx_t_1 = 0;
12728         goto __pyx_L0;
12729
12730         /* function exit code */
12731         __pyx_L1_error:;
12732         __Pyx_XDECREF(__pyx_t_1);
12733         __Pyx_AddTraceback("PyClical.imag", __pyx_clineno, __pyx_lineno, __pyx_filename);
12734         __pyx_r = NULL;
12735         __pyx_L0:;
12736         __Pyx_XGIVEREF(__pyx_r);
12737         __Pyx_RefNannyFinishContext();
12738         return __pyx_r;
12739     }
12740
12741     /* "PyClical.pyx":1426
12742     *     return 0.0
12743     *
12744     * cpdef inline pure(obj):          # ««««««««
12745     *     """
12746     *     Pure part
12747     */
12748
12749     static PyObject *__pyx_pw_8PyClical_25pure(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12750     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_pure(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12751         PyObject *__pyx_r = NULL;
12752         __Pyx_RefNannyDeclarations
12753         PyObject *__pyx_t_1 = NULL;
12754         PyObject *__pyx_t_2 = NULL;
12755         PyObject *__pyx_t_3 = NULL;
12756         int __pyx_lineno = 0;
12757         const char *__pyx_filename = NULL;
12758         int __pyx_clineno = 0;
12759         __Pyx_RefNannySetupContext("pure", 0);
12760
12761         /* "PyClical.pyx":1435
12762     *     {1,2}
12763     *     """
12764     *     return clifford(obj).pure()          # ««««««««
12765     *
12766     * cpdef inline even(obj):
12767     */
12768         __Pyx_XDECREF(__pyx_r);
12769         __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1435, __pyx_L1_error)
12770         __Pyx_GOTREF(__pyx_t_2);
12771         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_pure); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1435, __pyx_L1_error)
12772         __Pyx_GOTREF(__pyx_t_3);
12773         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12774         __pyx_t_2 = NULL;
12775         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12776             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12777             if (likely(__pyx_t_2)) {
12778                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12779                 __Pyx_INCREF(__pyx_t_2);
12780                 __Pyx_INCREF(function);
12781                 __Pyx_DECREF_SET(__pyx_t_3, function);

```

```

12782     }
12783     }
12784     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12785     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12786     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1435, __pyx_L1_error)
12787     __Pyx_GOTREF(__pyx_t_1);
12788     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12789     __pyx_r = __pyx_t_1;
12790     __pyx_t_1 = 0;
12791     goto __pyx_L0;
12792
12793     /* "PyClical.pyx":1426
12794     *     return 0.0
12795     *
12796     * cpdef inline pure(obj):          # ««««««««
12797     *     """
12798     *     Pure part
12799     */
12800
12801     /* function exit code */
12802     __pyx_L1_error:;
12803     __Pyx_XDECREF(__pyx_t_1);
12804     __Pyx_XDECREF(__pyx_t_2);
12805     __Pyx_XDECREF(__pyx_t_3);
12806     __Pyx_AddTraceback("PyClical.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
12807     __pyx_r = 0;
12808     __pyx_L0:;
12809     __Pyx_XGIVEREF(__pyx_r);
12810     __Pyx_RefNannyFinishContext();
12811     return __pyx_r;
12812 }
12813
12814     /* Python wrapper */
12815     static PyObject * __pyx_pw_8PyClical_25pure(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
12816     static char __pyx_doc_8PyClical_24pure[] = "\n    Pure part\n\n    >>
print(pure(clifford(\"1+{1}+{1,2}\")))\n    {1}+{1,2}\n    >> print(pure(clifford(\"{1,2}\")))\n
{1,2}\n    ";
12817     static PyObject * __pyx_pw_8PyClical_25pure(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
12818         PyObject * __pyx_r = 0;
12819         __Pyx_RefNannyDeclarations
12820         __Pyx_RefNannySetupContext("pure (wrapper)", 0);
12821         __pyx_r = __pyx_pf_8PyClical_24pure(__pyx_self, ((PyObject *) __pyx_v_obj));
12822
12823         /* function exit code */
12824         __Pyx_RefNannyFinishContext();
12825         return __pyx_r;
12826     }
12827
12828     static PyObject * __pyx_pf_8PyClical_24pure(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
12829         PyObject * __pyx_r = NULL;
12830         __Pyx_RefNannyDeclarations
12831         PyObject * __pyx_t_1 = NULL;
12832         int __pyx_lineno = 0;
12833         const char * __pyx_filename = NULL;
12834         int __pyx_clineno = 0;
12835         __Pyx_RefNannySetupContext("pure", 0);
12836         __Pyx_XDECREF(__pyx_r);
12837         __pyx_t_1 = __pyx_f_8PyClical_pure(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1426, __pyx_L1_error)
12838         __Pyx_GOTREF(__pyx_t_1);
12839         __pyx_r = __pyx_t_1;
12840         __pyx_t_1 = 0;
12841         goto __pyx_L0;
12842
12843         /* function exit code */
12844         __pyx_L1_error:;
12845         __Pyx_XDECREF(__pyx_t_1);
12846         __Pyx_AddTraceback("PyClical.pure", __pyx_clineno, __pyx_lineno, __pyx_filename);
12847         __pyx_r = NULL;
12848         __pyx_L0:;
12849         __Pyx_XGIVEREF(__pyx_r);
12850         __Pyx_RefNannyFinishContext();
12851         return __pyx_r;
12852     }
12853
12854     /* "PyClical.pyx":1437
12855     *     return clifford(obj).pure()
12856     *
12857     * cpdef inline even(obj):          # ««««««««
12858     *     """
12859     *     Even part of multivector, sum of even grade terms.
12860     */
12861

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12862         static PyObject *__pyx_pw_8PyClical_27even(PyObject *__pyx_self, PyObject
12863         *__pyx_v_obj); /*proto*/
12863         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_even(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
12864             PyObject *__pyx_r = NULL;
12865             __Pyx_RefNannyDeclarations
12866             PyObject *__pyx_t_1 = NULL;
12867             PyObject *__pyx_t_2 = NULL;
12868             PyObject *__pyx_t_3 = NULL;
12869             int __pyx_lineno = 0;
12870             const char *__pyx_filename = NULL;
12871             int __pyx_clineno = 0;
12872             __Pyx_RefNannySetupContext("even", 0);
12873
12874             /* "PyClical.pyx":1444
12875             *      1+{1,2}
12876             *      """
12877             *      return clifford(obj).even()          # ««««««««
12878             *
12879             * cpdef inline odd(obj):
12880             */
12881             __Pyx_XDECREF(__pyx_r);
12882             __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1444, __pyx_L1_error)
12883             __Pyx_GOTREF(__pyx_t_2);
12884             __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_even); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1444, __pyx_L1_error)
12885             __Pyx_GOTREF(__pyx_t_3);
12886             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
12887             __pyx_t_2 = NULL;
12888             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
12889                 __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
12890                 if (likely(__pyx_t_2)) {
12891                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
12892                     __Pyx_INCREF(__pyx_t_2);
12893                     __Pyx_INCREF(function);
12894                     __Pyx_DECREF_SET(__pyx_t_3, function);
12895                 }
12896             }
12897             __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
12898             __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
12899             if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1444, __pyx_L1_error)
12900             __Pyx_GOTREF(__pyx_t_1);
12901             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
12902             __pyx_r = __pyx_t_1;
12903             __pyx_t_1 = 0;
12904             goto __pyx_L0;
12905
12906             /* "PyClical.pyx":1437
12907             *      return clifford(obj).pure()
12908             *
12909             * cpdef inline even(obj):          # ««««««««
12910             *      """
12911             *      Even part of multivector, sum of even grade terms.
12912             */
12913
12914             /* function exit code */
12915             __pyx_L1_error;;
12916             __Pyx_XDECREF(__pyx_t_1);
12917             __Pyx_XDECREF(__pyx_t_2);
12918             __Pyx_XDECREF(__pyx_t_3);
12919             __Pyx_AddTraceback("PyClical.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
12920             __pyx_r = 0;
12921             __pyx_L0;;
12922             __Pyx_XGIVEREF(__pyx_r);
12923             __Pyx_RefNannyFinishContext();
12924             return __pyx_r;
12925         }
12926
12927         /* Python wrapper */
12928         static PyObject *__pyx_pw_8PyClical_27even(PyObject *__pyx_self, PyObject
12929         *__pyx_v_obj); /*proto*/
12929         static char __pyx_doc_8PyClical_26even[] = "\n      Even part of multivector, sum of
even grade terms.\n\n      >> print(even(clifford(\"1+{1}+{1,2}\")))\n      1+{1,2}\n      ";
12930         static PyObject *__pyx_pw_8PyClical_27even(PyObject *__pyx_self, PyObject
__pyx_v_obj) {
12931             PyObject *__pyx_r = 0;
12932             __Pyx_RefNannyDeclarations
12933             __Pyx_RefNannySetupContext("even (wrapper)", 0);
12934             __pyx_r = __pyx_f_8PyClical_26even(__pyx_self, ((PyObject *)__pyx_v_obj));
12935
12936             /* function exit code */
12937             __Pyx_RefNannyFinishContext();
12938             return __pyx_r;
12939         }
12940

```



```

12941         static PyObject *__pyx_pf_8PyClical_26even(CYTHON_UNUSED PyObject *__pyx_self,
12942         PyObject *__pyx_v_obj) {
12943             PyObject *__pyx_r = NULL;
12944             __Pyx_RefNannyDeclarations
12945             PyObject *__pyx_t_1 = NULL;
12946             int __pyx_lineno = 0;
12947             const char *__pyx_filename = NULL;
12948             int __pyx_clineno = 0;
12949             __Pyx_RefNannySetupContext("even", 0);
12950             __Pyx_XDECREF(__pyx_r);
12951             __pyx_t_1 = __pyx_f_8PyClical_even(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
12952                 __PYX_ERR(0, 1437, __pyx_L1_error)
12953             __Pyx_GOTREF(__pyx_t_1);
12954             __pyx_r = __pyx_t_1;
12955             __pyx_t_1 = 0;
12956             goto __pyx_L0;
12957
12958             /* function exit code */
12959             __Pyx_XDECREF(__pyx_t_1);
12960             __Pyx_AddTraceback("PyClical.even", __pyx_clineno, __pyx_lineno, __pyx_filename);
12961             __pyx_r = NULL;
12962             __Pyx_L0:;
12963             __Pyx_XGIVEREF(__pyx_r);
12964             __Pyx_RefNannyFinishContext();
12965             return __pyx_r;
12966         }
12967
12968         /* "PyClical.pyx":1446
12969         *      return clifford(obj).even()
12970         *
12971         * cpdef inline odd(obj):
12972         *      """
12973         *      Odd part of multivector, sum of odd grade terms.
12974         */
12975         static PyObject *__pyx_pw_8PyClical_29odd(PyObject *__pyx_self, PyObject
12976         *__pyx_v_obj); /*proto*/
12977         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_odd(PyObject *__pyx_v_obj,
12978         CYTHON_UNUSED int __pyx_skip_dispatch) {
12979             PyObject *__pyx_r = NULL;
12980             __Pyx_RefNannyDeclarations
12981             PyObject *__pyx_t_1 = NULL;
12982             PyObject *__pyx_t_2 = NULL;
12983             PyObject *__pyx_t_3 = NULL;
12984             int __pyx_lineno = 0;
12985             const char *__pyx_filename = NULL;
12986             int __pyx_clineno = 0;
12987             __Pyx_RefNannySetupContext("odd", 0);
12988
12989             /* "PyClical.pyx":1453
12990             *      {1}
12991             *      """
12992             *      return clifford(obj).odd()
12993             *      # ««««««««
12994             * cpdef inline involute(obj):
12995             */
12996             __Pyx_XDECREF(__pyx_r);
12997             __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
12998             __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1453, __pyx_L1_error)
12999             __Pyx_GOTREF(__pyx_t_2);
13000             __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_odd); if
13001             (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1453, __pyx_L1_error)
13002             __Pyx_GOTREF(__pyx_t_3);
13003             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13004             __pyx_t_2 = NULL;
13005             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13006                 __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13007                 if (likely(__pyx_t_2)) {
13008                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13009                     __Pyx_INCREF(__pyx_t_2);
13010                     __Pyx_INCREF(function);
13011                     __Pyx_DECREF_SET(__pyx_t_3, function);
13012                 }
13013             }
13014             __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
13015             __Pyx_PyObject_CallNoArg(__pyx_t_3);
13016             __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13017             if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1453, __pyx_L1_error)
13018             __Pyx_GOTREF(__pyx_t_1);
13019             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13020             __pyx_r = __pyx_t_1;
13021             __pyx_t_1 = 0;
13022             goto __pyx_L0;
13023
13024             /* "PyClical.pyx":1446
13025             *      return clifford(obj).even()

```

```

13021 *
13022 * cpdef inline odd(obj): # ««««««««
13023 * """
13024 * Odd part of multivector, sum of odd grade terms.
13025 */
13026
13027 /* function exit code */
13028 __pyx_L1_error;
13029 __Pyx_XDECREF(__pyx_t_1);
13030 __Pyx_XDECREF(__pyx_t_2);
13031 __Pyx_XDECREF(__pyx_t_3);
13032 __Pyx_AddTraceback("PyClical.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
13033 __pyx_r = 0;
13034 __pyx_L0;
13035 __Pyx_XGIVEREF(__pyx_r);
13036 __Pyx_RefNannyFinishContext();
13037 return __pyx_r;
13038 }
13039
13040 /* Python wrapper */
13041 static PyObject *__pyx_pw_8PyClical_29odd(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13042 static char __pyx_doc_8PyClical_28odd[] = "\n Odd part of multivector, sum of odd
grade terms.\n\n >> print(odd(clifford(\"1+{1}+{1,2}\"))\n {1}\n ");
13043 static PyObject *__pyx_pw_8PyClical_29odd(PyObject *__pyx_self, PyObject *__pyx_v_obj)
{
13044     PyObject *__pyx_r = 0;
13045     __Pyx_RefNannyDeclarations
13046     __Pyx_RefNannySetupContext("odd (wrapper)", 0);
13047     __pyx_r = __pyx_pf_8PyClical_28odd(__pyx_self, ((PyObject *)__pyx_v_obj));
13048
13049     /* function exit code */
13050     __Pyx_RefNannyFinishContext();
13051     return __pyx_r;
13052 }
13053
13054 static PyObject *__pyx_pf_8PyClical_28odd(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13055     PyObject *__pyx_r = NULL;
13056     __Pyx_RefNannyDeclarations
13057     PyObject *__pyx_t_1 = NULL;
13058     int __pyx_lineno = 0;
13059     const char *__pyx_filename = NULL;
13060     int __pyx_clineno = 0;
13061     __Pyx_RefNannySetupContext("odd", 0);
13062     __Pyx_XDECREF(__pyx_r);
13063     __pyx_t_1 = __pyx_f_8PyClical_odd(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1446, __pyx_L1_error)
13064     __Pyx_GOTREF(__pyx_t_1);
13065     __pyx_r = __pyx_t_1;
13066     __pyx_t_1 = 0;
13067     goto __pyx_L0;
13068
13069     /* function exit code */
13070     __pyx_L1_error;
13071     __Pyx_XDECREF(__pyx_t_1);
13072     __Pyx_AddTraceback("PyClical.odd", __pyx_clineno, __pyx_lineno, __pyx_filename);
13073     __pyx_r = NULL;
13074     __pyx_L0;
13075     __Pyx_XGIVEREF(__pyx_r);
13076     __Pyx_RefNannyFinishContext();
13077     return __pyx_r;
13078 }
13079
13080 /* "PyClical.pyx":1455
13081 * return clifford(obj).odd()
13082 *
13083 * cpdef inline involute(obj): # ««««««««
13084 * """
13085 * Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
13086 */
13087
13088 static PyObject *__pyx_pw_8PyClical_31involute(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13089 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_involute(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13090     PyObject *__pyx_r = NULL;
13091     __Pyx_RefNannyDeclarations
13092     PyObject *__pyx_t_1 = NULL;
13093     PyObject *__pyx_t_2 = NULL;
13094     PyObject *__pyx_t_3 = NULL;
13095     int __pyx_lineno = 0;
13096     const char *__pyx_filename = NULL;
13097     int __pyx_clineno = 0;
13098     __Pyx_RefNannySetupContext("involute", 0);
13099
13100     /* "PyClical.pyx":1468

```

```

13101 *      1-{{1}}+{{1,2}}
13102 *      """
13103 *      return clifford(obj).involute()          # ««««««««
13104 *
13105 * cpdef inline reverse(obj):
13106 */
13107         __Pyx_XDECREF(__pyx_r);
13108         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
13109         __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1468, __pyx_L1_error)
13109         __Pyx_GOTREF(__pyx_t_2);
13110         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_involute); if
13111         (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1468, __pyx_L1_error)
13111         __Pyx_GOTREF(__pyx_t_3);
13112         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13113         __pyx_t_2 = NULL;
13114         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13115             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13116             if (likely(__pyx_t_2)) {
13117                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13118                 __Pyx_INCREF(__pyx_t_2);
13119                 __Pyx_INCREF(function);
13120                 __Pyx_DECREF_SET(__pyx_t_3, function);
13121             }
13122         }
13123         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
13124         __Pyx_PyObject_CallNoArg(__pyx_t_3);
13124         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13125         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1468, __pyx_L1_error)
13126         __Pyx_GOTREF(__pyx_t_1);
13127         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13128         __pyx_r = __pyx_t_1;
13129         __pyx_t_1 = 0;
13130         goto __pyx_L0;
13131
13132         /* "PyClical.pyx":1455
13133 *      return clifford(obj).odd()
13134 *
13135 * cpdef inline involute(obj):          # ««««««««
13136 *      """
13137 *      Main involution, each {i} is replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})
13138 */
13139
13140         /* function exit code */
13141         __pyx_L1_error;
13142         __Pyx_XDECREF(__pyx_t_1);
13143         __Pyx_XDECREF(__pyx_t_2);
13144         __Pyx_XDECREF(__pyx_t_3);
13145         __Pyx_AddTraceback("PyClical.involute", __pyx_clineno, __pyx_lineno,
13146         __pyx_filename);
13146         __pyx_r = 0;
13147         __pyx_L0;
13148         __Pyx_XGIVEREF(__pyx_r);
13149         __Pyx_RefNannyFinishContext();
13150         return __pyx_r;
13151     }
13152
13153     /* Python wrapper */
13154     static PyObject * __pyx_pw_8PyClical_31involute(PyObject * __pyx_self, PyObject
13155     * __pyx_v_obj); /*proto*/
13155     static char __pyx_doc_8PyClical_30involute[] = "\n    Main involution, each {i} is
13156     replaced by -{i} in each term, eg. {1}*{2} -> (-{2})*(-{1})\n\n    >>
13157     print(involute(clifford(\"{1}\")))\n    -{1}\n    >> print(involute(clifford(\"{2}\")) *
13158     clifford(\"{1}\"))\n    -{1,2}\n    >> print(involute(clifford(\"{1}\") * clifford(\"{2}\")))\n
13159     {1,2}\n    >> print(involute(clifford(\"1+{1}+{1,2}\")))\n    1-{{1}}+{{1,2}}\n    ";
13156     static PyObject * __pyx_pf_8PyClical_31involute(PyObject * __pyx_self, PyObject
13157     * __pyx_v_obj) {
13157         PyObject * __pyx_r = 0;
13158         __Pyx_RefNannyDeclarations
13159         __Pyx_RefNannySetupContext("involute (wrapper)", 0);
13160         __pyx_r = __pyx_pf_8PyClical_30involute(__pyx_self, ((PyObject *) __pyx_v_obj));
13161
13162         /* function exit code */
13163         __Pyx_RefNannyFinishContext();
13164         return __pyx_r;
13165     }
13166
13167     static PyObject * __pyx_pf_8PyClical_30involute(CYTHON_UNUSED PyObject * __pyx_self,
13168     PyObject * __pyx_v_obj) {
13168         PyObject * __pyx_r = NULL;
13169         __Pyx_RefNannyDeclarations
13170         PyObject * __pyx_t_1 = NULL;
13171         int __pyx_lineno = 0;
13172         const char * __pyx_filename = NULL;
13173         int __pyx_clineno = 0;
13174         __Pyx_RefNannySetupContext("involute", 0);
13175         __Pyx_XDECREF(__pyx_r);
13176         __pyx_t_1 = __pyx_f_8PyClical_involute(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))

```

```

__PYX_ERR(0, 1455, __pyx_l1_error)
13177     __Pyx_GOTREF(__pyx_t_1);
13178     __pyx_r = __pyx_t_1;
13179     __pyx_t_1 = 0;
13180     goto __pyx_L0;
13181
13182     /* function exit code */
13183     __pyx_l1_error;
13184     __Pyx_XDECREF(__pyx_t_1);
13185     __Pyx_AddTraceback("PyClical.involute", __pyx_clineno, __pyx_lineno,
__pyx_filename);
13186     __pyx_r = NULL;
13187     __pyx_L0;
13188     __Pyx_XGIVEREF(__pyx_r);
13189     __Pyx_RefNannyFinishContext();
13190     return __pyx_r;
13191 }
13192
13193     /* "PyClical.pyx":1470
13194 *     return clifford(obj).involute()
13195 *
13196 * cpdef inline reverse(obj):
13197 *     """
13198 *     Reversion, eg. {1}*{2} -> {2}*{1}
13199 */
13200
13201     static PyObject * __pyx_pw_8PyClical_33reverse(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
13202     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_reverse(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13203     PyObject * __pyx_r = NULL;
13204     __Pyx_RefNannyDeclarations
13205     PyObject * __pyx_t_1 = NULL;
13206     PyObject * __pyx_t_2 = NULL;
13207     PyObject * __pyx_t_3 = NULL;
13208     int __pyx_lineno = 0;
13209     const char * __pyx_filename = NULL;
13210     int __pyx_clineno = 0;
13211     __Pyx_RefNannySetupContext("reverse", 0);
13212
13213     /* "PyClical.pyx":1483
13214 *     1+{1}-{1,2}
13215 *     """
13216 *     return clifford(obj).reverse()
13217 *
13218 * cpdef inline conj(obj):
13219 */
13220     __Pyx_XDECREF(__pyx_r);
13221     __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *) __pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1483, __pyx_l1_error)
13222     __Pyx_GOTREF(__pyx_t_2);
13223     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_reverse); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1483, __pyx_l1_error)
13224     __Pyx_GOTREF(__pyx_t_3);
13225     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13226     __pyx_t_2 = NULL;
13227     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13228         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13229         if (likely(__pyx_t_2)) {
13230             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13231             __Pyx_INCREF(__pyx_t_2);
13232             __Pyx_INCREF(function);
13233             __Pyx_DECREF_SET(__pyx_t_3, function);
13234         }
13235     }
13236     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13237     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13238     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1483, __pyx_l1_error)
13239     __Pyx_GOTREF(__pyx_t_1);
13240     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13241     __pyx_r = __pyx_t_1;
13242     __pyx_t_1 = 0;
13243     goto __pyx_L0;
13244
13245     /* "PyClical.pyx":1470
13246 *     return clifford(obj).involute()
13247 *
13248 * cpdef inline reverse(obj):
13249 *     """
13250 *     Reversion, eg. {1}*{2} -> {2}*{1}
13251 */
13252
13253     /* function exit code */
13254     __pyx_l1_error;
13255     __Pyx_XDECREF(__pyx_t_1);
13256     __Pyx_XDECREF(__pyx_t_2);

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__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1498, __pyx_L1_error)
13335     __Pyx_GOTREF(__pyx_t_2);
13336     __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_conj); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1498, __pyx_L1_error)
13337     __Pyx_GOTREF(__pyx_t_3);
13338     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13339     __pyx_t_2 = NULL;
13340     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13341         __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13342         if (likely(__pyx_t_2)) {
13343             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13344             __Pyx_INCREF(__pyx_t_2);
13345             __Pyx_INCREF(function);
13346             __Pyx_DECREF_SET(__pyx_t_3, function);
13347         }
13348     }
13349     __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13350     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13351     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1498, __pyx_L1_error)
13352     __Pyx_GOTREF(__pyx_t_1);
13353     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13354     __pyx_r = __pyx_t_1;
13355     __pyx_t_1 = 0;
13356     goto __pyx_L0;
13357
13358     /* "PyClical.pyx":1485
13359     *     return clifford(obj).reverse()
13360     *
13361     * cpdef inline conj(obj):          # ««««««««
13362     *     """
13363     *     Conjugation, reverse o involute == involute o reverse.
13364     */
13365
13366     /* function exit code */
13367     __pyx_L1_error;;
13368     __Pyx_XDECREF(__pyx_t_1);
13369     __Pyx_XDECREF(__pyx_t_2);
13370     __Pyx_XDECREF(__pyx_t_3);
13371     __Pyx_AddTraceback("PyClical.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
13372     __pyx_r = 0;
13373     __pyx_L0;;
13374     __Pyx_XGIVEREF(__pyx_r);
13375     __Pyx_RefNannyFinishContext();
13376     return __pyx_r;
13377 }
13378
13379 /* Python wrapper */
13380 static PyObject * __pyx_pw_8PyClical_35conj(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13381 static char __pyx_doc_8PyClical_34conj[] = "\n    Conjugation, reverse o involute ==
involute o reverse.\n\n    >> print(conj(clifford(\"{1}\")))\n    -{1}\n    >>
print(conj(clifford(\"{2}\")) * clifford(\"{1}\"))\n    {1,2}\n    >> print(conj(clifford(\"{1}\") *
clifford(\"{2}\")))\n    -{1,2}\n    >> print(conj(clifford(\"1+{1}+{1,2}\")))\n    1-{1}-{1,2}\n
";
13382 static PyObject * __pyx_pw_8PyClical_35conj(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
13383     PyObject * __pyx_r = 0;
13384     __Pyx_RefNannyDeclarations
13385     __Pyx_RefNannySetupContext("conj (wrapper)", 0);
13386     __pyx_r = __pyx_pf_8PyClical_34conj(__pyx_self, ((PyObject *) __pyx_v_obj));
13387
13388     /* function exit code */
13389     __Pyx_RefNannyFinishContext();
13390     return __pyx_r;
13391 }
13392
13393 static PyObject * __pyx_pf_8PyClical_34conj(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
13394     PyObject * __pyx_r = NULL;
13395     __Pyx_RefNannyDeclarations
13396     PyObject * __pyx_t_1 = NULL;
13397     int __pyx_lineno = 0;
13398     const char * __pyx_filename = NULL;
13399     int __pyx_clineno = 0;
13400     __Pyx_RefNannySetupContext("conj", 0);
13401     __Pyx_XDECREF(__pyx_r);
13402     __pyx_t_1 = __pyx_pf_8PyClical_conj(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1485, __pyx_L1_error)
13403     __Pyx_GOTREF(__pyx_t_1);
13404     __pyx_r = __pyx_t_1;
13405     __pyx_t_1 = 0;
13406     goto __pyx_L0;
13407
13408     /* function exit code */
13409     __pyx_L1_error;;
13410     __Pyx_XDECREF(__pyx_t_1);

```

```

13411         __Pyx_AddTraceback("PyClical.conj", __pyx_clineno, __pyx_lineno, __pyx_filename);
13412         __pyx_r = NULL;
13413         __pyx_L0;
13414         __Pyx_XGIVEREF(__pyx_r);
13415         __Pyx_RefNannyFinishContext();
13416         return __pyx_r;
13417     }
13418
13419     /* "PyClical.pyx":1500
13420  *     return clifford(obj).conj()
13421  *
13422  * cpdef inline quad(obj):                # ««««««««
13423  *     """
13424  *     Quadratic form == (rev(x)*x)(0).
13425  */
13426
13427     static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13428     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_quad(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13429         PyObject *__pyx_r = NULL;
13430         __Pyx_RefNannyDeclarations
13431         PyObject *__pyx_t_1 = NULL;
13432         PyObject *__pyx_t_2 = NULL;
13433         PyObject *__pyx_t_3 = NULL;
13434         int __pyx_lineno = 0;
13435         const char *__pyx_filename = NULL;
13436         int __pyx_clineno = 0;
13437         __Pyx_RefNannySetupContext("quad", 0);
13438
13439         /* "PyClical.pyx":1509
13440  *     2.0
13441  *     """
13442  *     return clifford(obj).quad()        # ««««««««
13443  *
13444  * cpdef inline norm(obj):
13445  */
13446         __Pyx_XDECREF(__pyx_r);
13447         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1509, __pyx_L1_error)
13448         __Pyx_GOTREF(__pyx_t_2);
13449         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_quad); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1509, __pyx_L1_error)
13450         __Pyx_GOTREF(__pyx_t_3);
13451         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13452         __pyx_t_2 = NULL;
13453         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13454             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13455             if (likely(__pyx_t_2)) {
13456                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
13457                 __Pyx_INCREF(__pyx_t_2);
13458                 __Pyx_INCREF(function);
13459                 __Pyx_DECREF_SET(__pyx_t_3, function);
13460             }
13461         }
13462         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13463         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13464         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1509, __pyx_L1_error)
13465         __Pyx_GOTREF(__pyx_t_1);
13466         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13467         __pyx_r = __pyx_t_1;
13468         __pyx_t_1 = 0;
13469         goto __pyx_L0;
13470
13471         /* "PyClical.pyx":1500
13472  *     return clifford(obj).conj()
13473  *
13474  * cpdef inline quad(obj):                # ««««««««
13475  *     """
13476  *     Quadratic form == (rev(x)*x)(0).
13477  */
13478
13479         /* function exit code */
13480         __pyx_L1_error;
13481         __Pyx_XDECREF(__pyx_t_1);
13482         __Pyx_XDECREF(__pyx_t_2);
13483         __Pyx_XDECREF(__pyx_t_3);
13484         __Pyx_AddTraceback("PyClical.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
13485         __pyx_r = 0;
13486         __pyx_L0;
13487         __Pyx_XGIVEREF(__pyx_r);
13488         __Pyx_RefNannyFinishContext();
13489         return __pyx_r;
13490     }
13491
13492     /* Python wrapper */

```

```

13493         static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
13494 *__pyx_v_obj); /*proto*/
13494         static char __pyx_doc_8PyClical_36quad[] = "\n    Quadratic form == (rev(x)*x)(0).\n\n
>> print(quad(clifford(\"1+{1}+{1,2}\"))\n    3.0\n    >>
print(quad(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n    2.0\n    ";
13495         static PyObject *__pyx_pw_8PyClical_37quad(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13496             PyObject *__pyx_r = 0;
13497             __Pyx_RefNannyDeclarations
13498             __Pyx_RefNannySetupContext("quad (wrapper)", 0);
13499             __pyx_r = __pyx_pf_8PyClical_36quad(__pyx_self, ((PyObject *)__pyx_v_obj));
13500
13501             /* function exit code */
13502             __Pyx_RefNannyFinishContext();
13503             return __pyx_r;
13504         }
13505
13506         static PyObject *__pyx_pf_8PyClical_36quad(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13507             PyObject *__pyx_r = NULL;
13508             __Pyx_RefNannyDeclarations
13509             PyObject *__pyx_t_1 = NULL;
13510             int __pyx_lineno = 0;
13511             const char *__pyx_filename = NULL;
13512             int __pyx_clineno = 0;
13513             __Pyx_RefNannySetupContext("quad", 0);
13514             __Pyx_XDECREF(__pyx_r);
13515             __pyx_t_1 = __pyx_f_8PyClical_quad(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1500, __pyx_L1_error)
13516             __Pyx_GOTREF(__pyx_t_1);
13517             __pyx_r = __pyx_t_1;
13518             __pyx_t_1 = 0;
13519             goto __pyx_L0;
13520
13521             /* function exit code */
13522             __pyx_L1_error:;
13523             __Pyx_XDECREF(__pyx_t_1);
13524             __Pyx_AddTraceback("PyClical.quad", __pyx_clineno, __pyx_lineno, __pyx_filename);
13525             __pyx_r = NULL;
13526             __pyx_L0:;
13527             __Pyx_XGIVEREF(__pyx_r);
13528             __Pyx_RefNannyFinishContext();
13529             return __pyx_r;
13530         }
13531
13532         /* "PyClical.pyx":1511
13533 *         return clifford(obj).quad()
13534 *
13535 * cpdef inline norm(obj):
13536 *     """
13537 *     norm == sum of squares of coordinates.
13538 */
13539
13540         static PyObject *__pyx_pw_8PyClical_39norm(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13541         static CYTHON_INLINE PyObject *__pyx_f_8PyClical_norm(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13542             PyObject *__pyx_r = NULL;
13543             __Pyx_RefNannyDeclarations
13544             PyObject *__pyx_t_1 = NULL;
13545             PyObject *__pyx_t_2 = NULL;
13546             PyObject *__pyx_t_3 = NULL;
13547             int __pyx_lineno = 0;
13548             const char *__pyx_filename = NULL;
13549             int __pyx_clineno = 0;
13550             __Pyx_RefNannySetupContext("norm", 0);
13551
13552             /* "PyClical.pyx":1520
13553 *         4.0
13554 *         """
13555 *         return clifford(obj).norm()
13556 *
13557 * cpdef inline abs(obj):
13558 */
13559             __Pyx_XDECREF(__pyx_r);
13560             __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1520, __pyx_L1_error)
13561             __Pyx_GOTREF(__pyx_t_2);
13562             __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_norm); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1520, __pyx_L1_error)
13563             __Pyx_GOTREF(__pyx_t_3);
13564             __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
13565             __pyx_t_2 = NULL;
13566             if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
13567                 __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
13568                 if (likely(__pyx_t_2)) {
13569                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);

```



```

13570         __Pyx_INCREF(__pyx_t_2);
13571         __Pyx_INCREF(function);
13572         __Pyx_DECREF_SET(__pyx_t_3, function);
13573     }
13574 }
13575 __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_t_2) :
__Pyx_PyObject_CallNoArg(__pyx_t_3);
13576 __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13577 if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1520, __pyx_L1_error)
13578 __Pyx_GOTREF(__pyx_t_1);
13579 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
13580 __pyx_r = __pyx_t_1;
13581 __pyx_t_1 = 0;
13582 goto __pyx_L0;
13583
13584 /* "PyClical.pyx":1511
13585 *     return clifford(obj).quad()
13586 *
13587 * cpdef inline norm(obj):           # ««««««««
13588 *     """
13589 *     norm == sum of squares of coordinates.
13590 */
13591
13592 /* function exit code */
13593 __pyx_L1_error;;
13594 __Pyx_XDECREF(__pyx_t_1);
13595 __Pyx_XDECREF(__pyx_t_2);
13596 __Pyx_XDECREF(__pyx_t_3);
13597 __Pyx_AddTraceback("PyClical.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
13598 __pyx_r = 0;
13599 __pyx_L0;;
13600 __Pyx_XGIVEREF(__pyx_r);
13601 __Pyx_RefNannyFinishContext();
13602 return __pyx_r;
13603 }
13604
13605 /* Python wrapper */
13606 static PyObject * __pyx_pw_8PyClical_39norm(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13607 static char __pyx_doc_8PyClical_38norm[] = "\n    norm == sum of squares of
coordinates.\n\n    >> norm(clifford(\"1+{1}+{1,2}\"))\n    3.0\n    >>
norm(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n    4.0\n    ";
13608 static PyObject * __pyx_pw_8PyClical_39norm(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
13609     PyObject * __pyx_r = 0;
13610     __Pyx_RefNannyDeclarations
13611     __Pyx_RefNannySetupContext("norm (wrapper)", 0);
13612     __pyx_r = __pyx_pf_8PyClical_38norm(__pyx_self, ((PyObject *) __pyx_v_obj));
13613
13614     /* function exit code */
13615     __Pyx_RefNannyFinishContext();
13616     return __pyx_r;
13617 }
13618
13619 static PyObject * __pyx_pf_8PyClical_38norm(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
13620     PyObject * __pyx_r = NULL;
13621     __Pyx_RefNannyDeclarations
13622     PyObject * __pyx_t_1 = NULL;
13623     int __pyx_lineno = 0;
13624     const char * __pyx_filename = NULL;
13625     int __pyx_clineno = 0;
13626     __Pyx_RefNannySetupContext("norm", 0);
13627     __Pyx_XDECREF(__pyx_r);
13628     __pyx_t_1 = __pyx_f_8PyClical_norm(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1511, __pyx_L1_error)
13629     __Pyx_GOTREF(__pyx_t_1);
13630     __pyx_r = __pyx_t_1;
13631     __pyx_t_1 = 0;
13632     goto __pyx_L0;
13633
13634     /* function exit code */
13635     __pyx_L1_error;;
13636     __Pyx_XDECREF(__pyx_t_1);
13637     __Pyx_AddTraceback("PyClical.norm", __pyx_clineno, __pyx_lineno, __pyx_filename);
13638     __pyx_r = NULL;
13639     __pyx_L0;;
13640     __Pyx_XGIVEREF(__pyx_r);
13641     __Pyx_RefNannyFinishContext();
13642     return __pyx_r;
13643 }
13644
13645 /* "PyClical.pyx":1522
13646 *     return clifford(obj).norm()
13647 *
13648 * cpdef inline abs(obj):           # ««««««««
13649 *     """

```

```

13650 *      Absolute value of multivector: multivector 2-norm.
13651 */
13652
13653     static PyObject *__pyx_pw_8PyClical_4labs(PyObject *__pyx_self, PyObject
13654 *__pyx_v_obj); /*proto*/
13655     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_abs(PyObject *__pyx_v_obj,
13656 CYTHON_UNUSED int __pyx_skip_dispatch) {
13657     PyObject *__pyx_r = NULL;
13658     __Pyx_RefNannyDeclarations
13659     PyObject *__pyx_t_1 = NULL;
13660     int __pyx_lineno = 0;
13661     const char *__pyx_filename = NULL;
13662     int __pyx_clineno = 0;
13663     __Pyx_RefNannySetupContext("abs", 0);
13664
13665     /* "PyClical.pyx":1529
13666 *      2.0
13667 *      """
13668 *      return glucat.abs(toClifford(obj))          # ««««««««
13669 *
13670 * cpdef inline max_abs(obj):
13671 */
13672     __Pyx_XDECREF(__pyx_r);
13673     __pyx_t_1 = PyFloat_FromDouble(abs(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
13674 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1529, __pyx_L1_error)
13675     __Pyx_GOTREF(__pyx_t_1);
13676     __pyx_r = __pyx_t_1;
13677     __pyx_t_1 = 0;
13678     goto __pyx_L0;
13679
13680     /* "PyClical.pyx":1522
13681 *      return clifford(obj).norm()
13682 *
13683 * cpdef inline abs(obj):          # ««««««««
13684 *      """
13685 *      Absolute value of multivector: multivector 2-norm.
13686 */
13687
13688     /* function exit code */
13689     __pyx_L1_error:;
13690     __Pyx_XDECREF(__pyx_t_1);
13691     __Pyx_AddTraceback("PyClical.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13692     __pyx_r = 0;
13693     __pyx_L0:;
13694     __Pyx_XGIVEREF(__pyx_r);
13695     __Pyx_RefNannyFinishContext();
13696     return __pyx_r;
13697 }
13698
13699 /* Python wrapper */
13700 static PyObject *__pyx_pw_8PyClical_4labs(PyObject *__pyx_self, PyObject
13701 *__pyx_v_obj); /*proto*/
13702 static char __pyx_doc_8PyClical_40abs[] = "\n      Absolute value of multivector:
13703 multivector 2-norm.\n      >> abs(clifford(\"1+{-1}+{1,2}+{1,2,3}\"))\n      2.0\n      ";
13704 static PyObject *__pyx_pw_8PyClical_4labs(PyObject *__pyx_self, PyObject *__pyx_v_obj)
13705 {
13706     PyObject *__pyx_r = 0;
13707     __Pyx_RefNannyDeclarations
13708     PyObject *__pyx_t_1 = NULL;
13709     int __pyx_lineno = 0;
13710     const char *__pyx_filename = NULL;
13711     int __pyx_clineno = 0;
13712     __Pyx_RefNannySetupContext("abs (wrapper)", 0);
13713     __pyx_r = __pyx_f_8PyClical_40abs(__pyx_self, ((PyObject *)__pyx_v_obj));
13714
13715     /* function exit code */
13716     __Pyx_RefNannyFinishContext();
13717     return __pyx_r;
13718 }
13719
13720 static PyObject *__pyx_pf_8PyClical_40abs(CYTHON_UNUSED PyObject *__pyx_self, PyObject
13721 *__pyx_v_obj) {
13722     PyObject *__pyx_r = NULL;
13723     __Pyx_RefNannyDeclarations
13724     PyObject *__pyx_t_1 = NULL;
13725     int __pyx_lineno = 0;
13726     const char *__pyx_filename = NULL;
13727     int __pyx_clineno = 0;
13728     __Pyx_RefNannySetupContext("abs", 0);
13729     __Pyx_XDECREF(__pyx_r);
13730     __pyx_t_1 = __pyx_f_8PyClical_abs(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
13731 __PYX_ERR(0, 1522, __pyx_L1_error)
13732     __Pyx_GOTREF(__pyx_t_1);
13733     __pyx_r = __pyx_t_1;
13734     __pyx_t_1 = 0;
13735     goto __pyx_L0;
13736
13737     /* function exit code */
13738     __pyx_L1_error:;
13739     __Pyx_XDECREF(__pyx_t_1);
13740     __Pyx_AddTraceback("PyClical.abs", __pyx_clineno, __pyx_lineno, __pyx_filename);

```

```

13729         __pyx_r = NULL;
13730         __pyx_L0:;
13731         __Pyx_XGIVEREF(__pyx_r);
13732         __Pyx_RefNannyFinishContext();
13733         return __pyx_r;
13734     }
13735
13736     /* "PyClical.pyx":1531
13737     *     return glucat.abs(toClifford(obj))
13738     *
13739     * cpdef inline max_abs(obj):
13740     *     """
13741     *     Maximum absolute value of coordinates multivector: multivector infinity-norm.
13742     */
13743
13744     static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13745     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_max_abs(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
13746         PyObject *__pyx_r = NULL;
13747         __Pyx_RefNannyDeclarations
13748         PyObject *__pyx_t_1 = NULL;
13749         int __pyx_lineno = 0;
13750         const char *__pyx_filename = NULL;
13751         int __pyx_clineno = 0;
13752         __Pyx_RefNannySetupContext("max_abs", 0);
13753
13754     /* "PyClical.pyx":1541
13755     *
13756     *     """
13757     *     return glucat.max_abs(toClifford(obj))
13758     *
13759     * cpdef inline pow(obj, m):
13760     */
13761         __Pyx_XDECREF(__pyx_r);
13762         __pyx_t_1 = PyFloat_FromDouble(max_abs(__pyx_f_8PyClical_toClifford(__pyx_v_obj)));
13763         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1541, __pyx_L1_error)
13764         __Pyx_GOTREF(__pyx_t_1);
13765         __pyx_r = __pyx_t_1;
13766         __pyx_t_1 = 0;
13767         goto __pyx_L0;
13768
13769     /* "PyClical.pyx":1531
13770     *     return glucat.abs(toClifford(obj))
13771     *
13772     * cpdef inline max_abs(obj):
13773     *     """
13774     *     Maximum absolute value of coordinates multivector: multivector infinity-norm.
13775     */
13776
13777     /* function exit code */
13778     __pyx_L1_error:;
13779     __Pyx_XDECREF(__pyx_t_1);
13780     __Pyx_AddTraceback("PyClical.max_abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13781     __pyx_r = 0;
13782     __pyx_L0:;
13783     __Pyx_XGIVEREF(__pyx_r);
13784     __Pyx_RefNannyFinishContext();
13785     return __pyx_r;
13786 }
13787
13788 /* Python wrapper */
13789 static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
13790 static char __pyx_doc_8PyClical_42max_abs[] = "\n    Maximum absolute value of
coordinates multivector: multivector infinity-norm.\n    >>
max_abs(clifford(\"1+{-1}+{1,2}+{1,2,3}\")\n    1.0\n    >> max_abs(clifford(\"3+2{1}+{1,2}\")\n
3.0\n\n    ";
13791 static PyObject *__pyx_pw_8PyClical_43max_abs(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
13792     PyObject *__pyx_r = 0;
13793     __Pyx_RefNannyDeclarations
13794     __Pyx_RefNannySetupContext("max_abs (wrapper)", 0);
13795     __pyx_r = __pyx_pf_8PyClical_42max_abs(__pyx_self, ((PyObject *)__pyx_v_obj));
13796
13797     /* function exit code */
13798     __Pyx_RefNannyFinishContext();
13799     return __pyx_r;
13800 }
13801
13802 static PyObject *__pyx_pf_8PyClical_42max_abs(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
13803     PyObject *__pyx_r = NULL;
13804     __Pyx_RefNannyDeclarations
13805     PyObject *__pyx_t_1 = NULL;
13806     int __pyx_lineno = 0;
13807     const char *__pyx_filename = NULL;

```

```

13807         int __pyx_clineno = 0;
13808         __Pyx_RefNannySetupContext("max_abs", 0);
13809         __Pyx_XDECREF(__pyx_r);
13810         __pyx_t_1 = __pyx_f_8PyClical_max_abs(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1531, __pyx_L1_error)
13811         __Pyx_GOTREF(__pyx_t_1);
13812         __pyx_r = __pyx_t_1;
13813         __pyx_t_1 = 0;
13814         goto __pyx_L0;
13815
13816         /* function exit code */
13817         __pyx_L1_error:;
13818         __Pyx_XDECREF(__pyx_t_1);
13819         __Pyx_AddTraceback("PyClical.max_abs", __pyx_clineno, __pyx_lineno, __pyx_filename);
13820         __pyx_r = NULL;
13821         __pyx_L0:;
13822         __Pyx_XGIVEREF(__pyx_r);
13823         __Pyx_RefNannyFinishContext();
13824         return __pyx_r;
13825     }
13826
13827     /* "PyClical.pyx":1543
13828     *     return glucat.max_abs(toClifford(obj))
13829     *
13830     * cpdef inline pow(obj, m):                # ««««««««
13831     *     """
13832     *     Integer power of multivector: obj to the m.
13833     */
13834
13835     static PyObject * __pyx_pw_8PyClical_45pow(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
13836     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_pow(PyObject * __pyx_v_obj, PyObject
__pyx_v_m, CYTHON_UNUSED int __pyx_skip_dispatch) {
13837         PyObject * __pyx_r = NULL;
13838         __Pyx_RefNannyDeclarations
13839         PyObject * __pyx_t_1 = NULL;
13840         PyObject * __pyx_t_2 = NULL;
13841         PyObject * __pyx_t_3 = NULL;
13842         PyObject * __pyx_t_4 = NULL;
13843         PyObject * __pyx_t_5 = NULL;
13844         PyObject * __pyx_t_6 = NULL;
13845         int __pyx_t_7;
13846         PyObject * __pyx_t_8 = NULL;
13847         PyObject * __pyx_t_9 = NULL;
13848         PyObject * __pyx_t_10 = NULL;
13849         int __pyx_lineno = 0;
13850         const char * __pyx_filename = NULL;
13851         int __pyx_clineno = 0;
13852         __Pyx_RefNannySetupContext("pow", 0);
13853
13854         /* "PyClical.pyx":1562
13855         *     1
13856         *     """
13857         *     try:                # ««««««««
13858         *         math.pow(obj, m)
13859         *     except:
13860         */
13861         {
13862             __Pyx_PyThreadState_declare
13863             __Pyx_PyThreadState_assign
13864             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
13865             __Pyx_XGOTREF(__pyx_t_1);
13866             __Pyx_XGOTREF(__pyx_t_2);
13867             __Pyx_XGOTREF(__pyx_t_3);
13868             /*try:*/ {
13869
13870                 /* "PyClical.pyx":1563
13871                 *     """
13872                 *     try:
13873                 *         math.pow(obj, m)                # ««««««««
13874                 *     except:
13875                 *         return clifford(obj).pow(m)
13876                 */
13877                 __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1563, __pyx_L3_error)
13878                 __Pyx_GOTREF(__pyx_t_5);
13879                 __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_pow); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1563, __pyx_L3_error)
13880                 __Pyx_GOTREF(__pyx_t_6);
13881                 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
13882                 __pyx_t_5 = NULL;
13883                 __pyx_t_7 = 0;
13884                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
13885                     __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
13886                     if (likely(__pyx_t_5)) {
13887                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
13888                         __Pyx_INCREF(__pyx_t_5);

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```

13889         __Pyx_INCREF(function);
13890         __Pyx_DECREF_SET(__pyx_t_6, function);
13891         __pyx_t_7 = 1;
13892     }
13893 }
13894 #if CYTHON_FAST_PYCALL
13895 if (PyFunction_Check(__pyx_t_6)) {
13896     PyObject *__pyx_temp[3] = {__pyx_t_5, __pyx_v_obj, __pyx_v_m};
13897     __pyx_t_4 = __Pyx_PyFunction_FastCall(__pyx_t_6, __pyx_temp+1-__pyx_t_7,
2+__pyx_t_7); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13898     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13899     __Pyx_GOTREF(__pyx_t_4);
13900 } else
13901 #endif
13902 #if CYTHON_FAST_PYCCALL
13903 if (__Pyx_PyFastCFunction_Check(__pyx_t_6)) {
13904     PyObject *__pyx_temp[3] = {__pyx_t_5, __pyx_v_obj, __pyx_v_m};
13905     __pyx_t_4 = __Pyx_PyCFunction_FastCall(__pyx_t_6, __pyx_temp+1-__pyx_t_7,
2+__pyx_t_7); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13906     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13907     __Pyx_GOTREF(__pyx_t_4);
13908 } else
13909 #endif
13910 {
13911     __pyx_t_8 = PyTuple_New(2+__pyx_t_7); if (unlikely(!__pyx_t_8)) __PYX_ERR(0,
1563, __pyx_L3_error)
13912     __Pyx_GOTREF(__pyx_t_8);
13913     if (__pyx_t_5) {
13914         __Pyx_GIVEREF(__pyx_t_5); PyTuple_SET_ITEM(__pyx_t_8, 0, __pyx_t_5);
__pyx_t_5 = NULL;
13915     }
13916     __Pyx_INCREF(__pyx_v_obj);
13917     __Pyx_GIVEREF(__pyx_v_obj);
13918     PyTuple_SET_ITEM(__pyx_t_8, 0+__pyx_t_7, __pyx_v_obj);
13919     __Pyx_INCREF(__pyx_v_m);
13920     __Pyx_GIVEREF(__pyx_v_m);
13921     PyTuple_SET_ITEM(__pyx_t_8, 1+__pyx_t_7, __pyx_v_m);
13922     __pyx_t_4 = __Pyx_PyObject_Call(__pyx_t_6, __pyx_t_8, NULL); if
(unlikely(!__pyx_t_4)) __PYX_ERR(0, 1563, __pyx_L3_error)
13923     __Pyx_GOTREF(__pyx_t_4);
13924     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
13925 }
13926 __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
13927 __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
13928
13929 /* "PyClical.pyx":1562
13930 *      1
13931 *      """
13932 *      try:                # ««««««
13933 *          math.pow(obj, m)
13934 *      except:
13935 */
13936     }
13937     __Pyx_XDECREF(__pyx_t_1); __pyx_t_1 = 0;
13938     __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
13939     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
13940     goto __pyx_L8_try_end;
13941     __pyx_L3_error:;
13942     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
13943     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
13944     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
13945     __Pyx_XDECREF(__pyx_t_8); __pyx_t_8 = 0;
13946
13947 /* "PyClical.pyx":1564
13948 *      try:
13949 *          math.pow(obj, m)
13950 *      except:                # ««««««
13951 *          return clifford(obj).pow(m)
13952 *
13953 */
13954     /*except:*/ {
13955         __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
13956         if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_8) < 0) __PYX_ERR(0,
1564, __pyx_L5_except_error)
13957         __Pyx_GOTREF(__pyx_t_4);
13958         __Pyx_GOTREF(__pyx_t_6);
13959         __Pyx_GOTREF(__pyx_t_8);
13960
13961         /* "PyClical.pyx":1565
13962 *          math.pow(obj, m)
13963 *      except:
13964 *          return clifford(obj).pow(m)                # ««««««
13965 *
13966 * cpdef inline outer_pow(obj, m):
13967 */
13968         __Pyx_XDECREF(__pyx_r);
13969         __pyx_t_9 = __Pyx_PyObject_CallOneArg((PyObject)

```

```

*)__pyx_ptype_8PyClical_clifford), __pyx_v_obj); if (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1565,
__pyx_L5_except_error)
13970     __Pyx_GOTREF(__pyx_t_9);
13971     __pyx_t_10 = __Pyx_PyObject_GetAttrStr(__pyx_t_9, __pyx_n_s_pow); if
(unlikely(!__pyx_t_10)) __PYX_ERR(0, 1565, __pyx_L5_except_error)
13972     __Pyx_GOTREF(__pyx_t_10);
13973     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
13974     __pyx_t_9 = NULL;
13975     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_10))) {
13976         __pyx_t_9 = PyMethod_GET_SELF(__pyx_t_10);
13977         if (likely(__pyx_t_9)) {
13978             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_10);
13979             __Pyx_INCREF(__pyx_t_9);
13980             __Pyx_INCREF(function);
13981             __Pyx_DECREF_SET(__pyx_t_10, function);
13982         }
13983     }
13984     __pyx_t_5 = (__pyx_t_9) ? __Pyx_PyObject_Call2Args(__pyx_t_10, __pyx_t_9,
__pyx_v_m) : __Pyx_PyObject_CallOneArg(__pyx_t_10, __pyx_v_m);
13985     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
13986     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1565, __pyx_L5_except_error)
13987     __Pyx_GOTREF(__pyx_t_5);
13988     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
13989     __pyx_r = __pyx_t_5;
13990     __pyx_t_5 = 0;
13991     __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
13992     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
13993     __Pyx_DECREF(__pyx_t_8); __pyx_t_8 = 0;
13994     goto __pyx_L6_except_return;
13995 }
13996 __pyx_L5_except_error:;
13997
13998 /* "PyClical.pyx":1562
13999 *
14000 * """
14001 * try:                # ««««««««
14002 *     math.pow(obj, m)
14003 * except:
14004 */
14005     __Pyx_XGIVEREF(__pyx_t_1);
14006     __Pyx_XGIVEREF(__pyx_t_2);
14007     __Pyx_XGIVEREF(__pyx_t_3);
14008     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14009     goto __pyx_L1_error;
14010     __pyx_L6_except_return:;
14011     __Pyx_XGIVEREF(__pyx_t_1);
14012     __Pyx_XGIVEREF(__pyx_t_2);
14013     __Pyx_XGIVEREF(__pyx_t_3);
14014     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14015     goto __pyx_L0;
14016     __pyx_L8_try_end:;
14017 }
14018
14019 /* "PyClical.pyx":1543
14020 *     return glucat.max_abs(toClifford(obj))
14021 *
14022 * cpdef inline pow(obj, m):                # ««««««««
14023 *     """
14024 *     Integer power of multivector: obj to the m.
14025 */
14026
14027 /* function exit code */
14028 __pyx_r = Py_None; __Pyx_INCREF(Py_None);
14029 goto __pyx_L0;
14030 __pyx_L1_error:;
14031 __Pyx_XDECREF(__pyx_t_4);
14032 __Pyx_XDECREF(__pyx_t_5);
14033 __Pyx_XDECREF(__pyx_t_6);
14034 __Pyx_XDECREF(__pyx_t_8);
14035 __Pyx_XDECREF(__pyx_t_9);
14036 __Pyx_XDECREF(__pyx_t_10);
14037 __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14038 __pyx_r = 0;
14039 __pyx_L0:;
14040 __Pyx_XGIVEREF(__pyx_r);
14041 __Pyx_RefNannyFinishContext();
14042 return __pyx_r;
14043 }
14044
14045 /* Python wrapper */
14046 static PyObject * __pyx_pw_8PyClical_45pow(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
14047 static char __pyx_doc_8PyClical_44pow[] = "\n Integer power of multivector: obj to
the m.\n\n >> x=clifford(\"{1}\"); print(pow(x,2))\n 1\n >> x=clifford(\"2\");
print(pow(x,2))\n 4\n >> x=clifford(\"2+{1}\"); print(pow(x,0))\n 1\n >>
x=clifford(\"2+{1}\"); print(pow(x,1))\n 2+{1}\n >> x=clifford(\"2+{1}\"); print(pow(x,2))\n
5+4{1}\n >> print(pow(clifford(\"1+{1}+{1,2}\"),3))\n 1+3{1}+3{1,2}\n >>";

```

```

i=clifford("\{1,2}\"); print(exp(pi/2) * pow(i, i))\n    1\n    ";
14048     static PyObject *__pyx_pw_8PyClical_45pow(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
14049         PyObject *__pyx_v_obj = 0;
14050         PyObject *__pyx_v_m = 0;
14051         int __pyx_lineno = 0;
14052         const char *__pyx_filename = NULL;
14053         int __pyx_clineno = 0;
14054         PyObject *__pyx_r = 0;
14055         __Pyx_RefNannyDeclarations
14056         __Pyx_RefNannySetupContext("pow (wrapper)", 0);
14057         {
14058             static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_m,0};
14059             PyObject* values[2] = {0,0};
14060             if (unlikely(__pyx_kwds)) {
14061                 Py_ssize_t kw_args;
14062                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14063                 switch (pos_args) {
14064                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14065                         CYTHON_FALLTHROUGH;
14066                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14067                         CYTHON_FALLTHROUGH;
14068                     case 0: break;
14069                     default: goto __pyx_L5_argtuple_error;
14070                 }
14071                 kw_args = PyDict_Size(__pyx_kwds);
14072                 switch (pos_args) {
14073                     case 0:
14074                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14075                         else goto __pyx_L5_argtuple_error;
14076                         CYTHON_FALLTHROUGH;
14077                     case 1:
14078                         if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_m)) !=
0)) kw_args--;
14079                         else {
14080                             __Pyx_RaiseArgtupleInvalid("pow", 1, 2, 2, 1); __PYX_ERR(0, 1543,
__pyx_L3_error)
14081                         }
14082                     if (unlikely(kw_args > 0)) {
14083                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "pow") < 0)) __PYX_ERR(0, 1543, __pyx_L3_error)
14084                     }
14085                     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
14086                         goto __pyx_L5_argtuple_error;
14087                     } else {
14088                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14089                         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14090                     }
14091                     __pyx_v_obj = values[0];
14092                     __pyx_v_m = values[1];
14093                 }
14094                 goto __pyx_L4_argument_unpacking_done;
14095                 __pyx_L5_argtuple_error:;
14096                 __Pyx_RaiseArgtupleInvalid("pow", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
14097                 __PYX_ERR(0, 1543, __pyx_L3_error)
14098                 __pyx_L3_error:;
14099                 __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14100                 __Pyx_RefNannyFinishContext();
14101                 return NULL;
14102                 __pyx_L4_argument_unpacking_done:;
14103                 __pyx_r = __pyx_pf_8PyClical_44pow(__pyx_self, __pyx_v_obj, __pyx_v_m);
14104
14105                 /* function exit code */
14106                 __Pyx_RefNannyFinishContext();
14107                 return __pyx_r;
14108             }
14109
14110             static PyObject *__pyx_pf_8PyClical_44pow(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj, PyObject *__pyx_v_m) {
14111                 PyObject *__pyx_r = NULL;
14112                 __Pyx_RefNannyDeclarations
14113                 PyObject *__pyx_t_1 = NULL;
14114                 int __pyx_lineno = 0;
14115                 const char *__pyx_filename = NULL;
14116                 int __pyx_clineno = 0;
14117                 __Pyx_RefNannySetupContext("pow", 0);
14118                 __Pyx_XDECREF(__pyx_r);
14119                 __pyx_t_1 = __pyx_pf_8PyClical_pow(__pyx_v_obj, __pyx_v_m, 0); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1543, __pyx_L1_error)
14120                 __Pyx_GOTREF(__pyx_t_1);
14121                 __pyx_r = __pyx_t_1;
14122                 __pyx_t_1 = 0;
14123                 goto __pyx_L0;
14124
14125                 /* function exit code */

```

```

14126         __pyx_L1_error;;
14127         __Pyx_XDECREF(__pyx_t_1);
14128         __Pyx_AddTraceback("PyClical.pow", __pyx_clineno, __pyx_lineno, __pyx_filename);
14129         __pyx_r = NULL;
14130         __pyx_L0;;
14131         __Pyx_XGIVEREF(__pyx_r);
14132         __Pyx_RefNannyFinishContext();
14133         return __pyx_r;
14134     }
14135
14136     /* "PyClical.pyx":1567
14137     *         return clifford(obj).pow(m)
14138     *
14139     * cpdef inline outer_pow(obj, m):
14140     *     """
14141     *     Outer product power of multivector.
14142     */
14143
14144     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
14145 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
14146     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_outer_pow(PyObject *__pyx_v_obj,
14147 PyObject *__pyx_v_m, CYTHON_UNUSED int __pyx_skip_dispatch) {
14148         PyObject *__pyx_r = NULL;
14149         __Pyx_RefNannyDeclarations
14150         PyObject *__pyx_t_1 = NULL;
14151         PyObject *__pyx_t_2 = NULL;
14152         PyObject *__pyx_t_3 = NULL;
14153         int __pyx_lineno = 0;
14154         const char *__pyx_filename = NULL;
14155         int __pyx_clineno = 0;
14156         __Pyx_RefNannySetupContext("outer_pow", 0);
14157
14158     /* "PyClical.pyx":1574
14159     *         1+3{1}+3{1,2}
14160     *         return clifford(obj).outer_pow(m)
14161     *     """
14162     *     cpdef inline complexifier(obj):
14163     */
14164         __Pyx_XDECREF(__pyx_r);
14165         __pyx_t_2 = __Pyx_PyObject_CallOneArg((PyObject *)__pyx_ptype_8PyClical_clifford),
14166 __pyx_v_obj); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1574, __pyx_L1_error)
14167         __Pyx_GOTREF(__pyx_t_2);
14168         __pyx_t_3 = __Pyx_PyObject_GetAttrStr(__pyx_t_2, __pyx_n_s_outer_pow); if
14169 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1574, __pyx_L1_error)
14170         __Pyx_GOTREF(__pyx_t_3);
14171         __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
14172         __pyx_t_2 = NULL;
14173         if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_3))) {
14174             __pyx_t_2 = PyMethod_GET_SELF(__pyx_t_3);
14175             if (likely(__pyx_t_2)) {
14176                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_3);
14177                 __Pyx_INCREF(__pyx_t_2);
14178                 __Pyx_INCREF(function);
14179                 __Pyx_DECREF_SET(__pyx_t_3, function);
14180             }
14181         }
14182         __pyx_t_1 = (__pyx_t_2) ? __Pyx_PyObject_Call2Args(__pyx_t_3, __pyx_t_2, __pyx_v_m)
14183 : __Pyx_PyObject_CallOneArg(__pyx_t_3, __pyx_v_m);
14184         __Pyx_XDECREF(__pyx_t_2); __pyx_t_2 = 0;
14185         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1574, __pyx_L1_error)
14186         __Pyx_GOTREF(__pyx_t_1);
14187         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14188         __pyx_r = __pyx_t_1;
14189         __pyx_t_1 = 0;
14190         goto __pyx_L0;
14191
14192     /* "PyClical.pyx":1567
14193     *         return clifford(obj).pow(m)
14194     *
14195     * cpdef inline outer_pow(obj, m):
14196     *     """
14197     *     Outer product power of multivector.
14198     */
14199
14200     /* function exit code */
14201     __pyx_L1_error;;
14202     __Pyx_XDECREF(__pyx_t_1);
14203     __Pyx_XDECREF(__pyx_t_2);
14204     __Pyx_XDECREF(__pyx_t_3);
14205     __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
14206 __pyx_filename);
14207     __pyx_r = 0;
14208     __pyx_L0;;
14209     __Pyx_XGIVEREF(__pyx_r);
14210     __Pyx_RefNannyFinishContext();
14211     return __pyx_r;

```



```

14207     }
14208
14209     /* Python wrapper */
14210     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
14211 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
14212     static char __pyx_doc_8PyClical_46outer_pow[] = "\n    Outer product power of
multivector.\n\n    >> print(outer_pow(clifford(\"1+{1}+{1,2}\"),3))\n    1+3{1}+3{1,2}\n    ";
14213     static PyObject *__pyx_pw_8PyClical_47outer_pow(PyObject *__pyx_self, PyObject
14214 *__pyx_args, PyObject *__pyx_kwds) {
14215     PyObject *__pyx_v_obj = 0;
14216     PyObject *__pyx_v_m = 0;
14217     int __pyx_lineno = 0;
14218     const char *__pyx_filename = NULL;
14219     int __pyx_clineno = 0;
14220     PyObject *__pyx_r = 0;
14221     __Pyx_RefNannyDeclarations
14222     __Pyx_RefNannySetupContext("outer_pow (wrapper)", 0);
14223     {
14224     static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_m,0};
14225     PyObject* values[2] = {0,0};
14226     if (unlikely(__pyx_kwds)) {
14227     Py_ssize_t kw_args;
14228     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14229     switch (pos_args) {
14230     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14231     CYTHON_FALLTHROUGH;
14232     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14233     CYTHON_FALLTHROUGH;
14234     case 0: break;
14235     default: goto __pyx_L5_argtuple_error;
14236     }
14237     kw_args = PyDict_Size(__pyx_kwds);
14238     switch (pos_args) {
14239     case 0:
14240     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
14241     else goto __pyx_L5_argtuple_error;
14242     CYTHON_FALLTHROUGH;
14243     case 1:
14244     if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_m)) !=
0)) kw_args--;
14245     else {
14246     __Pyx_RaiseArgtupleInvalid("outer_pow", 1, 2, 2, 1); __PYX_ERR(0, 1567,
__pyx_L3_error)
14247     }
14248     if (unlikely(kw_args > 0)) {
14249     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "outer_pow") < 0)) __PYX_ERR(0, 1567, __pyx_L3_error)
14250     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
14251     goto __pyx_L5_argtuple_error;
14252     } else {
14253     values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14254     values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
14255     }
14256     __pyx_v_obj = values[0];
14257     __pyx_v_m = values[1];
14258     }
14259     goto __pyx_L4_argument_unpacking_done;
14260     __pyx_L5_argtuple_error:;
14261     __Pyx_RaiseArgtupleInvalid("outer_pow", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
14262     __PYX_ERR(0, 1567, __pyx_L3_error)
14263     __pyx_L3_error:;
14264     __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14265     __Pyx_RefNannyFinishContext();
14266     return NULL;
14267     __pyx_L4_argument_unpacking_done:;
14268     __pyx_r = __pyx_pf_8PyClical_46outer_pow(__pyx_self, __pyx_v_obj, __pyx_v_m);
14269
14270     /* function exit code */
14271     __Pyx_RefNannyFinishContext();
14272     return __pyx_r;
14273 }
14274
14275     static PyObject *__pyx_pf_8PyClical_46outer_pow(CYTHON_UNUSED PyObject *__pyx_self,
14276 PyObject *__pyx_v_obj, PyObject *__pyx_v_m) {
14277     PyObject *__pyx_r = NULL;
14278     __Pyx_RefNannyDeclarations
14279     PyObject *__pyx_t_1 = NULL;
14280     int __pyx_lineno = 0;
14281     const char *__pyx_filename = NULL;
14282     int __pyx_clineno = 0;
14283     __Pyx_RefNannySetupContext("outer_pow", 0);
14284     __Pyx_XDECREF(__pyx_r);
14285     __pyx_t_1 = __pyx_f_8PyClical_outer_pow(__pyx_v_obj, __pyx_v_m, 0); if

```

```

(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1567, __pyx_L1_error)
14284     __Pyx_GOTREF(__pyx_t_1);
14285     __pyx_r = __pyx_t_1;
14286     __pyx_t_1 = 0;
14287     goto __pyx_L0;
14288
14289     /* function exit code */
14290     __pyx_L1_error;
14291     __Pyx_XDECREF(__pyx_t_1);
14292     __Pyx_AddTraceback("PyClical.outer_pow", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14293     __pyx_r = NULL;
14294     __pyx_L0;
14295     __Pyx_XGIVEREF(__pyx_r);
14296     __Pyx_RefNannyFinishContext();
14297     return __pyx_r;
14298 }
14299
14300     /* "PyClical.pyx":1576
14301 *     return clifford(obj).outer_pow(m)
14302 *
14303 * cpdef inline complexifier(obj):          # ««««««««
14304 *     """
14305 *     Square root of -1 which commutes with all members of the frame of the given multivector.
14306 */
14307
14308     static PyObject * __pyx_pw_8PyClical_49complexifier(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
14309     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_complexifier(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
14310     PyObject * __pyx_r = NULL;
14311     __Pyx_RefNannyDeclarations
14312     PyObject * __pyx_t_1 = NULL;
14313     PyObject * __pyx_t_2 = NULL;
14314     int __pyx_lineno = 0;
14315     const char * __pyx_filename = NULL;
14316     int __pyx_clineno = 0;
14317     __Pyx_RefNannySetupContext("complexifier", 0);
14318
14319     /* "PyClical.pyx":1589
14320 *     {-1}
14321 *     """
14322 *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )          # ««««««««
14323 *
14324 * cpdef inline sqrt(obj, i = None):
14325 */
14326     __Pyx_XDECREF(__pyx_r);
14327     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
14328     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1589, __pyx_L1_error)
14329     __Pyx_GOTREF(__pyx_t_1);
14330     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_1), complexifier(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1589, __pyx_L1_error)
14331     __Pyx_GOTREF(__pyx_t_2);
14332     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
14333     __pyx_r = __pyx_t_2;
14334     __pyx_t_2 = 0;
14335     goto __pyx_L0;
14336
14337     /* "PyClical.pyx":1576
14338 *     return clifford(obj).outer_pow(m)
14339 *
14340 * cpdef inline complexifier(obj):          # ««««««««
14341 *     """
14342 *     Square root of -1 which commutes with all members of the frame of the given multivector.
14343 */
14344
14345     /* function exit code */
14346     __pyx_L1_error;
14347     __Pyx_XDECREF(__pyx_t_1);
14348     __Pyx_XDECREF(__pyx_t_2);
14349     __Pyx_AddTraceback("PyClical.complexifier", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14350     __pyx_r = 0;
14351     __pyx_L0;
14352     __Pyx_XGIVEREF(__pyx_r);
14353     __Pyx_RefNannyFinishContext();
14354     return __pyx_r;
14355 }
14356
14357     /* Python wrapper */
14358     static PyObject * __pyx_pw_8PyClical_49complexifier(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
14359     static char __pyx_doc_8PyClical_48complexifier[] = "\n    Square root of -1 which
commutes with all members of the frame of the given multivector.\n\n    >>
print(complexifier(clifford(index_set({1})))\n    {1,2,3}\n    >>
print(complexifier(clifford(index_set({-1})))\n    {-1}\n    >> print(complexifier(index_set({1})))\n

```

```

    {1,2,3}\n    >> print(complexifier(index_set({-1}))\n    {-1}\n    ";
14359     static PyObject *__pyx_pw_8PyClical_49complexifier(PyObject *__pyx_self, PyObject
    *__pyx_v_obj) {
14360         PyObject *__pyx_r = 0;
14361         __Pyx_RefNannyDeclarations
14362         __Pyx_RefNannySetupContext("complexifier (wrapper)", 0);
14363         __pyx_r = __pyx_pf_8PyClical_48complexifier(__pyx_self, ((PyObject *)__pyx_v_obj));
14364
14365         /* function exit code */
14366         __Pyx_RefNannyFinishContext();
14367         return __pyx_r;
14368     }
14369
14370     static PyObject *__pyx_pf_8PyClical_48complexifier(CYTHON_UNUSED PyObject *__pyx_self,
    PyObject *__pyx_v_obj) {
14371         PyObject *__pyx_r = NULL;
14372         __Pyx_RefNannyDeclarations
14373         PyObject *__pyx_t_1 = NULL;
14374         int __pyx_lineno = 0;
14375         const char *__pyx_filename = NULL;
14376         int __pyx_clineno = 0;
14377         __Pyx_RefNannySetupContext("complexifier", 0);
14378         __Pyx_XDECREF(__pyx_r);
14379         __pyx_t_1 = __pyx_f_8PyClical_complexifier(__pyx_v_obj, 0); if
    (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1576, __pyx_L1_error)
14380         __Pyx_GOTREF(__pyx_t_1);
14381         __pyx_r = __pyx_t_1;
14382         __pyx_t_1 = 0;
14383         goto __pyx_L0;
14384
14385         /* function exit code */
14386         __pyx_L1_error;
14387         __Pyx_XDECREF(__pyx_t_1);
14388         __Pyx_AddTraceback("PyClical.complexifier", __pyx_clineno, __pyx_lineno,
    __pyx_filename);
14389         __pyx_r = NULL;
14390         __pyx_L0;
14391         __Pyx_XGIVEREF(__pyx_r);
14392         __Pyx_RefNannyFinishContext();
14393         return __pyx_r;
14394     }
14395
14396     /* "PyClical.pyx":1591
14397     *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )
14398     *
14399     * cpdef inline sqrt(obj, i = None):
14400     *     """
14401     *     Square root of multivector with optional complexifier.
14402     */
14403
14404     static PyObject *__pyx_pw_8PyClical_51sqrt(PyObject *__pyx_self, PyObject *__pyx_args,
    PyObject *__pyx_kwds); /*proto*/
14405     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sqrt(PyObject *__pyx_v_obj,
    CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_sqrt *__pyx_optional_args) {
14406         PyObject *__pyx_v_i = ((PyObject *)Py_None);
14407         PyObject *__pyx_r = NULL;
14408         __Pyx_RefNannyDeclarations
14409         int __pyx_t_1;
14410         int __pyx_t_2;
14411         PyObject *__pyx_t_3 = NULL;
14412         Clifford __pyx_t_4;
14413         PyObject *__pyx_t_5 = NULL;
14414         PyObject *__pyx_t_6 = NULL;
14415         PyObject *__pyx_t_7 = NULL;
14416         PyObject *__pyx_t_8 = NULL;
14417         PyObject *__pyx_t_9 = NULL;
14418         PyObject *__pyx_t_10 = NULL;
14419         PyObject *__pyx_t_11 = NULL;
14420         int __pyx_lineno = 0;
14421         const char *__pyx_filename = NULL;
14422         int __pyx_clineno = 0;
14423         __Pyx_RefNannySetupContext("sqrt", 0);
14424         if (__pyx_optional_args) {
14425             if (__pyx_optional_args->__pyx_n > 0) {
14426                 __pyx_v_i = __pyx_optional_args->1;
14427             }
14428         }
14429
14430         /* "PyClical.pyx":1606
14431     *     -1
14432     *     """
14433     *     if not (i is None):
14434     *         return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14435     *     else:
14436     */
14437         __pyx_t_1 = (__pyx_v_i != Py_None);
14438         __pyx_t_2 = (__pyx_t_1 != 0);

```

```

14439         if (__pyx_t_2) {
14440
14441             /* "PyClicl.pyx":1607
14442             *
14443             * if not (i is None):
14444             *     return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) ) # ««««««««
14445             * else:
14446             *     try:
14447             */
14448             __Pyx_XDECREF(__pyx_r);
14449             __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
14450 *)__pyx_ptype_8PyClicl_clifford); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1607, __pyx_L1_error)
14451             __Pyx_GOTREF(__pyx_t_3);
14452             try {
14453                 __pyx_t_4 = sqrt(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
14454 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
14455             } catch (...) {
14456                 __Pyx_CppExn2PyErr();
14457                 __PYX_ERR(0, 1607, __pyx_L1_error)
14458             }
14459             __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
14460 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1607, __pyx_L1_error)
14461             __Pyx_GOTREF(__pyx_t_5);
14462             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14463             __pyx_r = __pyx_t_5;
14464             __pyx_t_5 = 0;
14465             goto __pyx_L0;
14466
14467             /* "PyClicl.pyx":1606
14468             *
14469             * -1
14470             *
14471             * if not (i is None): # ««««««««
14472             *     return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14473             * else:
14474             *
14475             */
14476             }
14477
14478             /* "PyClicl.pyx":1609
14479             *
14480             * return clifford().wrap( glucat.sqrt(toClifford(obj), toClifford(i)) )
14481             * else:
14482             *
14483             * try: # ««««««««
14484             *     return math.sqrt(obj)
14485             * except:
14486             */
14487             /*else*/ {
14488                 {
14489                     __Pyx_PyThreadState_declare
14490                     __Pyx_PyThreadState_assign
14491                     __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
14492                     __Pyx_XGOTREF(__pyx_t_6);
14493                     __Pyx_XGOTREF(__pyx_t_7);
14494                     __Pyx_XGOTREF(__pyx_t_8);
14495                     /*try:*/ {
14496
14497                         /* "PyClicl.pyx":1610
14498                         *
14499                         * else:
14500                         *
14501                         * try:
14502                         *     return math.sqrt(obj) # ««««««««
14503                         * except:
14504                         *     return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14505                         */
14506                         __Pyx_XDECREF(__pyx_r);
14507                         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
14508 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1610, __pyx_L4_error)
14509                         __Pyx_GOTREF(__pyx_t_3);
14510                         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sqrt); if
14511 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1610, __pyx_L4_error)
14512                         __Pyx_GOTREF(__pyx_t_9);
14513                         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14514                         __pyx_t_3 = NULL;
14515                         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
14516                             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
14517                             if (likely(__pyx_t_3)) {
14518                                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
14519                                 __Pyx_INCREF(__pyx_t_3);
14520                                 __Pyx_INCREF(function);
14521                                 __Pyx_DECREF_SET(__pyx_t_9, function);
14522                             }
14523                         }
14524                         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
14525 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
14526                         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
14527                         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1610, __pyx_L4_error)
14528                         __Pyx_GOTREF(__pyx_t_5);
14529                         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
14530                         __pyx_r = __pyx_t_5;
14531                         __pyx_t_5 = 0;

```

```

14520         goto __pyx_L8_try_return;
14521
14522         /* "PyClical.pyx":1609
14523  *         return clifford().wrap( glucat.sqrt(toClifford(obj)), toClifford(i)) )
14524  *     else:
14525  *         try:
14526  *             # ««««««««
14527  *             return math.sqrt(obj)
14528  *         except:
14529  */
14530     }
14531     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
14532     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14533     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
14534
14535     /* "PyClical.pyx":1611
14536  *     try:
14537  *         return math.sqrt(obj)
14538  *     except:
14539  *         # ««««««««
14540  *         return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14541  */
14542
14543     /*except:*/ {
14544         __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno,
__pyx_filename);
14545         if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1611, __pyx_L6_except_error)
14546         __Pyx_GOTREF(__pyx_t_5);
14547         __Pyx_GOTREF(__pyx_t_9);
14548         __Pyx_GOTREF(__pyx_t_3);
14549
14550         /* "PyClical.pyx":1612
14551  *         return math.sqrt(obj)
14552  *     except:
14553  *         return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14554  *         # ««««««««
14555  * cpdef inline exp(obj):
14556  */
14557         __Pyx_XDECREF(__pyx_r);
14558         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1612,
__pyx_L6_except_error)
14559         __Pyx_GOTREF(__pyx_t_10);
14560         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), sqrt(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1612, __pyx_L6_except_error)
14561         __Pyx_GOTREF(__pyx_t_11);
14562         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
14563         __pyx_r = __pyx_t_11;
14564         __pyx_t_11 = 0;
14565         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14566         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14567         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
14568         goto __pyx_L7_except_return;
14569     }
14570     __pyx_L6_except_error:;
14571
14572     /* "PyClical.pyx":1609
14573  *     return clifford().wrap( glucat.sqrt(toClifford(obj)), toClifford(i)) )
14574  * else:
14575  *     try:
14576  *         # ««««««««
14577  *         return math.sqrt(obj)
14578  *     except:
14579  */
14580     __Pyx_XGIVEREF(__pyx_t_6);
14581     __Pyx_XGIVEREF(__pyx_t_7);
14582     __Pyx_XGIVEREF(__pyx_t_8);
14583     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14584     goto __pyx_L1_error;
14585     __pyx_L8_try_return:;
14586     __Pyx_XGIVEREF(__pyx_t_6);
14587     __Pyx_XGIVEREF(__pyx_t_7);
14588     __Pyx_XGIVEREF(__pyx_t_8);
14589     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14590     goto __pyx_L0;
14591     __pyx_L7_except_return:;
14592     __Pyx_XGIVEREF(__pyx_t_6);
14593     __Pyx_XGIVEREF(__pyx_t_7);
14594     __Pyx_XGIVEREF(__pyx_t_8);
14595     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
14596     goto __pyx_L0;
14597 }
14598
14599     /* "PyClical.pyx":1591
14600  *     return clifford().wrap( glucat.complexifier(toClifford(obj)) )

```

```

14601 * cpdef inline sqrt(obj, i = None):          # ««««««««
14602 *      """
14603 *      Square root of multivector with optional complexifier.
14604 */
14605
14606         /* function exit code */
14607         __pyx_L1_error++;
14608         __Pyx_XDECREF(__pyx_t_3);
14609         __Pyx_XDECREF(__pyx_t_5);
14610         __Pyx_XDECREF(__pyx_t_9);
14611         __Pyx_XDECREF(__pyx_t_10);
14612         __Pyx_XDECREF(__pyx_t_11);
14613         __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14614         __pyx_r = 0;
14615         __pyx_L0;
14616         __Pyx_XGIVEREF(__pyx_r);
14617         __Pyx_RefNannyFinishContext();
14618         return __pyx_r;
14619     }
14620
14621     /* Python wrapper */
14622     static PyObject * __pyx_pw_8PyClical_5lsqrt(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
14623     static char __pyx_doc_8PyClical_50sqrt[] = "\n      Square root of multivector with
optional complexifier.\n\n      >> print(sqrt(-1))\n      {-1}\n      >> print(sqrt(clifford(\"2{-1}\")))\n
1+{-1}\n      >> j=sqrt(-1,complexifier(index_set({1}))); print(j); print(j*j)\n      {1,2,3}\n      -1\n
>> j=sqrt(-1,\"{1,2,3}\"); print(j); print(j*j)\n      {1,2,3}\n      -1\n      ";
14624     static PyObject * __pyx_pw_8PyClical_5lsqrt(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
14625         PyObject * __pyx_v_obj = 0;
14626         PyObject * __pyx_v_i = 0;
14627         int __pyx_lineno = 0;
14628         const char * __pyx_filename = NULL;
14629         int __pyx_clineno = 0;
14630         PyObject * __pyx_r = 0;
14631         __Pyx_RefNannyDeclarations
14632         __Pyx_RefNannySetupContext("sqrt (wrapper)", 0);
14633         {
14634             static PyObject ** __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
14635             PyObject* values[2] = {0,0};
14636             values[1] = ((PyObject *) Py_None);
14637             if (unlikely(__pyx_kwds)) {
14638                 Py_ssize_t kw_args;
14639                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
14640                 switch (pos_args) {
14641                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
14642                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
14643                     case 0: break;
14644                     default: goto __pyx_L5_argtuple_error;
14645                 }
14646                 kw_args = PyDict_Size(__pyx_kwds);
14647                 switch (pos_args) {
14648                     case 0:
14649                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
14650 0)) kw_args--;
14651                     else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
14652                     case 1:
14653                     if (kw_args > 0) {
14654                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
14655                         if (value) { values[1] = value; kw_args--; }
14656                     }
14657                     if (unlikely(kw_args > 0)) {
14658                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
14659 values, pos_args, "sqrt") < 0)) __PYX_ERR(0, 1591, __pyx_L3_error)
14660                     }
14661                     } else {
14662                         switch (PyTuple_GET_SIZE(__pyx_args)) {
14663                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
14664                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
14665                             break;
14666                             default: goto __pyx_L5_argtuple_error;
14667                         }
14668                     }
14669                     __pyx_v_obj = values[0];
14670                     __pyx_v_i = values[1];
14671                 }
14672                 goto __pyx_L4_argument_unpacking_done;
14673                 __pyx_L5_argtuple_error:
14674                 __Pyx_RaiseArgtupleInvalid("sqrt", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
14675                 __PYX_ERR(0, 1591, __pyx_L3_error)
14676                 __pyx_L3_error:
14677                 __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);

```

```

14680         __Pyx_RefNannyFinishContext();
14681         return NULL;
14682         __pyx_L4_argument_unpacking_done:;
14683         __pyx_r = __pyx_pf_8PyClical_50sqrt(__pyx_self, __pyx_v_obj, __pyx_v_i);
14684
14685         /* function exit code */
14686         __Pyx_RefNannyFinishContext();
14687         return __pyx_r;
14688     }
14689
14690     static PyObject *__pyx_pf_8PyClical_50sqrt(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
14691         PyObject *__pyx_r = NULL;
14692         __Pyx_RefNannyDeclarations
14693         PyObject *__pyx_t_1 = NULL;
14694         struct __pyx_opt_args_8PyClical_sqrt __pyx_t_2;
14695         int __pyx_lineno = 0;
14696         const char *__pyx_filename = NULL;
14697         int __pyx_clineno = 0;
14698         __Pyx_RefNannySetupContext("sqrt", 0);
14699         __Pyx_XDECREF(__pyx_r);
14700         __pyx_t_2.__pyx_n = 1;
14701         __pyx_t_2.i = __pyx_v_i;
14702         __pyx_t_1 = __pyx_f_8PyClical_sqrt(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1591, __pyx_L1_error)
14703         __Pyx_GOTREF(__pyx_t_1);
14704         __pyx_r = __pyx_t_1;
14705         __pyx_t_1 = 0;
14706         goto __pyx_L0;
14707
14708         /* function exit code */
14709         __pyx_L1_error:;
14710         __Pyx_XDECREF(__pyx_t_1);
14711         __Pyx_AddTraceback("PyClical.sqrt", __pyx_clineno, __pyx_lineno, __pyx_filename);
14712         __pyx_r = NULL;
14713         __pyx_L0:;
14714         __Pyx_XGIVEREF(__pyx_r);
14715         __Pyx_RefNannyFinishContext();
14716         return __pyx_r;
14717     }
14718
14719     /* "PyClical.pyx":1614
14720     *
14721     * return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14722     * cpdef inline exp(obj):
14723     *     """
14724     *     Exponential of multivector.
14725     */
14726
14727     static PyObject *__pyx_pw_8PyClical_53exp(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
14728     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_exp(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
14729         PyObject *__pyx_r = NULL;
14730         __Pyx_RefNannyDeclarations
14731         PyObject *__pyx_t_1 = NULL;
14732         PyObject *__pyx_t_2 = NULL;
14733         PyObject *__pyx_t_3 = NULL;
14734         PyObject *__pyx_t_4 = NULL;
14735         PyObject *__pyx_t_5 = NULL;
14736         PyObject *__pyx_t_6 = NULL;
14737         PyObject *__pyx_t_7 = NULL;
14738         PyObject *__pyx_t_8 = NULL;
14739         int __pyx_lineno = 0;
14740         const char *__pyx_filename = NULL;
14741         int __pyx_clineno = 0;
14742         __Pyx_RefNannySetupContext("exp", 0);
14743
14744         /* "PyClical.pyx":1623
14745     *     {1,2}
14746     *     """
14747     *     try:
14748     *         return math.exp(obj)
14749     *     except:
14750     */
14751         {
14752             __Pyx_PyThreadState_declare
14753             __Pyx_PyThreadState_assign
14754             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
14755             __Pyx_XGOTREF(__pyx_t_1);
14756             __Pyx_XGOTREF(__pyx_t_2);
14757             __Pyx_XGOTREF(__pyx_t_3);
14758             /*try:*/ {
14759
14760                 /* "PyClical.pyx":1624
14761     *     """
14762     *     try:

```

```

14763 *         return math.exp(obj)                # ««««««««
14764 *     except:
14765 *         return clifford().wrap( glucat.exp(toClifford(obj)) )
14766 */
14767         __Pyx_XDECREF(__pyx_r);
14768         __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1624, __pyx_L3_error)
14769         __Pyx_GOTREF(__pyx_t_5);
14770         __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_exp); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1624, __pyx_L3_error)
14771         __Pyx_GOTREF(__pyx_t_6);
14772         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14773         __pyx_t_5 = NULL;
14774         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
14775             __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
14776             if (likely(__pyx_t_5)) {
14777                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
14778                 __Pyx_INCREF(__pyx_t_5);
14779                 __Pyx_INCREF(function);
14780                 __Pyx_DECREF_SET(__pyx_t_6, function);
14781             }
14782         }
14783         __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
14784         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14785         if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1624, __pyx_L3_error)
14786         __Pyx_GOTREF(__pyx_t_4);
14787         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
14788         __pyx_r = __pyx_t_4;
14789         __pyx_t_4 = 0;
14790         goto __pyx_L7_try_return;
14791
14792         /* "PyClicl.pyx":1623
14793  *     {1,2}
14794  *     """
14795  *     try:                # ««««««««
14796  *         return math.exp(obj)
14797  *     except:
14798  */
14799         }
14800         __pyx_L3_error;;
14801         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
14802         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
14803         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
14804
14805         /* "PyClicl.pyx":1625
14806  *     try:
14807  *         return math.exp(obj)
14808  *     except:                # ««««««««
14809  *         return clifford().wrap( glucat.exp(toClifford(obj)) )
14810  *
14811  */
14812         /*except:*/ {
14813             __Pyx_AddTraceback("PyClicl.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14814             if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1625, __pyx_L5_except_error)
14815             __Pyx_GOTREF(__pyx_t_4);
14816             __Pyx_GOTREF(__pyx_t_6);
14817             __Pyx_GOTREF(__pyx_t_5);
14818
14819             /* "PyClicl.pyx":1626
14820  *         return math.exp(obj)
14821  *     except:
14822  *         return clifford().wrap( glucat.exp(toClifford(obj)) )                # ««««««««
14823  *
14824  * cpdef inline log(obj,i = None):
14825  */
14826             __Pyx_XDECREF(__pyx_r);
14827             __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1626,
__pyx_L5_except_error)
14828             __Pyx_GOTREF(__pyx_t_7);
14829             __pyx_t_8 = __pyx_f_8PyClicl_8clifford_wrap(((struct
__pyx_obj_8PyClicl_clifford *)__pyx_t_7), exp(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1626, __pyx_L5_except_error)
14830             __Pyx_GOTREF(__pyx_t_8);
14831             __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
14832             __pyx_r = __pyx_t_8;
14833             __pyx_t_8 = 0;
14834             __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
14835             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
14836             __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
14837             goto __pyx_L6_except_return;
14838         }
14839         __pyx_L5_except_error;;
14840
14841         /* "PyClicl.pyx":1623

```



```

14842 *      {1,2}
14843 *      """
14844 *      try:          # ««««««««
14845 *          return math.exp(obj)
14846 *      except:
14847 */
14848
14848     __Pyx_XGIVEREF(__pyx_t_1);
14849     __Pyx_XGIVEREF(__pyx_t_2);
14850     __Pyx_XGIVEREF(__pyx_t_3);
14851     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14852     goto __pyx_L1_error;
14853     __pyx_L7_try_return:;
14854     __Pyx_XGIVEREF(__pyx_t_1);
14855     __Pyx_XGIVEREF(__pyx_t_2);
14856     __Pyx_XGIVEREF(__pyx_t_3);
14857     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14858     goto __pyx_L0;
14859     __pyx_L6_except_return:;
14860     __Pyx_XGIVEREF(__pyx_t_1);
14861     __Pyx_XGIVEREF(__pyx_t_2);
14862     __Pyx_XGIVEREF(__pyx_t_3);
14863     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
14864     goto __pyx_L0;
14865 }
14866
14867 /* "PyClical.pyx":1614
14868 *      return clifford().wrap( glucat.sqrt(toClifford(obj)) )
14869 *
14870 * cpdef inline exp(obj):          # ««««««««
14871 *      """
14872 *      Exponential of multivector.
14873 */
14874
14875     /* function exit code */
14876     __pyx_L1_error:;
14877     __Pyx_XDECREF(__pyx_t_4);
14878     __Pyx_XDECREF(__pyx_t_5);
14879     __Pyx_XDECREF(__pyx_t_6);
14880     __Pyx_XDECREF(__pyx_t_7);
14881     __Pyx_XDECREF(__pyx_t_8);
14882     __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);
14883     __pyx_r = 0;
14884     __pyx_L0:;
14885     __Pyx_XGIVEREF(__pyx_r);
14886     __Pyx_RefNannyFinishContext();
14887     return __pyx_r;
14888 }
14889
14890 /* Python wrapper */
14891 static PyObject * __pyx_pw_8PyClical_53exp(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
14892 static char __pyx_doc_8PyClical_52exp[] = "\n    Exponential of multivector.\n\n    >>
x=clifford(\"{1,2}\") * pi/4; print(exp(x))\n    0.7071+0.7071j{1,2}\n    >> x=clifford(\"{1,2}\") *
pi/2; print(exp(x))\n    {1,2}\n    ";
14893 static PyObject * __pyx_pw_8PyClical_53exp(PyObject * __pyx_self, PyObject * __pyx_v_obj)
{
14894     PyObject * __pyx_r = 0;
14895     __Pyx_RefNannyDeclarations
14896     __Pyx_RefNannySetupContext("exp (wrapper)", 0);
14897     __pyx_r = __pyx_pf_8PyClical_52exp(__pyx_self, ((PyObject *) __pyx_v_obj));
14898
14899     /* function exit code */
14900     __Pyx_RefNannyFinishContext();
14901     return __pyx_r;
14902 }
14903
14904 static PyObject * __pyx_pf_8PyClical_52exp(CYTHON_UNUSED PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
14905     PyObject * __pyx_r = NULL;
14906     __Pyx_RefNannyDeclarations
14907     PyObject * __pyx_t_1 = NULL;
14908     int __pyx_lineno = 0;
14909     const char * __pyx_filename = NULL;
14910     int __pyx_clineno = 0;
14911     __Pyx_RefNannySetupContext("exp", 0);
14912     __Pyx_XDECREF(__pyx_r);
14913     __pyx_t_1 = __pyx_f_8PyClical_exp(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1614, __pyx_L1_error)
14914     __Pyx_GOTREF(__pyx_t_1);
14915     __pyx_r = __pyx_t_1;
14916     __pyx_t_1 = 0;
14917     goto __pyx_L0;
14918
14919     /* function exit code */
14920     __pyx_L1_error:;
14921     __Pyx_XDECREF(__pyx_t_1);
14922     __Pyx_AddTraceback("PyClical.exp", __pyx_clineno, __pyx_lineno, __pyx_filename);

```

```

14923         __pyx_r = NULL;
14924         __pyx_L0:;
14925         __Pyx_XGIVEREF(__pyx_r);
14926         __Pyx_RefNannyFinishContext();
14927         return __pyx_r;
14928     }
14929
14930     /* "PyClicl.pyx":1628
14931     *         return clifford().wrap( glucat.exp(toClifford(obj)) )
14932     *
14933     * cpdef inline log(obj,i = None):
14934     *     """
14935     *     Natural logarithm of multivector with optional complexifier.
14936     */
14937
14938     static PyObject *__pyx_pw_8PyClicl_55log(PyObject *__pyx_self, PyObject *__pyx_args,
14939     PyObject *__pyx_kwds); /*proto*/
14940     static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_log(PyObject *__pyx_v_obj,
14941     CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_log *__pyx_optional_args) {
14942         PyObject *__pyx_v_i = ((PyObject *)Py_None);
14943         PyObject *__pyx_r = NULL;
14944         __Pyx_RefNannyDeclarations
14945         int __pyx_t_1;
14946         int __pyx_t_2;
14947         PyObject *__pyx_t_3 = NULL;
14948         Clifford __pyx_t_4;
14949         PyObject *__pyx_t_5 = NULL;
14950         PyObject *__pyx_t_6 = NULL;
14951         PyObject *__pyx_t_7 = NULL;
14952         PyObject *__pyx_t_8 = NULL;
14953         PyObject *__pyx_t_9 = NULL;
14954         PyObject *__pyx_t_10 = NULL;
14955         PyObject *__pyx_t_11 = NULL;
14956         int __pyx_lineno = 0;
14957         const char *__pyx_filename = NULL;
14958         int __pyx_clineno = 0;
14959         __Pyx_RefNannySetupContext("log", 0);
14960         if (__pyx_optional_args) {
14961             if (__pyx_optional_args->__pyx_n > 0) {
14962                 __pyx_v_i = __pyx_optional_args->i;
14963             }
14964         }
14965         /* "PyClicl.pyx":1643
14966     *     RuntimeError: check_complex(val, i): i is not a valid complexifier for val
14967     *     """
14968     *     if not (i is None):
14969     *         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
14970     *     else:
14971     */
14972         __pyx_t_1 = (__pyx_v_i != Py_None);
14973         __pyx_t_2 = (__pyx_t_1 != 0);
14974         if (__pyx_t_2) {
14975             /* "PyClicl.pyx":1644
14976     *     """
14977     *     if not (i is None):
14978     *         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
14979     *     else:
14980     *         try:
14981     */
14982             __Pyx_XDECREF(__pyx_r);
14983             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
14984     *)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1644, __pyx_L1_error)
14985             __Pyx_GOTREF(__pyx_t_3);
14986             try {
14987                 __pyx_t_4 = log(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
14988                 __pyx_f_8PyClicl_toClifford(__pyx_v_i));
14989             } catch (...) {
14990                 __Pyx_CppExn2PyErr();
14991                 __PYX_ERR(0, 1644, __pyx_L1_error)
14992             }
14993             __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
14994     *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1644, __pyx_L1_error)
14995             __Pyx_GOTREF(__pyx_t_5);
14996             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
14997             __pyx_r = __pyx_t_5;
14998             __pyx_t_5 = 0;
14999             goto __pyx_L0;
15000         }
15001         /* "PyClicl.pyx":1643
15002     *     RuntimeError: check_complex(val, i): i is not a valid complexifier for val
15003     *     """
15004     *     if not (i is None):
15005     *         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15006     *     else:
15007     */

```

```

15005         }
15006
15007         /* "PyClical.pyx":1646
15008 *         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15009 *     else:
15010 *         try:             # ««««««««
15011 *             return math.log(obj)
15012 *         except:
15013 */
15014         /*else*/ {
15015             {
15016                 __Pyx_PyThreadState_declare
15017                 __Pyx_PyThreadState_assign
15018                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15019                 __Pyx_XGOTREF(__pyx_t_6);
15020                 __Pyx_XGOTREF(__pyx_t_7);
15021                 __Pyx_XGOTREF(__pyx_t_8);
15022             /*try:*/ {
15023
15024                 /* "PyClical.pyx":1647
15025 *         else:
15026 *         try:
15027 *             return math.log(obj)             # ««««««««
15028 *         except:
15029 *             return clifford().wrap( glucat.log(toClifford(obj)) )
15030 */
15031                 __Pyx_XDECREF(__pyx_r);
15032                 __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1647, __pyx_L4_error)
15033                 __Pyx_GOTREF(__pyx_t_3);
15034                 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_log); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1647, __pyx_L4_error)
15035                 __Pyx_GOTREF(__pyx_t_9);
15036                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15037                 __pyx_t_3 = NULL;
15038                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15039                     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15040                     if (likely(__pyx_t_3)) {
15041                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15042                         __Pyx_INCREF(__pyx_t_3);
15043                         __Pyx_INCREF(function);
15044                         __Pyx_DECREF_SET(__pyx_t_9, function);
15045                     }
15046                 }
15047                 __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15048                 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15049                 if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1647, __pyx_L4_error)
15050                 __Pyx_GOTREF(__pyx_t_5);
15051                 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15052                 __pyx_r = __pyx_t_5;
15053                 __pyx_t_5 = 0;
15054                 goto __pyx_L8_try_return;
15055
15056                 /* "PyClical.pyx":1646
15057 *         return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15058 *     else:
15059 *         try:             # ««««««««
15060 *             return math.log(obj)
15061 *         except:
15062 */
15063             }
15064             __pyx_L4_error:;
15065             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15066             __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15067             __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15068
15069             /* "PyClical.pyx":1648
15070 *         try:
15071 *             return math.log(obj)
15072 *         except:             # ««««««««
15073 *             return clifford().wrap( glucat.log(toClifford(obj)) )
15074 *
15075 */
15076             /*except:*/ {
15077                 __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15078                 if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1648, __pyx_L6_except_error)
15079                 __Pyx_GOTREF(__pyx_t_5);
15080                 __Pyx_GOTREF(__pyx_t_9);
15081                 __Pyx_GOTREF(__pyx_t_3);
15082
15083                 /* "PyClical.pyx":1649
15084 *         return math.log(obj)
15085 *     except:
15086 *         return clifford().wrap( glucat.log(toClifford(obj)) )             # ««««««««

```

```

15087 *
15088 * cpdef inline cos(obj,i = None):
15089 */
15090
15091         __Pyx_XDECREF(__pyx_r);
15092         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
15093 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1649,
__pyx_L6_except_error)
15092         __Pyx_GOTREF(__pyx_t_10);
15093         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), log(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1649, __pyx_L6_except_error)
15094         __Pyx_GOTREF(__pyx_t_11);
15095         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15096         __pyx_r = __pyx_t_11;
15097         __pyx_t_11 = 0;
15098         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15099         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15100         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15101         goto __pyx_L7_except_return;
15102     }
15103     __pyx_L6_except_error;;
15104
15105     /* "PyClical.pyx":1646
15106     return clifford().wrap( glucat.log(toClifford(obj), toClifford(i)) )
15107     else:
15108     try:
15109         # ««««««««
15109         return math.log(obj)
15110     except:
15111 */
15112         __Pyx_XGIVEREF(__pyx_t_6);
15113         __Pyx_XGIVEREF(__pyx_t_7);
15114         __Pyx_XGIVEREF(__pyx_t_8);
15115         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15116         goto __pyx_L1_error;
15117         __pyx_L8_try_return;;
15118         __Pyx_XGIVEREF(__pyx_t_6);
15119         __Pyx_XGIVEREF(__pyx_t_7);
15120         __Pyx_XGIVEREF(__pyx_t_8);
15121         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15122         goto __pyx_L0;
15123         __pyx_L7_except_return;;
15124         __Pyx_XGIVEREF(__pyx_t_6);
15125         __Pyx_XGIVEREF(__pyx_t_7);
15126         __Pyx_XGIVEREF(__pyx_t_8);
15127         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15128         goto __pyx_L0;
15129     }
15130 }
15131
15132     /* "PyClical.pyx":1628
15133     return clifford().wrap( glucat.exp(toClifford(obj)) )
15134 */
15135 * cpdef inline log(obj,i = None):
15136 *     """
15137 *     Natural logarithm of multivector with optional complexifier.
15138 */
15139
15140     /* function exit code */
15141     __pyx_L1_error;;
15142     __Pyx_XDECREF(__pyx_t_3);
15143     __Pyx_XDECREF(__pyx_t_5);
15144     __Pyx_XDECREF(__pyx_t_9);
15145     __Pyx_XDECREF(__pyx_t_10);
15146     __Pyx_XDECREF(__pyx_t_11);
15147     __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15148     __pyx_r = 0;
15149     __pyx_L0;;
15150     __Pyx_XGIVEREF(__pyx_r);
15151     __Pyx_RefNannyFinishContext();
15152     return __pyx_r;
15153 }
15154
15155     /* Python wrapper */
15156     static PyObject *__pyx_pw_8PyClical_55log(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws); /*proto*/
15157     static char __pyx_doc_8PyClical_54log[] = "\n    Natural logarithm of multivector with
optional complexifier.\n\n    >> x=clifford(\"{-1}\"); print((log(x,\"{-1}\") * 2/pi))\n    {-1}\n
>> x=clifford(\"{1,2}\"); print((log(x,\"{1,2,3}\") * 2/pi))\n    {1,2}\n    >> x=clifford(\"{1,2}\");
print((log(x) * 2/pi))\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print((log(x,\"{1,2}\") * 2/pi))\n
Traceback (most recent call last):\n    ...\n    RuntimeError: check_complex(val, i): i is not a valid
complexifier for val\n    ";
15158     static PyObject *__pyx_pw_8PyClical_55log(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws) {
15159         PyObject *__pyx_v_obj = 0;
15160         PyObject *__pyx_v_i = 0;
15161         int __pyx_lineno = 0;
15162         const char *__pyx_filename = NULL;

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15163         int __pyx_clineno = 0;
15164         PyObject *__pyx_r = 0;
15165         __Pyx_RefNannyDeclarations
15166         __Pyx_RefNannySetupContext("log (wrapper)", 0);
15167     {
15168         static PyObject *__pyx_pyargnames[] = {&__pyx_n_s_obj, &__pyx_n_s_i, 0};
15169         PyObject* values[2] = {0, 0};
15170         values[1] = ((PyObject *)Py_None);
15171         if (unlikely(__pyx_kwds)) {
15172             Py_ssize_t kw_args;
15173             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15174             switch (pos_args) {
15175                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
15176                     CYTHON_FALLTHROUGH;
15177                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15178                     CYTHON_FALLTHROUGH;
15179                 case 0: break;
15180                 default: goto __pyx_L5_argtuple_error;
15181             }
15182             kw_args = PyDict_Size(__pyx_kwds);
15183             switch (pos_args) {
15184                 case 0:
15185                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
15186                     else goto __pyx_L5_argtuple_error;
15187                     CYTHON_FALLTHROUGH;
15188                 case 1:
15189                     if (kw_args > 0) {
15190                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15191                         if (value) { values[1] = value; kw_args--; }
15192                     }
15193                     if (unlikely(kw_args > 0)) {
15194                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "log") < 0)) __PYX_ERR(0, 1628, __pyx_L3_error)
15196                     } else {
15197                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15198                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
15199                                 CYTHON_FALLTHROUGH;
15200                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15201                                 break;
15202                             default: goto __pyx_L5_argtuple_error;
15203                         }
15204                     }
15205                     __pyx_v_obj = values[0];
15206                     __pyx_v_i = values[1];
15207             }
15208             goto __pyx_L4_argument_unpacking_done;
15209             __pyx_L5_argtuple_error:;
15210             __Pyx_RaiseArgtupleInvalid("log", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
15211             __PYX_ERR(0, 1628, __pyx_L3_error)
15212             __pyx_L3_error:;
15213             __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15214             __Pyx_RefNannyFinishContext();
15215             return NULL;
15216             __pyx_L4_argument_unpacking_done:;
15217             __pyx_r = __pyx_pf_8PyClical_54log(__pyx_self, __pyx_v_obj, __pyx_v_i);
15218
15219             /* function exit code */
15220             __Pyx_RefNannyFinishContext();
15221             return __pyx_r;
15222         }
15223     }
15224     static PyObject *__pyx_pf_8PyClical_54log(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj, PyObject *__pyx_v_i) {
15225         PyObject *__pyx_r = NULL;
15226         __Pyx_RefNannyDeclarations
15227         PyObject *__pyx_t_1 = NULL;
15228         struct __pyx_opt_args_8PyClical_log __pyx_t_2;
15229         int __pyx_lineno = 0;
15230         const char *__pyx_filename = NULL;
15231         int __pyx_clineno = 0;
15232         __Pyx_RefNannySetupContext("log", 0);
15233         __Pyx_XDECREF(__pyx_r);
15234         __pyx_t_2.__pyx_n = 1;
15235         __pyx_t_2.i = __pyx_v_i;
15236         __pyx_t_1 = __pyx_pf_8PyClical_log(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1628, __pyx_L1_error)
15237         __Pyx_GOTREF(__pyx_t_1);
15238         __pyx_r = __pyx_t_1;
15239         __pyx_t_1 = 0;
15240         goto __pyx_L0;
15241
15242         /* function exit code */
15243         __pyx_L1_error:;
15244         __Pyx_XDECREF(__pyx_t_1);

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```

15245         __Pyx_AddTraceback("PyClical.log", __pyx_clineno, __pyx_lineno, __pyx_filename);
15246         __pyx_r = NULL;
15247         __pyx_L0;
15248         __Pyx_XGIVEREF(__pyx_r);
15249         __Pyx_RefNannyFinishContext();
15250         return __pyx_r;
15251     }
15252
15253     /* "PyClical.pyx":1651
15254     *         return clifford().wrap( glucat.log(toClifford(obj)) )
15255     *
15256     * cpdef inline cos(obj,i = None):                # ««««««««
15257     *         """
15258     *         Cosine of multivector with optional complexifier.
15259     */
15260
15261     static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15262     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cos(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_cos *__pyx_optional_args) {
15263     PyObject *__pyx_v_i = ((PyObject *)Py_None);
15264     PyObject *__pyx_r = NULL;
15265     __Pyx_RefNannyDeclarations
15266     int __pyx_t_1;
15267     int __pyx_t_2;
15268     PyObject *__pyx_t_3 = NULL;
15269     Clifford __pyx_t_4;
15270     PyObject *__pyx_t_5 = NULL;
15271     PyObject *__pyx_t_6 = NULL;
15272     PyObject *__pyx_t_7 = NULL;
15273     PyObject *__pyx_t_8 = NULL;
15274     PyObject *__pyx_t_9 = NULL;
15275     PyObject *__pyx_t_10 = NULL;
15276     PyObject *__pyx_t_11 = NULL;
15277     int __pyx_lineno = 0;
15278     const char *__pyx_filename = NULL;
15279     int __pyx_clineno = 0;
15280     __Pyx_RefNannySetupContext("cos", 0);
15281     if (__pyx_optional_args) {
15282         if (__pyx_optional_args->__pyx_n > 0) {
15283             __pyx_v_i = __pyx_optional_args->i;
15284         }
15285     }
15286
15287     /* "PyClical.pyx":1660
15288     *         {1,2}
15289     *         """
15290     *         if not (i is None):                # ««««««««
15291     *             return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15292     *         else:
15293     */
15294         __pyx_t_1 = (__pyx_v_i != Py_None);
15295         __pyx_t_2 = (__pyx_t_1 != 0);
15296         if (__pyx_t_2) {
15297
15298             /* "PyClical.pyx":1661
15299             *
15300             *         if not (i is None):
15301             *             return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15302             *         else:
15303             *             try:
15304             */
15305             __Pyx_XDECREF(__pyx_r);
15306             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1661, __pyx_L1_error)
15307             __Pyx_GOTREF(__pyx_t_3);
15308             try {
15309                 __pyx_t_4 = cos(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
15310             } catch (...) {
15311                 __Pyx_CppExn2PyErr();
15312                 __PYX_ERR(0, 1661, __pyx_L1_error)
15313             }
15314             __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1661, __pyx_L1_error)
15315             __Pyx_GOTREF(__pyx_t_5);
15316             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15317             __pyx_r = __pyx_t_5;
15318             __pyx_t_5 = 0;
15319             goto __pyx_L0;
15320
15321             /* "PyClical.pyx":1660
15322             *         {1,2}
15323             *         """
15324             *         if not (i is None):                # ««««««««
15325             *             return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15326             *         else:

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15327 */
15328     }
15329
15330     /* "PyClical.pyx":1663
15331     *     return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15332     *     else:
15333     *         try:
15334     *             # ««««««««
15335     *             return math.cos(obj)
15336     *         except:
15337     */
15338     /*else*/ {
15339     {
15340         __Pyx_PyThreadState_declare
15341         __Pyx_PyThreadState_assign
15342         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15343         __Pyx_XGOTREF(__pyx_t_6);
15344         __Pyx_XGOTREF(__pyx_t_7);
15345         __Pyx_XGOTREF(__pyx_t_8);
15346     /*try:*/ {
15347
15348         /* "PyClical.pyx":1664
15349         *     try:
15350         *         return math.cos(obj)
15351         *         except:
15352         *             return clifford().wrap( glucat.cos(toClifford(obj)) )
15353         */
15354         __Pyx_XDECREF(__pyx_r);
15355         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1664, __pyx_L4_error)
15356         __Pyx_GOTREF(__pyx_t_3);
15357         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_cos); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1664, __pyx_L4_error)
15358         __Pyx_GOTREF(__pyx_t_9);
15359         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15360         __pyx_t_3 = NULL;
15361         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15362             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15363             if (likely(__pyx_t_3)) {
15364                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15365                 __Pyx_INCREF(__pyx_t_3);
15366                 __Pyx_INCREF(function);
15367                 __Pyx_DECREF_SET(__pyx_t_9, function);
15368             }
15369         }
15370         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15371         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15372         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1664, __pyx_L4_error)
15373         __Pyx_GOTREF(__pyx_t_5);
15374         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15375         __pyx_r = __pyx_t_5;
15376         __pyx_t_5 = 0;
15377         goto __pyx_L8_try_return;
15378
15379         /* "PyClical.pyx":1663
15380         *     return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15381         *     else:
15382         *         try:
15383         *             # ««««««««
15384         *             return math.cos(obj)
15385         *         except:
15386         */
15387         }
15388         __pyx_L4_error:;
15389         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15390         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15391         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15392
15393         /* "PyClical.pyx":1665
15394         *     try:
15395         *         return math.cos(obj)
15396         *         except:
15397         *             # ««««««««
15398         *             return clifford().wrap( glucat.cos(toClifford(obj)) )
15399         */
15400         /*except:*/ {
15401             __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15402             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1665, __pyx_L6_except_error)
15403             __Pyx_GOTREF(__pyx_t_5);
15404             __Pyx_GOTREF(__pyx_t_9);
15405             __Pyx_GOTREF(__pyx_t_3);
15406
15407             /* "PyClical.pyx":1666
15408             *     return math.cos(obj)
15409             *     except:

```

```

15409 *          return clifford().wrap( glucat.cos(toClifford(obj)) )          # ««««««««
15410 *
15411 * cpdef inline acos(obj,i = None):          # ««««««««
15412 */
15413         __Pyx_XDECREF(__pyx_r);
15414         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1666,
__pyx_L6_except_error)
15415         __Pyx_GOTREF(__pyx_t_10);
15416         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), cos(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1666, __pyx_L6_except_error)
15417         __Pyx_GOTREF(__pyx_t_11);
15418         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15419         __pyx_r = __pyx_t_11;
15420         __pyx_t_11 = 0;
15421         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15422         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15423         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15424         goto __pyx_L7_except_return;
15425     }
15426     __pyx_L6_except_error;;
15427
15428     /* "PyClical.pyx":1663
15429     *          return clifford().wrap( glucat.cos(toClifford(obj), toClifford(i)) )
15430     *     else:
15431     *         try:          # ««««««««
15432     *             return math.cos(obj)
15433     *         except:
15434     */
15435         __Pyx_XGIVEREF(__pyx_t_6);
15436         __Pyx_XGIVEREF(__pyx_t_7);
15437         __Pyx_XGIVEREF(__pyx_t_8);
15438         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15439         goto __pyx_L1_error;
15440         __pyx_L8_try_return;;
15441         __Pyx_XGIVEREF(__pyx_t_6);
15442         __Pyx_XGIVEREF(__pyx_t_7);
15443         __Pyx_XGIVEREF(__pyx_t_8);
15444         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15445         goto __pyx_L0;
15446         __pyx_L7_except_return;;
15447         __Pyx_XGIVEREF(__pyx_t_6);
15448         __Pyx_XGIVEREF(__pyx_t_7);
15449         __Pyx_XGIVEREF(__pyx_t_8);
15450         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15451         goto __pyx_L0;
15452     }
15453 }
15454
15455     /* "PyClical.pyx":1651
15456     *          return clifford().wrap( glucat.log(toClifford(obj)) )
15457     *
15458     * cpdef inline cos(obj,i = None):          # ««««««««
15459     *     """
15460     *     Cosine of multivector with optional complexifier.
15461     */
15462
15463     /* function exit code */
15464     __pyx_L1_error;;
15465     __Pyx_XDECREF(__pyx_t_3);
15466     __Pyx_XDECREF(__pyx_t_5);
15467     __Pyx_XDECREF(__pyx_t_9);
15468     __Pyx_XDECREF(__pyx_t_10);
15469     __Pyx_XDECREF(__pyx_t_11);
15470     __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15471     __pyx_r = 0;
15472     __pyx_L0;;
15473     __Pyx_XGIVEREF(__pyx_r);
15474     __Pyx_RefNannyFinishContext();
15475     return __pyx_r;
15476 }
15477
15478 /* Python wrapper */
15479 static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
15480 static char __pyx_doc_8PyClical_56cos[] = "\n    Cosine of multivector with optional
complexifier.\n\n    >> x=clifford('{1,2}'); print(cos(acos(x), '{1,2,3}')\n    {1,2}\n    >>
x=clifford('{1,2}'); print(cos(acos(x))\n    {1,2}\n    ";
15481 static PyObject *__pyx_pw_8PyClical_57cos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
15482     PyObject *__pyx_v_obj = 0;
15483     PyObject *__pyx_v_i = 0;
15484     int __pyx_lineno = 0;
15485     const char *__pyx_filename = NULL;
15486     int __pyx_clineno = 0;
15487     PyObject *__pyx_r = 0;

```



```

15488     __Pyx_RefNannyDeclarations
15489     __Pyx_RefNannySetupContext("cos (wrapper)", 0);
15490     {
15491         static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj, &__pyx_n_s_i, 0};
15492         PyObject* values[2] = {0, 0};
15493         values[1] = (PyObject*)Py_None;
15494         if (unlikely(__pyx_kwds)) {
15495             Py_ssize_t kw_args;
15496             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15497             switch (pos_args) {
15498                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
15499                     CYTHON_FALLTHROUGH;
15500                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15501                     CYTHON_FALLTHROUGH;
15502                 case 0: break;
15503                 default: goto __pyx_L5_argtuple_error;
15504             }
15505             kw_args = PyDict_Size(__pyx_kwds);
15506             switch (pos_args) {
15507                 case 0:
15508                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
15509 0)) kw_args--;
15509                     else goto __pyx_L5_argtuple_error;
15510                     CYTHON_FALLTHROUGH;
15511                 case 1:
15512                     if (kw_args > 0) {
15513                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15514                         if (value) { values[1] = value; kw_args--; }
15515                     }
15516                 }
15517                 if (unlikely(kw_args > 0)) {
15518                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
15519 values, pos_args, "cos") < 0)) __PYX_ERR(0, 1651, __pyx_L3_error)
15520                 }
15521                 else {
15522                     switch (PyTuple_GET_SIZE(__pyx_args)) {
15523                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
15524                             CYTHON_FALLTHROUGH;
15525                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15526                             break;
15527                         default: goto __pyx_L5_argtuple_error;
15528                     }
15529                     __pyx_v_obj = values[0];
15530                     __pyx_v_i = values[1];
15531                 }
15532                 goto __pyx_L4_argument_unpacking_done;
15533                 __pyx_L5_argtuple_error:;
15534                 __Pyx_RaiseArgtupleInvalid("cos", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
15535                 __PYX_ERR(0, 1651, __pyx_L3_error)
15536                 __pyx_L3_error:;
15537                 __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15538                 __Pyx_RefNannyFinishContext();
15539                 return NULL;
15540                 __pyx_L4_argument_unpacking_done:;
15541                 __pyx_r = __pyx_pf_8PyClical_56cos(__pyx_self, __pyx_v_obj, __pyx_v_i);
15542                 /* function exit code */
15543                 __Pyx_RefNannyFinishContext();
15544                 return __pyx_r;
15545             }
15546         }
15547         static PyObject* __pyx_pf_8PyClical_56cos(CYTHON_UNUSED PyObject* __pyx_self, PyObject
*__pyx_v_obj, PyObject* __pyx_v_i) {
15548             PyObject* __pyx_r = NULL;
15549             __Pyx_RefNannyDeclarations
15550             PyObject* __pyx_t_1 = NULL;
15551             struct __pyx_opt_args_8PyClical_cos __pyx_t_2;
15552             int __pyx_lineno = 0;
15553             const char* __pyx_filename = NULL;
15554             int __pyx_clineno = 0;
15555             __Pyx_RefNannySetupContext("cos", 0);
15556             __Pyx_XDECREF(__pyx_r);
15557             __pyx_t_2.__pyx_n = 1;
15558             __pyx_t_2.i = __pyx_v_i;
15559             __pyx_t_1 = __pyx_f_8PyClical_cos(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1651, __pyx_L1_error)
15560             __Pyx_GOTREF(__pyx_t_1);
15561             __pyx_r = __pyx_t_1;
15562             __pyx_t_1 = 0;
15563             goto __pyx_L0;
15564         }
15565         /* function exit code */
15566         __pyx_L1_error:;
15567         __Pyx_XDECREF(__pyx_t_1);
15568         __Pyx_AddTraceback("PyClical.cos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15569         __pyx_r = NULL;

```

```

15570         __pyx_L0;;
15571         __Pyx_XGIVEREF(__pyx_r);
15572         __Pyx_RefNannyFinishContext();
15573         return __pyx_r;
15574     }
15575
15576     /* "PyClicl.pyx":1668
15577     *         return clifford().wrap( glucat.cos(toClifford(obj)) )
15578     *
15579     * cpdef inline acos(obj,i = None):
15580     *         """
15581     *         Inverse cosine of multivector with optional complexifier.
15582     */
15583
15584     static PyObject * __pyx_pw_8PyClicl_59acos(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
15585     static CYTHON_INLINE PyObject * __pyx_f_8PyClicl_acos(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_acos * __pyx_optional_args) {
15586     PyObject * __pyx_v_i = ((PyObject *)Py_None);
15587     PyObject * __pyx_r = NULL;
15588     __Pyx_RefNannyDeclarations
15589     int __pyx_t_1;
15590     int __pyx_t_2;
15591     PyObject * __pyx_t_3 = NULL;
15592     Clifford __pyx_t_4;
15593     PyObject * __pyx_t_5 = NULL;
15594     PyObject * __pyx_t_6 = NULL;
15595     PyObject * __pyx_t_7 = NULL;
15596     PyObject * __pyx_t_8 = NULL;
15597     PyObject * __pyx_t_9 = NULL;
15598     PyObject * __pyx_t_10 = NULL;
15599     PyObject * __pyx_t_11 = NULL;
15600     int __pyx_lineno = 0;
15601     const char * __pyx_filename = NULL;
15602     int __pyx_clineno = 0;
15603     __Pyx_RefNannySetupContext("acos", 0);
15604     if (__pyx_optional_args) {
15605         if (__pyx_optional_args->__pyx_n > 0) {
15606             __pyx_v_i = __pyx_optional_args->i;
15607         }
15608     }
15609
15610     /* "PyClicl.pyx":1681
15611     *         {1,2}
15612     *         """
15613     *         if not (i is None):
15614     *             return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15615     *         else:
15616     */
15617     __pyx_t_1 = (__pyx_v_i != Py_None);
15618     __pyx_t_2 = (__pyx_t_1 != 0);
15619     if (__pyx_t_2) {
15620
15621         /* "PyClicl.pyx":1682
15622     *         """
15623     *         if not (i is None):
15624     *             return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15625     *         else:
15626     *             try:
15627     */
15628     __Pyx_XDECREF(__pyx_r);
15629     __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1682, __pyx_L1_error)
15630     __Pyx_GOTREF(__pyx_t_3);
15631     try {
15632         __pyx_t_4 = acos(__pyx_f_8PyClicl_toClifford(__pyx_v_obj),
__pyx_f_8PyClicl_toClifford(__pyx_v_i));
15633     } catch (...) {
15634         __Pyx_CppExn2PyErr();
15635         __PYX_ERR(0, 1682, __pyx_L1_error)
15636     }
15637     __pyx_t_5 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1682, __pyx_L1_error)
15638     __Pyx_GOTREF(__pyx_t_5);
15639     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15640     __pyx_r = __pyx_t_5;
15641     __pyx_t_5 = 0;
15642     goto __pyx_L0;
15643
15644     /* "PyClicl.pyx":1681
15645     *         {1,2}
15646     *         """
15647     *         if not (i is None):
15648     *             return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15649     *         else:
15650     */
15651     }

```

```

15652
15653         /* "PyClical.pyx":1684
15654         *         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15655         *     else:
15656         *         try:
15657         *             # ««««««««
15658         *             return math.acos(obj)
15659         *         except:
15660         */
15661         /*else*/ {
15662         {
15663             __Pyx_PyThreadState_declare
15664             __Pyx_PyThreadState_assign
15665             __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
15666             __Pyx_XGOTREF(__pyx_t_6);
15667             __Pyx_XGOTREF(__pyx_t_7);
15668             __Pyx_XGOTREF(__pyx_t_8);
15669             /*try:*/ {
15670
15671             /* "PyClical.pyx":1685
15672             *     else:
15673             *         try:
15674             *             return math.acos(obj)
15675             *             # ««««««««
15676             *         except:
15677             *             return clifford().wrap( glucat.acos(toClifford(obj)) )
15678             */
15679             __Pyx_XDECREF(__pyx_r);
15680             __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
15681 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1685, __pyx_L4_error)
15682             __Pyx_GOTREF(__pyx_t_3);
15683             __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_acos); if
15684 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1685, __pyx_L4_error)
15685             __Pyx_GOTREF(__pyx_t_9);
15686             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15687             __pyx_t_3 = NULL;
15688             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
15689                 __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
15690                 if (likely(__pyx_t_3)) {
15691                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
15692                     __Pyx_INCREF(__pyx_t_3);
15693                     __Pyx_INCREF(function);
15694                     __Pyx_DECREF_SET(__pyx_t_9, function);
15695                 }
15696             }
15697             __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
15698 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
15699             __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15700             if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1685, __pyx_L4_error)
15701             __Pyx_GOTREF(__pyx_t_5);
15702             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15703             __pyx_r = __pyx_t_5;
15704             __pyx_t_5 = 0;
15705             goto __pyx_L8_try_return;
15706
15707         /* "PyClical.pyx":1684
15708         *         return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15709         *     else:
15710         *         try:
15711         *             # ««««««««
15712         *             return math.acos(obj)
15713         *         except:
15714         */
15715         }
15716         __pyx_L4_error:;
15717         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
15718         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15719         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
15720
15721         /* "PyClical.pyx":1686
15722         *         try:
15723         *             return math.acos(obj)
15724         *         except:
15725         *             # ««««««««
15726         *             return clifford().wrap( glucat.acos(toClifford(obj)) )
15727         */
15728         /*except:*/ {
15729             __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno,
15730 __pyx_filename);
15731             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
15732 1686, __pyx_L6_except_error)
15733             __Pyx_GOTREF(__pyx_t_5);
15734             __Pyx_GOTREF(__pyx_t_9);
15735             __Pyx_GOTREF(__pyx_t_3);
15736
15737             /* "PyClical.pyx":1687
15738             *         return math.acos(obj)
15739             *     except:
15740             *         return clifford().wrap( glucat.acos(toClifford(obj)) )
15741             *             # ««««««««
15742             */

```

```

15734 * cpdef inline cosh(obj):
15735 */
15736         __Pyx_XDECREF(__pyx_r);
15737         __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1687,
__pyx_L6_except_error)
15738         __Pyx_GOTREF(__pyx_t_10);
15739         __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), acos(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1687, __pyx_L6_except_error)
15740         __Pyx_GOTREF(__pyx_t_11);
15741         __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
15742         __pyx_r = __pyx_t_11;
15743         __pyx_t_11 = 0;
15744         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
15745         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15746         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
15747         goto __pyx_L7_except_return;
15748     }
15749     __pyx_L6_except_error;;
15750
15751     /* "PyClical.pyx":1684
15752     * return clifford().wrap( glucat.acos(toClifford(obj), toClifford(i)) )
15753     * else:
15754     *     try: # ««««««
15755     *         return math.acos(obj)
15756     *     except:
15757     */
15758         __Pyx_XGIVEREF(__pyx_t_6);
15759         __Pyx_XGIVEREF(__pyx_t_7);
15760         __Pyx_XGIVEREF(__pyx_t_8);
15761         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15762         goto __pyx_L1_error;
15763         __pyx_L8_try_return;;
15764         __Pyx_XGIVEREF(__pyx_t_6);
15765         __Pyx_XGIVEREF(__pyx_t_7);
15766         __Pyx_XGIVEREF(__pyx_t_8);
15767         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15768         goto __pyx_L0;
15769         __pyx_L7_except_return;;
15770         __Pyx_XGIVEREF(__pyx_t_6);
15771         __Pyx_XGIVEREF(__pyx_t_7);
15772         __Pyx_XGIVEREF(__pyx_t_8);
15773         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
15774         goto __pyx_L0;
15775     }
15776 }
15777
15778     /* "PyClical.pyx":1668
15779     * return clifford().wrap( glucat.cos(toClifford(obj)) )
15780     *
15781     * cpdef inline acos(obj, i = None): # ««««««
15782     * """
15783     *     Inverse cosine of multivector with optional complexifier.
15784     */
15785
15786     /* function exit code */
15787     __pyx_L1_error;;
15788     __Pyx_XDECREF(__pyx_t_3);
15789     __Pyx_XDECREF(__pyx_t_5);
15790     __Pyx_XDECREF(__pyx_t_9);
15791     __Pyx_XDECREF(__pyx_t_10);
15792     __Pyx_XDECREF(__pyx_t_11);
15793     __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15794     __pyx_r = 0;
15795     __pyx_L0;;
15796     __Pyx_XGIVEREF(__pyx_r);
15797     __Pyx_RefNannyFinishContext();
15798     return __pyx_r;
15799 }
15800
15801     /* Python wrapper */
15802     static PyObject *__pyx_pw_8PyClical_59acos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws); /*proto*/
15803     static char __pyx_doc_8PyClical_58acos[] = "\n    Inverse cosine of multivector with
optional complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x), \"{1,2,3}\")\n    {1,2}\n
>> x=clifford(\"{1,2}\"); print(cos(acos(x), \"{1,2,3,4}\")\n    {1,2}\n    >> print(acos(0) /
pi)\n    0.5\n    >> x=clifford(\"{1,2}\"); print(cos(acos(x)))\n    {1,2}\n    ";
15804     static PyObject *__pyx_pw_8PyClical_59acos(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws) {
15805         PyObject *__pyx_v_obj = 0;
15806         PyObject *__pyx_v_i = 0;
15807         int __pyx_lineno = 0;
15808         const char *__pyx_filename = NULL;
15809         int __pyx_clineno = 0;
15810         PyObject *__pyx_r = 0;
15811         __Pyx_RefNannyDeclarations

```

```

15812     __Pyx_RefNannySetupContext("acos (wrapper)", 0);
15813     {
15814         static PyObject* *__pyx_pyargnames[] = {&__pyx_n_s_obj, &__pyx_n_s_i, 0};
15815         PyObject* values[2] = {0, 0};
15816         values[1] = ((PyObject*)Py_None);
15817         if (unlikely(__pyx_kwds)) {
15818             Py_ssize_t kw_args;
15819             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
15820             switch (pos_args) {
15821                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
15822                     CYTHON_FALLTHROUGH;
15823                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15824                     CYTHON_FALLTHROUGH;
15825                 case 0: break;
15826                 default: goto __pyx_L5_argtuple_error;
15827             }
15828             kw_args = PyDict_Size(__pyx_kwds);
15829             switch (pos_args) {
15830                 case 0:
15831                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
15832                                0)) kw_args--;
15833                     else goto __pyx_L5_argtuple_error;
15834                     CYTHON_FALLTHROUGH;
15835                 case 1:
15836                     if (kw_args > 0) {
15837                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
15838                         if (value) { values[1] = value; kw_args--; }
15839                     }
15840                     if (unlikely(kw_args > 0)) {
15841                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
15842                            values, pos_args, "acos") < 0)) __PYX_ERR(0, 1668, __pyx_L3_error)
15843                     } else {
15844                         switch (PyTuple_GET_SIZE(__pyx_args)) {
15845                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
15846                                 CYTHON_FALLTHROUGH;
15847                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
15848                                 break;
15849                             default: goto __pyx_L5_argtuple_error;
15850                         }
15851                     }
15852                     __pyx_v_obj = values[0];
15853                     __pyx_v_i = values[1];
15854                 }
15855                 goto __pyx_L4_argument_unpacking_done;
15856                 __pyx_L5_argtuple_error:;
15857                 __Pyx_RaiseArgtupleInvalid("acos", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
15858                 __PYX_ERR(0, 1668, __pyx_L3_error)
15859                 __pyx_L3_error:;
15860                 __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15861                 __Pyx_RefNannyFinishContext();
15862                 return NULL;
15863                 __pyx_L4_argument_unpacking_done:;
15864                 __pyx_r = __pyx_pf_8PyClical_58acos(__pyx_self, __pyx_v_obj, __pyx_v_i);
15865
15866                 /* function exit code */
15867                 __Pyx_RefNannyFinishContext();
15868                 return __pyx_r;
15869             }
15870
15871             static PyObject* __pyx_pf_8PyClical_58acos(CYTHON_UNUSED PyObject* __pyx_self,
15872                 PyObject* __pyx_v_obj, PyObject* __pyx_v_i) {
15873                 PyObject* __pyx_r = NULL;
15874                 __Pyx_RefNannyDeclarations
15875                 PyObject* __pyx_t_1 = NULL;
15876                 struct __pyx_opt_args_8PyClical_acos __pyx_t_2;
15877                 int __pyx_lineno = 0;
15878                 const char* __pyx_filename = NULL;
15879                 int __pyx_clineno = 0;
15880                 __Pyx_RefNannySetupContext("acos", 0);
15881                 __Pyx_XDECREF(__pyx_r);
15882                 __pyx_t_2.__pyx_n = 1;
15883                 __pyx_t_2.i = __pyx_v_i;
15884                 __pyx_t_1 = __pyx_pf_8PyClical_acos(__pyx_v_obj, 0, &__pyx_t_2); if
15885                 (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1668, __pyx_L1_error)
15886                 __Pyx_GOTREF(__pyx_t_1);
15887                 __pyx_r = __pyx_t_1;
15888                 __pyx_t_1 = 0;
15889                 goto __pyx_L0;
15890
15891                 /* function exit code */
15892                 __pyx_L1_error:;
15893                 __Pyx_XDECREF(__pyx_t_1);
15894                 __Pyx_AddTraceback("PyClical.acos", __pyx_clineno, __pyx_lineno, __pyx_filename);
15895                 __pyx_r = NULL;
15896                 __pyx_L0:;

```

```

15894         __Pyx_XGIVEREF(__pyx_r);
15895         __Pyx_RefNannyFinishContext();
15896         return __pyx_r;
15897     }
15898
15899     /* "PyClical.pyx":1689
15900     *         return clifford().wrap( glucat.acos(toClifford(obj)) )
15901     *
15902     * ctypedef inline cosh(obj):                # ««««««««
15903     *     """
15904     *     Hyperbolic cosine of multivector.
15905     */
15906
15907     static PyObject *__pyx_pw_8PyClical_61cosh(PyObject *__pyx_self, PyObject
15908 *__pyx_v_obj); /*proto*/
15909     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cosh(PyObject *__pyx_v_obj,
15910 CYTHON_UNUSED int __pyx_skip_dispatch) {
15911         PyObject *__pyx_r = NULL;
15912         __Pyx_RefNannyDeclarations
15913         PyObject *__pyx_t_1 = NULL;
15914         PyObject *__pyx_t_2 = NULL;
15915         PyObject *__pyx_t_3 = NULL;
15916         PyObject *__pyx_t_4 = NULL;
15917         PyObject *__pyx_t_5 = NULL;
15918         PyObject *__pyx_t_6 = NULL;
15919         PyObject *__pyx_t_7 = NULL;
15920         PyObject *__pyx_t_8 = NULL;
15921         int __pyx_lineno = 0;
15922         const char *__pyx_filename = NULL;
15923         int __pyx_clineno = 0;
15924         __Pyx_RefNannySetupContext("cosh", 0);
15925
15926         /* "PyClical.pyx":1700
15927         *     {1,2}
15928         *     """
15929         *     try:                # ««««««««
15930         *         return math.cosh(obj)
15931         *     except:
15932         */
15933         {
15934             __Pyx_PyThreadState_declare
15935             __Pyx_PyThreadState_assign
15936             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
15937             __Pyx_XGOTREF(__pyx_t_1);
15938             __Pyx_XGOTREF(__pyx_t_2);
15939             __Pyx_XGOTREF(__pyx_t_3);
15940             /*try:*/ {
15941
15942                 /* "PyClical.pyx":1701
15943                 *     """
15944                 *     try:
15945                 *         return math.cosh(obj)                # ««««««««
15946                 *     except:
15947                 *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
15948                 */
15949
15950                 __Pyx_XDECREF(__pyx_r);
15951                 __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
15952                 __PYX_ERR(0, 1701, __pyx_L3_error)
15953                 __Pyx_GOTREF(__pyx_t_5);
15954                 __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_cosh); if
15955                 (unlikely(!__pyx_t_6)) __PYX_ERR(0, 1701, __pyx_L3_error)
15956                 __Pyx_GOTREF(__pyx_t_6);
15957                 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
15958                 __pyx_t_5 = NULL;
15959                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
15960                     __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
15961                     if (likely(__pyx_t_5)) {
15962                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
15963                         __Pyx_INCREF(__pyx_t_5);
15964                         __Pyx_DECREF(function);
15965                         __Pyx_DECREF_SET(__pyx_t_6, function);
15966                     }
15967                 }
15968                 __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
15969 __pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
15970                 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15971                 if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1701, __pyx_L3_error)
15972                 __Pyx_GOTREF(__pyx_t_4);
15973                 __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
15974                 __pyx_r = __pyx_t_4;
15975                 __pyx_t_4 = 0;
15976                 goto __pyx_L7_try_return;
15977
15978                 /* "PyClical.pyx":1700
15979                 *     {1,2}
15980                 *     """
15981                 *     try:                # ««««««««

```

```

15976 *         return math.cosh(obj)
15977 *     except:
15978 */
15979         }
15980         __pyx_L3_error;;
15981         __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
15982         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
15983         __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
15984
15985         /* "PyClical.pyx":1702
15986 *     try:
15987 *         return math.cosh(obj)
15988 *     except: # ««««««««
15989 *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
15990 *
15991 */
15992         /*except:*/ {
15993         __Pyx_AddTraceback("PyClical.cosh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
15994         if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1702, __pyx_L5_except_error)
15995         __Pyx_GOTREF(__pyx_t_4);
15996         __Pyx_GOTREF(__pyx_t_6);
15997         __Pyx_GOTREF(__pyx_t_5);
15998
15999         /* "PyClical.pyx":1703
16000 *         return math.cosh(obj)
16001 *     except:
16002 *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
16003 *
16004 * cpdef inline acosh(obj,i = None):
16005 */
16006         __Pyx_XDECREF(__pyx_r);
16007         __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1703,
__pyx_L5_except_error)
16008         __Pyx_GOTREF(__pyx_t_7);
16009         __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), cosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1703, __pyx_L5_except_error)
16010         __Pyx_GOTREF(__pyx_t_8);
16011         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
16012         __pyx_r = __pyx_t_8;
16013         __pyx_t_8 = 0;
16014         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
16015         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16016         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
16017         goto __pyx_L6_except_return;
16018     }
16019     __pyx_L5_except_error;;
16020
16021     /* "PyClical.pyx":1700
16022 *     {1,2}
16023 *     """
16024 *     try: # ««««««««
16025 *         return math.cosh(obj)
16026 *     except:
16027 */
16028         __Pyx_XGIVEREF(__pyx_t_1);
16029         __Pyx_XGIVEREF(__pyx_t_2);
16030         __Pyx_XGIVEREF(__pyx_t_3);
16031         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16032         goto __pyx_L1_error;
16033         __pyx_L7_try_return;
16034         __Pyx_XGIVEREF(__pyx_t_1);
16035         __Pyx_XGIVEREF(__pyx_t_2);
16036         __Pyx_XGIVEREF(__pyx_t_3);
16037         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16038         goto __pyx_L0;
16039         __pyx_L6_except_return;
16040         __Pyx_XGIVEREF(__pyx_t_1);
16041         __Pyx_XGIVEREF(__pyx_t_2);
16042         __Pyx_XGIVEREF(__pyx_t_3);
16043         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
16044         goto __pyx_L0;
16045     }
16046
16047     /* "PyClical.pyx":1689
16048 *         return clifford().wrap( glucat.acos(toClifford(obj)) )
16049 *
16050 * cpdef inline cosh(obj): # ««««««««
16051 *     """
16052 *     Hyperbolic cosine of multivector.
16053 */
16054
16055     /* function exit code */
16056     __pyx_L1_error;;

```

```

16057         __Pyx_XDECREFF(__pyx_t_4);
16058         __Pyx_XDECREFF(__pyx_t_5);
16059         __Pyx_XDECREFF(__pyx_t_6);
16060         __Pyx_XDECREFF(__pyx_t_7);
16061         __Pyx_XDECREFF(__pyx_t_8);
16062         __Pyx_AddTraceback("PyClicl.cosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16063         __pyx_r = 0;
16064         __pyx_L0:;
16065         __Pyx_XGIVEREF(__pyx_r);
16066         __Pyx_RefNannyFinishContext();
16067         return __pyx_r;
16068     }
16069
16070     /* Python wrapper */
16071     static PyObject * __pyx_pw_8PyClicl_61cosh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
16072     static char __pyx_doc_8PyClicl_60cosh[] = "\n    Hyperbolic cosine of
multivector.\n\n    >> x=clifford(\"{1,2}\") * pi; print(cosh(x))\n    -1\n    >>
x=clifford(\"{1,2,3}\"); print(cosh(acosh(x)))\n    {1,2,3}\n    >> x=clifford(\"{1,2}\");
print(cosh(acosh(x)))\n    {1,2}\n    ";
16073     static PyObject * __pyx_pf_8PyClicl_61cosh(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
16074         PyObject * __pyx_r = 0;
16075         __Pyx_RefNannyDeclarations
16076         __Pyx_RefNannySetupContext("cosh (wrapper)", 0);
16077         __pyx_r = __pyx_pf_8PyClicl_60cosh(__pyx_self, ((PyObject *) __pyx_v_obj));
16078
16079         /* function exit code */
16080         __Pyx_RefNannyFinishContext();
16081         return __pyx_r;
16082     }
16083
16084     static PyObject * __pyx_pf_8PyClicl_60cosh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
16085         PyObject * __pyx_r = NULL;
16086         __Pyx_RefNannyDeclarations
16087         PyObject * __pyx_t_1 = NULL;
16088         int __pyx_lineno = 0;
16089         const char * __pyx_filename = NULL;
16090         int __pyx_clineno = 0;
16091         __Pyx_RefNannySetupContext("cosh", 0);
16092         __Pyx_XDECREFF(__pyx_r);
16093         __pyx_t_1 = __pyx_f_8PyClicl_cosh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1689, __pyx_L1_error)
16094         __Pyx_GOTREF(__pyx_t_1);
16095         __pyx_r = __pyx_t_1;
16096         __pyx_t_1 = 0;
16097         goto __pyx_L0;
16098
16099         /* function exit code */
16100         __pyx_L1_error:;
16101         __Pyx_XDECREFF(__pyx_t_1);
16102         __Pyx_AddTraceback("PyClicl.cosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16103         __pyx_r = NULL;
16104         __pyx_L0:;
16105         __Pyx_XGIVEREF(__pyx_r);
16106         __Pyx_RefNannyFinishContext();
16107         return __pyx_r;
16108     }
16109
16110     /* "PyClicl.pyx":1705
16111     *         return clifford().wrap( glucat.cosh(toClifford(obj)) )
16112     *
16113     * cpdef inline acosh(obj,i = None):                                # ««««««««
16114     *         """
16115     *         Inverse hyperbolic cosine of multivector with optional complexifier.
16116     */
16117
16118     static PyObject * __pyx_pw_8PyClicl_63acosh(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kwds); /*proto*/
16119     static CYTHON_INLINE PyObject * __pyx_f_8PyClicl_acosh(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_acosh * __pyx_optional_args) {
16120         PyObject * __pyx_v_i = ((PyObject *) Py_None);
16121         PyObject * __pyx_r = NULL;
16122         __Pyx_RefNannyDeclarations
16123         int __pyx_t_1;
16124         int __pyx_t_2;
16125         PyObject * __pyx_t_3 = NULL;
16126         Clifford __pyx_t_4;
16127         PyObject * __pyx_t_5 = NULL;
16128         PyObject * __pyx_t_6 = NULL;
16129         PyObject * __pyx_t_7 = NULL;
16130         PyObject * __pyx_t_8 = NULL;
16131         PyObject * __pyx_t_9 = NULL;
16132         PyObject * __pyx_t_10 = NULL;
16133         PyObject * __pyx_t_11 = NULL;
16134         int __pyx_lineno = 0;

```



```

16135         const char *__pyx_filename = NULL;
16136         int __pyx_clineno = 0;
16137         __Pyx_RefNannySetupContext("acosh", 0);
16138         if (__pyx_optional_args) {
16139             if (__pyx_optional_args->__pyx_n > 0) {
16140                 __pyx_v_i = __pyx_optional_args->i;
16141             }
16142         }
16143
16144         /* "PyClical.pyx":1720
16145  *      {1,2}
16146  *      """
16147  *      if not (i is None):          # ««««««««
16148  *          return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16149  *      else:
16150  */
16151         __pyx_t_1 = (__pyx_v_i != Py_None);
16152         __pyx_t_2 = (__pyx_t_1 != 0);
16153         if (__pyx_t_2) {
16154
16155             /* "PyClical.pyx":1721
16156  *      """
16157  *      if not (i is None):
16158  *          return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )      #
16159  *      else:
16160  *          try:
16161  */
16162         __Pyx_XDECREF(__pyx_r);
16163         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
16164 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1721, __pyx_L1_error)
16165         __Pyx_GOTREF(__pyx_t_3);
16166         try {
16167             __pyx_t_4 = acosh(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
16168 __pyx_f_8PyClical_toClifford(__pyx_v_i));
16169         } catch (...) {
16170             __Pyx_CppExn2PyErr();
16171             __PYX_ERR(0, 1721, __pyx_L1_error)
16172         }
16173         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
16174 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1721, __pyx_L1_error)
16175         __Pyx_GOTREF(__pyx_t_5);
16176         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16177         __pyx_r = __pyx_t_5;
16178         __pyx_t_5 = 0;
16179         goto __pyx_L0;
16180
16181         /* "PyClical.pyx":1720
16182  *      {1,2}
16183  *      """
16184  *      if not (i is None):          # ««««««««
16185  *          return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16186  *      else:
16187  */
16188         }
16189
16190         /* "PyClical.pyx":1723
16191  *      return clifford().wrap( glucat.acosh(toClifford(obj), toClifford(i)) )
16192  *      else:
16193  *          try:          # ««««««««
16194  *              return math.acosh(obj)
16195  *          except:
16196  */
16197         /*else*/ {
16198             {
16199                 __Pyx_PyThreadState_declare
16200                 __Pyx_PyThreadState_assign
16201                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16202                 __Pyx_XGOTREF(__pyx_t_6);
16203                 __Pyx_XGOTREF(__pyx_t_7);
16204                 __Pyx_XGOTREF(__pyx_t_8);
16205             }
16206             /*try:*/ {
16207
16208                 /* "PyClical.pyx":1724
16209  *      else:
16210  *          try:
16211  *              return math.acosh(obj)          # ««««««««
16212  *          except:
16213  *              return clifford().wrap( glucat.acosh(toClifford(obj)) )
16214  */
16215                 __Pyx_XDECREF(__pyx_r);
16216                 __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16217 (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1724, __pyx_L4_error)
16218                 __Pyx_GOTREF(__pyx_t_3);
16219                 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_acosh); if
16220 (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1724, __pyx_L4_error)
16221                 __Pyx_GOTREF(__pyx_t_9);

```

```

16216         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16217         __pyx_t_3 = NULL;
16218         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16219             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16220             if (likely(__pyx_t_3)) {
16221                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16222                 __Pyx_INCREF(__pyx_t_3);
16223                 __Pyx_INCREF(function);
16224                 __Pyx_DECREF_SET(__pyx_t_9, function);
16225             }
16226         }
16227         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16228         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16229         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1724, __pyx_L4_error)
16230         __Pyx_GOTREF(__pyx_t_5);
16231         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16232         __pyx_r = __pyx_t_5;
16233         __pyx_t_5 = 0;
16234         goto __pyx_L8_try_return;
16235
16236         /* "PyClicl.pyx":1723
16237  *         return clifford().wrap( glucat.acosh(toClifford(obj)), toClifford(i)) )
16238  *     else:
16239  *         try:
16240  *             # ««««««««
16241  *             return math.acosh(obj)
16242  *         except:
16243  */
16244         __pyx_L4_error:;
16245         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16246         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16247         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16248
16249         /* "PyClicl.pyx":1725
16250  *     try:
16251  *         return math.acosh(obj)
16252  *     except:
16253  *         # ««««««««
16254  *         return clifford().wrap( glucat.acosh(toClifford(obj)) )
16255  *
16256  */
16257         /*except:*/ {
            __Pyx_AddTraceback("PyClicl.acosh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
16258             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1725, __pyx_L6_except_error)
16259             __Pyx_GOTREF(__pyx_t_5);
16260             __Pyx_GOTREF(__pyx_t_9);
16261             __Pyx_GOTREF(__pyx_t_3);
16262
16263             /* "PyClicl.pyx":1726
16264  *         return math.acosh(obj)
16265  *     except:
16266  *         return clifford().wrap( glucat.acosh(toClifford(obj)) )
16267  *
16268  * cpdef inline sin(obj,i = None):
16269  */
16270             __Pyx_XDECREF(__pyx_r);
16271             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1726,
__pyx_L6_except_error)
16272             __Pyx_GOTREF(__pyx_t_10);
16273             __pyx_t_11 = __pyx_f_8PyClicl_8clifford_wrap(((struct
__pyx_obj_8PyClicl_clifford *)__pyx_t_10), acosh(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1726, __pyx_L6_except_error)
16274             __Pyx_GOTREF(__pyx_t_11);
16275             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16276             __pyx_r = __pyx_t_11;
16277             __pyx_t_11 = 0;
16278             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16279             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16280             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16281             goto __pyx_L7_except_return;
16282         }
16283         __pyx_L6_except_error:;
16284
16285         /* "PyClicl.pyx":1723
16286  *         return clifford().wrap( glucat.acosh(toClifford(obj)), toClifford(i)) )
16287  *     else:
16288  *         try:
16289  *             # ««««««««
16290  *             return math.acosh(obj)
16291  *         except:
16292  */
16293             __Pyx_XGIVEREF(__pyx_t_6);
16294             __Pyx_XGIVEREF(__pyx_t_7);
16295             __Pyx_XGIVEREF(__pyx_t_8);
            __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);

```

```

16296         goto __pyx_L1_error;
16297     __pyx_L8_try_return:;
16298     __Pyx_XGIVEREF(__pyx_t_6);
16299     __Pyx_XGIVEREF(__pyx_t_7);
16300     __Pyx_XGIVEREF(__pyx_t_8);
16301     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16302     goto __pyx_L0;
16303     __pyx_L7_except_return:;
16304     __Pyx_XGIVEREF(__pyx_t_6);
16305     __Pyx_XGIVEREF(__pyx_t_7);
16306     __Pyx_XGIVEREF(__pyx_t_8);
16307     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16308     goto __pyx_L0;
16309 }
16310 }
16311
16312     /* "PyClical.pyx":1705
16313 *         return clifford().wrap( gluocat.cosh(toClifford(obj)) )
16314 *
16315 * cpdef inline acosh(obj,i = None):           # ««««««««
16316 *     """
16317 *     Inverse hyperbolic cosine of multivector with optional complexifier.
16318 */
16319
16320     /* function exit code */
16321     __pyx_L1_error:;
16322     __Pyx_XDECREF(__pyx_t_3);
16323     __Pyx_XDECREF(__pyx_t_5);
16324     __Pyx_XDECREF(__pyx_t_9);
16325     __Pyx_XDECREF(__pyx_t_10);
16326     __Pyx_XDECREF(__pyx_t_11);
16327     __Pyx_AddTraceback("PyClical.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16328     __pyx_r = 0;
16329     __pyx_L0:;
16330     __Pyx_XGIVEREF(__pyx_r);
16331     __Pyx_RefNannyFinishContext();
16332     return __pyx_r;
16333 }
16334
16335     /* Python wrapper */
16336     static PyObject* __pyx_pw_8PyClical_63acosh(PyObject* __pyx_self, PyObject
16337 *__pyx_args, PyObject* __pyx_kwds); /*proto*/
16338     static char __pyx_doc_8PyClical_62acosh[] = "\n    Inverse hyperbolic cosine of
16339 multivector with optional complexifier.\n\n    >> print(acosh(0, \{-2,-1,1\}\n
16340 >> x=clifford(\{1,2,3\}\n); print(cosh(acosh(x, \{-1,1,2,3,4\}\n)\n    {1,2,3}\n
16341 print(acosh(0))\n    1.571{-1}\n    >> x=clifford(\{1,2,3\}\n); print(cosh(acosh(x))\n    {1,2,3}\n
16342 >> x=clifford(\{1,2\}\n); print(cosh(acosh(x))\n    {1,2}\n    ";
16343     static PyObject* __pyx_pw_8PyClical_63acosh(PyObject* __pyx_self, PyObject
16344 *__pyx_args, PyObject* __pyx_kwds) {
16345     PyObject* __pyx_v_obj = 0;
16346     PyObject* __pyx_v_i = 0;
16347     int __pyx_lineno = 0;
16348     const char* __pyx_filename = NULL;
16349     int __pyx_clineno = 0;
16350     PyObject* __pyx_r = 0;
16351     __Pyx_RefNannyDeclarations
16352     __Pyx_RefNannySetupContext("acosh (wrapper)", 0);
16353     {
16354         static PyObject* __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16355         PyObject* values[2] = {0,0};
16356         values[1] = ((PyObject*)Py_None);
16357         if (unlikely(__pyx_kwds)) {
16358             Py_ssize_t kw_args;
16359             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
16360             switch (pos_args) {
16361                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16362                     CYTHON_FALLTHROUGH;
16363                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16364                     CYTHON_FALLTHROUGH;
16365                 case 0: break;
16366                 default: goto __pyx_L5_argtuple_error;
16367             }
16368             kw_args = PyDict_Size(__pyx_kwds);
16369             switch (pos_args) {
16370                 case 0:
16371                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
16372 0)) kw_args--;
16373                     else goto __pyx_L5_argtuple_error;
16374                     CYTHON_FALLTHROUGH;
16375                 case 1:
16376                     if (kw_args > 0) {
16377                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
16378                         if (value) { values[1] = value; kw_args--; }
16379                     }
16380             }
16381             if (unlikely(kw_args > 0)) {
16382                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,

```

```

        values, pos_args, "acosh") < 0)) __PYX_ERR(0, 1705, __pyx_L3_error)
16376     }
16377     } else {
16378         switch (PyTuple_GET_SIZE(__pyx_args)) {
16379             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16380                 CYTHON_FALLTHROUGH;
16381             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16382                 break;
16383             default: goto __pyx_L5_argtuple_error;
16384         }
16385     }
16386     __pyx_v_obj = values[0];
16387     __pyx_v_i = values[1];
16388 }
16389 goto __pyx_L4_argument_unpacking_done;
16390 __pyx_L5_argtuple_error:;
16391 __Pyx_RaiseArgtupleInvalid("acosh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1705, __pyx_L3_error)
16392 __pyx_L3_error:;
16393 __Pyx_AddTraceback("PyClicl.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16394 __Pyx_RefNannyFinishContext();
16395 return NULL;
16396 __pyx_L4_argument_unpacking_done:;
16397 __pyx_r = __pyx_pf_8PyClicl_62acosh(__pyx_self, __pyx_v_obj, __pyx_v_i);
16398
16399 /* function exit code */
16400 __Pyx_RefNannyFinishContext();
16401 return __pyx_r;
16402 }
16403
16404 static PyObject *__pyx_pf_8PyClicl_62acosh(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
16405     PyObject *__pyx_r = NULL;
16406     __Pyx_RefNannyDeclarations
16407     PyObject *__pyx_t_1 = NULL;
16408     struct __pyx_opt_args_8PyClicl_acosh __pyx_t_2;
16409     int __pyx_lineno = 0;
16410     const char *__pyx_filename = NULL;
16411     int __pyx_clineno = 0;
16412     __Pyx_RefNannySetupContext("acosh", 0);
16413     __Pyx_XDECREF(__pyx_r);
16414     __pyx_t_2.__pyx_n = 1;
16415     __pyx_t_2.i = __pyx_v_i;
16416     __pyx_t_1 = __pyx_f_8PyClicl_acosh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1705, __pyx_L1_error)
16417     __Pyx_GOTREF(__pyx_t_1);
16418     __pyx_r = __pyx_t_1;
16419     __pyx_t_1 = 0;
16420     goto __pyx_L0;
16421
16422 /* function exit code */
16423 __pyx_L1_error:;
16424 __Pyx_XDECREF(__pyx_t_1);
16425 __Pyx_AddTraceback("PyClicl.acosh", __pyx_clineno, __pyx_lineno, __pyx_filename);
16426 __pyx_r = NULL;
16427 __pyx_L0:;
16428 __Pyx_XGIVEREF(__pyx_r);
16429 __Pyx_RefNannyFinishContext();
16430 return __pyx_r;
16431 }
16432
16433 /* "PyClicl.pyx":1728
16434 *      return clifford().wrap( glucat.acosh(toClifford(obj)) )
16435 *
16436 * cpdef inline sin(obj,i = None): # «««««««
16437 *     """
16438 *     Sine of multivector with optional complexifier.
16439 */
16440
16441 static PyObject *__pyx_pw_8PyClicl_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws); /*proto*/
16442 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_sin(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_sin *__pyx_optional_args) {
16443     PyObject *__pyx_v_i = ((PyObject *)Py_None);
16444     PyObject *__pyx_r = NULL;
16445     __Pyx_RefNannyDeclarations
16446     int __pyx_t_1;
16447     int __pyx_t_2;
16448     PyObject *__pyx_t_3 = NULL;
16449     Clifford __pyx_t_4;
16450     PyObject *__pyx_t_5 = NULL;
16451     PyObject *__pyx_t_6 = NULL;
16452     PyObject *__pyx_t_7 = NULL;
16453     PyObject *__pyx_t_8 = NULL;
16454     PyObject *__pyx_t_9 = NULL;
16455     PyObject *__pyx_t_10 = NULL;
16456     PyObject *__pyx_t_11 = NULL;

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```

16457         int __pyx_lineno = 0;
16458         const char *__pyx_filename = NULL;
16459         int __pyx_clineno = 0;
16460         __Pyx_RefNannySetupContext("sin", 0);
16461         if (__pyx_optional_args) {
16462             if (__pyx_optional_args->__pyx_n > 0) {
16463                 __pyx_v_i = __pyx_optional_args->i;
16464             }
16465         }
16466
16467         /* "PyClical.pyx":1739
16468         * {1,2,3}
16469         * """
16470         * if not (i is None): # ««««««««
16471         *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16472         * else:
16473         */
16474         __pyx_t_1 = (__pyx_v_i != Py_None);
16475         __pyx_t_2 = (__pyx_t_1 != 0);
16476         if (__pyx_t_2) {
16477             /* "PyClical.pyx":1740
16478             * """
16479             * if not (i is None):
16480             *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) ) # ««««««««
16481             * else:
16482             *     try:
16483             */
16484             /*
16485             *     __Pyx_XDECREF(__pyx_r);
16486             *     __pyx_t_3 = __Pyx_PyObject_CallNoArg((PyObject
16487             *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1740, __pyx_L1_error)
16488             *     __Pyx_GOTREF(__pyx_t_3);
16489             *     try {
16490             *         __pyx_t_4 = sin(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
16491             *         __pyx_f_8PyClical_toClifford(__pyx_v_i));
16492             *     } catch (...) {
16493             *         __Pyx_CppExn2PyErr();
16494             *         __PYX_ERR(0, 1740, __pyx_L1_error)
16495             *     }
16496             *     __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
16497             *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1740, __pyx_L1_error)
16498             *     __Pyx_GOTREF(__pyx_t_5);
16499             *     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16500             *     __pyx_r = __pyx_t_5;
16501             *     __pyx_t_5 = 0;
16502             *     goto __pyx_L0;
16503             */
16504             /* "PyClical.pyx":1739
16505             * {1,2,3}
16506             * """
16507             * if not (i is None): # ««««««««
16508             *     return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16509             * else:
16510             */
16511             /*
16512             *     __Pyx_XDECREF(__pyx_r);
16513             *     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16514             (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1743, __pyx_L4_error)
16515             *     __Pyx_GOTREF(__pyx_t_3);
16516             *     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sin); if
16517             (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1743, __pyx_L4_error)
16518             *     __Pyx_GOTREF(__pyx_t_9);
16519             */
16520             /* "PyClical.pyx":1742
16521             * return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16522             * else:
16523             *     try: # ««««««««
16524             *         return math.sin(obj)
16525             *     except:
16526             */
16527             /*else*/ {
16528             {
16529                 __Pyx_PyThreadState_declare
16530                 __Pyx_PyThreadState_assign
16531                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16532                 __Pyx_XGOTREF(__pyx_t_6);
16533                 __Pyx_XGOTREF(__pyx_t_7);
16534                 __Pyx_XGOTREF(__pyx_t_8);
16535                 /*try:*/ {
16536                     /* "PyClical.pyx":1743
16537                     * else:
16538                     *     try:
16539                     *         return math.sin(obj) # ««««««««
16540                     *     except:
16541                     *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16542                     */
16543                     __Pyx_XDECREF(__pyx_r);
16544                     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16545                     (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1743, __pyx_L4_error)
16546                     __Pyx_GOTREF(__pyx_t_3);
16547                     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_sin); if
16548                     (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1743, __pyx_L4_error)
16549                     __Pyx_GOTREF(__pyx_t_9);

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16539         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16540         __pyx_t_3 = NULL;
16541         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16542             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16543             if (likely(__pyx_t_3)) {
16544                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16545                 __Pyx_INCREF(__pyx_t_3);
16546                 __Pyx_INCREF(function);
16547                 __Pyx_DECREF_SET(__pyx_t_9, function);
16548             }
16549         }
16550         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16551         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16552         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1743, __pyx_L4_error)
16553         __Pyx_GOTREF(__pyx_t_5);
16554         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16555         __pyx_r = __pyx_t_5;
16556         __pyx_t_5 = 0;
16557         goto __pyx_L8_try_return;
16558
16559         /* "PyClicl.pyx":1742
16560  *         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16561  *     else:
16562  *         try:
16563  *             # ««««««
16564  *             return math.sin(obj)
16565  *         except:
16566  */
16567         __pyx_L4_error:;
16568         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16569         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16570         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16571
16572         /* "PyClicl.pyx":1744
16573  *     try:
16574  *         return math.sin(obj)
16575  *     except:
16576  *         # ««««««
16577  *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16578  */
16579         /*except:*/ {
16580             __Pyx_AddTraceback("PyClicl.sin", __pyx_clineno, __pyx_lineno,
__pyx_filename);
16581             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1744, __pyx_L6_except_error)
16582             __Pyx_GOTREF(__pyx_t_5);
16583             __Pyx_GOTREF(__pyx_t_9);
16584             __Pyx_GOTREF(__pyx_t_3);
16585
16586             /* "PyClicl.pyx":1745
16587  *         return math.sin(obj)
16588  *     except:
16589  *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16590  *         # ««««««
16591  * cpdef inline asin(obj,i = None):
16592  */
16593             __Pyx_XDECREF(__pyx_r);
16594             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1745,
__pyx_L6_except_error)
16595             __Pyx_GOTREF(__pyx_t_10);
16596             __pyx_t_11 = __pyx_f_8PyClicl_8clifford_wrap(((struct
__pyx_obj_8PyClicl_clifford *)__pyx_t_10), sin(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1745, __pyx_L6_except_error)
16597             __Pyx_GOTREF(__pyx_t_11);
16598             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16599             __pyx_r = __pyx_t_11;
16600             __pyx_t_11 = 0;
16601             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16602             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16603             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16604             goto __pyx_L7_except_return;
16605         }
16606         __pyx_L6_except_error:;
16607
16608         /* "PyClicl.pyx":1742
16609  *         return clifford().wrap( glucat.sin(toClifford(obj), toClifford(i)) )
16610  *     else:
16611  *         try:
16612  *             # ««««««
16613  *             return math.sin(obj)
16614  *         except:
16615  */
16616             __Pyx_XGIVEREF(__pyx_t_6);
16617             __Pyx_XGIVEREF(__pyx_t_7);
16618             __Pyx_XGIVEREF(__pyx_t_8);
16619             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);

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```

16619         goto __pyx_L1_error;
16620         __pyx_L8_try_return:;
16621         __Pyx_XGIVEREF(__pyx_t_6);
16622         __Pyx_XGIVEREF(__pyx_t_7);
16623         __Pyx_XGIVEREF(__pyx_t_8);
16624         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16625         goto __pyx_L0;
16626         __pyx_L7_except_return:;
16627         __Pyx_XGIVEREF(__pyx_t_6);
16628         __Pyx_XGIVEREF(__pyx_t_7);
16629         __Pyx_XGIVEREF(__pyx_t_8);
16630         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16631         goto __pyx_L0;
16632     }
16633 }
16634
16635     /* "PyClical.pyx":1728
16636     *
16637     * cpdef inline sin(obj,i = None):
16638     *     """
16639     *     Sine of multivector with optional complexifier.
16640     */
16641
16642     /* function exit code */
16643     __pyx_L1_error:;
16644     __Pyx_XDECREF(__pyx_t_3);
16645     __Pyx_XDECREF(__pyx_t_5);
16646     __Pyx_XDECREF(__pyx_t_9);
16647     __Pyx_XDECREF(__pyx_t_10);
16648     __Pyx_XDECREF(__pyx_t_11);
16649     __Pyx_AddTraceback("PyClical.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16650     __pyx_r = 0;
16651     __pyx_L0:;
16652     __Pyx_XGIVEREF(__pyx_r);
16653     __Pyx_RefNannyFinishContext();
16654     return __pyx_r;
16655 }
16656
16657
16658     /* Python wrapper */
16659     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
16660     static char __pyx_doc_8PyClical_64sin[] = "\n    Sine of multivector with optional
complexifier.\n\n    >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>
s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n    {-1}\n    >>
x=clifford(\"{1,2,3}\"); print(asin(sin(x))\n    {1,2,3}\n    ";
16661     static PyObject *__pyx_pw_8PyClical_65sin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds) {
16662         PyObject *__pyx_v_obj = 0;
16663         PyObject *__pyx_v_i = 0;
16664         int __pyx_lineno = 0;
16665         const char *__pyx_filename = NULL;
16666         int __pyx_clineno = 0;
16667         PyObject *__pyx_r = 0;
16668         __Pyx_RefNannyDeclarations
16669         __Pyx_RefNannySetupContext("sin (wrapper)", 0);
16670         {
16671             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16672             PyObject* values[2] = {0,0};
16673             values[1] = ((PyObject *)Py_None);
16674             if (unlikely(__pyx_kwds)) {
16675                 Py_ssize_t kw_args;
16676                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
16677                 switch (pos_args) {
16678                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
16679                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
16680                     case 0: break;
16681                     default: goto __pyx_L5_argtuple_error;
16682                 }
16683                 kw_args = PyDict_Size(__pyx_kwds);
16684                 switch (pos_args) {
16685                     case 0:
16686                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
16689                     else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
16690                     case 1:
16691                         if (kw_args > 0) {
16692                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
16693                             if (value) { values[1] = value; kw_args--; }
16694                         }
16695                     }
16696                 if (unlikely(kw_args > 0)) {
16697                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "sin") < 0)) __PYX_ERR(0, 1728, __pyx_L3_error)

```

```

16699     }
16700     } else {
16701         switch (PyTuple_GET_SIZE(__pyx_args)) {
16702             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
16703             CYTHON_FALLTHROUGH;
16704             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
16705             break;
16706             default: goto __pyx_L5_argtuple_error;
16707         }
16708     }
16709     __pyx_v_obj = values[0];
16710     __pyx_v_i = values[1];
16711 }
16712 goto __pyx_L4_argument_unpacking_done;
16713 __pyx_L5_argtuple_error:;
16714 __Pyx_RaiseArgtupleInvalid("sin", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
16715 __PYX_ERR(0, 1728, __pyx_L3_error)
16716 __pyx_L3_error:;
16717 __Pyx_AddTraceback("PyClicl.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16718 __Pyx_RefNannyFinishContext();
16719 return NULL;
16720 __pyx_L4_argument_unpacking_done:;
16721 __pyx_r = __pyx_pf_8PyClicl_64sin(__pyx_self, __pyx_v_obj, __pyx_v_i);
16722
16723 /* function exit code */
16724 __Pyx_RefNannyFinishContext();
16725 return __pyx_r;
16726 }
16727
16728 static PyObject *__pyx_pf_8PyClicl_64sin(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj, PyObject *__pyx_v_i) {
16729     PyObject *__pyx_r = NULL;
16730     __Pyx_RefNannyDeclarations
16731     PyObject *__pyx_t_1 = NULL;
16732     struct __pyx_opt_args_8PyClicl_sin __pyx_t_2;
16733     int __pyx_lineno = 0;
16734     const char *__pyx_filename = NULL;
16735     int __pyx_clineno = 0;
16736     __Pyx_RefNannySetupContext("sin", 0);
16737     __Pyx_XDECREF(__pyx_r);
16738     __pyx_t_2.__pyx_n = 1;
16739     __pyx_t_2.i = __pyx_v_i;
16740     __pyx_t_1 = __pyx_f_8PyClicl_sin(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1728, __pyx_L1_error)
16741     __Pyx_GOTREF(__pyx_t_1);
16742     __pyx_r = __pyx_t_1;
16743     __pyx_t_1 = 0;
16744     goto __pyx_L0;
16745
16746 /* function exit code */
16747 __pyx_L1_error:;
16748 __Pyx_XDECREF(__pyx_t_1);
16749 __Pyx_AddTraceback("PyClicl.sin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16750 __pyx_r = NULL;
16751 __pyx_L0:;
16752 __Pyx_XGIVEREF(__pyx_r);
16753 __Pyx_RefNannyFinishContext();
16754 return __pyx_r;
16755 }
16756
16757 /* "PyClicl.pyx":1747
16758 *
16759 * cpdef inline asin(obj,i = None): # ««««««««
16760 *     """
16761 *     Inverse sine of multivector with optional complexifier.
16762 */
16763
16764 static PyObject *__pyx_pw_8PyClicl_67asin(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kws); /*proto*/
16765 static CYTHON_INLINE PyObject *__pyx_f_8PyClicl_asin(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClicl_asin *__pyx_optional_args) {
16766     PyObject *__pyx_v_i = (PyObject *)Py_None;
16767     PyObject *__pyx_r = NULL;
16768     __Pyx_RefNannyDeclarations
16769     int __pyx_t_1;
16770     int __pyx_t_2;
16771     PyObject *__pyx_t_3 = NULL;
16772     Clifford __pyx_t_4;
16773     PyObject *__pyx_t_5 = NULL;
16774     PyObject *__pyx_t_6 = NULL;
16775     PyObject *__pyx_t_7 = NULL;
16776     PyObject *__pyx_t_8 = NULL;
16777     PyObject *__pyx_t_9 = NULL;
16778     PyObject *__pyx_t_10 = NULL;
16779     PyObject *__pyx_t_11 = NULL;
16780     int __pyx_lineno = 0;

```



```

16781         const char *__pyx_filename = NULL;
16782         int __pyx_clineno = 0;
16783         __Pyx_RefNannySetupContext("asin", 0);
16784         if (__pyx_optional_args) {
16785             if (__pyx_optional_args->__pyx_n > 0) {
16786                 __pyx_v_i = __pyx_optional_args->i;
16787             }
16788         }
16789
16790         /* "PyClical.pyx":1760
16791         * {1,2,3}
16792         * """
16793         * if not (i is None): # ««««««««
16794         *     return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16795         * else:
16796         */
16797         __pyx_t_1 = (__pyx_v_i != Py_None);
16798         __pyx_t_2 = (__pyx_t_1 != 0);
16799         if (__pyx_t_2) {
16800
16801             /* "PyClical.pyx":1761
16802             * """
16803             * if not (i is None):
16804             *     return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) ) # ««««««««
16805             * else:
16806             *     try:
16807             */
16808             __Pyx_XDECREF(__pyx_r);
16809             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
16810 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1761, __pyx_L1_error)
16811             __Pyx_GOTREF(__pyx_t_3);
16812             try {
16813                 __pyx_t_4 = asin(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
16814 __pyx_f_8PyClical_toClifford(__pyx_v_i));
16815             } catch (...) {
16816                 __Pyx_CppExn2PyErr();
16817                 __PYX_ERR(0, 1761, __pyx_L1_error)
16818             }
16819             __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
16820 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1761, __pyx_L1_error)
16821             __Pyx_GOTREF(__pyx_t_5);
16822             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16823             __pyx_r = __pyx_t_5;
16824             __pyx_t_5 = 0;
16825             goto __pyx_L0;
16826
16827             /* "PyClical.pyx":1760
16828             * {1,2,3}
16829             * """
16830             * if not (i is None): # ««««««««
16831             *     return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16832             * else:
16833             */
16834             }
16835
16836             /* "PyClical.pyx":1763
16837             * return clifford().wrap( glucat.asin(toClifford(obj), toClifford(i)) )
16838             * else:
16839             *     try: # ««««««««
16840             *         return math.asin(obj)
16841             *     except:
16842             */
16843             /*else*/ {
16844                 {
16845                     __Pyx_PyThreadState_declare
16846                     __Pyx_PyThreadState_assign
16847                     __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
16848                     __Pyx_XGOTREF(__pyx_t_6);
16849                     __Pyx_XGOTREF(__pyx_t_7);
16850                     __Pyx_XGOTREF(__pyx_t_8);
16851                     /*try:*/ {
16852
16853                         /* "PyClical.pyx":1764
16854                         * else:
16855                         *     try:
16856                         *         return math.asin(obj) # ««««««««
16857                         *     except:
16858                         *         return clifford().wrap( glucat.asin(toClifford(obj)) )
16859                         */
16860                         __Pyx_XDECREF(__pyx_r);
16861                         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
16862                         (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1764, __pyx_L4_error)
16863                         __Pyx_GOTREF(__pyx_t_3);
16864                         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_asin); if
16865                         (unlikely(!__pyx_t_9)) __PYX_ERR(0, 1764, __pyx_L4_error)
16866                         __Pyx_GOTREF(__pyx_t_9);
16867                         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;

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16863         __pyx_t_3 = NULL;
16864         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
16865             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
16866             if (likely(__pyx_t_3)) {
16867                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
16868                 __Pyx_INCREF(__pyx_t_3);
16869                 __Pyx_INCREF(function);
16870                 __Pyx_DECREF_SET(__pyx_t_9, function);
16871             }
16872         }
16873         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
16874         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16875         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1764, __pyx_L4_error)
16876         __Pyx_GOTREF(__pyx_t_5);
16877         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16878         __pyx_r = __pyx_t_5;
16879         __pyx_t_5 = 0;
16880         goto __pyx_L8_try_return;
16881
16882         /* "PyClicl.pyx":1763
16883  *         return clifford().wrap( glucat.asin(toClifford(obj)), toClifford(i)) )
16884  *     else:
16885  *         try:
16886  *             # ««««««««
16887  *             return math.asin(obj)
16888  *         except:
16889  */
16890         }
16891         __pyx_L4_error:;
16892         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
16893         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
16894         __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
16895
16896         /* "PyClicl.pyx":1765
16897  *         try:
16898  *             return math.asin(obj)
16899  *         except:
16900  *             # ««««««««
16901  *             return clifford().wrap( glucat.asin(toClifford(obj)) )
16902  */
16903         /*except:*/ {
16904             __Pyx_AddTraceback("PyClicl.asin", __pyx_clineno, __pyx_lineno,
__pyx_filename);
16905             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1765, __pyx_L6_except_error)
16906             __Pyx_GOTREF(__pyx_t_5);
16907             __Pyx_GOTREF(__pyx_t_9);
16908             __Pyx_GOTREF(__pyx_t_3);
16909
16910             /* "PyClicl.pyx":1766
16911  *             return math.asin(obj)
16912  *         except:
16913  *             return clifford().wrap( glucat.asin(toClifford(obj)) )
16914  *             # ««««««««
16915  * cpdef inline sinh(obj):
16916  */
16917             __Pyx_XDECREF(__pyx_r);
16918             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClicl_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1766,
__pyx_L6_except_error)
16919             __Pyx_GOTREF(__pyx_t_10);
16920             __pyx_t_11 = __pyx_f_8PyClicl_8clifford_wrap(((struct
__pyx_obj_8PyClicl_clifford *)__pyx_t_10), asin(__pyx_f_8PyClicl_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1766, __pyx_L6_except_error)
16921             __Pyx_GOTREF(__pyx_t_11);
16922             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
16923             __pyx_r = __pyx_t_11;
16924             __pyx_t_11 = 0;
16925             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
16926             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
16927             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
16928             goto __pyx_L7_except_return;
16929         }
16930         __pyx_L6_except_error:;
16931
16932         /* "PyClicl.pyx":1763
16933  *         return clifford().wrap( glucat.asin(toClifford(obj)), toClifford(i)) )
16934  *     else:
16935  *         try:
16936  *             # ««««««««
16937  *             return math.asin(obj)
16938  *         except:
16939  */
16940         __Pyx_XGIVEREF(__pyx_t_6);
16941         __Pyx_XGIVEREF(__pyx_t_7);
16942         __Pyx_XGIVEREF(__pyx_t_8);
16943         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16944         goto __pyx_L1_error;

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16943         __pyx_L8_try_return;;
16944         __Pyx_XGIVEREF(__pyx_t_6);
16945         __Pyx_XGIVEREF(__pyx_t_7);
16946         __Pyx_XGIVEREF(__pyx_t_8);
16947         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16948         goto __pyx_L0;
16949         __pyx_L7_except_return;;
16950         __Pyx_XGIVEREF(__pyx_t_6);
16951         __Pyx_XGIVEREF(__pyx_t_7);
16952         __Pyx_XGIVEREF(__pyx_t_8);
16953         __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
16954         goto __pyx_L0;
16955     }
16956 }
16957
16958     /* "PyClical.pyx":1747
16959     *         return clifford().wrap( glucat.sin(toClifford(obj)) )
16960     *
16961     * cpdef inline asin(obj,i = None):                # ««««««««
16962     *         """
16963     *         Inverse sine of multivector with optional complexifier.
16964     */
16965
16966     /* function exit code */
16967     __pyx_L1_error;;
16968     __Pyx_XDECREF(__pyx_t_3);
16969     __Pyx_XDECREF(__pyx_t_5);
16970     __Pyx_XDECREF(__pyx_t_9);
16971     __Pyx_XDECREF(__pyx_t_10);
16972     __Pyx_XDECREF(__pyx_t_11);
16973     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
16974     __pyx_r = 0;
16975     __pyx_L0;;
16976     __Pyx_XGIVEREF(__pyx_r);
16977     __Pyx_RefNannyFinishContext();
16978     return __pyx_r;
16979 }
16980
16981     /* Python wrapper */
16982     static PyObject * __pyx_pw_8PyClical_67asin(PyObject * __pyx_self, PyObject * __pyx_args,
16983     PyObject * __pyx_kwds); /*proto*/
16984     static char __pyx_doc_8PyClical_66asin[] = "\n    Inverse sine of multivector with\n    optional complexifier.\n\n    >> s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),s))\n    {-1}\n    >>\n    s=\"{-1}\"; x=clifford(s); print(asin(sin(x,s),\"{-2,-1,1}\"))\n    {-1}\n    >> print(asin(1) / pi)\n    0.5\n    >> x=clifford(\"{1,2,3}\"); print(asin(sin(x)))\n    {1,2,3}\n    "
16985     static PyObject * __pyx_pw_8PyClical_67asin(PyObject * __pyx_self, PyObject * __pyx_args,
16986     PyObject * __pyx_kwds) {
16987         PyObject * __pyx_v_obj = 0;
16988         PyObject * __pyx_v_i = 0;
16989         int __pyx_lineno = 0;
16990         const char * __pyx_filename = NULL;
16991         int __pyx_clineno = 0;
16992         PyObject * __pyx_r = 0;
16993         __Pyx_RefNannyDeclarations
16994         __Pyx_RefNannySetupContext("asin (wrapper)", 0);
16995         {
16996             static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
16997             PyObject* values[2] = {0,0};
16998             values[1] = ((PyObject *)Py_None);
16999             if (unlikely(__pyx_kwds)) {
17000                 Py_ssize_t kw_args;
17001                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17002                 switch (pos_args) {
17003                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17004                         CYTHON_FALLTHROUGH;
17005                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17006                         CYTHON_FALLTHROUGH;
17007                     case 0: break;
17008                     default: goto __pyx_L5_argtuple_error;
17009                 }
17010                 kw_args = PyDict_Size(__pyx_kwds);
17011                 switch (pos_args) {
17012                     case 0:
17013                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
17014                             0)) kw_args--;
17015                         else goto __pyx_L5_argtuple_error;
17016                         CYTHON_FALLTHROUGH;
17017                     case 1:
17018                         if (kw_args > 0) {
17019                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17020                             if (value) { values[1] = value; kw_args--; }
17021                         }
17022                     if (unlikely(kw_args > 0)) {
17023                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
17024                             values, pos_args, "asin") < 0)) __PYX_ERR(0, 1747, __pyx_L3_error)
17025                     }

```

```

17023         } else {
17024             switch (PyTuple_GET_SIZE(__pyx_args)) {
17025                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17026                     CYTHON_FALLTHROUGH;
17027                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17028                     break;
17029                 default: goto __pyx_L5_argtuple_error;
17030             }
17031         }
17032         __pyx_v_obj = values[0];
17033         __pyx_v_i = values[1];
17034     }
17035     goto __pyx_L4_argument_unpacking_done;
17036     __pyx_L5_argtuple_error:;
17037     __Pyx_RaiseArgtupleInvalid("asin", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
17038     __PYX_ERR(0, 1747, __pyx_L3_error)
17039     __pyx_L3_error:;
17040     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
17041     __Pyx_RefNannyFinishContext();
17042     return NULL;
17043     __pyx_L4_argument_unpacking_done:;
17044     __pyx_r = __pyx_pf_8PyClical_66asin(__pyx_self, __pyx_v_obj, __pyx_v_i);
17045
17046     /* function exit code */
17047     __Pyx_RefNannyFinishContext();
17048     return __pyx_r;
17049 }
17050
17051 static PyObject *__pyx_pf_8PyClical_66asin(CYTHON_UNUSED PyObject *__pyx_self,
17052 PyObject *__pyx_v_obj, PyObject *__pyx_v_i) {
17053     PyObject *__pyx_r = NULL;
17054     __Pyx_RefNannyDeclarations
17055     PyObject *__pyx_t_1 = NULL;
17056     struct __pyx_opt_args_8PyClical_asin __pyx_t_2;
17057     int __pyx_lineno = 0;
17058     const char *__pyx_filename = NULL;
17059     int __pyx_clineno = 0;
17060     __Pyx_RefNannySetupContext("asin", 0);
17061     __Pyx_XDECREF(__pyx_r);
17062     __pyx_t_2.__pyx_n = 1;
17063     __pyx_t_2.i = __pyx_v_i;
17064     __pyx_t_1 = __pyx_f_8PyClical_asin(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1747, __pyx_L1_error)
17065     __Pyx_GOTREF(__pyx_t_1);
17066     __pyx_r = __pyx_t_1;
17067     __pyx_t_1 = 0;
17068     goto __pyx_L0;
17069
17070     /* function exit code */
17071     __pyx_L1_error:;
17072     __Pyx_XDECREF(__pyx_t_1);
17073     __Pyx_AddTraceback("PyClical.asin", __pyx_clineno, __pyx_lineno, __pyx_filename);
17074     __pyx_r = NULL;
17075     __pyx_L0:;
17076     __Pyx_XGIVEREF(__pyx_r);
17077     __Pyx_RefNannyFinishContext();
17078     return __pyx_r;
17079 }
17080
17081 /* "PyClical.pyx":1768
17082 *
17083 * return clifford().wrap( glucat.asin(toClifford(obj)) )
17084 *
17085 * cpdef inline sinh(obj):
17086 *     # ««««««««
17087 *     """
17088 *     Hyperbolic sine of multivector.
17089 * */
17090
17091 static PyObject *__pyx_pw_8PyClical_69sinh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
17092 static CYTHON_INLINE PyObject *__pyx_f_8PyClical_sinh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
17093     PyObject *__pyx_r = NULL;
17094     __Pyx_RefNannyDeclarations
17095     PyObject *__pyx_t_1 = NULL;
17096     PyObject *__pyx_t_2 = NULL;
17097     PyObject *__pyx_t_3 = NULL;
17098     PyObject *__pyx_t_4 = NULL;
17099     PyObject *__pyx_t_5 = NULL;
17100     PyObject *__pyx_t_6 = NULL;
17101     PyObject *__pyx_t_7 = NULL;
17102     PyObject *__pyx_t_8 = NULL;
17103     int __pyx_lineno = 0;
17104     const char *__pyx_filename = NULL;
17105     int __pyx_clineno = 0;
17106     __Pyx_RefNannySetupContext("sinh", 0);
17107
17108     /* "PyClical.pyx":1777

```

```

17105 *      0.5{1,2}
17106 *      """
17107 *      try:          # ««««««««
17108 *          return math.sinh(obj)
17109 *      except:
17110 */
17111 {
17112     __Pyx_PyThreadState_declare
17113     __Pyx_PyThreadState_assign
17114     __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
17115     __Pyx_XGOTREF(__pyx_t_1);
17116     __Pyx_XGOTREF(__pyx_t_2);
17117     __Pyx_XGOTREF(__pyx_t_3);
17118     /*try:*/ {
17119
17120         /* "PyClical.pyx":1778
17121 *      """
17122 *      try:
17123 *          return math.sinh(obj)          # ««««««««
17124 *      except:
17125 *          return clifford().wrap( glucat.sinh(toClifford(obj)) )
17126 */
17127         __Pyx_XDECREF(__pyx_r);
17128         __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1778, __pyx_L3_error)
17129         __Pyx_GOTREF(__pyx_t_5);
17130         __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_sinh); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1778, __pyx_L3_error)
17131         __Pyx_GOTREF(__pyx_t_6);
17132         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17133         __pyx_t_5 = NULL;
17134         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
17135             __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
17136             if (likely(__pyx_t_5)) {
17137                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
17138                 __Pyx_INCREF(__pyx_t_5);
17139                 __Pyx_INCREF(function);
17140                 __Pyx_DECREF_SET(__pyx_t_6, function);
17141             }
17142         }
17143         __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
17144         __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17145         if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1778, __pyx_L3_error)
17146         __Pyx_GOTREF(__pyx_t_4);
17147         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
17148         __pyx_r = __pyx_t_4;
17149         __pyx_t_4 = 0;
17150         goto __pyx_L7_try_return;
17151
17152         /* "PyClical.pyx":1777
17153 *      0.5{1,2}
17154 *      """
17155 *      try:          # ««««««««
17156 *          return math.sinh(obj)
17157 *      except:
17158 */
17159     }
17160     __pyx_L3_error:;
17161     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
17162     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17163     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
17164
17165     /* "PyClical.pyx":1779
17166 *      try:
17167 *          return math.sinh(obj)
17168 *      except:          # ««««««««
17169 *          return clifford().wrap( glucat.sinh(toClifford(obj)) )
17170 *
17171 */
17172     /*except:*/ {
17173         __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
17174         if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1779, __pyx_L5_except_error)
17175         __Pyx_GOTREF(__pyx_t_4);
17176         __Pyx_GOTREF(__pyx_t_6);
17177         __Pyx_GOTREF(__pyx_t_5);
17178
17179         /* "PyClical.pyx":1780
17180 *          return math.sinh(obj)
17181 *      except:
17182 *          return clifford().wrap( glucat.sinh(toClifford(obj)) )          # ««««««««
17183 *
17184 * cpdef inline asinh(obj,i = None):
17185 */
17186         __Pyx_XDECREF(__pyx_r);

```

```

17187         __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyCliclal_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1780,
__pyx_L5_except_error)
17188         __Pyx_GOTREF(__pyx_t_7);
17189         __pyx_t_8 = __pyx_f_8PyCliclal_8clifford_wrap(((struct
__pyx_obj_8PyCliclal_clifford *)__pyx_t_7), sinh(__pyx_f_8PyCliclal_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1780, __pyx_L5_except_error)
17190         __Pyx_GOTREF(__pyx_t_8);
17191         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
17192         __pyx_r = __pyx_t_8;
17193         __pyx_t_8 = 0;
17194         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
17195         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17196         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
17197         goto __pyx_L6_except_return;
17198     }
17199     __pyx_L5_except_error::;
17200
17201     /* "PyCliclal.pyx":1777
17202     *      0.5{1,2}
17203     *      """
17204     *      try:                # ««««««««
17205     *          return math.sinh(obj)
17206     *      except:
17207     */
17208         __Pyx_XGIVEREF(__pyx_t_1);
17209         __Pyx_XGIVEREF(__pyx_t_2);
17210         __Pyx_XGIVEREF(__pyx_t_3);
17211         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17212         goto __pyx_L1_error;
17213         __pyx_L7_try_return::;
17214         __Pyx_XGIVEREF(__pyx_t_1);
17215         __Pyx_XGIVEREF(__pyx_t_2);
17216         __Pyx_XGIVEREF(__pyx_t_3);
17217         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17218         goto __pyx_L0;
17219         __pyx_L6_except_return::;
17220         __Pyx_XGIVEREF(__pyx_t_1);
17221         __Pyx_XGIVEREF(__pyx_t_2);
17222         __Pyx_XGIVEREF(__pyx_t_3);
17223         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
17224         goto __pyx_L0;
17225     }
17226
17227     /* "PyCliclal.pyx":1768
17228     *      return clifford().wrap( glucat.asin(toClifford(obj)) )
17229     *
17230     * cpdef inline sinh(obj):                # ««««««««
17231     *      """
17232     *      Hyperbolic sine of multivector.
17233     */
17234
17235     /* function exit code */
17236     __pyx_L1_error::;
17237     __Pyx_XDECREF(__pyx_t_4);
17238     __Pyx_XDECREF(__pyx_t_5);
17239     __Pyx_XDECREF(__pyx_t_6);
17240     __Pyx_XDECREF(__pyx_t_7);
17241     __Pyx_XDECREF(__pyx_t_8);
17242     __Pyx_AddTraceback("PyCliclal.sinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17243     __pyx_r = 0;
17244     __pyx_L0::;
17245     __Pyx_XGIVEREF(__pyx_r);
17246     __Pyx_RefNannyFinishContext();
17247     return __pyx_r;
17248 }
17249
17250 /* Python wrapper */
17251 static PyObject *__pyx_pw_8PyCliclal_69sinh(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
17252 static char __pyx_doc_8PyCliclal_68sinh[] = "\n    Hyperbolic sine of multivector.\n\n
>> x=clifford(\"{1,2}\") * pi/2; print(sinh(x))\n    {1,2}\n    >> x=clifford(\"{1,2}\") * pi/6;
print(sinh(x))\n    0.5{1,2}\n    ";
17253 static PyObject *__pyx_pw_8PyCliclal_69sinh(PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
17254     PyObject *__pyx_r = 0;
17255     __Pyx_RefNannyDeclarations
17256     __Pyx_RefNannySetupContext("sinh (wrapper)", 0);
17257     __pyx_r = __pyx_pf_8PyCliclal_68sinh(__pyx_self, ((PyObject *)__pyx_v_obj));
17258
17259     /* function exit code */
17260     __Pyx_RefNannyFinishContext();
17261     return __pyx_r;
17262 }
17263
17264 static PyObject *__pyx_pf_8PyCliclal_68sinh(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {

```

```

17265         PyObject *__pyx_r = NULL;
17266         __Pyx_RefNannyDeclarations
17267         PyObject *__pyx_t_1 = NULL;
17268         int __pyx_lineno = 0;
17269         const char *__pyx_filename = NULL;
17270         int __pyx_clineno = 0;
17271         __Pyx_RefNannySetupContext("sinh", 0);
17272         __Pyx_XDECREF(__pyx_r);
17273         __pyx_t_1 = __pyx_f_8PyClical_sinh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1768, __pyx_L1_error)
17274         __Pyx_GOTREF(__pyx_t_1);
17275         __pyx_r = __pyx_t_1;
17276         __pyx_t_1 = 0;
17277         goto __pyx_L0;
17278
17279         /* function exit code */
17280         __pyx_L1_error:;
17281         __Pyx_XDECREF(__pyx_t_1);
17282         __Pyx_AddTraceback("PyClical.sinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17283         __pyx_r = NULL;
17284         __pyx_L0:;
17285         __Pyx_XGIVEREF(__pyx_r);
17286         __Pyx_RefNannyFinishContext();
17287         return __pyx_r;
17288     }
17289
17290     /* "PyClical.pyx":1782
17291     *         return clifford().wrap( glucat.sinh(toClifford(obj)) )
17292     *
17293     * cpdef inline asinh(obj,i = None):          # ««««««
17294     *     """
17295     *     Inverse hyperbolic sine of multivector with optional complexifier.
17296     */
17297
17298     static PyObject *__pyx_pw_8PyClical_7lasinh(PyObject *__pyx_self, PyObject
__pyx_args, PyObject *__pyx_kwds); /*proto*/
17299     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_asinh(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_asinh *__pyx_optional_args) {
17300         PyObject *__pyx_v_i = ((PyObject *)Py_None);
17301         PyObject *__pyx_r = NULL;
17302         __Pyx_RefNannyDeclarations
17303         int __pyx_t_1;
17304         int __pyx_t_2;
17305         PyObject *__pyx_t_3 = NULL;
17306         Clifford __pyx_t_4;
17307         PyObject *__pyx_t_5 = NULL;
17308         PyObject *__pyx_t_6 = NULL;
17309         PyObject *__pyx_t_7 = NULL;
17310         PyObject *__pyx_t_8 = NULL;
17311         PyObject *__pyx_t_9 = NULL;
17312         PyObject *__pyx_t_10 = NULL;
17313         PyObject *__pyx_t_11 = NULL;
17314         int __pyx_lineno = 0;
17315         const char *__pyx_filename = NULL;
17316         int __pyx_clineno = 0;
17317         __Pyx_RefNannySetupContext("asinh", 0);
17318         if (__pyx_optional_args) {
17319             if (__pyx_optional_args->__pyx_n > 0) {
17320                 __pyx_v_i = __pyx_optional_args->i;
17321             }
17322         }
17323
17324         /* "PyClical.pyx":1793
17325     *     {1,2}
17326     *     """
17327     *     if not (i is None):          # ««««««
17328     *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17329     *     else:
17330     */
17331         __pyx_t_1 = (__pyx_v_i != Py_None);
17332         __pyx_t_2 = (__pyx_t_1 != 0);
17333         if (__pyx_t_2) {
17334
17335             /* "PyClical.pyx":1794
17336     *     """
17337     *     if not (i is None):
17338     *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17339     *     else:
17340     *         try:
17341     */
17342         __Pyx_XDECREF(__pyx_r);
17343         __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1794, __pyx_L1_error)
17344         __Pyx_GOTREF(__pyx_t_3);
17345         try {
17346             __pyx_t_4 = asinh(__pyx_f_8PyClical_toClifford(__pyx_v_obj),

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__pyx_f_8PyClical_toClifford(__pyx_v_i));
17347         } catch(...) {
17348             __Pyx_CppExn2PyErr();
17349             __PYX_ERR(0, 1794, __pyx_L1_error)
17350         }
17351         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1794, __pyx_L1_error)
17352         __Pyx_GOTREF(__pyx_t_5);
17353         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17354         __pyx_r = __pyx_t_5;
17355         __pyx_t_5 = 0;
17356         goto __pyx_L0;
17357
17358         /* "PyClical.pyx":1793
17359         *
17360         * """
17361         * if not (i is None):
17362         *     return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17363         * else:
17364         */
17365         }
17366
17367         /* "PyClical.pyx":1796
17368         *     return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17369         * else:
17370         *     try:
17371         *         return math.asinh(obj)
17372         *     except:
17373         */
17374         /*else*/ {
17375             {
17376                 __Pyx_PyThreadState_declare
17377                 __Pyx_PyThreadState_assign
17378                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
17379                 __Pyx_XGOTREF(__pyx_t_6);
17380                 __Pyx_XGOTREF(__pyx_t_7);
17381                 __Pyx_XGOTREF(__pyx_t_8);
17382                 /*try:*/ {
17383
17384                 /* "PyClical.pyx":1797
17385                 *     else:
17386                 *         try:
17387                 *             return math.asinh(obj)
17388                 *         except:
17389                 *             return clifford().wrap( glucat.asinh(toClifford(obj)) )
17390                 */
17391                 __Pyx_XDECREF(__pyx_r);
17392                 __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1797, __pyx_L4_error)
17393                 __Pyx_GOTREF(__pyx_t_3);
17394                 __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_asinh); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1797, __pyx_L4_error)
17395                 __Pyx_GOTREF(__pyx_t_9);
17396                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17397                 __pyx_t_3 = NULL;
17398                 if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
17399                     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
17400                     if (likely(__pyx_t_3)) {
17401                         PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
17402                         __Pyx_INCREF(__pyx_t_3);
17403                         __Pyx_INCREF(function);
17404                         __Pyx_DECREF_SET(__pyx_t_9, function);
17405                     }
17406                 }
17407                 __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
17408                 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17409                 if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1797, __pyx_L4_error)
17410                 __Pyx_GOTREF(__pyx_t_5);
17411                 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17412                 __pyx_r = __pyx_t_5;
17413                 __pyx_t_5 = 0;
17414                 goto __pyx_L8_try_return;
17415
17416                 /* "PyClical.pyx":1796
17417                 *     return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17418                 * else:
17419                 *     try:
17420                 *         return math.asinh(obj)
17421                 *     except:
17422                 */
17423                 }
17424                 __pyx_L4_error:;
17425                 __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17426                 __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17427                 __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
17428

```



```

17429             /* "PyClical.pyx":1798
17430 *         try:
17431 *             return math.asinh(obj)
17432 *         except:
17433 *             # ««««««««
17434 *             return clifford().wrap( glucat.asinh(toClifford(obj)) )
17435 */
17436
17437             /*except:*/ {
17438                 __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
17439                 if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
17440 1798, __pyx_L6_except_error)
17441                 __Pyx_GOTREF(__pyx_t_5);
17442                 __Pyx_GOTREF(__pyx_t_9);
17443                 __Pyx_GOTREF(__pyx_t_3);
17444             /* "PyClical.pyx":1799
17445 *         return math.asinh(obj)
17446 *         except:
17447 *             return clifford().wrap( glucat.asinh(toClifford(obj)) )
17448 *             # ««««««««
17449 * cpdef inline tan(obj,i = None):
17450 */
17451                 __Pyx_XDECREF(__pyx_r);
17452                 __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1799,
__pyx_L6_except_error)
17453                 __Pyx_GOTREF(__pyx_t_10);
17454                 __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), asinh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1799, __pyx_L6_except_error)
17455                 __Pyx_GOTREF(__pyx_t_11);
17456                 __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
17457                 __pyx_r = __pyx_t_11;
17458                 __pyx_t_11 = 0;
17459                 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17460                 __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17461                 __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17462                 goto __pyx_L7_except_return;
17463             }
17464             __pyx_L6_except_error;
17465
17466             /* "PyClical.pyx":1796
17467 *         return clifford().wrap( glucat.asinh(toClifford(obj), toClifford(i)) )
17468 *         else:
17469 *             try:
17470 *                 # ««««««««
17471 *                 return math.asinh(obj)
17472 *             except:
17473 *                 # ««««««««
17474 *                 return clifford().wrap( glucat.asinh(toClifford(obj)) )
17475 */
17476                 __Pyx_XDECREF(__pyx_t_6);
17477                 __Pyx_XDECREF(__pyx_t_7);
17478                 __Pyx_XDECREF(__pyx_t_8);
17479                 __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17480                 goto __pyx_L1_error;
17481                 __pyx_L8_try_return;
17482                 __Pyx_XDECREF(__pyx_t_6);
17483                 __Pyx_XDECREF(__pyx_t_7);
17484                 __Pyx_XDECREF(__pyx_t_8);
17485                 __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17486                 goto __pyx_L0;
17487                 __pyx_L7_except_return;
17488                 __Pyx_XDECREF(__pyx_t_6);
17489                 __Pyx_XDECREF(__pyx_t_7);
17490                 __Pyx_XDECREF(__pyx_t_8);
17491                 __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17492                 goto __pyx_L0;
17493             }
17494
17495             /* "PyClical.pyx":1782
17496 *         return clifford().wrap( glucat.sinh(toClifford(obj)) )
17497 *         cpdef inline asinh(obj,i = None):
17498 *             # ««««««««
17499 *             """
17500 *             Inverse hyperbolic sine of multivector with optional complexifier.
17501 */
17502                 /* function exit code */
17503                 __Pyx_L1_error;
17504                 __Pyx_XDECREF(__pyx_t_3);
17505                 __Pyx_XDECREF(__pyx_t_5);
17506                 __Pyx_XDECREF(__pyx_t_9);
17507                 __Pyx_XDECREF(__pyx_t_10);
17508                 __Pyx_XDECREF(__pyx_t_11);
17509                 __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17510                 __pyx_r = 0;
17511                 __pyx_L0;

```

```

17510         __Pyx_XGIVEREF(__pyx_r);
17511         __Pyx_RefNannyFinishContext();
17512         return __pyx_r;
17513     }
17514
17515     /* Python wrapper */
17516     static PyObject * __pyx_pw_8PyClical_7lasinh(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kwds); /*proto*/
17517     static char __pyx_doc_8PyClical_70asinh[] = "\n    Inverse hyperbolic sine of
multivector with optional complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(asinh(x,\"{1,2,3}\") *
2/pi)\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print(asinh(x) * 2/pi)\n    {1,2}\n    >>
x=clifford(\"{1,2}\") / 2; print(asinh(x) * 6/pi)\n    {1,2}\n    ";
17518     static PyObject * __pyx_pw_8PyClical_7lasinh(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kwds) {
17519         PyObject * __pyx_v_obj = 0;
17520         PyObject * __pyx_v_i = 0;
17521         int __pyx_lineno = 0;
17522         const char * __pyx_filename = NULL;
17523         int __pyx_clineno = 0;
17524         PyObject * __pyx_r = 0;
17525         __Pyx_RefNannyDeclarations
17526         __Pyx_RefNannySetupContext("asinh (wrapper)", 0);
17527         {
17528             static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
17529             PyObject* values[2] = {0,0};
17530             values[1] = ((PyObject *)Py_None);
17531             if (unlikely(__pyx_kwds)) {
17532                 Py_ssize_t kw_args;
17533                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17534                 switch (pos_args) {
17535                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
17536                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
17537                     case 0: break;
17538                     default: goto __pyx_L5_argtuple_error;
17539                 }
17540                 kw_args = PyDict_Size(__pyx_kwds);
17541                 switch (pos_args) {
17542                     case 0:
17543                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
17544 0)) kw_args--;
17545                         else goto __pyx_L5_argtuple_error;
CYTHON_FALLTHROUGH;
17546                     case 1:
17547                         if (kw_args > 0) {
17548                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17549                             if (value) { values[1] = value; kw_args--; }
17550                         }
17551                     if (unlikely(kw_args > 0)) {
17552                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "asinh") < 0)) __PYX_ERR(0, 1782, __pyx_L3_error)
17553                     }
17554                     else {
17555                         switch (PyTuple_GET_SIZE(__pyx_args)) {
17556                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
17557                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
break;
17558                             default: goto __pyx_L5_argtuple_error;
17559                         }
17560                     }
17561                     __pyx_v_obj = values[0];
17562                     __pyx_v_i = values[1];
17563                 }
17564                 goto __pyx_L4_argument_unpacking_done;
17565                 __pyx_L5_argtuple_error:;
17566                 __Pyx_RaiseArgtupleInvalid("asinh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
17567                 __PYX_ERR(0, 1782, __pyx_L3_error)
17568                 __pyx_L3_error:;
17569                 __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17570                 __Pyx_RefNannyFinishContext();
17571                 return NULL;
17572                 __pyx_L4_argument_unpacking_done:;
17573                 __pyx_r = __pyx_pf_8PyClical_70asinh(__pyx_self, __pyx_v_obj, __pyx_v_i);
17574
17575                 /* function exit code */
17576                 __Pyx_RefNannyFinishContext();
17577                 return __pyx_r;
17578             }
17579
17580             static PyObject * __pyx_pf_8PyClical_70asinh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj, PyObject * __pyx_v_i) {
17581                 PyObject * __pyx_r = NULL;
17582                 __Pyx_RefNannyDeclarations
17583                 PyObject * __pyx_t_1 = NULL;

```

```

17588         struct __pyx_opt_args_8PyClical_asinh __pyx_t_2;
17589         int __pyx_lineno = 0;
17590         const char *__pyx_filename = NULL;
17591         int __pyx_clineno = 0;
17592         __Pyx_RefNannySetupContext("asinh", 0);
17593         __Pyx_XDECREF(__pyx_r);
17594         __pyx_t_2.__pyx_n = 1;
17595         __pyx_t_2.i = __pyx_v_i;
17596         __pyx_t_1 = __pyx_f_8PyClical_asinh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1782, __pyx_L1_error)
17597         __Pyx_GOTREF(__pyx_t_1);
17598         __pyx_r = __pyx_t_1;
17599         __pyx_t_1 = 0;
17600         goto __pyx_L0;
17601
17602         /* function exit code */
17603         __pyx_L1_error:;
17604         __Pyx_XDECREF(__pyx_t_1);
17605         __Pyx_AddTraceback("PyClical.asinh", __pyx_clineno, __pyx_lineno, __pyx_filename);
17606         __pyx_r = NULL;
17607         __pyx_L0:;
17608         __Pyx_XGIVEREF(__pyx_r);
17609         __Pyx_RefNannyFinishContext();
17610         return __pyx_r;
17611     }
17612
17613     /* "PyClical.pyx":1801
17614     *
17615     * return clifford().wrap( glucat.asinh(toClifford(obj)) )
17616     *
17617     * cpdef inline tan(obj,i = None):
17618     *     # ««««««««
17619     *     """
17620     *     Tangent of multivector with optional complexifier.
17621     */
17622     static PyObject *__pyx_pw_8PyClical_73tan(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
17623     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_tan(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_tan *__pyx_optional_args) {
17624         PyObject *__pyx_v_i = ((PyObject *)Py_None);
17625         PyObject *__pyx_r = NULL;
17626         __Pyx_RefNannyDeclarations
17627         int __pyx_t_1;
17628         int __pyx_t_2;
17629         PyObject *__pyx_t_3 = NULL;
17630         Clifford __pyx_t_4;
17631         PyObject *__pyx_t_5 = NULL;
17632         PyObject *__pyx_t_6 = NULL;
17633         PyObject *__pyx_t_7 = NULL;
17634         PyObject *__pyx_t_8 = NULL;
17635         PyObject *__pyx_t_9 = NULL;
17636         PyObject *__pyx_t_10 = NULL;
17637         PyObject *__pyx_t_11 = NULL;
17638         int __pyx_lineno = 0;
17639         const char *__pyx_filename = NULL;
17640         int __pyx_clineno = 0;
17641         __Pyx_RefNannySetupContext("tan", 0);
17642         if (__pyx_optional_args) {
17643             if (__pyx_optional_args->__pyx_n > 0) {
17644                 __pyx_v_i = __pyx_optional_args->i;
17645             }
17646         }
17647
17648         /* "PyClical.pyx":1810
17649         *
17650         * if not (i is None):
17651         *     return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17652         * else:
17653         */
17654         __pyx_t_1 = (__pyx_v_i != Py_None);
17655         __pyx_t_2 = (__pyx_t_1 != 0);
17656         if (__pyx_t_2) {
17657
17658             /* "PyClical.pyx":1811
17659             *
17660             * if not (i is None):
17661             *     return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17662             * else:
17663             *     try:
17664             */
17665             __Pyx_XDECREF(__pyx_r);
17666             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1811, __pyx_L1_error)
17667             __Pyx_GOTREF(__pyx_t_3);
17668             try {
17669                 __pyx_t_4 = tan(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));

```

```

17670         } catch(...) {
17671             __Pyx_CppExn2PyErr();
17672             __PYX_ERR(0, 1811, __pyx_L1_error)
17673         }
17674         __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1811, __pyx_L1_error)
17675         __Pyx_GOTREF(__pyx_t_5);
17676         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17677         __pyx_r = __pyx_t_5;
17678         __pyx_t_5 = 0;
17679         goto __pyx_L0;
17680
17681         /* "PyClical.pyx":1810
17682  *      0.7616{1,2}
17683  *      """
17684  *      if not (i is None):          # ««««««««
17685  *          return clifford().wrap( glucat.tan(toClifford(obj)), toClifford(i)) )
17686  *      else:
17687  */
17688     }
17689
17690     /* "PyClical.pyx":1813
17691  *      return clifford().wrap( glucat.tan(toClifford(obj)), toClifford(i)) )
17692  *      else:
17693  *          try:          # ««««««««
17694  *              return math.tan(obj)
17695  *          except:
17696  */
17697         /*else*/ {
17698             {
17699                 __Pyx_PyThreadState_declare
17700                 __Pyx_PyThreadState_assign
17701                 __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
17702                 __Pyx_XGOTREF(__pyx_t_6);
17703                 __Pyx_XGOTREF(__pyx_t_7);
17704                 __Pyx_XGOTREF(__pyx_t_8);
17705                 /*try:*/ {
17706
17707                     /* "PyClical.pyx":1814
17708  *          else:
17709  *              try:
17710  *                  return math.tan(obj)          # ««««««««
17711  *              except:
17712  *                  return clifford().wrap( glucat.tan(toClifford(obj)) )
17713  */
17714                     __Pyx_XDECREF(__pyx_r);
17715                     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1814, __pyx_L4_error)
17716                     __Pyx_GOTREF(__pyx_t_3);
17717                     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_tan); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1814, __pyx_L4_error)
17718                     __Pyx_GOTREF(__pyx_t_9);
17719                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17720                     __pyx_t_3 = NULL;
17721                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
17722                         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
17723                         if (likely(__pyx_t_3)) {
17724                             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
17725                             __Pyx_INCREF(__pyx_t_3);
17726                             __Pyx_INCREF(function);
17727                             __Pyx_DECREF_SET(__pyx_t_9, function);
17728                         }
17729                     }
17730                     __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
17731                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17732                     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1814, __pyx_L4_error)
17733                     __Pyx_GOTREF(__pyx_t_5);
17734                     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17735                     __pyx_r = __pyx_t_5;
17736                     __pyx_t_5 = 0;
17737                     goto __pyx_L8_try_return;
17738
17739                     /* "PyClical.pyx":1813
17740  *          return clifford().wrap( glucat.tan(toClifford(obj)), toClifford(i)) )
17741  *          else:
17742  *              try:          # ««««««««
17743  *                  return math.tan(obj)
17744  *              except:
17745  */
17746                     }
17747                     __pyx_L4_error:;
17748                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
17749                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
17750                     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
17751
17752                     /* "PyClical.pyx":1815

```

```

17753 *      try:
17754 *          return math.tan(obj)
17755 *      except: # ««««««««
17756 *          return clifford().wrap( glucat.tan(toClifford(obj)) )
17757 *
17758 */
17759
17760 /*except:*/ {
17761     __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno,
17762         __pyx_filename);
17763     if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
17764 1815, __pyx_L6_except_error)
17765     __Pyx_GOTREF(__pyx_t_5);
17766     __Pyx_GOTREF(__pyx_t_9);
17767     __Pyx_GOTREF(__pyx_t_3);
17768
17769     /* "PyClical.pyx":1816
17770 *      return math.tan(obj)
17771 *      except:
17772 *          return clifford().wrap( glucat.tan(toClifford(obj)) ) # ««««««««
17773 *
17774 * cpdef inline atan(obj,i = None):
17775 */
17776     __Pyx_XDECREF(__pyx_r);
17777     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
17778 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1816,
17779 __pyx_L6_except_error)
17780     __Pyx_GOTREF(__pyx_t_10);
17781     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
17782 __pyx_obj_8PyClical_clifford *)__pyx_t_10), tan(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
17783 (unlikely(!__pyx_t_11)) __PYX_ERR(0, 1816, __pyx_L6_except_error)
17784     __Pyx_GOTREF(__pyx_t_11);
17785     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
17786     __pyx_r = __pyx_t_11;
17787     __pyx_t_11 = 0;
17788     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
17789     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
17790     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
17791     goto __pyx_L7_except_return;
17792 }
17793 __pyx_L6_except_error:;
17794
17795 /* "PyClical.pyx":1813
17796 *      return clifford().wrap( glucat.tan(toClifford(obj), toClifford(i)) )
17797 *
17798 * else:
17799 *      try: # ««««««««
17800 *          return math.tan(obj)
17801 *      except:
17802 */
17803     __Pyx_XGIVEREF(__pyx_t_6);
17804     __Pyx_XGIVEREF(__pyx_t_7);
17805     __Pyx_XGIVEREF(__pyx_t_8);
17806     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17807     goto __pyx_L1_error;
17808     __pyx_L8_try_return:;
17809     __Pyx_XGIVEREF(__pyx_t_6);
17810     __Pyx_XGIVEREF(__pyx_t_7);
17811     __Pyx_XGIVEREF(__pyx_t_8);
17812     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17813     goto __pyx_L0;
17814     __pyx_L7_except_return:;
17815     __Pyx_XGIVEREF(__pyx_t_6);
17816     __Pyx_XGIVEREF(__pyx_t_7);
17817     __Pyx_XGIVEREF(__pyx_t_8);
17818     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
17819     goto __pyx_L0;
17820 }
17821 }
17822
17823 /* "PyClical.pyx":1801
17824 *      return clifford().wrap( glucat.asinh(toClifford(obj)) )
17825 *
17826 * cpdef inline tan(obj,i = None): # ««««««««
17827 *      """
17828 *      Tangent of multivector with optional complexifier.
17829 */
17830
17831 /* function exit code */
17832 __pyx_L1_error:;
17833 __Pyx_XDECREF(__pyx_t_3);
17834 __Pyx_XDECREF(__pyx_t_5);
17835 __Pyx_XDECREF(__pyx_t_9);
17836 __Pyx_XDECREF(__pyx_t_10);
17837 __Pyx_XDECREF(__pyx_t_11);
17838 __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17839 __pyx_r = 0;
17840 __pyx_L0:;
17841 __Pyx_XGIVEREF(__pyx_r);

```

```

17834         __Pyx_RefNannyFinishContext();
17835         return __pyx_r;
17836     }
17837
17838     /* Python wrapper */
17839     static PyObject *__pyx_pw_8PyClical_73tan(PyObject *__pyx_self, PyObject *__pyx_args,
17840     PyObject *__pyx_kwds); /*proto*/
17841     static char __pyx_doc_8PyClical_72tan[] = "\n    Tangent of multivector with optional
17842     complexifier.\n\n    >> x=clifford(\"{1,2}\"); print(tan(x,\"{1,2,3}\"))\n    0.7616{1,2}\n    >>
17843     x=clifford(\"{1,2}\"); print(tan(x))\n    0.7616{1,2}\n    ";
17844     static PyObject *__pyx_pw_8PyClical_73tan(PyObject *__pyx_self, PyObject *__pyx_args,
17845     PyObject *__pyx_kwds) {
17846         PyObject *__pyx_v_obj = 0;
17847         PyObject *__pyx_v_i = 0;
17848         int __pyx_lineno = 0;
17849         const char *__pyx_filename = NULL;
17850         int __pyx_clineno = 0;
17851         PyObject *__pyx_r = 0;
17852         __Pyx_RefNannyDeclarations
17853         __Pyx_RefNannySetupContext("tan (wrapper)", 0);
17854         {
17855             static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
17856             PyObject* values[2] = {0,0};
17857             values[1] = (PyObject *)Py_None;
17858             if (unlikely(__pyx_kwds)) {
17859                 Py_ssize_t kw_args;
17860                 const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
17861                 switch (pos_args) {
17862                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17863                     CYTHON_FALLTHROUGH;
17864                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17865                     CYTHON_FALLTHROUGH;
17866                     case 0: break;
17867                     default: goto __pyx_L5_argtuple_error;
17868                 }
17869                 kw_args = PyDict_Size(__pyx_kwds);
17870                 switch (pos_args) {
17871                     case 0:
17872                         if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
17873     0)) kw_args--;
17874                         else goto __pyx_L5_argtuple_error;
17875                         CYTHON_FALLTHROUGH;
17876                     case 1:
17877                         if (kw_args > 0) {
17878                             PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
17879                             if (value) { values[1] = value; kw_args--; }
17880                         }
17881                         if (unlikely(kw_args > 0)) {
17882                             if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
17883     values, pos_args, "tan") < 0)) __PYX_ERR(0, 1801, __pyx_L3_error)
17884                         }
17885                     } else {
17886                         switch (PyTuple_GET_SIZE(__pyx_args)) {
17887                             case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
17888                             CYTHON_FALLTHROUGH;
17889                             case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
17890                             break;
17891                             default: goto __pyx_L5_argtuple_error;
17892                         }
17893                     }
17894                 __pyx_v_obj = values[0];
17895                 __pyx_v_i = values[1];
17896             }
17897             goto __pyx_L4_argument_unpacking_done;
17898             __pyx_L5_argtuple_error:;
17899             __Pyx_RaiseArgtupleInvalid("tan", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
17900             __PYX_ERR(0, 1801, __pyx_L3_error)
17901             __pyx_L3_error:;
17902             __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17903             __Pyx_RefNannyFinishContext();
17904             return NULL;
17905             __pyx_L4_argument_unpacking_done:;
17906             __pyx_r = __pyx_pf_8PyClical_72tan(__pyx_self, __pyx_v_obj, __pyx_v_i);
17907
17908             /* function exit code */
17909             __Pyx_RefNannyFinishContext();
17910             return __pyx_r;
17911         }
17912
17913         static PyObject *__pyx_pf_8PyClical_72tan(CYTHON_UNUSED PyObject *__pyx_self, PyObject
17914     *__pyx_v_obj, PyObject *__pyx_v_i) {
17915             PyObject *__pyx_r = NULL;
17916             __Pyx_RefNannyDeclarations
17917             PyObject *__pyx_t_1 = NULL;
17918             struct __pyx_opt_args_8PyClical_tan __pyx_t_2;
17919             int __pyx_lineno = 0;

```

```

17913         const char *__pyx_filename = NULL;
17914         int __pyx_clineno = 0;
17915         __Pyx_RefNannySetupContext("tan", 0);
17916         __Pyx_XDECREF(__pyx_r);
17917         __pyx_t_2.__pyx_n = 1;
17918         __pyx_t_2.i = __pyx_v_i;
17919         __pyx_t_1 = __pyx_f_8PyClical_tan(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1801, __pyx_L1_error)
17920         __Pyx_GOTREF(__pyx_t_1);
17921         __pyx_r = __pyx_t_1;
17922         __pyx_t_1 = 0;
17923         goto __pyx_L0;
17924
17925         /* function exit code */
17926         __pyx_L1_error:;
17927         __Pyx_XDECREF(__pyx_t_1);
17928         __Pyx_AddTraceback("PyClical.tan", __pyx_clineno, __pyx_lineno, __pyx_filename);
17929         __pyx_r = NULL;
17930         __pyx_L0:;
17931         __Pyx_XGIVEREF(__pyx_r);
17932         __Pyx_RefNannyFinishContext();
17933         return __pyx_r;
17934     }
17935
17936     /* "PyClical.pyx":1818
17937     *
17938     *
17939     * cpdef inline atan(obj, i = None):          # ««««««««
17940     *     """
17941     *     Inverse tangent of multivector with optional complexifier.
17942     */
17943
17944     static PyObject *__pyx_pw_8PyClical_75atan(PyObject *__pyx_self, PyObject *__pyx_args,
PyObject *__pyx_kwds); /*proto*/
17945     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_atan(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_atan *__pyx_optional_args) {
17946         PyObject *__pyx_v_i = (PyObject *)Py_None;
17947         PyObject *__pyx_r = NULL;
17948         __Pyx_RefNannyDeclarations
17949         int __pyx_t_1;
17950         int __pyx_t_2;
17951         PyObject *__pyx_t_3 = NULL;
17952         Clifford __pyx_t_4;
17953         PyObject *__pyx_t_5 = NULL;
17954         PyObject *__pyx_t_6 = NULL;
17955         PyObject *__pyx_t_7 = NULL;
17956         PyObject *__pyx_t_8 = NULL;
17957         PyObject *__pyx_t_9 = NULL;
17958         PyObject *__pyx_t_10 = NULL;
17959         PyObject *__pyx_t_11 = NULL;
17960         int __pyx_lineno = 0;
17961         const char *__pyx_filename = NULL;
17962         int __pyx_clineno = 0;
17963         __Pyx_RefNannySetupContext("atan", 0);
17964         if (__pyx_optional_args) {
17965             if (__pyx_optional_args->__pyx_n > 0) {
17966                 __pyx_v_i = __pyx_optional_args->i;
17967             }
17968         }
17969
17970         /* "PyClical.pyx":1827
17971         * {1}
17972         * """
17973         * if not (i is None):          # ««««««««
17974         *     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
17975         * else:
17976         */
17977         __pyx_t_1 = (__pyx_v_i != Py_None);
17978         __pyx_t_2 = (__pyx_t_1 != 0);
17979         if (__pyx_t_2) {
17980
17981             /* "PyClical.pyx":1828
17982             * """
17983             * if not (i is None):
17984             *     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )          # ««««««««
17985             * else:
17986             *     try:
17987             */
17988             __Pyx_XDECREF(__pyx_r);
17989             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1828, __pyx_L1_error)
17990             __Pyx_GOTREF(__pyx_t_3);
17991             try {
17992                 __pyx_t_4 = atan(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
__pyx_f_8PyClical_toClifford(__pyx_v_i));
17993             } catch (...) {
17994                 __Pyx_CppExn2PyErr();

```

```

17995         __PYX_ERR(0, 1828, __pyx_L1_error)
17996     }
17997     __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1828, __pyx_L1_error)
17998     __Pyx_GOTREF(__pyx_t_5);
17999     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18000     __pyx_r = __pyx_t_5;
18001     __pyx_t_5 = 0;
18002     goto __pyx_L0;
18003
18004     /* "PyClical.pyx":1827
18005  *
18006  * """
18007  * if not (i is None): # ««««««
18008  *     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
18009  * else:
18010  */
18011     }
18012
18013     /* "PyClical.pyx":1830
18014  *     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
18015  * else:
18016  *     try: # ««««««
18017  *         return math.atan(obj)
18018  *     except:
18019  */
18020     /*else*/ {
18021     {
18022         __Pyx_PyThreadState_declare
18023         __Pyx_PyThreadState_assign
18024         __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
18025         __Pyx_XGOTREF(__pyx_t_6);
18026         __Pyx_XGOTREF(__pyx_t_7);
18027         __Pyx_XGOTREF(__pyx_t_8);
18028         /*try:*/ {
18029
18030         /* "PyClical.pyx":1831
18031  *     else:
18032  *     try:
18033  *         return math.atan(obj) # ««««««
18034  *     except:
18035  *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18036  */
18037         __Pyx_XDECREF(__pyx_r);
18038         __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1831, __pyx_L4_error)
18039         __Pyx_GOTREF(__pyx_t_3);
18040         __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_atan); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1831, __pyx_L4_error)
18041         __Pyx_GOTREF(__pyx_t_9);
18042         __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18043         __pyx_t_3 = NULL;
18044         if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
18045             __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
18046             if (likely(__pyx_t_3)) {
18047                 PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
18048                 __Pyx_INCREF(__pyx_t_3);
18049                 __Pyx_INCREF(function);
18050                 __Pyx_DECREF_SET(__pyx_t_9, function);
18051             }
18052         }
18053         __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
18054         __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18055         if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1831, __pyx_L4_error)
18056         __Pyx_GOTREF(__pyx_t_5);
18057         __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18058         __pyx_r = __pyx_t_5;
18059         __pyx_t_5 = 0;
18060         goto __pyx_L8_try_return;
18061
18062     /* "PyClical.pyx":1830
18063  *     return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
18064  * else:
18065  *     try: # ««««««
18066  *         return math.atan(obj)
18067  *     except:
18068  */
18069     }
18070     __pyx_L4_error:;
18071     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18072     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18073     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
18074
18075     /* "PyClical.pyx":1832
18076  *     try:
18077  *         return math.atan(obj)

```



```

18078 *         except:             # ««««««««
18079 *             return clifford().wrap( glucat.atan(toClifford(obj)) )
18080 *
18081 */
18082
18083         /*except:*/ {
18084             __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18085             if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1832, __pyx_L6_except_error)
18086                 __Pyx_GOTREF(__pyx_t_5);
18087                 __Pyx_GOTREF(__pyx_t_9);
18088                 __Pyx_GOTREF(__pyx_t_3);
18089
18090             /* "PyClical.pyx":1833
18091 *             return math.atan(obj)
18092 *         except:
18093 *             return clifford().wrap( glucat.atan(toClifford(obj)) )           # ««««««««
18094 * cpdef inline tanh(obj):
18095 */
18096             __Pyx_XDECREF(__pyx_r);
18097             __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1833,
__pyx_L6_except_error)
18098             __Pyx_GOTREF(__pyx_t_10);
18099             __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), atan(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1833, __pyx_L6_except_error)
18100             __Pyx_GOTREF(__pyx_t_11);
18101             __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
18102             __pyx_r = __pyx_t_11;
18103             __pyx_t_11 = 0;
18104             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18105             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18106             __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18107             goto __pyx_L7_except_return;
18108         }
18109         __pyx_L6_except_error;;
18110
18111         /* "PyClical.pyx":1830
18112 *         return clifford().wrap( glucat.atan(toClifford(obj), toClifford(i)) )
18113 *     else:
18114 *         try:             # ««««««««
18115 *             return math.atan(obj)
18116 *         except:
18117 */
18118             __Pyx_XGIVEREF(__pyx_t_6);
18119             __Pyx_XGIVEREF(__pyx_t_7);
18120             __Pyx_XGIVEREF(__pyx_t_8);
18121             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18122             goto __pyx_L1_error;
18123             __pyx_L8_try_return:;
18124             __Pyx_XGIVEREF(__pyx_t_6);
18125             __Pyx_XGIVEREF(__pyx_t_7);
18126             __Pyx_XGIVEREF(__pyx_t_8);
18127             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18128             goto __pyx_L0;
18129             __pyx_L7_except_return:;
18130             __Pyx_XGIVEREF(__pyx_t_6);
18131             __Pyx_XGIVEREF(__pyx_t_7);
18132             __Pyx_XGIVEREF(__pyx_t_8);
18133             __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18134             goto __pyx_L0;
18135         }
18136     }
18137
18138     /* "PyClical.pyx":1818
18139 *     return clifford().wrap( glucat.tan(toClifford(obj)) )
18140 *
18141 * cpdef inline atan(obj,i = None):             # ««««««««
18142 *     """
18143 *     Inverse tangent of multivector with optional complexifier.
18144 */
18145
18146     /* function exit code */
18147     __pyx_L1_error:;
18148     __Pyx_XDECREF(__pyx_t_3);
18149     __Pyx_XDECREF(__pyx_t_5);
18150     __Pyx_XDECREF(__pyx_t_9);
18151     __Pyx_XDECREF(__pyx_t_10);
18152     __Pyx_XDECREF(__pyx_t_11);
18153     __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18154     __pyx_r = 0;
18155     __pyx_L0:;
18156     __Pyx_XGIVEREF(__pyx_r);
18157     __Pyx_RefNannyFinishContext();
18158     return __pyx_r;

```

```

18159         }
18160
18161         /* Python wrapper */
18162         static PyObject * __pyx_pw_8PyClical_75atan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds); /*proto*/
18163         static char __pyx_doc_8PyClical_74atan[] = "\n    Inverse tangent of multivector with
optional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford(\"{1}\"); print(tan(atan(x,s),s))\n
{1}\n    >> x=clifford(\"{1}\"); print(tan(atan(x)))\n    {1}\n    ";
18164         static PyObject * __pyx_pw_8PyClical_75atan(PyObject * __pyx_self, PyObject * __pyx_args,
PyObject * __pyx_kwds) {
18165             PyObject * __pyx_v_obj = 0;
18166             PyObject * __pyx_v_i = 0;
18167             int __pyx_lineno = 0;
18168             const char * __pyx_filename = NULL;
18169             int __pyx_clineno = 0;
18170             PyObject * __pyx_r = 0;
18171             __Pyx_RefNannyDeclarations
18172             __Pyx_RefNannySetupContext("atan (wrapper)", 0);
18173             {
18174                 static PyObject ** __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
18175                 PyObject* values[2] = {0,0};
18176                 values[1] = ((PyObject *)Py_None);
18177                 if (unlikely(__pyx_kwds)) {
18178                     Py_ssize_t kw_args;
18179                     const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18180                     switch (pos_args) {
18181                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18182                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
18183                         case 0: break;
18184                         default: goto __pyx_L5_argtuple_error;
18185                     }
18186                     kw_args = PyDict_Size(__pyx_kwds);
18187                     switch (pos_args) {
18188                         case 0:
18189                             if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
18190
0)) kw_args--;
18192                             else goto __pyx_L5_argtuple_error;
18193                             CYTHON_FALLTHROUGH;
18194                             case 1:
18195                                 if (kw_args > 0) {
18196                                     PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
18197                                     if (value) { values[1] = value; kw_args--; }
18198                                 }
18199                             }
18200                             if (unlikely(kw_args > 0)) {
18201                                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "atan") < 0)) __PYX_ERR(0, 1818, __pyx_L3_error)
18202                             }
18203                             } else {
18204                                 switch (PyTuple_GET_SIZE(__pyx_args)) {
18205                                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18206                                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18207                                     break;
18208                                     default: goto __pyx_L5_argtuple_error;
18209                                 }
18210                             }
18211                             __pyx_v_obj = values[0];
18212                             __pyx_v_i = values[1];
18213                         }
18214                     goto __pyx_L4_argument_unpacking_done;
18215                     __pyx_L5_argtuple_error:;
18216                     __Pyx_RaiseArgtupleInvalid("atan", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
18217                     __PYX_ERR(0, 1818, __pyx_L3_error)
18218                     __pyx_L3_error:;
18219                     __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18220                     __Pyx_RefNannyFinishContext();
18221                     return NULL;
18222                     __pyx_L4_argument_unpacking_done:;
18223                     __pyx_r = __pyx_pf_8PyClical_74atan(__pyx_self, __pyx_v_obj, __pyx_v_i);
18224
18225                     /* function exit code */
18226                     __Pyx_RefNannyFinishContext();
18227                     return __pyx_r;
18228                 }
18229
18230                 static PyObject * __pyx_pf_8PyClical_74atan(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj, PyObject * __pyx_v_i) {
18231                     PyObject * __pyx_r = NULL;
18232                     __Pyx_RefNannyDeclarations
18233                     PyObject * __pyx_t_1 = NULL;
18234                     struct __pyx_opt_args_8PyClical_atan __pyx_t_2;
18235                     int __pyx_lineno = 0;
18236                     const char * __pyx_filename = NULL;
18237                     int __pyx_clineno = 0;

```

```

18238         __Pyx_RefNannySetupContext("atan", 0);
18239         __Pyx_XDECREF(__pyx_r);
18240         __pyx_t_2.__pyx_n = 1;
18241         __pyx_t_2.i = __pyx_v_i;
18242         __pyx_t_1 = __pyx_f_8PyClical_atan(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1818, __pyx_L1_error)
18243         __Pyx_GOTREF(__pyx_t_1);
18244         __pyx_r = __pyx_t_1;
18245         __pyx_t_1 = 0;
18246         goto __pyx_L0;
18247
18248         /* function exit code */
18249         __pyx_L1_error++;
18250         __Pyx_XDECREF(__pyx_t_1);
18251         __Pyx_AddTraceback("PyClical.atan", __pyx_clineno, __pyx_lineno, __pyx_filename);
18252         __pyx_r = NULL;
18253         __pyx_L0;
18254         __Pyx_XGIVEREF(__pyx_r);
18255         __Pyx_RefNannyFinishContext();
18256         return __pyx_r;
18257     }
18258
18259     /* "PyClical.pyx":1835
18260     *
18261     * return clifford().wrap( glucat.atan(toClifford(obj)) )
18262     * cpdef inline tanh(obj):
18263     *     """
18264     *     Hyperbolic tangent of multivector.
18265     */
18266
18267     static PyObject * __pyx_pw_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18268     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_tanh(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
18269         PyObject * __pyx_r = NULL;
18270         __Pyx_RefNannyDeclarations
18271         PyObject * __pyx_t_1 = NULL;
18272         PyObject * __pyx_t_2 = NULL;
18273         PyObject * __pyx_t_3 = NULL;
18274         PyObject * __pyx_t_4 = NULL;
18275         PyObject * __pyx_t_5 = NULL;
18276         PyObject * __pyx_t_6 = NULL;
18277         PyObject * __pyx_t_7 = NULL;
18278         PyObject * __pyx_t_8 = NULL;
18279         int __pyx_lineno = 0;
18280         const char * __pyx_filename = NULL;
18281         int __pyx_clineno = 0;
18282         __Pyx_RefNannySetupContext("tanh", 0);
18283
18284         /* "PyClical.pyx":1842
18285     *     {1,2}
18286     *     """
18287     *     try:
18288     *         return math.tanh(obj)
18289     *     except:
18290     */
18291         {
18292             __Pyx_PyThreadState_declare
18293             __Pyx_PyThreadState_assign
18294             __Pyx_ExceptionSave(&__pyx_t_1, &__pyx_t_2, &__pyx_t_3);
18295             __Pyx_XGOTREF(__pyx_t_1);
18296             __Pyx_XGOTREF(__pyx_t_2);
18297             __Pyx_XGOTREF(__pyx_t_3);
18298             /*try:*/ {
18299
18300                 /* "PyClical.pyx":1843
18301     *     """
18302     *     try:
18303     *         return math.tanh(obj)
18304     *     except:
18305     *         return clifford().wrap( glucat.tanh(toClifford(obj)) )
18306     */
18307             __Pyx_XDECREF(__pyx_r);
18308             __Pyx_GetModuleGlobalName(__pyx_t_5, __pyx_n_s_math); if (unlikely(!__pyx_t_5))
__PYX_ERR(0, 1843, __pyx_L3_error)
18309             __Pyx_GOTREF(__pyx_t_5);
18310             __pyx_t_6 = __Pyx_PyObject_GetAttrStr(__pyx_t_5, __pyx_n_s_tanh); if
(unlikely(!__pyx_t_6)) __PYX_ERR(0, 1843, __pyx_L3_error)
18311             __Pyx_GOTREF(__pyx_t_6);
18312             __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18313             __pyx_t_5 = NULL;
18314             if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_6))) {
18315                 __pyx_t_5 = PyMethod_GET_SELF(__pyx_t_6);
18316                 if (likely(__pyx_t_5)) {
18317                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_6);
18318                     __Pyx_INCREF(__pyx_t_5);
18319                     __Pyx_INCREF(function);

```

```

18320         __Pyx_DECREF_SET(__pyx_t_6, function);
18321     }
18322 }
18323     __pyx_t_4 = (__pyx_t_5) ? __Pyx_PyObject_Call2Args(__pyx_t_6, __pyx_t_5,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_6, __pyx_v_obj);
18324     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18325     if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1843, __pyx_L3_error)
18326     __Pyx_GOTREF(__pyx_t_4);
18327     __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
18328     __pyx_r = __pyx_t_4;
18329     __pyx_t_4 = 0;
18330     goto __pyx_L7_try_return;
18331
18332     /* "PyClical.pyx":1842
18333     *
18334     * {1,2}
18335     * """
18336     * try:
18337     *     # ««««««««
18338     *     return math.tanh(obj)
18339     * except:
18340     */
18341     }
18342     __pyx_L3_error:;
18343     __Pyx_XDECREF(__pyx_t_4); __pyx_t_4 = 0;
18344     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18345     __Pyx_XDECREF(__pyx_t_6); __pyx_t_6 = 0;
18346
18347     /* "PyClical.pyx":1844
18348     *
18349     * try:
18350     *     return math.tanh(obj)
18351     * except:
18352     *     # ««««««««
18353     *     return clifford().wrap( glucat.tanh(toClifford(obj)) )
18354     */
18355     /*except:*/ {
18356         __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18357         if (__Pyx_GetException(&__pyx_t_4, &__pyx_t_6, &__pyx_t_5) < 0) __PYX_ERR(0,
1844, __pyx_L5_except_error)
18358         __Pyx_GOTREF(__pyx_t_4);
18359         __Pyx_GOTREF(__pyx_t_6);
18360         __Pyx_GOTREF(__pyx_t_5);
18361
18362         /* "PyClical.pyx":1845
18363         *
18364         * return math.tanh(obj)
18365         * except:
18366         *     return clifford().wrap( glucat.tanh(toClifford(obj)) )
18367         *     # ««««««««
18368         * cpdef inline atanh(obj,i = None):
18369         */
18370         __Pyx_XDECREF(__pyx_r);
18371         __pyx_t_7 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_7)) __PYX_ERR(0, 1845,
__pyx_L5_except_error)
18372         __Pyx_GOTREF(__pyx_t_7);
18373         __pyx_t_8 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_7), tanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_8)) __PYX_ERR(0, 1845, __pyx_L5_except_error)
18374         __Pyx_GOTREF(__pyx_t_8);
18375         __Pyx_DECREF(__pyx_t_7); __pyx_t_7 = 0;
18376         __pyx_r = __pyx_t_8;
18377         __pyx_t_8 = 0;
18378         __Pyx_DECREF(__pyx_t_4); __pyx_t_4 = 0;
18379         __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18380         __Pyx_DECREF(__pyx_t_6); __pyx_t_6 = 0;
18381         goto __pyx_L6_except_return;
18382     }
18383     __pyx_L5_except_error:;
18384
18385     /* "PyClical.pyx":1842
18386     *
18387     * {1,2}
18388     * """
18389     * try:
18390     *     # ««««««««
18391     *     return math.tanh(obj)
18392     * except:
18393     */
18394     __Pyx_XGIVEREF(__pyx_t_1);
18395     __Pyx_XGIVEREF(__pyx_t_2);
18396     __Pyx_XGIVEREF(__pyx_t_3);
18397     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18398     goto __pyx_L1_error;
18399     __pyx_L7_try_return:;
18400     __Pyx_XGIVEREF(__pyx_t_1);
18401     __Pyx_XGIVEREF(__pyx_t_2);
18402     __Pyx_XGIVEREF(__pyx_t_3);
18403     __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18404     goto __pyx_L0;
18405     __pyx_L6_except_return:;

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```

18400         __Pyx_XGIVEREF(__pyx_t_1);
18401         __Pyx_XGIVEREF(__pyx_t_2);
18402         __Pyx_XGIVEREF(__pyx_t_3);
18403         __Pyx_ExceptionReset(__pyx_t_1, __pyx_t_2, __pyx_t_3);
18404         goto __pyx_L0;
18405     }
18406
18407     /* "PyClical.pyx":1835
18408     *         return clifford().wrap( glucat.atan(toClifford(obj)) )
18409     *
18410     * cpdef inline tanh(obj):
18411     *     """
18412     *     Hyperbolic tangent of multivector.
18413     */
18414
18415     /* function exit code */
18416     __pyx_L1_error:;
18417     __Pyx_XDECREF(__pyx_t_4);
18418     __Pyx_XDECREF(__pyx_t_5);
18419     __Pyx_XDECREF(__pyx_t_6);
18420     __Pyx_XDECREF(__pyx_t_7);
18421     __Pyx_XDECREF(__pyx_t_8);
18422     __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18423     __pyx_r = 0;
18424     __pyx_L0:;
18425     __Pyx_XGIVEREF(__pyx_r);
18426     __Pyx_RefNannyFinishContext();
18427     return __pyx_r;
18428 }
18429
18430 /* Python wrapper */
18431 static PyObject * __pyx_pw_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
18432 static char __pyx_doc_8PyClical_76tanh[] = "\n Hyperbolic tangent of
multivector.\n\n >> x=clifford(\"{1,2}\") * pi/4; print(tanh(x))\n {1,2}\n ";
18433 static PyObject * __pyx_pf_8PyClical_77tanh(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
18434     PyObject * __pyx_r = 0;
18435     __Pyx_RefNannyDeclarations
18436     __Pyx_RefNannySetupContext("tanh (wrapper)", 0);
18437     __pyx_r = __pyx_pf_8PyClical_76tanh(__pyx_self, ((PyObject *) __pyx_v_obj));
18438
18439     /* function exit code */
18440     __Pyx_RefNannyFinishContext();
18441     return __pyx_r;
18442 }
18443
18444 static PyObject * __pyx_pf_8PyClical_76tanh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
18445     PyObject * __pyx_r = NULL;
18446     __Pyx_RefNannyDeclarations
18447     PyObject * __pyx_t_1 = NULL;
18448     int __pyx_lineno = 0;
18449     const char * __pyx_filename = NULL;
18450     int __pyx_clineno = 0;
18451     __Pyx_RefNannySetupContext("tanh", 0);
18452     __Pyx_XDECREF(__pyx_r);
18453     __pyx_t_1 = __pyx_f_8PyClical_tanh(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1835, __pyx_L1_error)
18454     __Pyx_GOTREF(__pyx_t_1);
18455     __pyx_r = __pyx_t_1;
18456     __pyx_t_1 = 0;
18457     goto __pyx_L0;
18458
18459     /* function exit code */
18460     __pyx_L1_error:;
18461     __Pyx_XDECREF(__pyx_t_1);
18462     __Pyx_AddTraceback("PyClical.tanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18463     __pyx_r = NULL;
18464     __pyx_L0:;
18465     __Pyx_XGIVEREF(__pyx_r);
18466     __Pyx_RefNannyFinishContext();
18467     return __pyx_r;
18468 }
18469
18470 /* "PyClical.pyx":1847
18471 *         return clifford().wrap( glucat.tanh(toClifford(obj)) )
18472 *
18473 * cpdef inline atanh(obj,i = None):
18474 *     """
18475 *     Inverse hyperbolic tangent of multivector with optional complexifier.
18476 */
18477
18478 static PyObject * __pyx_pw_8PyClical_79atanh(PyObject * __pyx_self, PyObject
*__pyx_args, PyObject * __pyx_kws); /*proto*/
18479 static CYTHON_INLINE PyObject * __pyx_f_8PyClical_atanh(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch, struct __pyx_opt_args_8PyClical_atanh * __pyx_optional_args) {

```

```

18480         PyObject *__pyx_v_i = ((PyObject *)Py_None);
18481         PyObject *__pyx_r = NULL;
18482         __Pyx_RefNannyDeclarations
18483         int __pyx_t_1;
18484         int __pyx_t_2;
18485         PyObject *__pyx_t_3 = NULL;
18486         Clifford __pyx_t_4;
18487         PyObject *__pyx_t_5 = NULL;
18488         PyObject *__pyx_t_6 = NULL;
18489         PyObject *__pyx_t_7 = NULL;
18490         PyObject *__pyx_t_8 = NULL;
18491         PyObject *__pyx_t_9 = NULL;
18492         PyObject *__pyx_t_10 = NULL;
18493         PyObject *__pyx_t_11 = NULL;
18494         int __pyx_lineno = 0;
18495         const char *__pyx_filename = NULL;
18496         int __pyx_clineno = 0;
18497         __Pyx_RefNannySetupContext("atanh", 0);
18498         if (__pyx_optional_args) {
18499             if (__pyx_optional_args->__pyx_n > 0) {
18500                 __pyx_v_i = __pyx_optional_args->i;
18501             }
18502         }
18503
18504         /* "PyClical.pyx":1856
18505  *     {1,2}
18506  *     """
18507  *     if not (i is None):          # ««««««««
18508  *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18509  *     else:
18510  */
18511         __pyx_t_1 = (__pyx_v_i != Py_None);
18512         __pyx_t_2 = (__pyx_t_1 != 0);
18513         if (__pyx_t_2) {
18514
18515             /* "PyClical.pyx":1857
18516  *     """
18517  *     if not (i is None):
18518  *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18519  *     else:
18520  *         try:
18521  */
18522             __Pyx_XDECREF(__pyx_r);
18523             __pyx_t_3 = __Pyx_PyObject_CallNoArg(((PyObject
18524 *)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1857, __pyx_L1_error)
18525             __Pyx_GOTREF(__pyx_t_3);
18526             __pyx_t_4 = atanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj),
18527 __pyx_f_8PyClical_toClifford(__pyx_v_i));
18528             } catch (...) {
18529                 __Pyx_CppExn2PyErr();
18530                 __PYX_ERR(0, 1857, __pyx_L1_error)
18531             }
18532             __pyx_t_5 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
18533 *)__pyx_t_3), __pyx_t_4); if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1857, __pyx_L1_error)
18534             __Pyx_GOTREF(__pyx_t_5);
18535             __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18536             __pyx_r = __pyx_t_5;
18537             __pyx_t_5 = 0;
18538             goto __pyx_L0;
18539
18540             /* "PyClical.pyx":1856
18541  *     {1,2}
18542  *     """
18543  *     if not (i is None):          # ««««««««
18544  *         return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18545  *     else:
18546  */
18547             }
18548
18549             /* "PyClical.pyx":1859
18550  *     return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18551  *     else:
18552  *         try:
18553  *             # ««««««««
18554  *             return math.atanh(obj)
18555  *         except:
18556  */
18557             /*else*/ {
18558                 {
18559                     __Pyx_PyThreadState_declare
18560                     __Pyx_PyThreadState_assign
18561                     __Pyx_ExceptionSave(&__pyx_t_6, &__pyx_t_7, &__pyx_t_8);
18562                     __Pyx_XGOTREF(__pyx_t_6);
18563                     __Pyx_XGOTREF(__pyx_t_7);
18564                     __Pyx_XGOTREF(__pyx_t_8);
18565                 }
18566                 /*try:*/ {

```

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18563
18564                                     /* "PyClical.pyx":1860
18565 *      else:
18566 *          try:
18567 *              return math.atanh(obj)                # ««««««««
18568 *          except:
18569 *              return clifford().wrap( glucat.atanh(toClifford(obj)) )
18570 */
18571                                     __Pyx_XDECREF(__pyx_r);
18572                                     __Pyx_GetModuleGlobalName(__pyx_t_3, __pyx_n_s_math); if
(unlikely(!__pyx_t_3)) __PYX_ERR(0, 1860, __pyx_L4_error)
18573                                     __Pyx_GOTREF(__pyx_t_3);
18574                                     __pyx_t_9 = __Pyx_PyObject_GetAttrStr(__pyx_t_3, __pyx_n_s_atanh); if
(unlikely(!__pyx_t_9)) __PYX_ERR(0, 1860, __pyx_L4_error)
18575                                     __Pyx_GOTREF(__pyx_t_9);
18576                                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18577                                     __pyx_t_3 = NULL;
18578                                     if (CYTHON_UNPACK_METHODS && unlikely(PyMethod_Check(__pyx_t_9))) {
18579                                     __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_9);
18580                                     if (likely(__pyx_t_3)) {
18581                                     PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_9);
18582                                     __Pyx_INCREF(__pyx_t_3);
18583                                     __Pyx_INCREF(function);
18584                                     __Pyx_DECREF_SET(__pyx_t_9, function);
18585                                     }
18586                                     }
18587                                     __pyx_t_5 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_9, __pyx_t_3,
__pyx_v_obj) : __Pyx_PyObject_CallOneArg(__pyx_t_9, __pyx_v_obj);
18588                                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18589                                     if (unlikely(!__pyx_t_5)) __PYX_ERR(0, 1860, __pyx_L4_error)
18590                                     __Pyx_GOTREF(__pyx_t_5);
18591                                     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;
18592                                     __pyx_r = __pyx_t_5;
18593                                     __pyx_t_5 = 0;
18594                                     goto __pyx_L8_try_return;
18595
18596                                     /* "PyClical.pyx":1859
18597 *      return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18598 *      else:
18599 *          try:
18600 *              return math.atanh(obj)
18601 *          except:
18602 */
18603                                     }
18604                                     __pyx_L4_error:;
18605                                     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
18606                                     __Pyx_XDECREF(__pyx_t_5); __pyx_t_5 = 0;
18607                                     __Pyx_XDECREF(__pyx_t_9); __pyx_t_9 = 0;
18608
18609                                     /* "PyClical.pyx":1861
18610 *      try:
18611 *          return math.atanh(obj)
18612 *      except:
18613 *          return clifford().wrap( glucat.atanh(toClifford(obj)) )
18614 *
18615 */
18616                                     /*except:*/ {
18617                                     __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18618                                     if (__Pyx_GetException(&__pyx_t_5, &__pyx_t_9, &__pyx_t_3) < 0) __PYX_ERR(0,
1861, __pyx_L6_except_error)
18619                                     __Pyx_GOTREF(__pyx_t_5);
18620                                     __Pyx_GOTREF(__pyx_t_9);
18621                                     __Pyx_GOTREF(__pyx_t_3);
18622
18623                                     /* "PyClical.pyx":1862
18624 *      return math.atanh(obj)
18625 *      except:
18626 *          return clifford().wrap( glucat.atanh(toClifford(obj)) )                # ««««««««
18627 *
18628 * cpdef inline random_clifford(index_set ixt, fill = 1.0):
18629 */
18630                                     __Pyx_XDECREF(__pyx_r);
18631                                     __pyx_t_10 = __Pyx_PyObject_CallNoArg(((PyObject
*)__pyx_ptype_8PyClical_clifford)); if (unlikely(!__pyx_t_10)) __PYX_ERR(0, 1862,
__pyx_L6_except_error)
18632                                     __Pyx_GOTREF(__pyx_t_10);
18633                                     __pyx_t_11 = __pyx_f_8PyClical_8clifford_wrap(((struct
__pyx_obj_8PyClical_clifford *)__pyx_t_10), atanh(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if
(unlikely(!__pyx_t_11)) __PYX_ERR(0, 1862, __pyx_L6_except_error)
18634                                     __Pyx_GOTREF(__pyx_t_11);
18635                                     __Pyx_DECREF(__pyx_t_10); __pyx_t_10 = 0;
18636                                     __pyx_r = __pyx_t_11;
18637                                     __pyx_t_11 = 0;
18638                                     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
18639                                     __Pyx_DECREF(__pyx_t_5); __pyx_t_5 = 0;
18640                                     __Pyx_DECREF(__pyx_t_9); __pyx_t_9 = 0;

```

```

18641         goto __pyx_L7_except_return;
18642     }
18643     __pyx_L6_except_error;;
18644
18645     /* "PyClical.pyx":1859
18646 *     return clifford().wrap( glucat.atanh(toClifford(obj), toClifford(i)) )
18647 * else:
18648 *     try:
18649 *         return math.atanh(obj)
18650 *     except:
18651 */
18652     __Pyx_XGIVEREF(__pyx_t_6);
18653     __Pyx_XGIVEREF(__pyx_t_7);
18654     __Pyx_XGIVEREF(__pyx_t_8);
18655     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18656     goto __pyx_L1_error;
18657     __pyx_L8_try_return;;
18658     __Pyx_XGIVEREF(__pyx_t_6);
18659     __Pyx_XGIVEREF(__pyx_t_7);
18660     __Pyx_XGIVEREF(__pyx_t_8);
18661     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18662     goto __pyx_L0;
18663     __pyx_L7_except_return;;
18664     __Pyx_XGIVEREF(__pyx_t_6);
18665     __Pyx_XGIVEREF(__pyx_t_7);
18666     __Pyx_XGIVEREF(__pyx_t_8);
18667     __Pyx_ExceptionReset(__pyx_t_6, __pyx_t_7, __pyx_t_8);
18668     goto __pyx_L0;
18669 }
18670
18671
18672     /* "PyClical.pyx":1847
18673 *     return clifford().wrap( glucat.tanh(toClifford(obj)) )
18674 *
18675 * cpdef inline atanh(obj,i = None):
18676 *     """
18677 *     Inverse hyperbolic tangent of multivector with optional complexifier.
18678 */
18679
18680     /* function exit code */
18681     __pyx_L1_error;;
18682     __Pyx_XDECREF(__pyx_t_3);
18683     __Pyx_XDECREF(__pyx_t_5);
18684     __Pyx_XDECREF(__pyx_t_9);
18685     __Pyx_XDECREF(__pyx_t_10);
18686     __Pyx_XDECREF(__pyx_t_11);
18687     __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18688     __pyx_r = 0;
18689     __pyx_L0;;
18690     __Pyx_XGIVEREF(__pyx_r);
18691     __Pyx_RefNannyFinishContext();
18692     return __pyx_r;
18693 }
18694
18695     /* Python wrapper */
18696     static PyObject * __pyx_pw_8PyClical_79atanh(PyObject * __pyx_self, PyObject
18697 * __pyx_args, PyObject * __pyx_kwds); /*proto*/
18698     static char __pyx_doc_8PyClical_78atanh[] = "\n    Inverse hyperbolic tangent of
multivector with optional complexifier.\n\n    >> s=index_set({1,2,3}); x=clifford(\"{1,2}\");
print(tanh(atanh(x,s)))\n    {1,2}\n    >> x=clifford(\"{1,2}\"); print(tanh(atanh(x)))\n    {1,2}\n
";
18699     static PyObject * __pyx_pw_8PyClical_79atanh(PyObject * __pyx_self, PyObject
18700 * __pyx_args, PyObject * __pyx_kwds) {
18701     PyObject * __pyx_v_obj = 0;
18702     PyObject * __pyx_v_i = 0;
18703     int __pyx_lineno = 0;
18704     const char * __pyx_filename = NULL;
18705     int __pyx_clineno = 0;
18706     PyObject * __pyx_r = 0;
18707     __Pyx_RefNannyDeclarations
18708     __Pyx_RefNannySetupContext("atanh (wrapper)", 0);
18709     {
18710         static PyObject ** __pyx_pyargnames[] = {&__pyx_n_s_obj,&__pyx_n_s_i,0};
18711         PyObject* values[2] = {0,0};
18712         values[1] = ((PyObject *)Py_None);
18713         if (unlikely(__pyx_kwds)) {
18714             Py_ssize_t kw_args;
18715             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18716             switch (pos_args) {
18717                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
CYTHON_FALLTHROUGH;
18718                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
CYTHON_FALLTHROUGH;
18719                 case 0: break;
18720                 default: goto __pyx_L5_argtuple_error;
18721             }
18722             kw_args = PyDict_Size(__pyx_kwds);

```



```

18723         switch (pos_args) {
18724             case 0:
18725                 if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_obj)) !=
0)) kw_args--;
18726                 else goto __pyx_L5_argtuple_error;
18727                 CYTHON_FALLTHROUGH;
18728             case 1:
18729                 if (kw_args > 0) {
18730                     PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_i);
18731                     if (value) { values[1] = value; kw_args--; }
18732                 }
18733             }
18734             if (unlikely(kw_args > 0)) {
18735                 if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "atanh") < 0)) __PYX_ERR(0, 1847, __pyx_L3_error)
18736             }
18737             } else {
18738                 switch (PyTuple_GET_SIZE(__pyx_args)) {
18739                     case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18740                     CYTHON_FALLTHROUGH;
18741                     case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18742                     break;
18743                     default: goto __pyx_L5_argtuple_error;
18744                 }
18745             }
18746             __pyx_v_obj = values[0];
18747             __pyx_v_i = values[1];
18748         }
18749         goto __pyx_L4_argument_unpacking_done;
18750         __pyx_L5_argtuple_error:;
18751         __Pyx_RaiseArgtupleInvalid("atanh", 0, 1, 2, PyTuple_GET_SIZE(__pyx_args));
__PYX_ERR(0, 1847, __pyx_L3_error)
18752         __pyx_L3_error:;
18753         __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18754         __Pyx_RefNannyFinishContext();
18755         return NULL;
18756         __pyx_L4_argument_unpacking_done:;
18757         __pyx_r = __pyx_pf_8PyClical_78atanh(__pyx_self, __pyx_v_obj, __pyx_v_i);
18758
18759         /* function exit code */
18760         __Pyx_RefNannyFinishContext();
18761         return __pyx_r;
18762     }
18763
18764     static PyObject * __pyx_pf_8PyClical_78atanh(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj, PyObject * __pyx_v_i) {
18765         PyObject * __pyx_r = NULL;
18766         __Pyx_RefNannyDeclarations
18767         PyObject * __pyx_t_1 = NULL;
18768         struct __pyx_opt_args_8PyClical_atanh __pyx_t_2;
18769         int __pyx_lineno = 0;
18770         const char * __pyx_filename = NULL;
18771         int __pyx_clineno = 0;
18772         __Pyx_RefNannySetupContext("atanh", 0);
18773         __Pyx_XDECREF(__pyx_r);
18774         __pyx_t_2.__pyx_n = 1;
18775         __pyx_t_2.i = __pyx_v_i;
18776         __pyx_t_1 = __pyx_f_8PyClical_atanh(__pyx_v_obj, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1847, __pyx_L1_error)
18777         __Pyx_GOTREF(__pyx_t_1);
18778         __pyx_r = __pyx_t_1;
18779         __pyx_t_1 = 0;
18780         goto __pyx_L0;
18781
18782         /* function exit code */
18783         __pyx_L1_error:;
18784         __Pyx_XDECREF(__pyx_t_1);
18785         __Pyx_AddTraceback("PyClical.atanh", __pyx_clineno, __pyx_lineno, __pyx_filename);
18786         __pyx_r = NULL;
18787         __pyx_L0:;
18788         __Pyx_XGIVEREF(__pyx_r);
18789         __Pyx_RefNannyFinishContext();
18790         return __pyx_r;
18791     }
18792
18793     /* "PyClical.pyx":1864
18794     *
18795     * return clifford().wrap( glucat.atanh(toClifford(obj)) )
18796     *
18797     * cpdef inline random_clifford(index_set ixt, fill = 1.0): # ««««««««
18798     *     """
18799     *     Random multivector within a frame.
18800     */
18801     static PyObject * __pyx_pw_8PyClical_81random_clifford(PyObject * __pyx_self, PyObject
* __pyx_args, PyObject * __pyx_kwds); /*proto*/
18802     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_random_clifford(struct
__pyx_obj_8PyClical_index_set * __pyx_v_ixt, CYTHON_UNUSED int __pyx_skip_dispatch, struct

```

```

__pyx_opt_args_8PyClicl_random_clifford *__pyx_optional_args) {
18803     PyObject *__pyx_v_fill = ((PyObject *)__pyx_float_1_0);
18804     PyObject *__pyx_r = NULL;
18805     __Pyx_RefNannyDeclarations
18806     PyObject *__pyx_t_1 = NULL;
18807     PyObject *__pyx_t_2 = NULL;
18808     scalar_t __pyx_t_3;
18809     PyObject *__pyx_t_4 = NULL;
18810     int __pyx_lineno = 0;
18811     const char *__pyx_filename = NULL;
18812     int __pyx_clineno = 0;
18813     __Pyx_RefNannySetupContext("random_clifford", 0);
18814     if (__pyx_optional_args) {
18815         if (__pyx_optional_args->__pyx_n > 0) {
18816             __pyx_v_fill = __pyx_optional_args->fill;
18817         }
18818     }
18819
18820     /* "PyClicl.pyx":1871
18821     *     {-3,-1,2}
18822     *     """
18823     *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
18824     *
18825     * cpdef inline cga3(obj):
18826     */
18827     __Pyx_XDECREF(__pyx_r);
18828     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford));
18829     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1871, __pyx_L1_error)
18830     __Pyx_GOTREF(__pyx_t_1);
18831     __pyx_t_2 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClicl_clifford));
18832     if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1871, __pyx_L1_error)
18833     __Pyx_GOTREF(__pyx_t_2);
18834     __pyx_t_3 = __pyx_PyFloat_AsDouble(__pyx_v_fill); if (unlikely((__pyx_t_3 ==
18835     ((scalar_t)-1)) && PyErr_Occurred())) __PYX_ERR(0, 1871, __pyx_L1_error)
18836     __pyx_t_4 = __pyx_f_8PyClicl_8clifford_wrap(((struct __pyx_obj_8PyClicl_clifford
18837     *)__pyx_t_1), ((struct __pyx_obj_8PyClicl_clifford
18838     *)__pyx_t_2)->instance->random(__pyx_f_8PyClicl_9index_set_unwrap(__pyx_v_ixt),
18839     ((scalar_t)__pyx_t_3))); if (unlikely(!__pyx_t_4)) __PYX_ERR(0, 1871, __pyx_L1_error)
18840     __Pyx_GOTREF(__pyx_t_4);
18841     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
18842     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
18843     __pyx_r = __pyx_t_4;
18844     __pyx_t_4 = 0;
18845     goto __pyx_L0;
18846
18847     /* "PyClicl.pyx":1864
18848     *     return clifford().wrap( glucat.atanh(toClifford(obj)) )
18849     *
18850     * cpdef inline random_clifford(index_set ixt, fill = 1.0):
18851     *     """
18852     *     Random multivector within a frame.
18853     */
18854
18855     /* function exit code */
18856     __pyx_L1_error:;
18857     __Pyx_XDECREF(__pyx_t_1);
18858     __Pyx_XDECREF(__pyx_t_2);
18859     __Pyx_XDECREF(__pyx_t_4);
18860     __Pyx_AddTraceback("PyClicl.random_clifford", __pyx_clineno, __pyx_lineno,
18861     __pyx_filename);
18862     __pyx_r = 0;
18863     __pyx_L0:;
18864     __Pyx_XGIVEREF(__pyx_r);
18865     __Pyx_RefNannyFinishContext();
18866     return __pyx_r;
18867 }
18868
18869 /* Python wrapper */
18870 static PyObject *__pyx_pw_8PyClicl_81random_clifford(PyObject *__pyx_self, PyObject
18871 *__pyx_args, PyObject *__pyx_kwds); /*proto*/
18872 static char __pyx_doc_8PyClicl_80random_clifford[] = "\n    Random multivector within
18873 a frame.\n\n    >> print(random_clifford(index_set({-3,-1,2})).frame())\n    {-3,-1,2}\n    ";
18874 static PyObject *__pyx_pw_8PyClicl_81random_clifford(PyObject *__pyx_self, PyObject
18875 *__pyx_args, PyObject *__pyx_kwds) {
18876     struct __pyx_obj_8PyClicl_index_set *__pyx_v_ixt = 0;
18877     PyObject *__pyx_v_fill = 0;
18878     int __pyx_lineno = 0;
18879     const char *__pyx_filename = NULL;
18880     int __pyx_clineno = 0;
18881     PyObject *__pyx_r = 0;
18882     __Pyx_RefNannyDeclarations
18883     __Pyx_RefNannySetupContext("random_clifford (wrapper)", 0);
18884     {
18885         static PyObject **__pyx_pyargnames[] = {&__pyx_n_s_ixt,&__pyx_n_s_fill,0};
18886         PyObject* values[2] = {0,0};
18887         values[1] = ((PyObject *)__pyx_float_1_0);

```

```

18878         if (unlikely(__pyx_kwds)) {
18879             Py_ssize_t kw_args;
18880             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
18881             switch (pos_args) {
18882                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18883                     CYTHON_FALLTHROUGH;
18884                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18885                     CYTHON_FALLTHROUGH;
18886                 case 0: break;
18887                 default: goto __pyx_L5_argtuple_error;
18888             }
18889             kw_args = PyDict_Size(__pyx_kwds);
18890             switch (pos_args) {
18891                 case 0:
18892                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_ixt)) !=
0)) kw_args--;
18893                     else goto __pyx_L5_argtuple_error;
18894                     CYTHON_FALLTHROUGH;
18895                 case 1:
18896                     if (kw_args > 0) {
18897                         PyObject* value = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_fill);
18898                         if (value) { values[1] = value; kw_args--; }
18899                     }
18900                 }
18901                 if (unlikely(kw_args > 0)) {
18902                     if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
values, pos_args, "random_clifford") < 0)) __PYX_ERR(0, 1864, __pyx_L3_error)
18903                 }
18904                 } else {
18905                     switch (PyTuple_GET_SIZE(__pyx_args)) {
18906                         case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
18907                             CYTHON_FALLTHROUGH;
18908                         case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
18909                             break;
18910                         default: goto __pyx_L5_argtuple_error;
18911                     }
18912                 }
18913                 __pyx_v_ixt = ((struct __pyx_obj_8PyClical_index_set *)values[0]);
18914                 __pyx_v_fill = values[1];
18915             }
18916             goto __pyx_L4_argument_unpacking_done;
18917             __pyx_L5_argtuple_error:;
18918             __Pyx_RaiseArgtupleInvalid("random_clifford", 0, 1, 2,
PyTuple_GET_SIZE(__pyx_args)); __PYX_ERR(0, 1864, __pyx_L3_error)
18919             __pyx_L3_error:;
18920             __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18921             __Pyx_RefNannyFinishContext();
18922             return NULL;
18923             __pyx_L4_argument_unpacking_done:;
18924             if (unlikely(!__Pyx_ArgTypeTest(((PyObject *)__pyx_v_ixt),
__pyx_ptype_8PyClical_index_set, 1, "ixt", 0))) __PYX_ERR(0, 1864, __pyx_L1_error)
18925             __pyx_r = __pyx_pf_8PyClical_80random_clifford(__pyx_self, __pyx_v_ixt,
__pyx_v_fill);
18926
18927             /* function exit code */
18928             goto __pyx_L0;
18929             __pyx_L1_error:;
18930             __pyx_r = NULL;
18931             __pyx_L0:;
18932             __Pyx_RefNannyFinishContext();
18933             return __pyx_r;
18934         }
18935
18936         static PyObject *__pyx_pf_8PyClical_80random_clifford(CYTHON_UNUSED PyObject
*__pyx_self, struct __pyx_obj_8PyClical_index_set *__pyx_v_ixt, PyObject *__pyx_v_fill) {
18937             PyObject *__pyx_r = NULL;
18938             __Pyx_RefNannyDeclarations
18939             PyObject *__pyx_t_1 = NULL;
18940             struct __pyx_opt_args_8PyClical_random_clifford __pyx_t_2;
18941             int __pyx_lineno = 0;
18942             const char *__pyx_filename = NULL;
18943             int __pyx_clineno = 0;
18944             __Pyx_RefNannySetupContext("random_clifford", 0);
18945             __Pyx_XDECREF(__pyx_r);
18946             __pyx_t_2.__pyx_n = 1;
18947             __pyx_t_2.fill = __pyx_v_fill;
18948             __pyx_t_1 = __pyx_f_8PyClical_random_clifford(__pyx_v_ixt, 0, &__pyx_t_2); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1864, __pyx_L1_error)
18949             __Pyx_GOTREF(__pyx_t_1);
18950             __pyx_r = __pyx_t_1;
18951             __pyx_t_1 = 0;
18952             goto __pyx_L0;
18953
18954             /* function exit code */
18955             __pyx_L1_error:;
18956             __Pyx_XDECREF(__pyx_t_1);

```

```

18957     __Pyx_AddTraceback("PyClical.random_clifford", __pyx_clineno, __pyx_lineno,
__pyx_filename);
18958     __pyx_r = NULL;
18959     __pyx_L0:;
18960     __Pyx_XGIVEREF(__pyx_r);
18961     __Pyx_RefNannyFinishContext();
18962     return __pyx_r;
18963 }
18964
18965     /* "PyClical.pyx":1873
18966     *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
18967     *
18968     * cpdef inline cga3(obj):
18969     *     """
18970     *     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
definition.
18971     */
18972
18973     static PyObject * __pyx_pw_8PyClical_83cga3(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
18974     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_cga3(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
18975     PyObject * __pyx_r = NULL;
18976     __Pyx_RefNannyDeclarations
18977     PyObject * __pyx_t_1 = NULL;
18978     PyObject * __pyx_t_2 = NULL;
18979     int __pyx_lineno = 0;
18980     const char * __pyx_filename = NULL;
18981     int __pyx_clineno = 0;
18982     __Pyx_RefNannySetupContext("cga3", 0);
18983
18984     /* "PyClical.pyx":1880
18985     *     87{-1}+4{1}+18{2}+2{3}+85{4}
18986     *     """
18987     *     return clifford().wrap( glucat.cga3(toClifford(obj)) )
18988     *
18989     * cpdef inline cga3std(obj):
18990     */
18991     __Pyx_XDECREF(__pyx_r);
18992     __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
18993     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1880, __pyx_L1_error)
18994     __Pyx_GOTREF(__pyx_t_1);
18995     __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_1), cga3::cga3(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1880, __pyx_L1_error)
18996     __Pyx_GOTREF(__pyx_t_2);
18997     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
18998     __pyx_r = __pyx_t_2;
18999     __pyx_t_2 = 0;
19000     goto __pyx_L0;
19001
19002     /* "PyClical.pyx":1873
19003     *     return clifford().wrap( clifford().instance.random(ixt.unwrap(), <scalar_t>fill) )
19004     *
19005     * cpdef inline cga3(obj):
19006     *     """
19007     *     Convert Euclidean 3D multivector to Conformal Geometric Algebra using Doran and Lasenby
definition.
19008     */
19009
19010     /* function exit code */
19011     __pyx_L1_error:;
19012     __Pyx_XDECREF(__pyx_t_1);
19013     __Pyx_XDECREF(__pyx_t_2);
19014     __Pyx_AddTraceback("PyClical.cga3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19015     __pyx_r = 0;
19016     __pyx_L0:;
19017     __Pyx_XGIVEREF(__pyx_r);
19018     __Pyx_RefNannyFinishContext();
19019     return __pyx_r;
19020 }
19021
19022     /* Python wrapper */
19023     static PyObject * __pyx_pw_8PyClical_83cga3(PyObject * __pyx_self, PyObject
* __pyx_v_obj); /*proto*/
19024     static char __pyx_doc_8PyClical_82cga3[] = "\n    Convert Euclidean 3D multivector to
Conformal Geometric Algebra using Doran and Lasenby definition.\n\n    >>
x=clifford(\"2{1}+9{2}+{3}\"); print(cga3(x))\n    87{-1}+4{1}+18{2}+2{3}+85{4}\n    ";
19025     static PyObject * __pyx_pw_8PyClical_83cga3(PyObject * __pyx_self, PyObject
* __pyx_v_obj) {
19026     PyObject * __pyx_r = 0;
19027     __Pyx_RefNannyDeclarations
19028     __Pyx_RefNannySetupContext("cga3 (wrapper)", 0);
19029     __pyx_r = __pyx_f_8PyClical_82cga3(__pyx_self, ((PyObject *) __pyx_v_obj));
19030
19031     /* function exit code */
__Pyx_RefNannyFinishContext();

```

```

19032         return __pyx_r;
19033     }
19034
19035     static PyObject *__pyx_pf_8PyClical_82cga3(CYTHON_UNUSED PyObject *__pyx_self,
PyObject *__pyx_v_obj) {
19036         PyObject *__pyx_r = NULL;
19037         __Pyx_RefNannyDeclarations
19038         PyObject *__pyx_t_1 = NULL;
19039         int __pyx_lineno = 0;
19040         const char *__pyx_filename = NULL;
19041         int __pyx_clineno = 0;
19042         __Pyx_RefNannySetupContext("cga3", 0);
19043         __Pyx_XDECREF(__pyx_r);
19044         __pyx_t_1 = __pyx_f_8PyClical_cga3(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1873, __pyx_L1_error)
19045         __Pyx_GOTREF(__pyx_t_1);
19046         __pyx_r = __pyx_t_1;
19047         __pyx_t_1 = 0;
19048         goto __pyx_L0;
19049
19050         /* function exit code */
19051         __pyx_L1_error++;
19052         __Pyx_XDECREF(__pyx_t_1);
19053         __Pyx_AddTraceback("PyClical.cga3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19054         __pyx_r = NULL;
19055         __pyx_L0++;
19056         __Pyx_XGIVEREF(__pyx_r);
19057         __Pyx_RefNannyFinishContext();
19058         return __pyx_r;
19059     }
19060
19061     /* "PyClical.pyx":1882
19062 *     return clifford().wrap( glucat.cga3(toClifford(obj)) )
19063 *
19064 * cpdef inline cga3std(obj):
19065 *     """
19066 *     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
19067 */
19068
19069     static PyObject *__pyx_pw_8PyClical_85cga3std(PyObject *__pyx_self, PyObject
*__pyx_v_obj); /*proto*/
19070     static CYTHON_INLINE PyObject *__pyx_f_8PyClical_cga3std(PyObject *__pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
19071         PyObject *__pyx_r = NULL;
19072         __Pyx_RefNannyDeclarations
19073         PyObject *__pyx_t_1 = NULL;
19074         PyObject *__pyx_t_2 = NULL;
19075         int __pyx_lineno = 0;
19076         const char *__pyx_filename = NULL;
19077         int __pyx_clineno = 0;
19078         __Pyx_RefNannySetupContext("cga3std", 0);
19079
19080         /* "PyClical.pyx":1891
19081 *     0
19082 *     """
19083 *     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19084 *
19085 * cpdef inline agc3(obj):
19086 */
19087         __Pyx_XDECREF(__pyx_r);
19088         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *)__pyx_ptype_8PyClical_clifford));
19089         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1891, __pyx_L1_error)
19090         __Pyx_GOTREF(__pyx_t_1);
19091         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*)__pyx_t_1), cga3::cga3std(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1891, __pyx_L1_error)
19091         __Pyx_GOTREF(__pyx_t_2);
19092         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19093         __pyx_r = __pyx_t_2;
19094         __pyx_t_2 = 0;
19095         goto __pyx_L0;
19096
19097         /* "PyClical.pyx":1882
19098 *     return clifford().wrap( glucat.cga3(toClifford(obj)) )
19099 *
19100 * cpdef inline cga3std(obj):
19101 *     """
19102 *     Convert CGA3 null vector to standard conformal null vector using Doran and Lasenby definition.
19103 */
19104
19105         /* function exit code */
19106         __pyx_L1_error++;
19107         __Pyx_XDECREF(__pyx_t_1);
19108         __Pyx_XDECREF(__pyx_t_2);
19109         __Pyx_AddTraceback("PyClical.cga3std", __pyx_clineno, __pyx_lineno, __pyx_filename);
19110         __pyx_r = 0;
19111         __pyx_L0++;

```

```

19112         __Pyx_XGIVEREF(__pyx_r);
19113         __Pyx_RefNannyFinishContext();
19114         return __pyx_r;
19115     }
19116
19117     /* Python wrapper */
19118     static PyObject * __pyx_pw_8PyClical_85cga3std(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
19119     static char __pyx_doc_8PyClical_84cga3std[] = "\n    Convert CGA3 null vector to
standard conformal null vector using Doran and Lasenby definition.\n\n    >>
x=clifford(\"2{1}+9{2}+{3}\"); print(cga3std(cga3(x)))\n    87{-1}+4{1}+18{2}+2{3}+85{4}\n    >>
x=clifford(\"2{1}+9{2}+{3}\"); print(cga3std(cga3(x))-cga3(x))\n    0\n    ";
19120     static PyObject * __pyx_pw_8PyClical_85cga3std(PyObject * __pyx_self, PyObject
*__pyx_v_obj) {
19121         PyObject * __pyx_r = 0;
19122         __Pyx_RefNannyDeclarations
19123         __Pyx_RefNannySetupContext("cga3std (wrapper)", 0);
19124         __pyx_r = __pyx_pf_8PyClical_84cga3std(__pyx_self, ((PyObject *) __pyx_v_obj));
19125
19126         /* function exit code */
19127         __Pyx_RefNannyFinishContext();
19128         return __pyx_r;
19129     }
19130
19131     static PyObject * __pyx_pf_8PyClical_84cga3std(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
19132         PyObject * __pyx_r = NULL;
19133         __Pyx_RefNannyDeclarations
19134         PyObject * __pyx_t_1 = NULL;
19135         int __pyx_lineno = 0;
19136         const char * __pyx_filename = NULL;
19137         int __pyx_clineno = 0;
19138         __Pyx_RefNannySetupContext("cga3std", 0);
19139         __Pyx_XDECREf(__pyx_r);
19140         __pyx_t_1 = __pyx_f_8PyClical_cga3std(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1882, __pyx_L1_error)
19141         __Pyx_GOTREF(__pyx_t_1);
19142         __pyx_r = __pyx_t_1;
19143         __pyx_t_1 = 0;
19144         goto __pyx_L0;
19145
19146         /* function exit code */
19147         __pyx_L1_error:;
19148         __Pyx_XDECREf(__pyx_t_1);
19149         __Pyx_AddTraceback("PyClical.cga3std", __pyx_clineno, __pyx_lineno, __pyx_filename);
19150         __pyx_r = NULL;
19151         __pyx_L0:;
19152         __Pyx_XGIVEREF(__pyx_r);
19153         __Pyx_RefNannyFinishContext();
19154         return __pyx_r;
19155     }
19156
19157     /* "PyClical.pyx":1893
19158 *     return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19159 *
19160 * cpdef inline agc3(obj): # «««««««
19161 *     """
19162 *     Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
19163 */
19164
19165     static PyObject * __pyx_pw_8PyClical_87agc3(PyObject * __pyx_self, PyObject
*__pyx_v_obj); /*proto*/
19166     static CYTHON_INLINE PyObject * __pyx_f_8PyClical_agc3(PyObject * __pyx_v_obj,
CYTHON_UNUSED int __pyx_skip_dispatch) {
19167         PyObject * __pyx_r = NULL;
19168         __Pyx_RefNannyDeclarations
19169         PyObject * __pyx_t_1 = NULL;
19170         PyObject * __pyx_t_2 = NULL;
19171         int __pyx_lineno = 0;
19172         const char * __pyx_filename = NULL;
19173         int __pyx_clineno = 0;
19174         __Pyx_RefNannySetupContext("agc3", 0);
19175
19176         /* "PyClical.pyx":1902
19177 *     0
19178 *     """
19179 *     return clifford().wrap( glucat.agc3(toClifford(obj)) ) # «««««««
19180 *
19181 * # Some abbreviations.
19182 */
19183         __Pyx_XDECREf(__pyx_r);
19184         __pyx_t_1 = __Pyx_PyObject_CallNoArg(((PyObject *) __pyx_ptype_8PyClical_clifford));
19185         if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1902, __pyx_L1_error)
19186         __Pyx_GOTREF(__pyx_t_1);
19187         __pyx_t_2 = __pyx_f_8PyClical_8clifford_wrap(((struct __pyx_obj_8PyClical_clifford
*) __pyx_t_1), cga3::agc3(__pyx_f_8PyClical_toClifford(__pyx_v_obj))); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1902, __pyx_L1_error)

```

```

19187         __Pyx_GOTREF(__pyx_t_2);
19188         __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19189         __pyx_r = __pyx_t_2;
19190         __pyx_t_2 = 0;
19191         goto __pyx_L0;
19192
19193         /* "PyClical.pyx":1893
19194  *      return clifford().wrap( glucat.cga3std(toClifford(obj)) )
19195  *
19196  * cpdef inline agc3(obj):          # ««««««««
19197  *      """
19198  *      Convert CGA3 null vector to Euclidean 3D vector using Doran and Lasenby definition.
19199  */
19200
19201         /* function exit code */
19202         __pyx_L1_error++;
19203         __Pyx_XDECREF(__pyx_t_1);
19204         __Pyx_XDECREF(__pyx_t_2);
19205         __Pyx_AddTraceback("PyClical.agc3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19206         __pyx_r = 0;
19207         __pyx_L0:;
19208         __Pyx_XGIVEREF(__pyx_r);
19209         __Pyx_RefNannyFinishContext();
19210         return __pyx_r;
19211     }
19212
19213     /* Python wrapper */
19214     static PyObject * __pyx_pw_8PyClical_87agc3(PyObject * __pyx_self, PyObject
19215 * __pyx_v_obj); /*proto*/
19216     static char __pyx_doc_8PyClical_86agc3[] = "\n    Convert CGA3 null vector to
Euclidean 3D vector using Doran and Lasenby definition.\n\n    >> x=clifford(\"2{1}+9{2}+{3}\");
print(agc3(cga3(x)))\n    2{1}+9{2}+{3}\n    >> x=clifford(\"2{1}+9{2}+{3}\");
print(agc3(cga3(x))-x)\n    0\n    ";
19217     static PyObject * __pyx_pw_8PyClical_87agc3(PyObject * __pyx_self, PyObject
19218 * __pyx_v_obj) {
19219         PyObject * __pyx_r = 0;
19220         __Pyx_RefNannyDeclarations
19221         __Pyx_RefNannySetupContext("agc3 (wrapper)", 0);
19222         __pyx_r = __pyx_pf_8PyClical_86agc3(__pyx_self, ((PyObject *) __pyx_v_obj));
19223
19224         /* function exit code */
19225         __Pyx_RefNannyFinishContext();
19226         return __pyx_r;
19227     }
19228     static PyObject * __pyx_pf_8PyClical_86agc3(CYTHON_UNUSED PyObject * __pyx_self,
PyObject * __pyx_v_obj) {
19229         PyObject * __pyx_r = NULL;
19230         __Pyx_RefNannyDeclarations
19231         PyObject * __pyx_t_1 = NULL;
19232         int __pyx_lineno = 0;
19233         const char * __pyx_filename = NULL;
19234         int __pyx_clineno = 0;
19235         __Pyx_RefNannySetupContext("agc3", 0);
19236         __Pyx_XDECREF(__pyx_r);
19237         __pyx_t_1 = __pyx_f_8PyClical_agc3(__pyx_v_obj, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1893, __pyx_L1_error)
19238         __Pyx_GOTREF(__pyx_t_1);
19239         __pyx_r = __pyx_t_1;
19240         __pyx_t_1 = 0;
19241         goto __pyx_L0;
19242
19243         /* function exit code */
19244         __pyx_L1_error++;
19245         __Pyx_XDECREF(__pyx_t_1);
19246         __Pyx_AddTraceback("PyClical.agc3", __pyx_clineno, __pyx_lineno, __pyx_filename);
19247         __pyx_r = NULL;
19248         __pyx_L0:;
19249         __Pyx_XGIVEREF(__pyx_r);
19250         __Pyx_RefNannyFinishContext();
19251         return __pyx_r;
19252     }
19253
19254     /* "PyClical.pyx":1936
19255  *      """
19256  *      def e(obj):          # ««««««««
19257  *      """
19258  *      Abbreviation for clifford(index_set(obj)).
19259  */
19260
19261     /* Python wrapper */
19262     static PyObject * __pyx_pw_8PyClical_89e(PyObject * __pyx_self, PyObject * __pyx_v_obj);
19263     /*proto*/
19264     static char __pyx_doc_8PyClical_88e[] = "\n    Abbreviation for
clifford(index_set(obj)).\n\n    >> print(e(1))\n    {1}\n    >> print(e(-1))\n    {-1}\n    >>
print(e(0))\n    1\n    ";

```

```

19264         static PyMethodDef __pyx_mdef_8PyClical_89e = {"e",
(PyCFunction)__pyx_pw_8PyClical_89e, METH_O, __pyx_doc_8PyClical_88e};
19265         static PyObject *__pyx_pw_8PyClical_89e(PyObject *__pyx_self, PyObject *__pyx_v_obj) {
19266             PyObject *__pyx_r = 0;
19267             __Pyx_RefNannyDeclarations
19268             __Pyx_RefNannySetupContext("e (wrapper)", 0);
19269             __pyx_r = __pyx_pf_8PyClical_88e(__pyx_self, ((PyObject *)__pyx_v_obj));
19270
19271             /* function exit code */
19272             __Pyx_RefNannyFinishContext();
19273             return __pyx_r;
19274         }
19275
19276         static PyObject *__pyx_pf_8PyClical_88e(CYTHON_UNUSED PyObject *__pyx_self, PyObject
*__pyx_v_obj) {
19277             PyObject *__pyx_r = NULL;
19278             __Pyx_RefNannyDeclarations
19279             PyObject *__pyx_t_1 = NULL;
19280             PyObject *__pyx_t_2 = NULL;
19281             int __pyx_lineno = 0;
19282             const char *__pyx_filename = NULL;
19283             int __pyx_clineno = 0;
19284             __Pyx_RefNannySetupContext("e", 0);
19285
19286             /* "PyClical.pyx":1947
19287             *      1
19288             *      """
19289             *      return clifford(index_set(obj))          # ««««««««
19290             *
19291             * def istpq(p, q):
19292             */
19293             __Pyx_XDECREF(__pyx_r);
19294             __pyx_t_1 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_index_set),
__pyx_v_obj); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1947, __pyx_L1_error)
19295             __Pyx_GOTREF(__pyx_t_1);
19296             __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_clifford),
__pyx_t_1); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1947, __pyx_L1_error)
19297             __Pyx_GOTREF(__pyx_t_2);
19298             __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
19299             __pyx_r = __pyx_t_2;
19300             __pyx_t_2 = 0;
19301             goto __pyx_L0;
19302
19303             /* "PyClical.pyx":1936
19304             *      """
19305             *
19306             * def e(obj):          # ««««««««
19307             *      """
19308             *      Abbreviation for clifford(index_set(obj)).
19309             */
19310
19311             /* function exit code */
19312             __pyx_L1_error;;
19313             __Pyx_XDECREF(__pyx_t_1);
19314             __Pyx_XDECREF(__pyx_t_2);
19315             __Pyx_AddTraceback("PyClical.e", __pyx_clineno, __pyx_lineno, __pyx_filename);
19316             __pyx_r = NULL;
19317             __pyx_L0;
19318             __Pyx_XGIVEREF(__pyx_r);
19319             __Pyx_RefNannyFinishContext();
19320             return __pyx_r;
19321         }
19322
19323             /* "PyClical.pyx":1949
19324             *      return clifford(index_set(obj))
19325             *
19326             * def istpq(p, q):          # ««««««««
19327             *      """
19328             *      Abbreviation for index_set({-q,...p}).
19329             */
19330
19331             /* Python wrapper */
19332             static PyObject *__pyx_pw_8PyClical_91istpq(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds); /*proto*/
19333             static char __pyx_doc_8PyClical_90istpq[] = "\n    Abbreviation for
index_set({-q,...p}).\n\n    >> print(istpq(2,3))\n    {-3,-2,-1,1,2}\n    ";
19334             static PyMethodDef __pyx_mdef_8PyClical_91istpq = {"istpq",
(PyCFunction)(void*) (PyCFunctionWithKeywords)__pyx_pw_8PyClical_91istpq, METH_VARARGS|METH_KEYWORDS,
__pyx_doc_8PyClical_90istpq};
19335             static PyObject *__pyx_pw_8PyClical_91istpq(PyObject *__pyx_self, PyObject
*__pyx_args, PyObject *__pyx_kwds) {
19336                 PyObject *__pyx_v_p = 0;
19337                 PyObject *__pyx_v_q = 0;
19338                 int __pyx_lineno = 0;
19339                 const char *__pyx_filename = NULL;
19340                 int __pyx_clineno = 0;
19341                 PyObject *__pyx_r = 0;

```



```

19342     __Pyx_RefNannyDeclarations
19343     __Pyx_RefNannySetupContext("istpq (wrapper)", 0);
19344     {
19345         static PyObject * __pyx_pyargnames[] = {&__pyx_n_s_p, &__pyx_n_s_q, 0};
19346         PyObject* values[2] = {0, 0};
19347         if (unlikely(__pyx_kwds)) {
19348             Py_ssize_t kw_args;
19349             const Py_ssize_t pos_args = PyTuple_GET_SIZE(__pyx_args);
19350             switch (pos_args) {
19351                 case 2: values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
19352                     CYTHON_FALLTHROUGH;
19353                 case 1: values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
19354                     CYTHON_FALLTHROUGH;
19355                 case 0: break;
19356                 default: goto __pyx_L5_argtuple_error;
19357             }
19358             kw_args = PyDict_Size(__pyx_kwds);
19359             switch (pos_args) {
19360                 case 0:
19361                     if (likely((values[0] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_p)) !=
19362 0)) kw_args--;
19363                     else goto __pyx_L5_argtuple_error;
19364                     CYTHON_FALLTHROUGH;
19365                 case 1:
19366                     if (likely((values[1] = __Pyx_PyDict_GetItemStr(__pyx_kwds, __pyx_n_s_q)) !=
19367 0)) kw_args--;
19368                     else {
19369                         __Pyx_RaiseArgtupleInvalid("istpq", 1, 2, 2, 1); __PYX_ERR(0, 1949,
19370 __pyx_L3_error)
19371                     }
19372                     if (unlikely(kw_args > 0)) {
19373                         if (unlikely(__Pyx_ParseOptionalKeywords(__pyx_kwds, __pyx_pyargnames, 0,
19374 values, pos_args, "istpq") < 0)) __PYX_ERR(0, 1949, __pyx_L3_error)
19375                     } else if (PyTuple_GET_SIZE(__pyx_args) != 2) {
19376                         goto __pyx_L5_argtuple_error;
19377                     } else {
19378                         values[0] = PyTuple_GET_ITEM(__pyx_args, 0);
19379                         values[1] = PyTuple_GET_ITEM(__pyx_args, 1);
19380                     }
19381                     __pyx_v_p = values[0];
19382                     __pyx_v_q = values[1];
19383                 }
19384                 goto __pyx_L4_argument_unpacking_done;
19385                 __pyx_L5_argtuple_error:;
19386                 __Pyx_RaiseArgtupleInvalid("istpq", 1, 2, 2, PyTuple_GET_SIZE(__pyx_args));
19387                 __PYX_ERR(0, 1949, __pyx_L3_error)
19388                 __pyx_L3_error:;
19389                 __Pyx_AddTraceback("PyClical.istpq", __pyx_clineno, __pyx_lineno, __pyx_filename);
19390                 __Pyx_RefNannyFinishContext();
19391                 return NULL;
19392                 __pyx_L4_argument_unpacking_done:;
19393                 __pyx_r = __pyx_pf_8PyClical_90istpq(__pyx_self, __pyx_v_p, __pyx_v_q);
19394
19395                 /* function exit code */
19396                 __Pyx_RefNannyFinishContext();
19397                 return __pyx_r;
19398             }
19399
19400             static PyObject * __pyx_pf_8PyClical_90istpq(CYTHON_UNUSED PyObject * __pyx_self,
19401 PyObject * __pyx_v_p, PyObject * __pyx_v_q) {
19402                 PyObject * __pyx_r = NULL;
19403                 __Pyx_RefNannyDeclarations
19404                 PyObject * __pyx_t_1 = NULL;
19405                 PyObject * __pyx_t_2 = NULL;
19406                 PyObject * __pyx_t_3 = NULL;
19407                 int __pyx_lineno = 0;
19408                 const char * __pyx_filename = NULL;
19409                 int __pyx_clineno = 0;
19410                 __Pyx_RefNannySetupContext("istpq", 0);
19411
19412                 /* "PyClical.pyx":1956
19413 *      {-3,-2,-1,1,2}
19414 *      """
19415 *      return index_set(set(range(-q,p+1))) # ««««««««
19416 *      ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
19417 */
19418                 __Pyx_XDECREF(__pyx_r);
19419                 __pyx_t_1 = PyNumber_Negative(__pyx_v_q); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
19420 1956, __pyx_L1_error)
19421                 __Pyx_GOTREF(__pyx_t_1);
19422                 __pyx_t_2 = __Pyx_PyInt_AddObjC(__pyx_v_p, __pyx_t_1, 1, 0, 0); if
19423 (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19424                 __Pyx_GOTREF(__pyx_t_2);
19425                 __pyx_t_3 = PyTuple_New(2); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1956,

```

```

__pyx_L1_error)
19421     __Pyx_GOTREF(__pyx_t_3);
19422     __Pyx_GIVEREF(__pyx_t_1);
19423     PyTuple_SET_ITEM(__pyx_t_3, 0, __pyx_t_1);
19424     __Pyx_GIVEREF(__pyx_t_2);
19425     PyTuple_SET_ITEM(__pyx_t_3, 1, __pyx_t_2);
19426     __pyx_t_1 = 0;
19427     __pyx_t_2 = 0;
19428     __pyx_t_2 = __Pyx_PyObject_Call(__pyx_builtin_range, __pyx_t_3, NULL); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19429     __Pyx_GOTREF(__pyx_t_2);
19430     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
19431     __pyx_t_3 = PySet_New(__pyx_t_2); if (unlikely(!__pyx_t_3)) __PYX_ERR(0, 1956,
__pyx_L1_error)
19432     __Pyx_GOTREF(__pyx_t_3);
19433     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
19434     __pyx_t_2 = __Pyx_PyObject_CallOneArg(((PyObject *)__pyx_ptype_8PyClical_index_set),
__pyx_t_3); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1956, __pyx_L1_error)
19435     __Pyx_GOTREF(__pyx_t_2);
19436     __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
19437     __pyx_r = __pyx_t_2;
19438     __pyx_t_2 = 0;
19439     goto __pyx_L0;
19440
19441     /* "PyClical.pyx":1949
19442     *     return clifford(index_set(obj))
19443     *
19444     * def istpq(p, q):           # ««««««««
19445     *     """
19446     *     Abbreviation for index_set({-q,...p}).
19447     */
19448
19449     /* function exit code */
19450     __pyx_L1_error;;
19451     __Pyx_XDECREF(__pyx_t_1);
19452     __Pyx_XDECREF(__pyx_t_2);
19453     __Pyx_XDECREF(__pyx_t_3);
19454     __Pyx_AddTraceback("PyClical.istpq", __pyx_clineno, __pyx_lineno, __pyx_filename);
19455     __pyx_r = NULL;
19456     __pyx_L0;
19457     __Pyx_XGIVEREF(__pyx_r);
19458     __Pyx_RefNannyFinishContext();
19459     return __pyx_r;
19460 }
19461
19462     /* "PyClical.pyx":1962
19463     *
19464     * # Doctest interface.
19465     * def _test():           # ««««««««
19466     *     import PyClical, doctest
19467     *     return doctest.testmod(PyClical)
19468     */
19469
19470     /* Python wrapper */
19471     static PyObject * __pyx_pw_8PyClical_93_test(PyObject * __pyx_self, CYTHON_UNUSED
PyObject *unused); /*proto*/
19472     static PyMethodDef __pyx_mdef_8PyClical_93_test = {"_test",
(PyCFunction) __pyx_pw_8PyClical_93_test, METH_NOARGS, 0};
19473     static PyObject * __pyx_pw_8PyClical_93_test(PyObject * __pyx_self, CYTHON_UNUSED
PyObject *unused) {
19474         PyObject * __pyx_r = 0;
19475         __Pyx_RefNannyDeclarations
19476         __Pyx_RefNannySetupContext("_test (wrapper)", 0);
19477         __pyx_r = __pyx_pf_8PyClical_92_test(__pyx_self);
19478
19479         /* function exit code */
19480         __Pyx_RefNannyFinishContext();
19481         return __pyx_r;
19482     }
19483
19484     static PyObject * __pyx_pf_8PyClical_92_test(CYTHON_UNUSED PyObject * __pyx_self) {
19485         PyObject * __pyx_v_PyClical = NULL;
19486         PyObject * __pyx_v_doctest = NULL;
19487         PyObject * __pyx_r = NULL;
19488         __Pyx_RefNannyDeclarations
19489         PyObject * __pyx_t_1 = NULL;
19490         PyObject * __pyx_t_2 = NULL;
19491         PyObject * __pyx_t_3 = NULL;
19492         int __pyx_lineno = 0;
19493         const char * __pyx_filename = NULL;
19494         int __pyx_clineno = 0;
19495         __Pyx_RefNannySetupContext("_test", 0);
19496
19497         /* "PyClical.pyx":1963
19498     * # Doctest interface.
19499     * def _test():
19500     *     import PyClical, doctest           # ««««««««

```

```

19501 *      return doctest.testmod(PyClical)
19502 *
19503 */
19504     __pyx_t_1 = __Pyx_Import(__pyx_n_s_PyClical, 0, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1963, __pyx_L1_error)
19505     __Pyx_GOTREF(__pyx_t_1);
19506     __pyx_v_PyClical = __pyx_t_1;
19507     __pyx_t_1 = 0;
19508     __pyx_t_1 = __Pyx_Import(__pyx_n_s_doctest, 0, 0); if (unlikely(!__pyx_t_1))
__PYX_ERR(0, 1963, __pyx_L1_error)
19509     __Pyx_GOTREF(__pyx_t_1);
19510     __pyx_v_doctest = __pyx_t_1;
19511     __pyx_t_1 = 0;
19512
19513     /* "PyClical.pyx":1964
19514 * def _test():
19515 *     import PyClical, doctest
19516 *     return doctest.testmod(PyClical)                # ««««««««
19517 *
19518 * if __name__ == "__main__":
19519 */
19520     __Pyx_XDECREF(__pyx_r);
19521     __pyx_t_2 = __Pyx_PyObject_GetAttrStr(__pyx_v_doctest, __pyx_n_s_testmod); if
(unlikely(!__pyx_t_2)) __PYX_ERR(0, 1964, __pyx_L1_error)
19522     __Pyx_GOTREF(__pyx_t_2);
19523     __pyx_t_3 = NULL;
19524     if (CYTHON_UNPACK_METHODS && likely(PyMethod_Check(__pyx_t_2))) {
19525         __pyx_t_3 = PyMethod_GET_SELF(__pyx_t_2);
19526         if (likely(__pyx_t_3)) {
19527             PyObject* function = PyMethod_GET_FUNCTION(__pyx_t_2);
19528             __Pyx_INCREF(__pyx_t_3);
19529             __Pyx_INCREF(function);
19530             __Pyx_DECREF_SET(__pyx_t_2, function);
19531         }
19532     }
19533     __pyx_t_1 = (__pyx_t_3) ? __Pyx_PyObject_Call2Args(__pyx_t_2, __pyx_t_3,
__pyx_v_PyClical) : __Pyx_PyObject_CallOneArg(__pyx_t_2, __pyx_v_PyClical);
19534     __Pyx_XDECREF(__pyx_t_3); __pyx_t_3 = 0;
19535     if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1964, __pyx_L1_error)
19536     __Pyx_GOTREF(__pyx_t_1);
19537     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
19538     __pyx_r = __pyx_t_1;
19539     __pyx_t_1 = 0;
19540     goto __pyx_L0;
19541
19542     /* "PyClical.pyx":1962
19543 *
19544 * # Doctest interface.
19545 * def _test():                # ««««««««
19546 *     import PyClical, doctest
19547 *     return doctest.testmod(PyClical)
19548 */
19549
19550     /* function exit code */
19551     __pyx_L1_error;
19552     __Pyx_XDECREF(__pyx_t_1);
19553     __Pyx_XDECREF(__pyx_t_2);
19554     __Pyx_XDECREF(__pyx_t_3);
19555     __Pyx_AddTraceback("PyClical._test", __pyx_clineno, __pyx_lineno, __pyx_filename);
19556     __pyx_r = NULL;
19557     __pyx_L0;
19558     __Pyx_XDECREF(__pyx_v_PyClical);
19559     __Pyx_XDECREF(__pyx_v_doctest);
19560     __Pyx_XGIVEREF(__pyx_r);
19561     __Pyx_RefNannyFinishContext();
19562     return __pyx_r;
19563 }
19564
19565     /* "string.to_py":31
19566 *
19567 * @cname("__pyx_convert_PyObject_string_to_py_std_in_string")
19568 * cdef inline object __pyx_convert_PyObject_string_to_py_std_in_string(const string& s):
19569 *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19570 * cdef extern from *:
19571 */
19572
19573     static CYTHON_INLINE PyObject
*__pyx_convert_PyObject_string_to_py_std_in_string(std::string const &__pyx_v_s) {
19574         PyObject *__pyx_r = NULL;
19575         __Pyx_RefNannyDeclarations
19576         PyObject *__pyx_t_1 = NULL;
19577         int __pyx_lineno = 0;
19578         const char *__pyx_filename = NULL;
19579         int __pyx_clineno = 0;
19580         __Pyx_RefNannySetupContext("__pyx_convert_PyObject_string_to_py_std_in_string", 0);
19581

```

```

19582             /* "string.to_py":32
19583 * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19584 * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19585 *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size()) # ««««««««
19586 * cdef extern from *:
19587 *     cdef object __Pyx_PyUnicode_FromStringAndSize(const char*, size_t)
19588 */
19589         __Pyx_XDECREF(__pyx_r);
19590         __pyx_t_1 = __Pyx_PyObject_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 32, __pyx_L1_error)
19591         __Pyx_GOTREF(__pyx_t_1);
19592         __pyx_r = __pyx_t_1;
19593         __pyx_t_1 = 0;
19594         goto __pyx_L0;
19595
19596             /* "string.to_py":31
19597 *
19598 * @cname("__pyx_convert_PyObject_string_to_py_std__in_string")
19599 * cdef inline object __pyx_convert_PyObject_string_to_py_std__in_string(const string& s):
19600 # ««««««««
19601 *     return __Pyx_PyObject_FromStringAndSize(s.data(), s.size())
19602 * cdef extern from *:
19603 */
19604
19605             /* function exit code */
19606         __pyx_L1_error;;
19607         __Pyx_XDECREF(__pyx_t_1);
19608
19609         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyObject_string_to_py_std__in_string", __pyx_clineno,
__pyx_lineno, __pyx_filename);
19610         __pyx_r = 0;
19611         __pyx_L0;;
19612         __Pyx_XGIVEREF(__pyx_r);
19613         __Pyx_RefNannyFinishContext();
19614         return __pyx_r;
19615     }
19616
19617             /* "string.to_py":37
19618 *
19619 * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19620 * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19621 # ««««««««
19622 *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size())
19623 * cdef extern from *:
19624 */
19625
19626         static CYTHON_INLINE PyObject
19627 *__pyx_convert_PyUnicode_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19628     PyObject *__pyx_r = NULL;
19629     __Pyx_RefNannyDeclarations
19630     PyObject *__pyx_t_1 = NULL;
19631     int __pyx_lineno = 0;
19632     const char *__pyx_filename = NULL;
19633     int __pyx_clineno = 0;
19634     __Pyx_RefNannySetupContext("__pyx_convert_PyUnicode_string_to_py_std__in_string",
0);
19635
19636             /* "string.to_py":38
19637 * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19638 * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19639 *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size()) # ««««««««
19640 * cdef extern from *:
19641 *     cdef object __Pyx_PyStr_FromStringAndSize(const char*, size_t)
19642 */
19643         __Pyx_XDECREF(__pyx_r);
19644         __pyx_t_1 = __Pyx_PyUnicode_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size());
19645         if (unlikely(!__pyx_t_1)) __PYX_ERR(1, 38, __pyx_L1_error)
19646         __Pyx_GOTREF(__pyx_t_1);
19647         __pyx_r = __pyx_t_1;
19648         __pyx_t_1 = 0;
19649         goto __pyx_L0;
19650
19651             /* "string.to_py":37
19652 *
19653 * @cname("__pyx_convert_PyUnicode_string_to_py_std__in_string")
19654 * cdef inline object __pyx_convert_PyUnicode_string_to_py_std__in_string(const string& s):
19655 # ««««««««
19656 *     return __Pyx_PyUnicode_FromStringAndSize(s.data(), s.size())
19657 * cdef extern from *:
19658 */
19659
19660             /* function exit code */
19661         __pyx_L1_error;;
19662         __Pyx_XDECREF(__pyx_t_1);
19663
19664         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyUnicode_string_to_py_std__in_string", __pyx_clineno,
__pyx_lineno, __pyx_filename);

```

```

19658         __pyx_r = 0;
19659         __pyx_L0;
19660         __Pyx_XGIVEREF(__pyx_r);
19661         __Pyx_RefNannyFinishContext();
19662         return __pyx_r;
19663     }
19664
19665     /* "string.to_py":43
19666  *
19667  * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19668  * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s): #
19669  * <<<<<<<<
19670  *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19671  * cdef extern from *:
19672  */
19673
19674     static CYTHON_INLINE PyObject
19675     *__pyx_convert_PyStr_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19676         PyObject *__pyx_r = NULL;
19677         __Pyx_RefNannyDeclarations
19678         PyObject *__pyx_t_1 = NULL;
19679         int __pyx_lineno = 0;
19680         const char *__pyx_filename = NULL;
19681         int __pyx_clineno = 0;
19682         __Pyx_RefNannySetupContext("__pyx_convert_PyStr_string_to_py_std__in_string", 0);
19683
19684     /* "string.to_py":44
19685  * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19686  * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s):
19687  *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size()) # <<<<<<<<
19688  * cdef extern from *:
19689  *     cdef object __Pyx_PyBytes_FromStringAndSize(const char*, size_t)
19690  */
19691         __Pyx_XDECREF(__pyx_r);
19692         __pyx_t_1 = __Pyx_PyStr_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
19693         (unlikely(!__pyx_t_1)) __PYX_ERR(1, 44, __pyx_L1_error)
19694         __Pyx_GOTREF(__pyx_t_1);
19695         __pyx_r = __pyx_t_1;
19696         __pyx_t_1 = 0;
19697         goto __pyx_L0;
19698
19699     /* "string.to_py":43
19700  *
19701  * @cname("__pyx_convert_PyStr_string_to_py_std__in_string")
19702  * cdef inline object __pyx_convert_PyStr_string_to_py_std__in_string(const string& s): #
19703  * <<<<<<<<
19704  *     return __Pyx_PyStr_FromStringAndSize(s.data(), s.size())
19705  * cdef extern from *:
19706  */
19707
19708     /* function exit code */
19709     __pyx_L1_error;
19710     __Pyx_XDECREF(__pyx_t_1);
19711     __Pyx_AddTraceback("string.to_py.__pyx_convert_PyStr_string_to_py_std__in_string",
19712     __pyx_clineno, __pyx_lineno, __pyx_filename);
19713     __pyx_r = 0;
19714     __pyx_L0;
19715     __Pyx_XGIVEREF(__pyx_r);
19716     __Pyx_RefNannyFinishContext();
19717     return __pyx_r;
19718 }
19719
19720 /* "string.to_py":49
19721  *
19722  * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19723  * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19724  * <<<<<<<<
19725  *     return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size())
19726  * cdef extern from *:
19727  */
19728
19729     static CYTHON_INLINE PyObject
19730     *__pyx_convert_PyBytes_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19731         PyObject *__pyx_r = NULL;
19732         __Pyx_RefNannyDeclarations
19733         PyObject *__pyx_t_1 = NULL;
19734         int __pyx_lineno = 0;
19735         const char *__pyx_filename = NULL;
19736         int __pyx_clineno = 0;
19737         __Pyx_RefNannySetupContext("__pyx_convert_PyBytes_string_to_py_std__in_string", 0);
19738
19739     /* "string.to_py":50
19740  * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19741  * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19742  *     return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size()) # <<<<<<<<
19743  * cdef extern from *:
19744  *     cdef object __Pyx_PyByteArray_FromStringAndSize(const char*, size_t)

```

```

19738 */
19739         __Pyx_XDECREF(__pyx_r);
19740         __pyx_t_1 = __Pyx_PyBytes_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size()); if
(unlikely(!__pyx_t_1)) __PYX_ERR(1, 50, __pyx_L1_error)
19741         __Pyx_GOTREF(__pyx_t_1);
19742         __pyx_r = __pyx_t_1;
19743         __pyx_t_1 = 0;
19744         goto __pyx_L0;
19745
19746         /* "string.to_py":49
19747  *
19748  * @cname("__pyx_convert_PyBytes_string_to_py_std__in_string")
19749  * cdef inline object __pyx_convert_PyBytes_string_to_py_std__in_string(const string& s):
19750  *     return __Pyx_PyBytes_FromStringAndSize(s.data(), s.size())
19751  * cdef extern from *:
19752  */
19753
19754         /* function exit code */
19755         __pyx_L1_error;
19756         __Pyx_XDECREF(__pyx_t_1);
19757         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyBytes_string_to_py_std__in_string",
__pyx_clineno, __pyx_lineno, __pyx_filename);
19758         __pyx_r = 0;
19759         __pyx_L0;
19760         __Pyx_XGIVEREF(__pyx_r);
19761         __Pyx_RefNannyFinishContext();
19762         return __pyx_r;
19763     }
19764
19765     /* "string.to_py":55
19766  *
19767  * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19768  * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19769  *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19770  *
19771  */
19772
19773     static CYTHON_INLINE PyObject
*__pyx_convert_PyByteArray_string_to_py_std__in_string(std::string const &__pyx_v_s) {
19774         PyObject *__pyx_r = NULL;
19775         __Pyx_RefNannyDeclarations
19776         PyObject *__pyx_t_1 = NULL;
19777         int __pyx_lineno = 0;
19778         const char *__pyx_filename = NULL;
19779         int __pyx_clineno = 0;
19780         __Pyx_RefNannySetupContext("__pyx_convert_PyByteArray_string_to_py_std__in_string",
0);
19781
19782         /* "string.to_py":56
19783  * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19784  * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19785  *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size()) # ««««««««
19786  *
19787  */
19788         __Pyx_XDECREF(__pyx_r);
19789         __pyx_t_1 = __Pyx_PyByteArray_FromStringAndSize(__pyx_v_s.data(), __pyx_v_s.size());
19790         if (unlikely(!__pyx_t_1)) __PYX_ERR(1, 56, __pyx_L1_error)
19791         __Pyx_GOTREF(__pyx_t_1);
19792         __pyx_r = __pyx_t_1;
19793         __pyx_t_1 = 0;
19794         goto __pyx_L0;
19795
19796         /* "string.to_py":55
19797  *
19798  * @cname("__pyx_convert_PyByteArray_string_to_py_std__in_string")
19799  * cdef inline object __pyx_convert_PyByteArray_string_to_py_std__in_string(const string& s):
19800  *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
19801  *
19802  */
19803         /* function exit code */
19804         __pyx_L1_error;
19805         __Pyx_XDECREF(__pyx_t_1);
19806         __Pyx_AddTraceback("string.to_py.__pyx_convert_PyByteArray_string_to_py_std__in_string",
__pyx_clineno, __pyx_lineno, __pyx_filename);
19807         __pyx_r = 0;
19808         __pyx_L0;
19809         __Pyx_XGIVEREF(__pyx_r);
19810         __Pyx_RefNannyFinishContext();
19811         return __pyx_r;
19812     }
19813     static struct __pyx_vtabstruct_8PyClical_index_set __pyx_vtable_8PyClical_index_set;
19814

```

```

19815     static PyObject *__pyx_tp_new_8PyClical_index_set(PyTypeObject *t, PyObject *a,
19816     PyObject *k) {
19817         struct __pyx_obj_8PyClical_index_set *p;
19818         PyObject *o;
19819         if (likely((t->tp_flags & Py_TPFLAGS_IS_ABSTRACT) == 0)) {
19820             o = (*t->tp_alloc)(t, 0);
19821         } else {
19822             o = (PyObject *) PyBaseObject_Type.tp_new(t, __pyx_empty_tuple, 0);
19823         }
19824         if (unlikely(!o)) return 0;
19825         p = ((struct __pyx_obj_8PyClical_index_set *)o);
19826         p->__pyx_vtab = __pyx_vtabptr_8PyClical_index_set;
19827         if (unlikely(__pyx_pw_8PyClical_9index_set_3__cinit__(o, a, k) < 0)) goto bad;
19828         return o;
19829     bad:
19830         Py_DECREF(o); o = 0;
19831         return NULL;
19832     }
19833
19834     static void __pyx_tp_dealloc_8PyClical_index_set(PyObject *o) {
19835         #if CYTHON_USE_TP_FINALIZE
19836         if (unlikely(PyType_HasFeature(Py_TYPE(o), Py_TPFLAGS_HAVE_FINALIZE) &&
19837         (!PyType_IS_GC(Py_TYPE(o)) || !PyGC_FINALIZED(o))) {
19838             if (PyObject_CallFinalizerFromDealloc(o)) return;
19839         }
19840         #endif
19841         {
19842             PyObject *etype, *eval, *etb;
19843             PyErr_Fetch(&etype, &eval, &etb);
19844             __Pyx_SET_REFCNT(o, Py_REFCNT(o) + 1);
19845             __pyx_pw_8PyClical_9index_set_5__dealloc__(o);
19846             __Pyx_SET_REFCNT(o, Py_REFCNT(o) - 1);
19847             PyErr_Restore(etype, eval, etb);
19848         }
19849         (*Py_TYPE(o)->tp_free)(o);
19850     }
19851     static PyObject *__pyx_sq_item_8PyClical_index_set(PyObject *o, Py_ssize_t i) {
19852         PyObject *r;
19853         PyObject *x = PyInt_FromSsize_t(i); if(!x) return 0;
19854         r = Py_TYPE(o)->tp_as_mapping->mp_subscript(o, x);
19855         Py_DECREF(x);
19856         return r;
19857     }
19858     static int __pyx_mp_ass_subscript_8PyClical_index_set(PyObject *o, PyObject *i,
19859     PyObject *v) {
19860         if (v) {
19861             return __pyx_pw_8PyClical_9index_set_9__setitem__(o, i, v);
19862         }
19863         else {
19864             PyErr_Format(PyExc_NotImplementedError,
19865             "Subscript deletion not supported by %.200s", Py_TYPE(o)->tp_name);
19866             return -1;
19867         }
19868     }
19869     static PyMethodDef __pyx_methods_8PyClical_index_set[] = {
19870         {"copy", (PyCFunction) __pyx_pw_8PyClical_9index_set_1copy, METH_NOARGS,
19871         __pyx_doc_8PyClical_9index_set_copy},
19872         {"count", (PyCFunction) __pyx_pw_8PyClical_9index_set_32count, METH_NOARGS,
19873         __pyx_doc_8PyClical_9index_set_31count},
19874         {"count_neg", (PyCFunction) __pyx_pw_8PyClical_9index_set_34count_neg, METH_NOARGS,
19875         __pyx_doc_8PyClical_9index_set_33count_neg},
19876         {"count_pos", (PyCFunction) __pyx_pw_8PyClical_9index_set_36count_pos, METH_NOARGS,
19877         __pyx_doc_8PyClical_9index_set_35count_pos},
19878         {"min", (PyCFunction) __pyx_pw_8PyClical_9index_set_38min, METH_NOARGS,
19879         __pyx_doc_8PyClical_9index_set_37min},
19880         {"max", (PyCFunction) __pyx_pw_8PyClical_9index_set_40max, METH_NOARGS,
19881         __pyx_doc_8PyClical_9index_set_39max},
19882         {"hash_fn", (PyCFunction) __pyx_pw_8PyClical_9index_set_42hash_fn, METH_NOARGS,
19883         __pyx_doc_8PyClical_9index_set_41hash_fn},
19884         {"sign_of_mult", (PyCFunction) __pyx_pw_8PyClical_9index_set_44sign_of_mult, METH_O,
19885         __pyx_doc_8PyClical_9index_set_43sign_of_mult},
19886         {"sign_of_square", (PyCFunction) __pyx_pw_8PyClical_9index_set_46sign_of_square,
19887         METH_NOARGS, __pyx_doc_8PyClical_9index_set_45sign_of_square},
19888         {"__reduce_cython__",
19889         (PyCFunction) __pyx_pw_8PyClical_9index_set_52__reduce_cython__, METH_NOARGS, 0},
19890         {"__setstate_cython__",
19891         (PyCFunction) __pyx_pw_8PyClical_9index_set_54__setstate_cython__, METH_O, 0},
19892         {0, 0, 0, 0}
19893     };
19894
19895     static PyNumberMethods __pyx_tp_as_number_index_set = {
19896         0, /*nb_add*/
19897         0, /*nb_subtract*/
19898         0, /*nb_multiply*/
19899         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <

```

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0x03050000)
19888         0, /*nb_divide*/
19889         #endif
19890         0, /*nb_remainder*/
19891         0, /*nb_divmod*/
19892         0, /*nb_power*/
19893         0, /*nb_negative*/
19894         0, /*nb_positive*/
19895         0, /*nb_absolute*/
19896         0, /*nb_nonzero*/
19897         __pyx_pw_8PyClical_9index_set_18__invert__, /*nb_invert*/
19898         0, /*nb_lshift*/
19899         0, /*nb_rshift*/
19900         __pyx_pw_8PyClical_9index_set_24__and__, /*nb_and*/
19901         __pyx_pw_8PyClical_9index_set_20__xor__, /*nb_xor*/
19902         __pyx_pw_8PyClical_9index_set_28__or__, /*nb_or*/
19903         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19904         0, /*nb_coerce*/
19905         #endif
19906         0, /*nb_int*/
19907         #if PY_MAJOR_VERSION < 3
19908         0, /*nb_long*/
19909         #else
19910         0, /*reserved*/
19911         #endif
19912         0, /*nb_float*/
19913         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19914         0, /*nb_oct*/
19915         #endif
19916         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19917         0, /*nb_hex*/
19918         #endif
19919         0, /*nb_inplace_add*/
19920         0, /*nb_inplace_subtract*/
19921         0, /*nb_inplace_multiply*/
19922         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
19923         0, /*nb_inplace_divide*/
19924         #endif
19925         0, /*nb_inplace_remainder*/
19926         0, /*nb_inplace_power*/
19927         0, /*nb_inplace_lshift*/
19928         0, /*nb_inplace_rshift*/
19929         __pyx_pw_8PyClical_9index_set_26__iand__, /*nb_inplace_and*/
19930         __pyx_pw_8PyClical_9index_set_22__ixor__, /*nb_inplace_xor*/
19931         __pyx_pw_8PyClical_9index_set_30__ior__, /*nb_inplace_or*/
19932         0, /*nb_floor_divide*/
19933         0, /*nb_true_divide*/
19934         0, /*nb_inplace_floor_divide*/
19935         0, /*nb_inplace_true_divide*/
19936         0, /*nb_index*/
19937         #if PY_VERSION_HEX >= 0x03050000
19938         0, /*nb_matrix_multiply*/
19939         #endif
19940         #if PY_VERSION_HEX >= 0x03050000
19941         0, /*nb_inplace_matrix_multiply*/
19942         #endif
19943     };
19944
19945     static PySequenceMethods __pyx_tp_as_sequence_index_set = {
19946         0, /*sq_length*/
19947         0, /*sq_concat*/
19948         0, /*sq_repeat*/
19949         __pyx_sq_item_8PyClical_index_set, /*sq_item*/
19950         0, /*sq_slice*/
19951         0, /*sq_ass_item*/
19952         0, /*sq_ass_slice*/
19953         __pyx_pw_8PyClical_9index_set_13__contains__, /*sq_contains*/
19954         0, /*sq_inplace_concat*/
19955         0, /*sq_inplace_repeat*/
19956     };
19957
19958     static PyMappingMethods __pyx_tp_as_mapping_index_set = {
19959         0, /*mp_length*/
19960         __pyx_pw_8PyClical_9index_set_11__getitem__, /*mp_subscript*/
19961         __pyx_mp_ass_subscript_8PyClical_index_set, /*mp_ass_subscript*/
19962     };
19963
19964     static PyTypeObject __pyx_type_8PyClical_index_set = {
19965         PyVarObject_HEAD_INIT(0, 0)
19966         "PyClical.index_set", /*tp_name*/
19967         sizeof(struct __pyx_obj_8PyClical_index_set), /*tp_basicsize*/
19968         0, /*tp_itemsize*/
19969         __pyx_tp_dealloc_8PyClical_index_set, /*tp_dealloc*/

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19970         #if PY_VERSION_HEX < 0x030800b4
19971         0, /*tp_print*/
19972         #endif
19973         #if PY_VERSION_HEX >= 0x030800b4
19974         0, /*tp_vectorcall_offset*/
19975         #endif
19976         0, /*tp_getattr*/
19977         0, /*tp_setattr*/
19978         #if PY_MAJOR_VERSION < 3
19979         0, /*tp_compare*/
19980         #endif
19981         #if PY_MAJOR_VERSION >= 3
19982         0, /*tp_as_async*/
19983         #endif
19984         __pyx_pw_8PyClical_9index_set_48__repr__, /*tp_repr*/
19985         &__pyx_tp_as_number_index_set, /*tp_as_number*/
19986         &__pyx_tp_as_sequence_index_set, /*tp_as_sequence*/
19987         &__pyx_tp_as_mapping_index_set, /*tp_as_mapping*/
19988         0, /*tp_hash*/
19989         0, /*tp_call*/
19990         __pyx_pw_8PyClical_9index_set_50__str__, /*tp_str*/
19991         0, /*tp_getattro*/
19992         0, /*tp_setattro*/
19993         0, /*tp_as_buffer*/
19994
19995         Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_BASETYPE,
19996         /*tp_flags*/
19997
19998         "\n    Python class index_set wraps C++ class IndexSet.\n    ", /*tp_doc*/
19999         0, /*tp_traverse*/
20000         0, /*tp_clear*/
20001         __pyx_pw_8PyClical_9index_set_7__richcmp__, /*tp_richcompare*/
20002         0, /*tp_weaklistoffset*/
20003         __pyx_pw_8PyClical_9index_set_15__iter__, /*tp_iter*/
20004         0, /*tp_iternext*/
20005         __pyx_methods_8PyClical_index_set, /*tp_methods*/
20006         0, /*tp_members*/
20007         0, /*tp_getset*/
20008         0, /*tp_base*/
20009         0, /*tp_dict*/
20010         0, /*tp_descr_get*/
20011         0, /*tp_descr_set*/
20012         0, /*tp_dictoffset*/
20013         0, /*tp_init*/
20014         0, /*tp_alloc*/
20015         __pyx_tp_new_8PyClical_index_set, /*tp_new*/
20016         0, /*tp_free*/
20017         0, /*tp_is_gc*/
20018         0, /*tp_bases*/
20019         0, /*tp_mro*/
20020         0, /*tp_cache*/
20021         0, /*tp_subclasses*/
20022         0, /*tp_weaklist*/
20023         0, /*tp_del*/
20024         0, /*tp_version_tag*/
20025         #if PY_VERSION_HEX >= 0x030400a1
20026         0, /*tp_finalize*/
20027         #endif
20028         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
20029         >= 0x07030800)
20030         0, /*tp_vectorcall*/
20031         #endif
20032         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20033         0, /*tp_print*/
20034         #endif
20035         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20036         0, /*tp_pypy_flags*/
20037         #endif
20038     };
20039     static struct __pyx_vtabstruct_8PyClical_clifford __pyx_vtable_8PyClical_clifford;
20040
20041     static PyObject *__pyx_tp_new_8PyClical_clifford(PyTypeObject *t, PyObject *a,
20042     PyObject *k) {
20043         struct __pyx_obj_8PyClical_clifford *p;
20044         PyObject *o;
20045         if (likely((t->tp_flags & Py_TPFLAGS_IS_ABSTRACT) == 0)) {
20046             o = (*t->tp_alloc)(t, 0);
20047         } else {
20048             o = (PyObject *) PyBaseObject_Type.tp_new(t, __pyx_empty_tuple, 0);
20049         }
20050         if (unlikely(!o)) return 0;
20051         p = ((struct __pyx_obj_8PyClical_clifford *)o);
20052         p->__pyx_vtab = __pyx_vtabptr_8PyClical_clifford;
20053         if (unlikely(__pyx_pw_8PyClical_8clifford_3__cinit__(o, a, k) < 0)) goto bad;
20054         return o;
20055     bad:
20056         Py_DECREF(o); o = 0;
20057         return NULL;

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20053     }
20054
20055     static void __pyx_tp_dealloc_8PyClical_clifford(PyObject *o) {
20056         #if CYTHON_USE_TP_FINALIZE
20057         if (unlikely(PyType_HasFeature(Py_TYPE(o), Py_TPFLAGS_HAVE_FINALIZE) &&
Py_TYPE(o)->tp_finalize) && (!PyType_IS_GC(Py_TYPE(o)) || !_PyGC_FINALIZED(o))) {
20058             if (PyObject_CallFinalizerFromDealloc(o)) return;
20059         }
20060         #endif
20061         {
20062             PyObject *etype, *eval, *etb;
20063             PyErr_Fetch(&etype, &eval, &etb);
20064             __Pyx_SET_REFCNT(o, Py_REFCNT(o) + 1);
20065             __pyx_pw_8PyClical_8clifford_5__dealloc__(o);
20066             __Pyx_SET_REFCNT(o, Py_REFCNT(o) - 1);
20067             PyErr_Restore(etype, eval, etb);
20068         }
20069         (*Py_TYPE(o)->tp_free)(o);
20070     }
20071
20072     static PyObject * __pyx_sq_item_8PyClical_clifford(PyObject *o, Py_ssize_t i) {
20073         PyObject *r;
20074         PyObject *x = PyInt_FromSsize_t(i); if(!x) return 0;
20075         r = Py_TYPE(o)->tp_as_mapping->mp_subscript(o, x);
20076         Py_DECREF(x);
20077         return r;
20078     }
20079
20080     static PyMethodDef __pyx_methods_8PyClical_clifford[] = {
20081         {"copy", (PyCFunction) __pyx_pw_8PyClical_8clifford_1copy, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_copy},
20082         {"reframe", (PyCFunction) __pyx_pw_8PyClical_8clifford_11reframe, METH_O,
__pyx_doc_8PyClical_8clifford_10reframe},
20083         {"inv", (PyCFunction) __pyx_pw_8PyClical_8clifford_49inv, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_48inv},
20084         {"pow", (PyCFunction) __pyx_pw_8PyClical_8clifford_57pow, METH_O,
__pyx_doc_8PyClical_8clifford_56pow},
20085         {"outer_pow", (PyCFunction) __pyx_pw_8PyClical_8clifford_59outer_pow, METH_O,
__pyx_doc_8PyClical_8clifford_58outer_pow},
20086         {"scalar", (PyCFunction) __pyx_pw_8PyClical_8clifford_63scalar, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_62scalar},
20087         {"pure", (PyCFunction) __pyx_pw_8PyClical_8clifford_65pure, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_64pure},
20088         {"even", (PyCFunction) __pyx_pw_8PyClical_8clifford_67even, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_66even},
20089         {"odd", (PyCFunction) __pyx_pw_8PyClical_8clifford_69odd, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_68odd},
20090         {"vector_part",
(PyCFunction) (void*) (PyCFunctionWithKeywords) __pyx_pw_8PyClical_8clifford_71vector_part,
METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_8clifford_70vector_part},
20091         {"involute", (PyCFunction) __pyx_pw_8PyClical_8clifford_73involute, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_72involute},
20092         {"reverse", (PyCFunction) __pyx_pw_8PyClical_8clifford_75reverse, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_74reverse},
20093         {"conj", (PyCFunction) __pyx_pw_8PyClical_8clifford_77conj, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_76conj},
20094         {"quad", (PyCFunction) __pyx_pw_8PyClical_8clifford_79quad, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_78quad},
20095         {"norm", (PyCFunction) __pyx_pw_8PyClical_8clifford_81norm, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_80norm},
20096         {"abs", (PyCFunction) __pyx_pw_8PyClical_8clifford_83abs, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_82abs},
20097         {"max_abs", (PyCFunction) __pyx_pw_8PyClical_8clifford_85max_abs, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_84max_abs},
20098         {"truncated", (PyCFunction) __pyx_pw_8PyClical_8clifford_87truncated, METH_O,
__pyx_doc_8PyClical_8clifford_86truncated},
20099         {"isinf", (PyCFunction) __pyx_pw_8PyClical_8clifford_89isinf, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_88isinf},
20100         {"isnan", (PyCFunction) __pyx_pw_8PyClical_8clifford_91isnan, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_90isnan},
20101         {"frame", (PyCFunction) __pyx_pw_8PyClical_8clifford_93frame, METH_NOARGS,
__pyx_doc_8PyClical_8clifford_92frame},
20102         {"__reduce_cython__", (PyCFunction) __pyx_pw_8PyClical_8clifford_99__reduce_cython__,
METH_NOARGS, 0},
20103         {"__setstate_cython__",
(PyCFunction) __pyx_pw_8PyClical_8clifford_101__setstate_cython__, METH_O, 0},
20104         {0, 0, 0, 0};
20105
20106     static PyNumberMethods __pyx_tp_as_number_clifford = {
20107         __pyx_pw_8PyClical_8clifford_21__add__, /*nb_add*/
20108         __pyx_pw_8PyClical_8clifford_25__sub__, /*nb_subtract*/
20109         __pyx_pw_8PyClical_8clifford_29__mul__, /*nb_multiply*/
20110         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20111         0, /*nb_divide*/
20112         #endif
20113         __pyx_pw_8PyClical_8clifford_33__mod__, /*nb_remainder*/

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```

20114         0, /*nb_divmod*/
20115         __pyx_pw_8PyClical_8clifford_55_pow__, /*nb_power*/
20116         __pyx_pw_8PyClical_8clifford_17_neg__, /*nb_negative*/
20117         __pyx_pw_8PyClical_8clifford_19_pos__, /*nb_positive*/
20118         0, /*nb_absolute*/
20119         0, /*nb_nonzero*/
20120         0, /*nb_invert*/
20121         0, /*nb_lshift*/
20122         0, /*nb_rshift*/
20123         __pyx_pw_8PyClical_8clifford_37_and__, /*nb_and*/
20124         __pyx_pw_8PyClical_8clifford_41_xor__, /*nb_xor*/
20125         __pyx_pw_8PyClical_8clifford_51_or__, /*nb_or*/
20126         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20127         0, /*nb_coerce*/
20128         #endif
20129         0, /*nb_int*/
20130         #if PY_MAJOR_VERSION < 3
20131         0, /*nb_long*/
20132         #else
20133         0, /*reserved*/
20134         #endif
20135         0, /*nb_float*/
20136         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20137         0, /*nb_oct*/
20138         #endif
20139         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20140         0, /*nb_hex*/
20141         #endif
20142         __pyx_pw_8PyClical_8clifford_23_iadd__, /*nb_inplace_add*/
20143         __pyx_pw_8PyClical_8clifford_27_isub__, /*nb_inplace_subtract*/
20144         __pyx_pw_8PyClical_8clifford_31_imul__, /*nb_inplace_multiply*/
20145         #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX <
0x03050000)
20146         __pyx_pw_8PyClical_8clifford_47_idiv__, /*nb_inplace_divide*/
20147         #endif
20148         __pyx_pw_8PyClical_8clifford_35_imod__, /*nb_inplace_remainder*/
20149         0, /*nb_inplace_power*/
20150         0, /*nb_inplace_lshift*/
20151         0, /*nb_inplace_rshift*/
20152         __pyx_pw_8PyClical_8clifford_39_iand__, /*nb_inplace_and*/
20153         __pyx_pw_8PyClical_8clifford_43_ixor__, /*nb_inplace_xor*/
20154         __pyx_pw_8PyClical_8clifford_53_ior__, /*nb_inplace_or*/
20155         0, /*nb_floor_divide*/
20156         __pyx_pw_8PyClical_8clifford_45_truediv__, /*nb_true_divide*/
20157         0, /*nb_inplace_floor_divide*/
20158         0, /*nb_inplace_true_divide*/
20159         0, /*nb_index*/
20160         #if PY_VERSION_HEX >= 0x03050000
20161         0, /*nb_matrix_multiply*/
20162         #endif
20163         #if PY_VERSION_HEX >= 0x03050000
20164         0, /*nb_inplace_matrix_multiply*/
20165         #endif
20166     };
20167
20168     static PySequenceMethods __pyx_tp_as_sequence_clifford = {
20169         0, /*sq_length*/
20170         0, /*sq_concat*/
20171         0, /*sq_repeat*/
20172         __pyx_sq_item_8PyClical_clifford, /*sq_item*/
20173         0, /*sq_slice*/
20174         0, /*sq_ass_item*/
20175         0, /*sq_ass_slice*/
20176         __pyx_pw_8PyClical_8clifford_7_contains__, /*sq_contains*/
20177         0, /*sq_inplace_concat*/
20178         0, /*sq_inplace_repeat*/
20179     };
20180
20181     static PyMappingMethods __pyx_tp_as_mapping_clifford = {
20182         0, /*mp_length*/
20183         __pyx_pw_8PyClical_8clifford_15_getitem__, /*mp_subscript*/
20184         0, /*mp_ass_subscript*/
20185     };
20186
20187     static PyTypeObject __pyx_type_8PyClical_clifford = {
20188         PyVarObject_HEAD_INIT(0, 0)
20189         "PyClical.clifford", /*tp_name*/
20190         sizeof(struct __pyx_obj_8PyClical_clifford), /*tp_basicsize*/
20191         0, /*tp_itemsize*/
20192         __pyx_tp_dealloc_8PyClical_clifford, /*tp_dealloc*/
20193         #if PY_VERSION_HEX < 0x030800b4
20194         0, /*tp_print*/
20195         #endif
20196         #if PY_VERSION_HEX >= 0x030800b4

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20197         0, /*tp_vectorcall_offset*/
20198         #endif
20199         0, /*tp_getattr*/
20200         0, /*tp_setattr*/
20201         #if PY_MAJOR_VERSION < 3
20202         0, /*tp_compare*/
20203         #endif
20204         #if PY_MAJOR_VERSION >= 3
20205         0, /*tp_as_async*/
20206         #endif
20207         __pyx_pw_8PyClical_8clifford_95__repr__, /*tp_repr*/
20208         &__pyx_tp_as_number_clifford, /*tp_as_number*/
20209         &__pyx_tp_as_sequence_clifford, /*tp_as_sequence*/
20210         &__pyx_tp_as_mapping_clifford, /*tp_as_mapping*/
20211         0, /*tp_hash*/
20212         __pyx_pw_8PyClical_8clifford_61__call__, /*tp_call*/
20213         __pyx_pw_8PyClical_8clifford_97__str__, /*tp_str*/
20214         0, /*tp_getattro*/
20215         0, /*tp_setattro*/
20216         0, /*tp_as_buffer*/
20217
Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_BASETYPE,
/*tp_flags*/
20218         "\n    Python class clifford wraps C++ class Clifford.\n    ", /*tp_doc*/
20219         0, /*tp_traverse*/
20220         0, /*tp_clear*/
20221         __pyx_pw_8PyClical_8clifford_13__richcmp__, /*tp_richcompare*/
20222         0, /*tp_weaklistoffset*/
20223         __pyx_pw_8PyClical_8clifford_9__iter__, /*tp_iter*/
20224         0, /*tp_iternext*/
20225         __pyx_methods_8PyClical_clifford, /*tp_methods*/
20226         0, /*tp_members*/
20227         0, /*tp_getset*/
20228         0, /*tp_base*/
20229         0, /*tp_dict*/
20230         0, /*tp_descr_get*/
20231         0, /*tp_descr_set*/
20232         0, /*tp_dictoffset*/
20233         0, /*tp_init*/
20234         0, /*tp_alloc*/
20235         __pyx_tp_new_8PyClical_clifford, /*tp_new*/
20236         0, /*tp_free*/
20237         0, /*tp_is_gc*/
20238         0, /*tp_bases*/
20239         0, /*tp_mro*/
20240         0, /*tp_cache*/
20241         0, /*tp_subclasses*/
20242         0, /*tp_weaklist*/
20243         0, /*tp_del*/
20244         0, /*tp_version_tag*/
20245         #if PY_VERSION_HEX >= 0x030400a1
20246         0, /*tp_finalize*/
20247         #endif
20248         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
>= 0x07030800)
20249         0, /*tp_vectorcall*/
20250         #endif
20251         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20252         0, /*tp_print*/
20253         #endif
20254         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20255         0, /*tp_pypy_flags*/
20256         #endif
20257     };
20258
20259     static struct __pyx_obj_8PyClical__pyx_scope_struct____iter__
*_pyx_freelist_8PyClical__pyx_scope_struct____iter__[8];
20260     static int __pyx_freecount_8PyClical__pyx_scope_struct____iter__ = 0;
20261
20262     static PyObject * __pyx_tp_new_8PyClical__pyx_scope_struct____iter__(PyObject *t,
CYTHON_UNUSED PyObject *a, CYTHON_UNUSED PyObject *k) {
20263         PyObject *o;
20264         if (CYTHON_COMPILING_IN_CPYTHON &&
likely((__pyx_freecount_8PyClical__pyx_scope_struct____iter__ > 0) & (t->tp_basicsize ==
sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__))) {
20265             o =
(PyObject*)__pyx_freelist_8PyClical__pyx_scope_struct____iter__[--__pyx_freecount_8PyClical__pyx_scope_struct____iter];
20266             memset(o, 0, sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__));
20267             (void) PyObject_INIT(o, t);
20268             PyObject_GC_Track(o);
20269         } else {
20270             o = (*t->tp_alloc)(t, 0);
20271             if (unlikely(!o)) return 0;
20272         }
20273         return o;
20274     }
20275

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20276         static void __pyx_tp_dealloc_8PyClical__pyx_scope_struct____iter__(PyObject *o) {
20277             struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *p = (struct
__pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o;
20278             PyObject_GC_UnTrack(o);
20279             Py_CLEAR(p->__pyx_v_idx);
20280             Py_CLEAR(p->__pyx_v_self);
20281             Py_CLEAR(p->__pyx_t_0);
20282             if (CYTHON_COMPILING_IN_CPYTHON &&
((__pyx_freecount_8PyClical__pyx_scope_struct____iter__ < 8) & (Py_TYPE(o)->tp_basicsize ==
sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__))) {
20283                 __pyx_freelist_8PyClical__pyx_scope_struct____iter__[__pyx_freecount_8PyClical__pyx_scope_struct____iter__++]
= ((struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o);
20284             } else {
20285                 (*Py_TYPE(o)->tp_free)(o);
20286             }
20287         }
20288
20289         static int __pyx_tp_traverse_8PyClical__pyx_scope_struct____iter__(PyObject *o,
visitproc v, void *a) {
20290             int e;
20291             struct __pyx_obj_8PyClical__pyx_scope_struct____iter__ *p = (struct
__pyx_obj_8PyClical__pyx_scope_struct____iter__ *)o;
20292             if (p->__pyx_v_idx) {
20293                 e = (*v)(p->__pyx_v_idx, a); if (e) return e;
20294             }
20295             if (p->__pyx_v_self) {
20296                 e = (*v)((PyObject *)p->__pyx_v_self, a); if (e) return e;
20297             }
20298             if (p->__pyx_t_0) {
20299                 e = (*v)(p->__pyx_t_0, a); if (e) return e;
20300             }
20301             return 0;
20302         }
20303
20304         static PyTypeObject __pyx_type_8PyClical__pyx_scope_struct____iter__ = {
20305             PyVarObject_HEAD_INIT(0, 0)
20306             "PyClical.__pyx_scope_struct____iter__", /*tp_name*/
20307             sizeof(struct __pyx_obj_8PyClical__pyx_scope_struct____iter__), /*tp_basicsize*/
20308             0, /*tp_itemsize*/
20309             __pyx_tp_dealloc_8PyClical__pyx_scope_struct____iter__, /*tp_dealloc*/
20310             #if PY_VERSION_HEX < 0x030800b4
20311             0, /*tp_print*/
20312             #endif
20313             #if PY_VERSION_HEX >= 0x030800b4
20314             0, /*tp_vectorcall_offset*/
20315             #endif
20316             0, /*tp_getattr*/
20317             0, /*tp_setattr*/
20318             #if PY_MAJOR_VERSION < 3
20319             0, /*tp_compare*/
20320             #endif
20321             #if PY_MAJOR_VERSION >= 3
20322             0, /*tp_as_async*/
20323             #endif
20324             0, /*tp_repr*/
20325             0, /*tp_as_number*/
20326             0, /*tp_as_sequence*/
20327             0, /*tp_as_mapping*/
20328             0, /*tp_hash*/
20329             0, /*tp_call*/
20330             0, /*tp_str*/
20331             0, /*tp_getattro*/
20332             0, /*tp_setattro*/
20333             0, /*tp_as_buffer*/
20334             Py_TPFLAGS_DEFAULT|Py_TPFLAGS_HAVE_VERSION_TAG|Py_TPFLAGS_CHECKTYPES|Py_TPFLAGS_HAVE_NEWBUFFER|Py_TPFLAGS_HAVE_GC,
/*tp_flags*/
20335             0, /*tp_doc*/
20336             __pyx_tp_traverse_8PyClical__pyx_scope_struct____iter__, /*tp_traverse*/
20337             0, /*tp_clear*/
20338             0, /*tp_richcompare*/
20339             0, /*tp_weaklistoffset*/
20340             0, /*tp_iter*/
20341             0, /*tp_iternext*/
20342             0, /*tp_methods*/
20343             0, /*tp_members*/
20344             0, /*tp_getset*/
20345             0, /*tp_base*/
20346             0, /*tp_dict*/
20347             0, /*tp_descr_get*/
20348             0, /*tp_descr_set*/
20349             0, /*tp_dictoffset*/
20350             0, /*tp_init*/
20351             0, /*tp_alloc*/
20352             __pyx_tp_new_8PyClical__pyx_scope_struct____iter__, /*tp_new*/
20353             0, /*tp_free*/

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20354         0, /*tp_is_gc*/
20355         0, /*tp_bases*/
20356         0, /*tp_mro*/
20357         0, /*tp_cache*/
20358         0, /*tp_subclasses*/
20359         0, /*tp_weaklist*/
20360         0, /*tp_del*/
20361         0, /*tp_version_tag*/
20362         #if PY_VERSION_HEX >= 0x030400a1
20363         0, /*tp_finalize*/
20364         #endif
20365         #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM
>= 0x07030800)
20366         0, /*tp_vectorcall*/
20367         #endif
20368         #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
20369         0, /*tp_print*/
20370         #endif
20371         #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
20372         0, /*tp_pypy_flags*/
20373         #endif
20374     };
20375
20376     static PyMethodDef __pyx_methods[] = {
20377         {"compare",
20378         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_3compare, METH_VARARGS|METH_KEYWORDS,
20379         __pyx_doc_8PyClicl_2compare},
20380         {"min_neg", (PyCFunction)__pyx_pw_8PyClicl_5min_neg, METH_O,
20381         __pyx_doc_8PyClicl_4min_neg},
20382         {"max_pos", (PyCFunction)__pyx_pw_8PyClicl_7max_pos, METH_O,
20383         __pyx_doc_8PyClicl_6max_pos},
20384         {"error_squared_tol", (PyCFunction)__pyx_pw_8PyClicl_11error_squared_tol, METH_O,
20385         __pyx_doc_8PyClicl_10error_squared_tol},
20386         {"error_squared",
20387         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_13error_squared,
20388         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_12error_squared},
20389         {"approx_equal",
20390         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_15approx_equal,
20391         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_14approx_equal},
20392         {"inv", (PyCFunction)__pyx_pw_8PyClicl_17inv, METH_O, __pyx_doc_8PyClicl_16inv},
20393         {"scalar", (PyCFunction)__pyx_pw_8PyClicl_19scalar, METH_O,
20394         __pyx_doc_8PyClicl_18scalar},
20395         {"real", (PyCFunction)__pyx_pw_8PyClicl_21real, METH_O,
20396         __pyx_doc_8PyClicl_20real},
20397         {"imag", (PyCFunction)__pyx_pw_8PyClicl_23imag, METH_O,
20398         __pyx_doc_8PyClicl_22imag},
20399         {"pure", (PyCFunction)__pyx_pw_8PyClicl_25pure, METH_O,
20400         __pyx_doc_8PyClicl_24pure},
20401         {"even", (PyCFunction)__pyx_pw_8PyClicl_27even, METH_O,
20402         __pyx_doc_8PyClicl_26even},
20403         {"odd", (PyCFunction)__pyx_pw_8PyClicl_29odd, METH_O, __pyx_doc_8PyClicl_28odd},
20404         {"involute", (PyCFunction)__pyx_pw_8PyClicl_31involute, METH_O,
20405         __pyx_doc_8PyClicl_30involute},
20406         {"reverse", (PyCFunction)__pyx_pw_8PyClicl_33reverse, METH_O,
20407         __pyx_doc_8PyClicl_32reverse},
20408         {"conj", (PyCFunction)__pyx_pw_8PyClicl_35conj, METH_O,
20409         __pyx_doc_8PyClicl_34conj},
20410         {"quad", (PyCFunction)__pyx_pw_8PyClicl_37quad, METH_O,
20411         __pyx_doc_8PyClicl_36quad},
20412         {"norm", (PyCFunction)__pyx_pw_8PyClicl_39norm, METH_O,
20413         __pyx_doc_8PyClicl_38norm},
20414         {"abs", (PyCFunction)__pyx_pw_8PyClicl_41abs, METH_O, __pyx_doc_8PyClicl_40abs},
20415         {"max_abs", (PyCFunction)__pyx_pw_8PyClicl_43max_abs, METH_O,
20416         __pyx_doc_8PyClicl_42max_abs},
20417         {"pow", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_45pow,
20418         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_44pow},
20419         {"outer_pow",
20420         (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_47outer_pow,
20421         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_46outer_pow},
20422         {"complexifier", (PyCFunction)__pyx_pw_8PyClicl_49complexifier, METH_O,
20423         __pyx_doc_8PyClicl_48complexifier},
20424         {"sqrt", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_51sqrt,
20425         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_50sqrt},
20426         {"exp", (PyCFunction)__pyx_pw_8PyClicl_53exp, METH_O, __pyx_doc_8PyClicl_52exp},
20427         {"log", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_55log,
20428         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_54log},
20429         {"cos", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_57cos,
20430         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_56cos},
20431         {"acos", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_59acos,
20432         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_58acos},
20433         {"cosh", (PyCFunction)__pyx_pw_8PyClicl_61cosh, METH_O,
20434         __pyx_doc_8PyClicl_60cosh},
20435         {"acosh", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_63acosh,
20436         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_62acosh},
20437         {"sin", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_65sin,
20438         METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClicl_64sin},
20439         {"asin", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClicl_67asin,

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    METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_66asin},
20409     {"sinh", (PyCFunction)__pyx_pw_8PyClical_69sinh, METH_O,
    __pyx_doc_8PyClical_68sinh},
20410     {"asinh", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_71asinh,
    METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_70asinh},
20411     {"tan", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_73tan,
    METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_72tan},
20412     {"atan", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_75atan,
    METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_74atan},
20413     {"tanh", (PyCFunction)__pyx_pw_8PyClical_77tanh, METH_O,
    __pyx_doc_8PyClical_76tanh},
20414     {"atanh", (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_79atanh,
    METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_78atanh},
20415     {"random_clifford",
    (PyCFunction)(void*)(PyCFunctionWithKeywords)__pyx_pw_8PyClical_81random_clifford,
    METH_VARARGS|METH_KEYWORDS, __pyx_doc_8PyClical_80random_clifford},
20416     {"cga3", (PyCFunction)__pyx_pw_8PyClical_83cga3, METH_O,
    __pyx_doc_8PyClical_82cga3},
20417     {"cga3std", (PyCFunction)__pyx_pw_8PyClical_85cga3std, METH_O,
    __pyx_doc_8PyClical_84cga3std},
20418     {"agc3", (PyCFunction)__pyx_pw_8PyClical_87agc3, METH_O,
    __pyx_doc_8PyClical_86agc3},
20419     {0, 0, 0, 0}
20420 };
20421
20422 #if PY_MAJOR_VERSION >= 3
20423 #if CYTHON_PEP489_MULTI_PHASE_INIT
20424 static PyObject* __pyx_pymod_create(PyObject *spec, PyModuleDef *def); /*proto*/
20425 static int __pyx_pymod_exec_PyClical(PyObject* module); /*proto*/
20426 static PyModuleDef_Slot __pyx_moduledef_slots[] = {
20427     {Py_mod_create, (void*)__pyx_pymod_create},
20428     {Py_mod_exec, (void*)__pyx_pymod_exec_PyClical},
20429     {0, NULL}
20430 };
20431 #endif
20432
20433 static struct PyModuleDef __pyx_moduledef = {
20434     PyModuleDef_HEAD_INIT,
20435     "PyClical",
20436     0, /* m_doc */
20437     #if CYTHON_PEP489_MULTI_PHASE_INIT
20438     0, /* m_size */
20439     #else
20440     -1, /* m_size */
20441     #endif
20442     __pyx_methods /* m_methods */,
20443     #if CYTHON_PEP489_MULTI_PHASE_INIT
20444     __pyx_moduledef_slots, /* m_slots */
20445     #else
20446     NULL, /* m_reload */
20447     #endif
20448     NULL, /* m_traverse */
20449     NULL, /* m_clear */
20450     NULL /* m_free */
20451 };
20452 #endif
20453 #ifndef CYTHON_SMALL_CODE
20454 #if defined(__clang__)
20455     #define CYTHON_SMALL_CODE
20456 #elif defined(__GNUC__) && (__GNUC__ > 4 || (__GNUC__ == 4 && __GNUC_MINOR__ >= 3))
20457     #define CYTHON_SMALL_CODE __attribute__((cold))
20458 #else
20459     #define CYTHON_SMALL_CODE
20460 #endif
20461 #endif
20462
20463 static __Pyx_StringTabEntry __pyx_string_tab[] = {
20464     {&__pyx_kp_u_, __pyx_k_, sizeof(__pyx_k_), 0, 1, 0, 0},
20465     {&__pyx_kp_u_Abbreviation_for_clifford_index, __pyx_k_Abbreviation_for_clifford_index,
    sizeof(__pyx_k_Abbreviation_for_clifford_index), 0, 1, 0, 0},
20466     {&__pyx_kp_u_Abbreviation_for_index_set_q_p, __pyx_k_Abbreviation_for_index_set_q_p,
    sizeof(__pyx_k_Abbreviation_for_index_set_q_p), 0, 1, 0, 0},
20467     {&__pyx_kp_u_Absolute_value_of_multivector_m, __pyx_k_Absolute_value_of_multivector_m,
    sizeof(__pyx_k_Absolute_value_of_multivector_m), 0, 1, 0, 0},
20468     {&__pyx_kp_u_Absolute_value_square_root_of_n, __pyx_k_Absolute_value_square_root_of_n,
    sizeof(__pyx_k_Absolute_value_square_root_of_n), 0, 1, 0, 0},
20469     {&__pyx_kp_u_Cannot_initialize_clifford_objec, __pyx_k_Cannot_initialize_clifford_objec,
    sizeof(__pyx_k_Cannot_initialize_clifford_objec), 0, 1, 0, 0},
20470     {&__pyx_kp_u_Cannot_initialize_index_set_objec, __pyx_k_Cannot_initialize_index_set_objec,
    sizeof(__pyx_k_Cannot_initialize_index_set_objec), 0, 1, 0, 0},
20471     {&__pyx_kp_u_Cannot_reframe, __pyx_k_Cannot_reframe, sizeof(__pyx_k_Cannot_reframe), 0, 1, 0, 0},
20472     {&__pyx_kp_u_Cannot_take_vector_part_of, __pyx_k_Cannot_take_vector_part_of,
    sizeof(__pyx_k_Cannot_take_vector_part_of), 0, 1, 0, 0},
20473     {&__pyx_kp_u_Cardinality_Number_of_indices_i, __pyx_k_Cardinality_Number_of_indices_i,
    sizeof(__pyx_k_Cardinality_Number_of_indices_i), 0, 1, 0, 0},
20474     {&__pyx_kp_u_Check_if_a_multivector_contains, __pyx_k_Check_if_a_multivector_contains,
    sizeof(__pyx_k_Check_if_a_multivector_contains), 0, 1, 0, 0},

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20475     {&__pyx_kp_u_Check_if_a_multivector_contains_2, __pyx_k_Check_if_a_multivector_contains_2,
sizeof(__pyx_k_Check_if_a_multivector_contains_2), 0, 1, 0, 0},
20476     {&__pyx_kp_u_Conjugation_reverse_o_involute, __pyx_k_Conjugation_reverse_o_involute,
sizeof(__pyx_k_Conjugation_reverse_o_involute), 0, 1, 0, 0},
20477     {&__pyx_kp_u_Conjugation_reverse_o_involute_2, __pyx_k_Conjugation_reverse_o_involute_2,
sizeof(__pyx_k_Conjugation_reverse_o_involute_2), 0, 1, 0, 0},
20478     {&__pyx_kp_u_Contraction_print_clifford_1_cl, __pyx_k_Contraction_print_clifford_1_cl,
sizeof(__pyx_k_Contraction_print_clifford_1_cl), 0, 1, 0, 0},
20479     {&__pyx_kp_u_Contraction_x_clifford_1_x_clif, __pyx_k_Contraction_x_clifford_1_x_clif,
sizeof(__pyx_k_Contraction_x_clifford_1_x_clif), 0, 1, 0, 0},
20480     {&__pyx_kp_u_Convert_CGA3_null_vector_to_Euc, __pyx_k_Convert_CGA3_null_vector_to_Euc,
sizeof(__pyx_k_Convert_CGA3_null_vector_to_Euc), 0, 1, 0, 0},
20481     {&__pyx_kp_u_Convert_CGA3_null_vector_to_sta, __pyx_k_Convert_CGA3_null_vector_to_sta,
sizeof(__pyx_k_Convert_CGA3_null_vector_to_sta), 0, 1, 0, 0},
20482     {&__pyx_kp_u_Convert_Euclidean_3D_multivecto, __pyx_k_Convert_Euclidean_3D_multivecto,
sizeof(__pyx_k_Convert_Euclidean_3D_multivecto), 0, 1, 0, 0},
20483     {&__pyx_kp_u_Copy_this_clifford_object_x_cli, __pyx_k_Copy_this_clifford_object_x_cli,
sizeof(__pyx_k_Copy_this_clifford_object_x_cli), 0, 1, 0, 0},
20484     {&__pyx_kp_u_Copy_this_index_set_object_s_in, __pyx_k_Copy_this_index_set_object_s_in,
sizeof(__pyx_k_Copy_this_index_set_object_s_in), 0, 1, 0, 0},
20485     {&__pyx_kp_u_Cosine_of_multivector_with_opti, __pyx_k_Cosine_of_multivector_with_opti,
sizeof(__pyx_k_Cosine_of_multivector_with_opti), 0, 1, 0, 0},
20486     {&__pyx_kp_u_Even_part_of_multivector_sum_of, __pyx_k_Even_part_of_multivector_sum_of,
sizeof(__pyx_k_Even_part_of_multivector_sum_of), 0, 1, 0, 0},
20487     {&__pyx_kp_u_Even_part_of_multivector_sum_of_2, __pyx_k_Even_part_of_multivector_sum_of_2,
sizeof(__pyx_k_Even_part_of_multivector_sum_of_2), 0, 1, 0, 0},
20488     {&__pyx_kp_u_Exponential_of_multivector_x_cl, __pyx_k_Exponential_of_multivector_x_cl,
sizeof(__pyx_k_Exponential_of_multivector_x_cl), 0, 1, 0, 0},
20489     {&__pyx_kp_u_Geometric_difference_print_clif, __pyx_k_Geometric_difference_print_clif,
sizeof(__pyx_k_Geometric_difference_print_clif), 0, 1, 0, 0},
20490     {&__pyx_kp_u_Geometric_difference_x_clifford, __pyx_k_Geometric_difference_x_clifford,
sizeof(__pyx_k_Geometric_difference_x_clifford), 0, 1, 0, 0},
20491     {&__pyx_kp_u_Geometric_multiplicative_invers, __pyx_k_Geometric_multiplicative_invers,
sizeof(__pyx_k_Geometric_multiplicative_invers), 0, 1, 0, 0},
20492     {&__pyx_kp_u_Geometric_multiplicative_invers_2, __pyx_k_Geometric_multiplicative_invers_2,
sizeof(__pyx_k_Geometric_multiplicative_invers_2), 0, 1, 0, 0},
20493     {&__pyx_kp_u_Geometric_product_print_cliffor, __pyx_k_Geometric_product_print_cliffor,
sizeof(__pyx_k_Geometric_product_print_cliffor), 0, 1, 0, 0},
20494     {&__pyx_kp_u_Geometric_product_x_clifford_2, __pyx_k_Geometric_product_x_clifford_2,
sizeof(__pyx_k_Geometric_product_x_clifford_2), 0, 1, 0, 0},
20495     {&__pyx_kp_u_Geometric_quotient_print_cliffo, __pyx_k_Geometric_quotient_print_cliffo,
sizeof(__pyx_k_Geometric_quotient_print_cliffo), 0, 1, 0, 0},
20496     {&__pyx_kp_u_Geometric_quotient_x_clifford_1, __pyx_k_Geometric_quotient_x_clifford_1,
sizeof(__pyx_k_Geometric_quotient_x_clifford_1), 0, 1, 0, 0},
20497     {&__pyx_kp_u_Geometric_sum_print_clifford_1, __pyx_k_Geometric_sum_print_clifford_1,
sizeof(__pyx_k_Geometric_sum_print_clifford_1), 0, 1, 0, 0},
20498     {&__pyx_kp_u_Geometric_sum_x_clifford_1_x_cl, __pyx_k_Geometric_sum_x_clifford_1_x_cl,
sizeof(__pyx_k_Geometric_sum_x_clifford_1_x_cl), 0, 1, 0, 0},
20499     {&__pyx_kp_u_Get_the_value_of_an_index_set_o, __pyx_k_Get_the_value_of_an_index_set_o,
sizeof(__pyx_k_Get_the_value_of_an_index_set_o), 0, 1, 0, 0},
20500     {&__pyx_kp_u_Hyperbolic_cosine_of_multivecto, __pyx_k_Hyperbolic_cosine_of_multivecto,
sizeof(__pyx_k_Hyperbolic_cosine_of_multivecto), 0, 1, 0, 0},
20501     {&__pyx_kp_u_Hyperbolic_sine_of_multivector, __pyx_k_Hyperbolic_sine_of_multivector,
sizeof(__pyx_k_Hyperbolic_sine_of_multivector), 0, 1, 0, 0},
20502     {&__pyx_kp_u_Hyperbolic_tangent_of_multivect, __pyx_k_Hyperbolic_tangent_of_multivect,
sizeof(__pyx_k_Hyperbolic_tangent_of_multivect), 0, 1, 0, 0},
20503     {&__pyx_kp_u_Imaginary_part_deprecated_alway, __pyx_k_Imaginary_part_deprecated_alway,
sizeof(__pyx_k_Imaginary_part_deprecated_alway), 0, 1, 0, 0},
20504     {&__pyx_n_s_IndexError, __pyx_k_IndexError, sizeof(__pyx_k_IndexError), 0, 0, 1, 1},
20505     {&__pyx_kp_u_Inner_product_print_clifford_1, __pyx_k_Inner_product_print_clifford_1,
sizeof(__pyx_k_Inner_product_print_clifford_1), 0, 1, 0, 0},
20506     {&__pyx_kp_u_Inner_product_x_clifford_1_x_cl, __pyx_k_Inner_product_x_clifford_1_x_cl,
sizeof(__pyx_k_Inner_product_x_clifford_1_x_cl), 0, 1, 0, 0},
20507     {&__pyx_kp_u_Integer_power_of_multivector_ob, __pyx_k_Integer_power_of_multivector_ob,
sizeof(__pyx_k_Integer_power_of_multivector_ob), 0, 1, 0, 0},
20508     {&__pyx_n_s_Integral, __pyx_k_Integral, sizeof(__pyx_k_Integral), 0, 0, 1, 1},
20509     {&__pyx_kp_u_Inverse_cosine_of_multivector_w, __pyx_k_Inverse_cosine_of_multivector_w,
sizeof(__pyx_k_Inverse_cosine_of_multivector_w), 0, 1, 0, 0},
20510     {&__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu, __pyx_k_Inverse_hyperbolic_cosine_of_mu,
sizeof(__pyx_k_Inverse_hyperbolic_cosine_of_mu), 0, 1, 0, 0},
20511     {&__pyx_kp_u_Inverse_hyperbolic_sine_of_mult, __pyx_k_Inverse_hyperbolic_sine_of_mult,
sizeof(__pyx_k_Inverse_hyperbolic_sine_of_mult), 0, 1, 0, 0},
20512     {&__pyx_kp_u_Inverse_hyperbolic_tangent_of_m, __pyx_k_Inverse_hyperbolic_tangent_of_m,
sizeof(__pyx_k_Inverse_hyperbolic_tangent_of_m), 0, 1, 0, 0},
20513     {&__pyx_kp_u_Inverse_sine_of_multivector_wit, __pyx_k_Inverse_sine_of_multivector_wit,
sizeof(__pyx_k_Inverse_sine_of_multivector_wit), 0, 1, 0, 0},
20514     {&__pyx_kp_u_Inverse_tangent_of_multivector, __pyx_k_Inverse_tangent_of_multivector,
sizeof(__pyx_k_Inverse_tangent_of_multivector), 0, 1, 0, 0},
20515     {&__pyx_kp_u_Iterate_over_the_indices_of_an, __pyx_k_Iterate_over_the_indices_of_an,
sizeof(__pyx_k_Iterate_over_the_indices_of_an), 0, 1, 0, 0},
20516     {&__pyx_kp_u_Main_involution_each_i_is_repla, __pyx_k_Main_involution_each_i_is_repla,
sizeof(__pyx_k_Main_involution_each_i_is_repla), 0, 1, 0, 0},
20517     {&__pyx_kp_u_Main_involution_each_i_is_repla_2, __pyx_k_Main_involution_each_i_is_repla_2,
sizeof(__pyx_k_Main_involution_each_i_is_repla_2), 0, 1, 0, 0},
20518     {&__pyx_kp_u_Maximum_absolute_value_of_coord, __pyx_k_Maximum_absolute_value_of_coord,
sizeof(__pyx_k_Maximum_absolute_value_of_coord), 0, 1, 0, 0},
20519     {&__pyx_kp_u_Maximum_member_index_set_1_1_2, __pyx_k_Maximum_member_index_set_1_1_2,

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    sizeof(__pyx_k_Maximum_member_index_set_1_1_2), 0, 1, 0, 0),
20520 {&__pyx_kp_u_Maximum_of_absolute_values_of_c, __pyx_k_Maximum_of_absolute_values_of_c,
    sizeof(__pyx_k_Maximum_of_absolute_values_of_c), 0, 1, 0, 0),
20521 {&__pyx_kp_u_Maximum_positive_index_or_0_if, __pyx_k_Maximum_positive_index_or_0_if,
    sizeof(__pyx_k_Maximum_positive_index_or_0_if), 0, 1, 0, 0),
20522 {&__pyx_kp_u_Minimum_member_index_set_1_1_2, __pyx_k_Minimum_member_index_set_1_1_2,
    sizeof(__pyx_k_Minimum_member_index_set_1_1_2), 0, 1, 0, 0),
20523 {&__pyx_kp_u_Minimum_negative_index_or_0_if, __pyx_k_Minimum_negative_index_or_0_if,
    sizeof(__pyx_k_Minimum_negative_index_or_0_if), 0, 1, 0, 0),
20524 {&__pyx_kp_u_Natural_logarithm_of_multivecto, __pyx_k_Natural_logarithm_of_multivecto,
    sizeof(__pyx_k_Natural_logarithm_of_multivecto), 0, 1, 0, 0),
20525 {&__pyx_kp_u_Norm_sum_of_squares_of_coordina, __pyx_k_Norm_sum_of_squares_of_coordina,
    sizeof(__pyx_k_Norm_sum_of_squares_of_coordina), 0, 1, 0, 0),
20526 {&__pyx_n_s_NotImplemented, __pyx_k_NotImplemented, sizeof(__pyx_k_NotImplemented), 0, 0, 1, 1},
20527 {&__pyx_kp_u_Not_applicable, __pyx_k_Not_applicable, sizeof(__pyx_k_Not_applicable), 0, 1, 0, 0},
20528 {&__pyx_kp_u_Not_applicable_for_a_in_cliffor, __pyx_k_Not_applicable_for_a_in_cliffor,
    sizeof(__pyx_k_Not_applicable_for_a_in_cliffor), 0, 1, 0, 0),
20529 {&__pyx_kp_u_Number_of_negative_indices_incl, __pyx_k_Number_of_negative_indices_incl,
    sizeof(__pyx_k_Number_of_negative_indices_incl), 0, 1, 0, 0),
20530 {&__pyx_kp_u_Number_of_positive_indices_incl, __pyx_k_Number_of_positive_indices_incl,
    sizeof(__pyx_kp_u_Number_of_positive_indices_incl), 0, 1, 0, 0),
20531 {&__pyx_kp_u_Odd_part_of_multivector_sum_of, __pyx_k_Odd_part_of_multivector_sum_of,
    sizeof(__pyx_kp_u_Odd_part_of_multivector_sum_of), 0, 1, 0, 0),
20532 {&__pyx_kp_u_Odd_part_of_multivector_sum_of_2, __pyx_k_Odd_part_of_multivector_sum_of_2,
    sizeof(__pyx_k_Odd_part_of_multivector_sum_of_2), 0, 1, 0, 0),
20533 {&__pyx_kp_u_Outer_product_power_of_multivec, __pyx_k_Outer_product_power_of_multivec,
    sizeof(__pyx_kp_u_Outer_product_power_of_multivec), 0, 1, 0, 0),
20534 {&__pyx_kp_u_Outer_product_power_x_clifford, __pyx_k_Outer_product_power_x_clifford,
    sizeof(__pyx_kp_u_Outer_product_power_x_clifford), 0, 1, 0, 0),
20535 {&__pyx_kp_u_Outer_product_print_clifford_1, __pyx_k_Outer_product_print_clifford_1,
    sizeof(__pyx_kp_u_Outer_product_print_clifford_1), 0, 1, 0, 0),
20536 {&__pyx_kp_u_Outer_product_x_clifford_1_x_cl, __pyx_k_Outer_product_x_clifford_1_x_cl,
    sizeof(__pyx_kp_u_Outer_product_x_clifford_1_x_cl), 0, 1, 0, 0),
20537 {&__pyx_kp_u_Power_self_to_the_m_x_clifford, __pyx_k_Power_self_to_the_m_x_clifford,
    sizeof(__pyx_kp_u_Power_self_to_the_m_x_clifford), 0, 1, 0, 0),
20538 {&__pyx_kp_u_Power_self_to_the_m_x_clifford_2, __pyx_k_Power_self_to_the_m_x_clifford_2,
    sizeof(__pyx_kp_u_Power_self_to_the_m_x_clifford_2), 0, 1, 0, 0),
20539 {&__pyx_kp_u_Pure_grade_vector_part_print_cl, __pyx_k_Pure_grade_vector_part_print_cl,
    sizeof(__pyx_kp_u_Pure_grade_vector_part_print_cl), 0, 1, 0, 0),
20540 {&__pyx_kp_u_Pure_part_print_clifford_1_1_1, __pyx_k_Pure_part_print_clifford_1_1_1,
    sizeof(__pyx_kp_u_Pure_part_print_clifford_1_1_1), 0, 1, 0, 0),
20541 {&__pyx_kp_u_Pure_part_print_pure_clifford_1, __pyx_k_Pure_part_print_pure_clifford_1,
    sizeof(__pyx_kp_u_Pure_part_print_pure_clifford_1), 0, 1, 0, 0),
20542 {&__pyx_kp_u_Put_self_into_a_larger_frame_co, __pyx_k_Put_self_into_a_larger_frame_co,
    sizeof(__pyx_kp_u_Put_self_into_a_larger_frame_co), 0, 1, 0, 0),
20543 {&__pyx_n_s_PyClical, __pyx_k_PyClical, sizeof(__pyx_k_PyClical), 0, 0, 1, 1},
20544 {&__pyx_kp_s_PyClical_pyx, __pyx_k_PyClical_pyx, sizeof(__pyx_k_PyClical_pyx), 0, 0, 1, 0},
20545 {&__pyx_kp_u_Quadratic_form_rev_x_x_0_print, __pyx_k_Quadratic_form_rev_x_x_0_print,
    sizeof(__pyx_kp_u_Quadratic_form_rev_x_x_0_print), 0, 1, 0, 0),
20546 {&__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2, __pyx_k_Quadratic_form_rev_x_x_0_print_2,
    sizeof(__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2), 0, 1, 0, 0),
20547 {&__pyx_kp_u_Quadratic_norm_error_tolerance, __pyx_k_Quadratic_norm_error_tolerance,
    sizeof(__pyx_kp_u_Quadratic_norm_error_tolerance), 0, 1, 0, 0),
20548 {&__pyx_kp_u_Random_multivector_within_a_fra, __pyx_k_Random_multivector_within_a_fra,
    sizeof(__pyx_kp_u_Random_multivector_within_a_fra), 0, 1, 0, 0),
20549 {&__pyx_n_s_Real, __pyx_k_Real, sizeof(__pyx_k_Real), 0, 0, 1, 1},
20550 {&__pyx_kp_u_Real_part_synonym_for_scalar_pa, __pyx_k_Real_part_synonym_for_scalar_pa,
    sizeof(__pyx_kp_u_Real_part_synonym_for_scalar_pa), 0, 1, 0, 0),
20551 {&__pyx_kp_u_Relative_or_absolute_error_usin, __pyx_k_Relative_or_absolute_error_usin,
    sizeof(__pyx_kp_u_Relative_or_absolute_error_usin), 0, 1, 0, 0),
20552 {&__pyx_kp_u_Remove_all_terms_of_self_with_r, __pyx_k_Remove_all_terms_of_self_with_r,
    sizeof(__pyx_kp_u_Remove_all_terms_of_self_with_r), 0, 1, 0, 0),
20553 {&__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve, __pyx_k_Reversion_eg_1_2_2_1_print_reve,
    sizeof(__pyx_kp_u_Reversion_eg_1_2_2_1_print_reve), 0, 1, 0, 0),
20554 {&__pyx_kp_u_Reversion_eg_clifford_1_cliffor, __pyx_k_Reversion_eg_clifford_1_cliffor,
    sizeof(__pyx_kp_u_Reversion_eg_clifford_1_cliffor), 0, 1, 0, 0),
20555 {&__pyx_n_s_RuntimeError, __pyx_k_RuntimeError, sizeof(__pyx_k_RuntimeError), 0, 0, 1, 1},
20556 {&__pyx_kp_u_Scalar_part_clifford_1_1_1_2_sc, __pyx_k_Scalar_part_clifford_1_1_1_2_sc,
    sizeof(__pyx_kp_u_Scalar_part_clifford_1_1_1_2_sc), 0, 1, 0, 0),
20557 {&__pyx_kp_u_Scalar_part_scalar_clifford_1_1, __pyx_k_Scalar_part_scalar_clifford_1_1,
    sizeof(__pyx_kp_u_Scalar_part_scalar_clifford_1_1), 0, 1, 0, 0),
20558 {&__pyx_n_s_Sequence, __pyx_k_Sequence, sizeof(__pyx_k_Sequence), 0, 0, 1, 1},
20559 {&__pyx_kp_u_Set_complement_not_print_index, __pyx_k_Set_complement_not_print_index,
    sizeof(__pyx_kp_u_Set_complement_not_print_index), 0, 1, 0, 0),
20560 {&__pyx_kp_u_Set_intersection_and_print_inde, __pyx_k_Set_intersection_and_print_inde,
    sizeof(__pyx_kp_u_Set_intersection_and_print_inde), 0, 1, 0, 0),
20561 {&__pyx_kp_u_Set_intersection_and_x_index_se, __pyx_k_Set_intersection_and_x_index_se,
    sizeof(__pyx_kp_u_Set_intersection_and_x_index_se), 0, 1, 0, 0),
20562 {&__pyx_kp_u_Set_the_value_of_an_index_set_o, __pyx_k_Set_the_value_of_an_index_set_o,
    sizeof(__pyx_kp_u_Set_the_value_of_an_index_set_o), 0, 1, 0, 0),
20563 {&__pyx_kp_u_Set_union_or_print_index_set_1, __pyx_k_Set_union_or_print_index_set_1,
    sizeof(__pyx_kp_u_Set_union_or_print_index_set_1), 0, 1, 0, 0),
20564 {&__pyx_kp_u_Set_union_or_x_index_set_1_x_in, __pyx_k_Set_union_or_x_index_set_1_x_in,
    sizeof(__pyx_kp_u_Set_union_or_x_index_set_1_x_in), 0, 1, 0, 0),
20565 {&__pyx_kp_u_Sign_of_geometric_product_of_tw, __pyx_k_Sign_of_geometric_product_of_tw,
    sizeof(__pyx_kp_u_Sign_of_geometric_product_of_tw), 0, 1, 0, 0),
20566 {&__pyx_kp_u_Sign_of_geometric_square_of_a_C, __pyx_k_Sign_of_geometric_square_of_a_C,

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    sizeof(_pyx_k_Sign_of_geometric_square_of_a_C), 0, 1, 0, 0),
20567 {&_pyx_kp_u_Sine_of_multivector_with_option, _pyx_k_Sine_of_multivector_with_option,
    sizeof(_pyx_k_Sine_of_multivector_with_option), 0, 1, 0, 0),
20568 {&_pyx_kp_u_Square_root_of_1_which_commutates, _pyx_k_Square_root_of_1_which_commutates,
    sizeof(_pyx_k_Square_root_of_1_which_commutates), 0, 1, 0, 0),
20569 {&_pyx_kp_u_Square_root_of_multivector_with, _pyx_k_Square_root_of_multivector_with,
    sizeof(_pyx_k_Square_root_of_multivector_with), 0, 1, 0, 0),
20570 {&_pyx_kp_u_Subalgebra_generated_by_all_gen, _pyx_k_Subalgebra_generated_by_all_gen,
    sizeof(_pyx_k_Subalgebra_generated_by_all_gen), 0, 1, 0, 0),
20571 {&_pyx_kp_u_Subscripting_map_from_index_set, _pyx_k_Subscripting_map_from_index_set,
    sizeof(_pyx_k_Subscripting_map_from_index_set), 0, 1, 0, 0),
20572 {&_pyx_kp_u_Symmetric_set_difference_exclus, _pyx_k_Symmetric_set_difference_exclus,
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20573 {&_pyx_kp_u_Symmetric_set_difference_exclus_2, _pyx_k_Symmetric_set_difference_exclus_2,
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20575 {&_pyx_kp_u_Test_for_approximate_equality_o, _pyx_k_Test_for_approximate_equality_o,
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20576 {&_pyx_kp_u_Tests_for_functions_that_Doctes, _pyx_k_Tests_for_functions_that_Doctes,
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20577 {&_pyx_kp_u_Tests_for_functions_that_Doctes_2, _pyx_k_Tests_for_functions_that_Doctes_2,
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20578 {&_pyx_kp_u_The_informal_string_representat, _pyx_k_The_informal_string_representat,
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20579 {&_pyx_kp_u_The_informal_string_representat_2, _pyx_k_The_informal_string_representat_2,
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20580 {&_pyx_kp_u_The_official_string_representat, _pyx_k_The_official_string_representat,
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20582 {&_pyx_kp_u_This_comparison_operator_is_not, _pyx_k_This_comparison_operator_is_not,
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20583 {&_pyx_kp_u_Transform_left_hand_side_using, _pyx_k_Transform_left_hand_side_using,
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20584 {&_pyx_kp_u_Transform_left_hand_side_using_2, _pyx_k_Transform_left_hand_side_using_2,
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20586 {&_pyx_kp_u_UTF_8, _pyx_k_UTF_8, sizeof(_pyx_k_UTF_8), 0, 1, 0, 0),
20587 {&_pyx_kp_u_Unary_print_clifford_1_1, _pyx_k_Unary_print_clifford_1_1,
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20588 {&_pyx_kp_u_Unary_print_clifford_1_1_2, _pyx_k_Unary_print_clifford_1_1_2,
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20589 {&_pyx_n_s_ValueError, _pyx_k_ValueError, sizeof(_pyx_k_ValueError), 0, 0, 1, 1},
20590 {&_pyx_kp_u_Vector_part_of_multivector_as_a, _pyx_k_Vector_part_of_multivector_as_a,
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20591 {&_pyx_kp_u_2, _pyx_k_2, sizeof(_pyx_k_2), 0, 1, 0, 0),
20592 {&_pyx_kp_u_5, _pyx_k_5, sizeof(_pyx_k_5), 0, 1, 0, 0),
20593 {&_pyx_kp_u_6, _pyx_k_6, sizeof(_pyx_k_6), 0, 1, 0, 0),
20594 {&_pyx_kp_u_7, _pyx_k_7, sizeof(_pyx_k_7), 0, 1, 0, 0),
20595 {&_pyx_kp_u_8, _pyx_k_8, sizeof(_pyx_k_8), 0, 1, 0, 0),
20596 {&_pyx_kp_u_9, _pyx_k_9, sizeof(_pyx_k_9), 0, 1, 0, 0),
20597 {&_pyx_n_s_abc, _pyx_k_abc, sizeof(_pyx_k_abc), 0, 0, 1, 1},
20598 {&_pyx_kp_u_abs_line_1522, _pyx_k_abs_line_1522, sizeof(_pyx_k_abs_line_1522), 0, 1, 0, 0),
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20602 {&_pyx_kp_u_acosh_line_1705, _pyx_k_acosh_line_1705, sizeof(_pyx_k_acosh_line_1705), 0, 1, 0, 0),
20603 {&_pyx_kp_u_agc3_line_1893, _pyx_k_agc3_line_1893, sizeof(_pyx_k_agc3_line_1893), 0, 1, 0, 0),
20604 {&_pyx_kp_u_approx_equal_line_1359, _pyx_k_approx_equal_line_1359,
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20605 {&_pyx_n_s_args, _pyx_k_args, sizeof(_pyx_k_args), 0, 0, 1, 1},
20606 {&_pyx_kp_u_as_frame, _pyx_k_as_frame, sizeof(_pyx_k_as_frame), 0, 1, 0, 0),
20607 {&_pyx_n_s_asin, _pyx_k_asin, sizeof(_pyx_k_asin), 0, 0, 1, 1},
20608 {&_pyx_kp_u_asin_line_1747, _pyx_k_asin_line_1747, sizeof(_pyx_k_asin_line_1747), 0, 1, 0, 0),
20609 {&_pyx_n_s_asinh, _pyx_k_asinh, sizeof(_pyx_k_asinh), 0, 0, 1, 1},
20610 {&_pyx_kp_u_asinh_line_1782, _pyx_k_asinh_line_1782, sizeof(_pyx_k_asinh_line_1782), 0, 1, 0, 0),
20611 {&_pyx_n_s_atan, _pyx_k_atan, sizeof(_pyx_k_atan), 0, 0, 1, 1},
20612 {&_pyx_kp_u_atan_line_1818, _pyx_k_atan_line_1818, sizeof(_pyx_k_atan_line_1818), 0, 1, 0, 0),
20613 {&_pyx_n_s_atanh, _pyx_k_atanh, sizeof(_pyx_k_atanh), 0, 0, 1, 1},
20614 {&_pyx_kp_u_atanh_line_1847, _pyx_k_atanh_line_1847, sizeof(_pyx_k_atanh_line_1847), 0, 1, 0, 0),
20615 {&_pyx_kp_u_cga3_line_1873, _pyx_k_cga3_line_1873, sizeof(_pyx_k_cga3_line_1873), 0, 1, 0, 0),
20616 {&_pyx_kp_u_cga3std_line_1882, _pyx_k_cga3std_line_1882, sizeof(_pyx_k_cga3std_line_1882), 0, 1,
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20617 {&_pyx_n_s_cl, _pyx_k_cl, sizeof(_pyx_k_cl), 0, 0, 1, 1},
20618 {&_pyx_n_s_clifford, _pyx_k_clifford, sizeof(_pyx_k_clifford), 0, 0, 1, 1},
20619 {&_pyx_kp_u_clifford_add_line_740, _pyx_k_clifford_add_line_740,
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20621 {&_pyx_kp_u_clifford_call_line_1020, _pyx_k_clifford_call_line_1020,
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20622 {&_pyx_kp_u_clifford_getitem_line_707, _pyx_k_clifford_getitem_line_707,
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20633 {&_pyx_kp_u_clifford_mul_line_780, _pyx_k_clifford_mul_line_780,
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20634 {&_pyx_kp_u_clifford_neg_line_722, _pyx_k_clifford_neg_line_722,
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20635 {&_pyx_kp_u_clifford_or_line_939, _pyx_k_clifford_or_line_939,
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20637 {&_pyx_kp_u_clifford_pow_line_961, _pyx_k_clifford_pow_line_961,
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20638 {&_pyx_kp_u_clifford_repr_line_1235, _pyx_k_clifford_repr_line_1235,
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20639 {&_pyx_kp_u_clifford_str_line_1244, _pyx_k_clifford_str_line_1244,
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20640 {&_pyx_kp_u_clifford_sub_line_760, _pyx_k_clifford_sub_line_760,
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20644 {&_pyx_kp_u_clifford_conj_line_1138, _pyx_k_clifford_conj_line_1138,
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20645 {&_pyx_kp_u_clifford_copy_line_556, _pyx_k_clifford_copy_line_556,
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20646 {&_pyx_kp_u_clifford_even_line_1061, _pyx_k_clifford_even_line_1061,
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20647 {&_pyx_kp_u_clifford_frame_line_1224, _pyx_k_clifford_frame_line_1224,
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20648 {&_pyx_n_s_clifford_hidden_doctests, _pyx_k_clifford_hidden_doctests,
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20649 {&_pyx_kp_u_clifford_hidden_doctests_line_12, _pyx_k_clifford_hidden_doctests_line_12,
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20650 {&_pyx_kp_u_clifford_inv_line_926, _pyx_k_clifford_inv_line_926,
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20651 {&_pyx_kp_u_clifford_involute_line_1107, _pyx_k_clifford_involute_line_1107,
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20652 {&_pyx_kp_u_clifford_isinf_line_1206, _pyx_k_clifford_isinf_line_1206,
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20653 {&_pyx_kp_u_clifford_isnan_line_1215, _pyx_k_clifford_isnan_line_1215,
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20654 {&_pyx_kp_u_clifford_max_abs_line_1184, _pyx_k_clifford_max_abs_line_1184,
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20655 {&_pyx_kp_u_clifford_norm_line_1164, _pyx_k_clifford_norm_line_1164,
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20656 {&_pyx_kp_u_clifford_odd_line_1070, _pyx_k_clifford_odd_line_1070,
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20657 {&_pyx_kp_u_clifford_outer_pow_line_1004, _pyx_k_clifford_outer_pow_line_1004,
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20658 {&_pyx_kp_u_clifford_pow_line_980, _pyx_k_clifford_pow_line_980,
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20659 {&_pyx_kp_u_clifford_pure_line_1050, _pyx_k_clifford_pure_line_1050,
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20660 {&_pyx_kp_u_clifford_quad_line_1153, _pyx_k_clifford_quad_line_1153,
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20661 {&_pyx_kp_u_clifford_reframe_line_649, _pyx_k_clifford_reframe_line_649,
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20662 {&_pyx_kp_u_clifford_reverse_line_1123, _pyx_k_clifford_reverse_line_1123,
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20663 {&_pyx_kp_u_clifford_scalar_line_1039, _pyx_k_clifford_scalar_line_1039,
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20664 {&_pyx_kp_u_clifford_truncated_line_1195, _pyx_k_clifford_truncated_line_1195,
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20665 {&_pyx_kp_u_clifford_vector_part_line_1079, _pyx_k_clifford_vector_part_line_1079,
    sizeof(_pyx_k_clifford_vector_part_line_1079), 0, 1, 0, 0),
20666 {&_pyx_n_s_cline_in_traceback, _pyx_k_cline_in_traceback, sizeof(_pyx_k_cline_in_traceback), 0,
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20667 {&_pyx_n_s_close, _pyx_k_close, sizeof(_pyx_k_close), 0, 0, 1, 1),
20668 {&_pyx_n_s_collections, _pyx_k_collections, sizeof(_pyx_k_collections), 0, 0, 1, 1),

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20669  {&__pyx_kp_u_compare_line_492, __pyx_k_compare_line_492, sizeof(__pyx_k_compare_line_492), 0, 1, 0,
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20671  {&__pyx_kp_u_complexifier_line_1576, __pyx_k_complexifier_line_1576,
20672  sizeof(__pyx_k_complexifier_line_1576), 0, 1, 0, 0},
20673  {&__pyx_n_s_conj, __pyx_k_conj, sizeof(__pyx_k_conj), 0, 0, 1, 1},
20674  {&__pyx_kp_u_conj_line_1485, __pyx_k_conj_line_1485, sizeof(__pyx_k_conj_line_1485), 0, 1, 0, 0},
20675  {&__pyx_n_s_copy, __pyx_k_copy, sizeof(__pyx_k_copy), 0, 0, 1, 1},
20676  {&__pyx_n_s_cos, __pyx_k_cos, sizeof(__pyx_k_cos), 0, 0, 1, 1},
20677  {&__pyx_kp_u_cos_line_1651, __pyx_k_cos_line_1651, sizeof(__pyx_k_cos_line_1651), 0, 1, 0, 0},
20678  {&__pyx_n_s_cosh, __pyx_k_cosh, sizeof(__pyx_k_cosh), 0, 0, 1, 1},
20679  {&__pyx_kp_u_cosh_line_1689, __pyx_k_cosh_line_1689, sizeof(__pyx_k_cosh_line_1689), 0, 1, 0, 0},
20680  {&__pyx_n_s_doctest, __pyx_k_doctest, sizeof(__pyx_k_doctest), 0, 0, 1, 1},
20681  {&__pyx_n_s_e, __pyx_k_e, sizeof(__pyx_k_e), 0, 0, 1, 1},
20682  {&__pyx_kp_u_e_line_1936, __pyx_k_e_line_1936, sizeof(__pyx_k_e_line_1936), 0, 1, 0, 0},
20683  {&__pyx_n_s_encode, __pyx_k_encode, sizeof(__pyx_k_encode), 0, 0, 1, 1},
20684  {&__pyx_kp_u_error_squared_line_1346, __pyx_k_error_squared_line_1346,
20685  sizeof(__pyx_k_error_squared_line_1346), 0, 1, 0, 0},
20686  {&__pyx_kp_u_error_squared_tol_line_1337, __pyx_k_error_squared_tol_line_1337,
20687  sizeof(__pyx_k_error_squared_tol_line_1337), 0, 1, 0, 0},
20688  {&__pyx_n_s_even, __pyx_k_even, sizeof(__pyx_k_even), 0, 0, 1, 1},
20689  {&__pyx_kp_u_even_line_1437, __pyx_k_even_line_1437, sizeof(__pyx_k_even_line_1437), 0, 1, 0, 0},
20690  {&__pyx_n_s_exp, __pyx_k_exp, sizeof(__pyx_k_exp), 0, 0, 1, 1},
20691  {&__pyx_kp_u_exp_line_1614, __pyx_k_exp_line_1614, sizeof(__pyx_k_exp_line_1614), 0, 1, 0, 0},
20692  {&__pyx_n_s_fill, __pyx_k_fill, sizeof(__pyx_k_fill), 0, 0, 1, 1},
20693  {&__pyx_n_s_frm, __pyx_k_frm, sizeof(__pyx_k_frm), 0, 0, 1, 1},
20694  {&__pyx_kp_u_from, __pyx_k_from, sizeof(__pyx_k_from), 0, 1, 0, 0},
20695  {&__pyx_n_s_getstate, __pyx_k_getstate, sizeof(__pyx_k_getstate), 0, 0, 1, 1},
20696  {&__pyx_n_s_grade, __pyx_k_grade, sizeof(__pyx_k_grade), 0, 0, 1, 1},
20697  {&__pyx_n_s_i, __pyx_k_i, sizeof(__pyx_k_i), 0, 0, 1, 1},
20698  {&__pyx_kp_u_imag_line_1415, __pyx_k_imag_line_1415, sizeof(__pyx_k_imag_line_1415), 0, 1, 0, 0},
20699  {&__pyx_n_s_import, __pyx_k_import, sizeof(__pyx_k_import), 0, 0, 1, 1},
20700  {&__pyx_n_s_index_set, __pyx_k_index_set, sizeof(__pyx_k_index_set), 0, 0, 1, 1},
20701  {&__pyx_kp_u_index_set_and_line_271, __pyx_k_index_set_and_line_271,
20702  sizeof(__pyx_k_index_set_and_line_271), 0, 1, 0, 0},
20703  {&__pyx_kp_u_index_set_getitem_line_191, __pyx_k_index_set_getitem_line_191,
20704  sizeof(__pyx_k_index_set_getitem_line_191), 0, 1, 0, 0},
20705  {&__pyx_kp_u_index_set_iand_line_282, __pyx_k_index_set_iand_line_282,
20706  sizeof(__pyx_k_index_set_iand_line_282), 0, 1, 0, 0},
20707  {&__pyx_kp_u_index_set_invert_line_240, __pyx_k_index_set_invert_line_240,
20708  sizeof(__pyx_k_index_set_invert_line_240), 0, 1, 0, 0},
20709  {&__pyx_kp_u_index_set_ior_line_304, __pyx_k_index_set_ior_line_304,
20710  sizeof(__pyx_k_index_set_ior_line_304), 0, 1, 0, 0},
20711  {&__pyx_n_s_index_set_iter, __pyx_k_index_set_iter, sizeof(__pyx_k_index_set_iter), 0, 0, 1,
20712  1},
20713  {&__pyx_kp_u_index_set_iter_line_229, __pyx_k_index_set_iter_line_229,
20714  sizeof(__pyx_k_index_set_iter_line_229), 0, 1, 0, 0},
20715  {&__pyx_kp_u_index_set_ixor_line_260, __pyx_k_index_set_ixor_line_260,
20716  sizeof(__pyx_k_index_set_ixor_line_260), 0, 1, 0, 0},
20717  {&__pyx_kp_u_index_set_or_line_293, __pyx_k_index_set_or_line_293,
20718  sizeof(__pyx_k_index_set_or_line_293), 0, 1, 0, 0},
20719  {&__pyx_kp_u_index_set_repr_line_384, __pyx_k_index_set_repr_line_384,
20720  sizeof(__pyx_k_index_set_repr_line_384), 0, 1, 0, 0},
20721  {&__pyx_kp_u_index_set_setitem_line_179, __pyx_k_index_set_setitem_line_179,
20722  sizeof(__pyx_k_index_set_setitem_line_179), 0, 1, 0, 0},
20723  {&__pyx_kp_u_index_set_str_line_395, __pyx_k_index_set_str_line_395,
20724  sizeof(__pyx_k_index_set_str_line_395), 0, 1, 0, 0},
20725  {&__pyx_kp_u_index_set_xor_line_249, __pyx_k_index_set_xor_line_249,
20726  sizeof(__pyx_k_index_set_xor_line_249), 0, 1, 0, 0},
20727  {&__pyx_kp_u_index_set_copy_line_65, __pyx_k_index_set_copy_line_65,
20728  sizeof(__pyx_k_index_set_copy_line_65), 0, 1, 0, 0},
20729  {&__pyx_kp_u_index_set_count_line_315, __pyx_k_index_set_count_line_315,
20730  sizeof(__pyx_k_index_set_count_line_315), 0, 1, 0, 0},
20731  {&__pyx_kp_u_index_set_count_neg_line_324, __pyx_k_index_set_count_neg_line_324,
20732  sizeof(__pyx_k_index_set_count_neg_line_324), 0, 1, 0, 0},
20733  {&__pyx_kp_u_index_set_count_pos_line_333, __pyx_k_index_set_count_pos_line_333,
20734  sizeof(__pyx_k_index_set_count_pos_line_333), 0, 1, 0, 0},
20735  {&__pyx_n_s_index_set_hidden_doctests, __pyx_k_index_set_hidden_doctests,
20736  sizeof(__pyx_k_index_set_hidden_doctests), 0, 0, 1, 1},
20737  {&__pyx_kp_u_index_set_hidden_doctests_line_4, __pyx_k_index_set_hidden_doctests_line_4,
20738  sizeof(__pyx_k_index_set_hidden_doctests_line_4), 0, 1, 0, 0},
20739  {&__pyx_kp_u_index_set_max_line_351, __pyx_k_index_set_max_line_351,
20740  sizeof(__pyx_k_index_set_max_line_351), 0, 1, 0, 0},
20741  {&__pyx_kp_u_index_set_min_line_342, __pyx_k_index_set_min_line_342,
20742  sizeof(__pyx_k_index_set_min_line_342), 0, 1, 0, 0},
20743  {&__pyx_kp_u_index_set_sign_of_mult_line_366, __pyx_k_index_set_sign_of_mult_line_366,
20744  sizeof(__pyx_k_index_set_sign_of_mult_line_366), 0, 1, 0, 0},
20745  {&__pyx_kp_u_index_set_sign_of_square_line_37, __pyx_k_index_set_sign_of_square_line_37,
20746  sizeof(__pyx_k_index_set_sign_of_square_line_37), 0, 1, 0, 0},
20747  {&__pyx_n_s_inv, __pyx_k_inv, sizeof(__pyx_k_inv), 0, 0, 1, 1},
20748  {&__pyx_kp_u_inv_line_1378, __pyx_k_inv_line_1378, sizeof(__pyx_k_inv_line_1378), 0, 1, 0, 0},
20749  {&__pyx_kp_u_invalid, __pyx_k_invalid, sizeof(__pyx_k_invalid), 0, 1, 0, 0},
20750  {&__pyx_kp_u_invalid_string, __pyx_k_invalid_string, sizeof(__pyx_k_invalid_string), 0, 1, 0, 0},
20751  {&__pyx_n_s_involute, __pyx_k_involute, sizeof(__pyx_k_involute), 0, 0, 1, 1},
20752  {&__pyx_kp_u_involute_line_1455, __pyx_k_involute_line_1455, sizeof(__pyx_k_involute_line_1455), 0,
20753  1, 0, 0},
20754  {&__pyx_n_s_ist, __pyx_k_ist, sizeof(__pyx_k_ist), 0, 0, 1, 1},
20755  {&__pyx_n_s_istpq, __pyx_k_istpq, sizeof(__pyx_k_istpq), 0, 0, 1, 1},

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20728 {&_pyx_kp_u_istpq_line_1949, __pyx_k_istpq_line_1949, sizeof(__pyx_k_istpq_line_1949), 0, 1, 0, 0},
20729 {&_pyx_n_s_iter, __pyx_k_iter, sizeof(__pyx_k_iter), 0, 0, 1, 1},
20730 {&_pyx_n_s_ixt, __pyx_k_ixt, sizeof(__pyx_k_ixt), 0, 0, 1, 1},
20731 {&_pyx_kp_u_lexicographic_compare_eg_3_4_5, __pyx_k_lexicographic_compare_eg_3_4_5,
sizeof(__pyx_k_lexicographic_compare_eg_3_4_5), 0, 1, 0, 0},
20732 {&_pyx_n_s_lhs, __pyx_k_lhs, sizeof(__pyx_k_lhs), 0, 0, 1, 1},
20733 {&_pyx_n_s_log, __pyx_k_log, sizeof(__pyx_k_log), 0, 0, 1, 1},
20734 {&_pyx_kp_u_log_line_1628, __pyx_k_log_line_1628, sizeof(__pyx_k_log_line_1628), 0, 1, 0, 0},
20735 {&_pyx_n_s_m, __pyx_k_m, sizeof(__pyx_k_m), 0, 0, 1, 1},
20736 {&_pyx_n_s_main, __pyx_k_main, sizeof(__pyx_k_main), 0, 0, 1, 1},
20737 {&_pyx_n_u_main, __pyx_k_main, sizeof(__pyx_k_main), 0, 1, 0, 1},
20738 {&_pyx_n_s_math, __pyx_k_math, sizeof(__pyx_k_math), 0, 0, 1, 1},
20739 {&_pyx_n_s_max, __pyx_k_max, sizeof(__pyx_k_max), 0, 0, 1, 1},
20740 {&_pyx_kp_u_max_abs_line_1531, __pyx_k_max_abs_line_1531, sizeof(__pyx_k_max_abs_line_1531), 0, 1,
0, 0},
20741 {&_pyx_kp_u_max_pos_line_513, __pyx_k_max_pos_line_513, sizeof(__pyx_k_max_pos_line_513), 0, 1, 0,
0},
20742 {&_pyx_n_s_min, __pyx_k_min, sizeof(__pyx_k_min), 0, 0, 1, 1},
20743 {&_pyx_kp_u_min_neg_line_504, __pyx_k_min_neg_line_504, sizeof(__pyx_k_min_neg_line_504), 0, 1, 0,
0},
20744 {&_pyx_n_s_name, __pyx_k_name, sizeof(__pyx_k_name), 0, 0, 1, 1},
20745 {&_pyx_n_s_nbar3, __pyx_k_nbar3, sizeof(__pyx_k_nbar3), 0, 0, 1, 1},
20746 {&_pyx_n_s_ninf3, __pyx_k_ninf3, sizeof(__pyx_k_ninf3), 0, 0, 1, 1},
20747 {&_pyx_kp_u_no_default_reduce_due_to_non, __pyx_k_no_default_reduce_due_to_non,
sizeof(__pyx_k_no_default_reduce_due_to_non), 0, 0, 1, 0},
20748 {&_pyx_n_s_norm, __pyx_k_norm, sizeof(__pyx_k_norm), 0, 0, 1, 1},
20749 {&_pyx_kp_u_norm_line_1511, __pyx_k_norm_line_1511, sizeof(__pyx_k_norm_line_1511), 0, 1, 0, 0},
20750 {&_pyx_kp_u_norm_sum_of_squares_of_coordina, __pyx_k_norm_sum_of_squares_of_coordina,
sizeof(__pyx_k_norm_sum_of_squares_of_coordina), 0, 1, 0, 0},
20751 {&_pyx_n_s_numbers, __pyx_k_numbers, sizeof(__pyx_k_numbers), 0, 0, 1, 1},
20752 {&_pyx_n_s_obj, __pyx_k_obj, sizeof(__pyx_k_obj), 0, 0, 1, 1},
20753 {&_pyx_n_s_odd, __pyx_k_odd, sizeof(__pyx_k_odd), 0, 0, 1, 1},
20754 {&_pyx_kp_u_odd_line_1446, __pyx_k_odd_line_1446, sizeof(__pyx_k_odd_line_1446), 0, 1, 0, 0},
20755 {&_pyx_n_s_other, __pyx_k_other, sizeof(__pyx_k_other), 0, 0, 1, 1},
20756 {&_pyx_n_s_outer_pow, __pyx_k_outer_pow, sizeof(__pyx_k_outer_pow), 0, 0, 1, 1},
20757 {&_pyx_kp_u_outer_pow_line_1567, __pyx_k_outer_pow_line_1567, sizeof(__pyx_k_outer_pow_line_1567),
0, 1, 0, 0},
20758 {&_pyx_n_s_p, __pyx_k_p, sizeof(__pyx_k_p), 0, 0, 1, 1},
20759 {&_pyx_n_s_pi, __pyx_k_pi, sizeof(__pyx_k_pi), 0, 0, 1, 1},
20760 {&_pyx_n_s_pow, __pyx_k_pow, sizeof(__pyx_k_pow), 0, 0, 1, 1},
20761 {&_pyx_kp_u_pow_line_1543, __pyx_k_pow_line_1543, sizeof(__pyx_k_pow_line_1543), 0, 1, 0, 0},
20762 {&_pyx_n_s_pure, __pyx_k_pure, sizeof(__pyx_k_pure), 0, 0, 1, 1},
20763 {&_pyx_kp_u_pure_line_1426, __pyx_k_pure_line_1426, sizeof(__pyx_k_pure_line_1426), 0, 1, 0, 0},
20764 {&_pyx_n_s_pyx_vtable, __pyx_k_pyx_vtable, sizeof(__pyx_k_pyx_vtable), 0, 0, 1, 1},
20765 {&_pyx_n_s_q, __pyx_k_q, sizeof(__pyx_k_q), 0, 0, 1, 1},
20766 {&_pyx_n_s_quad, __pyx_k_quad, sizeof(__pyx_k_quad), 0, 0, 1, 1},
20767 {&_pyx_kp_u_quad_line_1500, __pyx_k_quad_line_1500, sizeof(__pyx_k_quad_line_1500), 0, 1, 0, 0},
20768 {&_pyx_kp_u_random_clifford_line_1864, __pyx_k_random_clifford_line_1864,
sizeof(__pyx_k_random_clifford_line_1864), 0, 1, 0, 0},
20769 {&_pyx_n_s_range, __pyx_k_range, sizeof(__pyx_k_range), 0, 0, 1, 1},
20770 {&_pyx_kp_u_real_line_1404, __pyx_k_real_line_1404, sizeof(__pyx_k_real_line_1404), 0, 1, 0, 0},
20771 {&_pyx_n_s_reduce, __pyx_k_reduce, sizeof(__pyx_k_reduce), 0, 0, 1, 1},
20772 {&_pyx_n_s_reduce_cython, __pyx_k_reduce_cython, sizeof(__pyx_k_reduce_cython), 0, 0, 1, 1},
20773 {&_pyx_n_s_reduce_ex, __pyx_k_reduce_ex, sizeof(__pyx_k_reduce_ex), 0, 0, 1, 1},
20774 {&_pyx_n_s_reverse, __pyx_k_reverse, sizeof(__pyx_k_reverse), 0, 0, 1, 1},
20775 {&_pyx_kp_u_reverse_line_1470, __pyx_k_reverse_line_1470, sizeof(__pyx_k_reverse_line_1470), 0, 1,
0, 0},
20776 {&_pyx_n_s_rhs, __pyx_k_rhs, sizeof(__pyx_k_rhs), 0, 0, 1, 1},
20777 {&_pyx_n_s_scalar, __pyx_k_scalar, sizeof(__pyx_k_scalar), 0, 0, 1, 1},
20778 {&_pyx_n_s_scalar_epsilon, __pyx_k_scalar_epsilon, sizeof(__pyx_k_scalar_epsilon), 0, 0, 1, 1},
20779 {&_pyx_kp_u_scalar_line_1393, __pyx_k_scalar_line_1393, sizeof(__pyx_k_scalar_line_1393), 0, 1, 0,
0},
20780 {&_pyx_n_s_send, __pyx_k_send, sizeof(__pyx_k_send), 0, 0, 1, 1},
20781 {&_pyx_n_s_setstate, __pyx_k_setstate, sizeof(__pyx_k_setstate), 0, 0, 1, 1},
20782 {&_pyx_n_s_setstate_cython, __pyx_k_setstate_cython, sizeof(__pyx_k_setstate_cython), 0, 0, 1, 1},
20783 {&_pyx_n_s_sin, __pyx_k_sin, sizeof(__pyx_k_sin), 0, 0, 1, 1},
20784 {&_pyx_kp_u_sin_line_1728, __pyx_k_sin_line_1728, sizeof(__pyx_k_sin_line_1728), 0, 1, 0, 0},
20785 {&_pyx_n_s_sinh, __pyx_k_sinh, sizeof(__pyx_k_sinh), 0, 0, 1, 1},
20786 {&_pyx_kp_u_sinh_line_1768, __pyx_k_sinh_line_1768, sizeof(__pyx_k_sinh_line_1768), 0, 1, 0, 0},
20787 {&_pyx_n_s_sqrt, __pyx_k_sqrt, sizeof(__pyx_k_sqrt), 0, 0, 1, 1},
20788 {&_pyx_kp_u_sqrt_line_1591, __pyx_k_sqrt_line_1591, sizeof(__pyx_k_sqrt_line_1591), 0, 1, 0, 0},
20789 {&_pyx_n_s_tan, __pyx_k_tan, sizeof(__pyx_k_tan), 0, 0, 1, 1},
20790 {&_pyx_kp_u_tan_line_1801, __pyx_k_tan_line_1801, sizeof(__pyx_k_tan_line_1801), 0, 1, 0, 0},
20791 {&_pyx_n_s_tanh, __pyx_k_tanh, sizeof(__pyx_k_tanh), 0, 0, 1, 1},
20792 {&_pyx_kp_u_tanh_line_1835, __pyx_k_tanh_line_1835, sizeof(__pyx_k_tanh_line_1835), 0, 1, 0, 0},
20793 {&_pyx_n_s_tau, __pyx_k_tau, sizeof(__pyx_k_tau), 0, 0, 1, 1},
20794 {&_pyx_n_s_test, __pyx_k_test, sizeof(__pyx_k_test), 0, 0, 1, 1},
20795 {&_pyx_n_s_test_2, __pyx_k_test_2, sizeof(__pyx_k_test_2), 0, 0, 1, 1},
20796 {&_pyx_n_s_testmod, __pyx_k_testmod, sizeof(__pyx_k_testmod), 0, 0, 1, 1},
20797 {&_pyx_n_s_threshold, __pyx_k_threshold, sizeof(__pyx_k_threshold), 0, 0, 1, 1},
20798 {&_pyx_n_s_throw, __pyx_k_throw, sizeof(__pyx_k_throw), 0, 0, 1, 1},
20799 {&_pyx_kp_u_to_frame, __pyx_k_to_frame, sizeof(__pyx_k_to_frame), 0, 1, 0, 0},
20800 {&_pyx_n_s_tol, __pyx_k_tol, sizeof(__pyx_k_tol), 0, 0, 1, 1},
20801 {&_pyx_kp_u_using, __pyx_k_using, sizeof(__pyx_k_using), 0, 1, 0, 0},
20802 {&_pyx_kp_u_using_invalid, __pyx_k_using_invalid, sizeof(__pyx_k_using_invalid), 0, 1, 0, 0},
20803 {&_pyx_kp_u_utf_8, __pyx_k_utf_8, sizeof(__pyx_k_utf_8), 0, 1, 0, 0},
20804 {&_pyx_kp_u_value, __pyx_k_value, sizeof(__pyx_k_value), 0, 1, 0, 0},

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20805     {&__pyx_n_s_version, __pyx_k_version, sizeof(__pyx_k_version), 0, 0, 1, 1},
20806     {&__pyx_n_s_xrange, __pyx_k_xrange, sizeof(__pyx_k_xrange), 0, 0, 1, 1},
20807     {0, 0, 0, 0, 0, 0, 0}
20808 };
20809 static CYTHON_SMALL_CODE int __Pyx_InitCachedBuiltins(void) {
20810     __pyx_builtin_IndexError = __Pyx_GetBuiltinName(__pyx_n_s_IndexError); if
20811     (!__pyx_builtin_IndexError) __PYX_ERR(0, 103, __pyx_L1_error)
20812     __pyx_builtin_RuntimeError = __Pyx_GetBuiltinName(__pyx_n_s_RuntimeError); if
20813     (!__pyx_builtin_RuntimeError) __PYX_ERR(0, 105, __pyx_L1_error)
20814     __pyx_builtin_TypeError = __Pyx_GetBuiltinName(__pyx_n_s_TypeError); if (!__pyx_builtin_TypeError)
20815     __PYX_ERR(0, 105, __pyx_L1_error)
20816     __pyx_builtin_ValueError = __Pyx_GetBuiltinName(__pyx_n_s_ValueError); if
20817     (!__pyx_builtin_ValueError) __PYX_ERR(0, 106, __pyx_L1_error)
20818     __pyx_builtin_NotImplemented = __Pyx_GetBuiltinName(__pyx_n_s_NotImplemented); if
20819     (!__pyx_builtin_NotImplemented) __PYX_ERR(0, 159, __pyx_L1_error)
20820     __pyx_builtin_range = __Pyx_GetBuiltinName(__pyx_n_s_range); if (!__pyx_builtin_range) __PYX_ERR(0,
20821     236, __pyx_L1_error)
20822     #if PY_MAJOR_VERSION >= 3
20823     __pyx_builtin_xrange = __Pyx_GetBuiltinName(__pyx_n_s_range); if (!__pyx_builtin_xrange)
20824     __PYX_ERR(0, 1099, __pyx_L1_error)
20825     #else
20826     __pyx_builtin_xrange = __Pyx_GetBuiltinName(__pyx_n_s_xrange); if (!__pyx_builtin_xrange)
20827     __PYX_ERR(0, 1099, __pyx_L1_error)
20828     #endif
20829     return 0;
20830     __pyx_L1_error:;
20831     return -1;
20832 }
20833 static CYTHON_SMALL_CODE int __Pyx_InitCachedConstants(void) {
20834     __Pyx_RefNannyDeclarations
20835     __Pyx_RefNannySetupContext("__Pyx_InitCachedConstants", 0);
20836
20837     /* "(tree fragment)":2
20838     * def __reduce_cython__(self):
20839     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")          # ««««««««
20840     * def __setstate_cython__(self, __pyx_state):
20841     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20842     */
20843     __pyx_tuple__3 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
20844     (unlikely(!__pyx_tuple__3)) __PYX_ERR(1, 2, __pyx_L1_error)
20845     __Pyx_GOTREF(__pyx_tuple__3);
20846     __Pyx_GIVEREF(__pyx_tuple__3);
20847
20848     /* "(tree fragment)":4
20849     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20850     * def __setstate_cython__(self, __pyx_state):
20851     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")          # ««««««««
20852     */
20853     __pyx_tuple__4 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
20854     (unlikely(!__pyx_tuple__4)) __PYX_ERR(1, 4, __pyx_L1_error)
20855     __Pyx_GOTREF(__pyx_tuple__4);
20856     __Pyx_GIVEREF(__pyx_tuple__4);
20857
20858     /* "PyClical.pyx":636
20859     *     TypeError: Not applicable.
20860     *     """
20861     *     raise TypeError("Not applicable.")          # ««««««««
20862     *
20863     *     def __iter__(self):
20864     */
20865     __pyx_tuple__10 = PyTuple_Pack(1, __pyx_kp_u_Not_applicable); if (unlikely(!__pyx_tuple__10))
20866     __PYX_ERR(0, 636, __pyx_L1_error)
20867     __Pyx_GOTREF(__pyx_tuple__10);
20868     __Pyx_GIVEREF(__pyx_tuple__10);
20869
20870     /* "(tree fragment)":2
20871     * def __reduce_cython__(self):
20872     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")          # ««««««««
20873     * def __setstate_cython__(self, __pyx_state):
20874     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20875     */
20876     __pyx_tuple__11 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
20877     (unlikely(!__pyx_tuple__11)) __PYX_ERR(1, 2, __pyx_L1_error)
20878     __Pyx_GOTREF(__pyx_tuple__11);
20879     __Pyx_GIVEREF(__pyx_tuple__11);
20880
20881     /* "(tree fragment)":4
20882     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")
20883     * def __setstate_cython__(self, __pyx_state):
20884     *     raise TypeError("no default __reduce__ due to non-trivial __cinit__")          # ««««««««
20885     */
20886     __pyx_tuple__12 = PyTuple_Pack(1, __pyx_kp_s_no_default__reduce__due_to_non); if
20887     (unlikely(!__pyx_tuple__12)) __PYX_ERR(1, 4, __pyx_L1_error)
20888     __Pyx_GOTREF(__pyx_tuple__12);
20889     __Pyx_GIVEREF(__pyx_tuple__12);
20890 }

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20879  /* "PyClical.pyx":406
20880  *      return index_set_to_str( self.unwrap() ).decode()
20881  *
20882  * def index_set_hidden_doctests():          # ««««««««
20883  *      """
20884  *      Tests for functions that Doctest cannot see.
20885  */
20886  __pyx_codeobj__13 = (PyObject*)__Pyx_PyCode_New(0, 0, 0, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_index_set_hidden_doctests, 406,
__pyx_empty_bytes); if (unlikely(!__pyx_codeobj__13)) __PYX_ERR(0, 406, __pyx_L1_error)

20887
20888  /* "PyClical.pyx":1253
20889  *      return clifford_to_str( self.unwrap() ).decode()
20890  *
20891  * def clifford_hidden_doctests():          # ««««««««
20892  *      """
20893  *      Tests for functions that Doctest cannot see.
20894  */
20895  __pyx_codeobj__14 = (PyObject*)__Pyx_PyCode_New(0, 0, 0, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_clifford_hidden_doctests, 1253,
__pyx_empty_bytes); if (unlikely(!__pyx_codeobj__14)) __PYX_ERR(0, 1253, __pyx_L1_error)

20896
20897  /* "PyClical.pyx":1907
20898  * scalar_epsilon = epsilon
20899  *
20900  * pi = atan(clifford(1.0)) * 4.0          # ««««««««
20901  * tau = atan(clifford(1.0)) * 8.0
20902  *
20903  */
20904  __pyx_tuple__15 = PyTuple_Pack(1, __pyx_float_1_0); if (unlikely(!__pyx_tuple__15)) __PYX_ERR(0,
1907, __pyx_L1_error)
20905  __Pyx_GOTREF(__pyx_tuple__15);
20906  __Pyx_GIVEREF(__pyx_tuple__15);
20907
20908  /* "PyClical.pyx":1936
20909  *      """
20910  *
20911  * def e(obj):          # ««««««««
20912  *      """
20913  *      Abbreviation for clifford(index_set(obj)).
20914  */
20915  __pyx_tuple__16 = PyTuple_Pack(1, __pyx_n_s_obj); if (unlikely(!__pyx_tuple__16)) __PYX_ERR(0, 1936,
__pyx_L1_error)
20916  __Pyx_GOTREF(__pyx_tuple__16);
20917  __Pyx_GIVEREF(__pyx_tuple__16);
20918  __pyx_codeobj__17 = (PyObject*)__Pyx_PyCode_New(1, 0, 1, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__16, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_e, 1936, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__17)) __PYX_ERR(0, 1936, __pyx_L1_error)

20919
20920  /* "PyClical.pyx":1949
20921  *      return clifford(index_set(obj))
20922  *
20923  * def istpq(p, q):          # ««««««««
20924  *      """
20925  *      Abbreviation for index_set({-q,...p}).
20926  */
20927  __pyx_tuple__18 = PyTuple_Pack(2, __pyx_n_s_p, __pyx_n_s_q); if (unlikely(!__pyx_tuple__18))
__PYX_ERR(0, 1949, __pyx_L1_error)
20928  __Pyx_GOTREF(__pyx_tuple__18);
20929  __Pyx_GIVEREF(__pyx_tuple__18);
20930  __pyx_codeobj__19 = (PyObject*)__Pyx_PyCode_New(2, 0, 2, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__18, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_istpq, 1949, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__19)) __PYX_ERR(0, 1949, __pyx_L1_error)

20931
20932  /* "PyClical.pyx":1958
20933  *      return index_set(set(range(-q,p+1)))
20934  *
20935  * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].          #
20936  * ««««««««
20937  * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
20938  *
20939  */
20939  __pyx_tuple__20 = PyTuple_Pack(1, __pyx_int_4); if (unlikely(!__pyx_tuple__20)) __PYX_ERR(0, 1958,
__pyx_L1_error)
20940  __Pyx_GOTREF(__pyx_tuple__20);
20941  __Pyx_GIVEREF(__pyx_tuple__20);
20942  __pyx_tuple__21 = PyTuple_Pack(1, __pyx_int_neg_1); if (unlikely(!__pyx_tuple__21)) __PYX_ERR(0,
1958, __pyx_L1_error)
20943  __Pyx_GOTREF(__pyx_tuple__21);
20944  __Pyx_GIVEREF(__pyx_tuple__21);
20945
20946  /* "PyClical.pyx":1962
20947  *

```

```

20948 * # Doctest interface.
20949 * def _test(): # ««««««
20950 *     import PyClical, doctest
20951 *     return doctest.testmod(PyClical)
20952 */
20953 __pyx_tuple__22 = PyTuple_Pack(2, __pyx_n_s_PyClical, __pyx_n_s_doctest); if
(unlikely(!__pyx_tuple__22)) __PYX_ERR(0, 1962, __pyx_L1_error)
20954 __Pyx_GOTREF(__pyx_tuple__22);
20955 __Pyx_GIVEREF(__pyx_tuple__22);
20956 __pyx_codeobj__23 = (PyObject*)__Pyx_PyCode_New(0, 0, 2, 0, CO_OPTIMIZED|CO_NEWLOCALS,
__pyx_empty_bytes, __pyx_empty_tuple, __pyx_empty_tuple, __pyx_tuple__22, __pyx_empty_tuple,
__pyx_empty_tuple, __pyx_kp_s_PyClical_pyx, __pyx_n_s_test, 1962, __pyx_empty_bytes); if
(unlikely(!__pyx_codeobj__23)) __PYX_ERR(0, 1962, __pyx_L1_error)
20957 __Pyx_RefNannyFinishContext();
20958 return 0;
20959 __pyx_L1_error:;
20960 __Pyx_RefNannyFinishContext();
20961 return -1;
20962 }
20963
20964 static CYTHON_SMALL_CODE int __Pyx_InitGlobals(void) {
20965     if (__Pyx_InitStrings(__pyx_string_tab) < 0) __PYX_ERR(0, 1, __pyx_L1_error);
20966     __pyx_float_0_0 = PyFloat_FromDouble(0.0); if (unlikely(!__pyx_float_0_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20967     __pyx_float_1_0 = PyFloat_FromDouble(1.0); if (unlikely(!__pyx_float_1_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20968     __pyx_float_4_0 = PyFloat_FromDouble(4.0); if (unlikely(!__pyx_float_4_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20969     __pyx_float_8_0 = PyFloat_FromDouble(8.0); if (unlikely(!__pyx_float_8_0)) __PYX_ERR(0, 1,
__pyx_L1_error)
20970     __pyx_int_0 = PyInt_FromLong(0); if (unlikely(!__pyx_int_0)) __PYX_ERR(0, 1, __pyx_L1_error)
20971     __pyx_int_1 = PyInt_FromLong(1); if (unlikely(!__pyx_int_1)) __PYX_ERR(0, 1, __pyx_L1_error)
20972     __pyx_int_4 = PyInt_FromLong(4); if (unlikely(!__pyx_int_4)) __PYX_ERR(0, 1, __pyx_L1_error)
20973     __pyx_int_neg_1 = PyInt_FromLong(-1); if (unlikely(!__pyx_int_neg_1)) __PYX_ERR(0, 1,
__pyx_L1_error)
20974     return 0;
20975     __pyx_L1_error:;
20976     return -1;
20977 }
20978
20979 static CYTHON_SMALL_CODE int __Pyx_modinit_global_init_code(void); /*proto*/
20980 static CYTHON_SMALL_CODE int __Pyx_modinit_variable_export_code(void); /*proto*/
20981 static CYTHON_SMALL_CODE int __Pyx_modinit_function_export_code(void); /*proto*/
20982 static CYTHON_SMALL_CODE int __Pyx_modinit_type_init_code(void); /*proto*/
20983 static CYTHON_SMALL_CODE int __Pyx_modinit_type_import_code(void); /*proto*/
20984 static CYTHON_SMALL_CODE int __Pyx_modinit_variable_import_code(void); /*proto*/
20985 static CYTHON_SMALL_CODE int __Pyx_modinit_function_import_code(void); /*proto*/
20986
20987 static int __Pyx_modinit_global_init_code(void) {
20988     __Pyx_RefNannyDeclarations
20989     __Pyx_RefNannySetupContext("__Pyx_modinit_global_init_code", 0);
20990     /*--- Global init code ---*/
20991     __Pyx_RefNannyFinishContext();
20992     return 0;
20993 }
20994
20995 static int __Pyx_modinit_variable_export_code(void) {
20996     __Pyx_RefNannyDeclarations
20997     __Pyx_RefNannySetupContext("__Pyx_modinit_variable_export_code", 0);
20998     /*--- Variable export code ---*/
20999     __Pyx_RefNannyFinishContext();
21000     return 0;
21001 }
21002
21003 static int __Pyx_modinit_function_export_code(void) {
21004     __Pyx_RefNannyDeclarations
21005     __Pyx_RefNannySetupContext("__Pyx_modinit_function_export_code", 0);
21006     /*--- Function export code ---*/
21007     __Pyx_RefNannyFinishContext();
21008     return 0;
21009 }
21010
21011 static int __Pyx_modinit_type_init_code(void) {
21012     __Pyx_RefNannyDeclarations
21013     int __pyx_lineno = 0;
21014     const char *__pyx_filename = NULL;
21015     int __pyx_clineno = 0;
21016     __Pyx_RefNannySetupContext("__Pyx_modinit_type_init_code", 0);
21017     /*--- Type init code ---*/
21018     __pyx_vtabptr_8PyClical_index_set = &__pyx_vtable_8PyClical_index_set;
21019     __pyx_vtable_8PyClical_index_set.wrap = (PyObject *) (struct __pyx_obj_8PyClical_index_set *,
IndexSet) __pyx_f_8PyClical_9index_set_wrap;
21020     __pyx_vtable_8PyClical_index_set.unwrap = (IndexSet *) (struct __pyx_obj_8PyClical_index_set
*) __pyx_f_8PyClical_9index_set_unwrap;
21021     __pyx_vtable_8PyClical_index_set.copy = (PyObject *) (struct __pyx_obj_8PyClical_index_set *, int
__pyx_skip_dispatch) __pyx_f_8PyClical_9index_set_copy;
21022     if (PyType_Ready(&__pyx_type_8PyClical_index_set) < 0) __PYX_ERR(0, 46, __pyx_L1_error)

```



```

21023     #if PY_VERSION_HEX < 0x030800B1
21024     __pyx_type_8PyClical_index_set.tp_print = 0;
21025     #endif
21026     if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
likely(!__pyx_type_8PyClical_index_set.tp_dictoffset && __pyx_type_8PyClical_index_set.tp_getattro ==
PyObject_GenericGetAttr)) {
21027         __pyx_type_8PyClical_index_set.tp_getattro = __Pyx_PyObject_GenericGetAttr;
21028     }
21029     #if CYTHON_COMPILING_IN_CPYTHON
21030     {
21031         PyObject *wrapper = PyObject_GetAttrString(PyObject *)&__pyx_type_8PyClical_index_set,
"__setitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21032         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21033             __pyx_wrapperbase_8PyClical_9index_set_8__setitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21034             __pyx_wrapperbase_8PyClical_9index_set_8__setitem__.doc =
__pyx_doc_8PyClical_9index_set_8__setitem__;
21035             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_8__setitem__;
21036         }
21037     }
21038     #endif
21039     #if CYTHON_COMPILING_IN_CPYTHON
21040     {
21041         PyObject *wrapper = PyObject_GetAttrString(PyObject *)&__pyx_type_8PyClical_index_set,
"__getitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21042         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21043             __pyx_wrapperbase_8PyClical_9index_set_10__getitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21044             __pyx_wrapperbase_8PyClical_9index_set_10__getitem__.doc =
__pyx_doc_8PyClical_9index_set_10__getitem__;
21045             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_10__getitem__;
21046         }
21047     }
21048     #endif
21049     #if CYTHON_COMPILING_IN_CPYTHON
21050     {
21051         PyObject *wrapper = PyObject_GetAttrString(PyObject *)&__pyx_type_8PyClical_index_set,
"__contains__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21052         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21053             __pyx_wrapperbase_8PyClical_9index_set_12__contains__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21054             __pyx_wrapperbase_8PyClical_9index_set_12__contains__.doc =
__pyx_doc_8PyClical_9index_set_12__contains__;
21055             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_12__contains__;
21056         }
21057     }
21058     #endif
21059     #if CYTHON_COMPILING_IN_CPYTHON
21060     {
21061         PyObject *wrapper = PyObject_GetAttrString(PyObject *)&__pyx_type_8PyClical_index_set,
"__iter__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21062         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21063             __pyx_wrapperbase_8PyClical_9index_set_14__iter__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21064             __pyx_wrapperbase_8PyClical_9index_set_14__iter__.doc =
__pyx_doc_8PyClical_9index_set_14__iter__;
21065             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_14__iter__;
21066         }
21067     }
21068     #endif
21069     #if CYTHON_COMPILING_IN_CPYTHON
21070     {
21071         PyObject *wrapper = PyObject_GetAttrString(PyObject *)&__pyx_type_8PyClical_index_set,
"__invert__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21072         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21073             __pyx_wrapperbase_8PyClical_9index_set_17__invert__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21074             __pyx_wrapperbase_8PyClical_9index_set_17__invert__.doc =
__pyx_doc_8PyClical_9index_set_17__invert__;
21075             ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_9index_set_17__invert__;
21076         }
21077     }
21078     #endif
21079     #if CYTHON_COMPILING_IN_CPYTHON
21080     {
21081         PyObject *wrapper = PyObject_GetAttrString(PyObject *)&__pyx_type_8PyClical_index_set,
"__xor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21082         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21083             __pyx_wrapperbase_8PyClical_9index_set_19__xor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21084             __pyx_wrapperbase_8PyClical_9index_set_19__xor__.doc = __pyx_doc_8PyClical_9index_set_19__xor__;
21085             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_19__xor__;
21086         }
21087     }
21088     #endif

```

```

21089     #if CYTHON_COMPILING_IN_CPYTHON
21090     {
21091         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21092             "__ixor__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21093         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21094             __pyx_wrapperbase_8PyClical_9index_set_21__ixor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21095             __pyx_wrapperbase_8PyClical_9index_set_21__ixor__.doc =
21096                 __pyx_doc_8PyClical_9index_set_21__ixor__;
21097             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_21__ixor__;
21098         }
21099     }
21100     #endif
21101     #if CYTHON_COMPILING_IN_CPYTHON
21102     {
21103         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21104             "__and__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21105         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21106             __pyx_wrapperbase_8PyClical_9index_set_23__and__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21107             __pyx_wrapperbase_8PyClical_9index_set_23__and__.doc = __pyx_doc_8PyClical_9index_set_23__and__;
21108             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_23__and__;
21109         }
21110     }
21111     #endif
21112     #if CYTHON_COMPILING_IN_CPYTHON
21113     {
21114         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21115             "__iand__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21116         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21117             __pyx_wrapperbase_8PyClical_9index_set_25__iand__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21118             __pyx_wrapperbase_8PyClical_9index_set_25__iand__.doc =
21119                 __pyx_doc_8PyClical_9index_set_25__iand__;
21120             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_25__iand__;
21121         }
21122     }
21123     #endif
21124     #if CYTHON_COMPILING_IN_CPYTHON
21125     {
21126         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set, "__ior__");
21127         if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21128         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21129             __pyx_wrapperbase_8PyClical_9index_set_27__ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21130             __pyx_wrapperbase_8PyClical_9index_set_27__ior__.doc = __pyx_doc_8PyClical_9index_set_27__ior__;
21131             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_27__ior__;
21132         }
21133     }
21134     #endif
21135     #if CYTHON_COMPILING_IN_CPYTHON
21136     {
21137         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21138             "__ior__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21139         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21140             __pyx_wrapperbase_8PyClical_9index_set_29__ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21141             __pyx_wrapperbase_8PyClical_9index_set_29__ior__.doc = __pyx_doc_8PyClical_9index_set_29__ior__;
21142             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_29__ior__;
21143         }
21144     }
21145     #endif
21146     #if CYTHON_COMPILING_IN_CPYTHON
21147     {
21148         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21149             "__repr__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21150         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21151             __pyx_wrapperbase_8PyClical_9index_set_47__repr__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21152             __pyx_wrapperbase_8PyClical_9index_set_47__repr__.doc =
21153                 __pyx_doc_8PyClical_9index_set_47__repr__;
21154             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_47__repr__;
21155         }
21156     }
21157     #endif
21158     #if CYTHON_COMPILING_IN_CPYTHON
21159     {
21160         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_index_set,
21161             "__str__"); if (unlikely(!wrapper)) __PYX_ERR(0, 46, __pyx_L1_error)
21162         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21163             __pyx_wrapperbase_8PyClical_9index_set_49__str__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21164             __pyx_wrapperbase_8PyClical_9index_set_49__str__.doc = __pyx_doc_8PyClical_9index_set_49__str__;
21165             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_9index_set_49__str__;
21166         }
21167     }
21168     #endif
21169     if (__Pyx_SetVtable(__pyx_type_8PyClical_index_set.tp_dict, __pyx_vtabptr_8PyClical_index_set) < 0)
21170         __PYX_ERR(0, 46, __pyx_L1_error)
21171     if (PyObject_SetAttr(__pyx_m, __pyx_n_s_index_set, (PyObject *) &__pyx_type_8PyClical_index_set) < 0)
21172         __PYX_ERR(0, 46, __pyx_L1_error)
21173     if (__Pyx_setup_reduce((PyObject *) &__pyx_type_8PyClical_index_set) < 0) __PYX_ERR(0, 46,
21174         __pyx_L1_error)
21175     __pyx_ptype_8PyClical_index_set = &__pyx_type_8PyClical_index_set;

```

```

21163 __pyx_vtabptr_8PyClical_clifford = &__pyx_vtable_8PyClical_clifford;
21164 __pyx_vtable_8PyClical_clifford.wrap = (PyObject (*)(struct __pyx_obj_8PyClical_clifford *,
Clifford))__pyx_f_8PyClical_8clifford_wrap;
21165 __pyx_vtable_8PyClical_clifford.unwrap = (Clifford (*)(struct __pyx_obj_8PyClical_clifford
*))__pyx_f_8PyClical_8clifford_unwrap;
21166 __pyx_vtable_8PyClical_clifford.copy = (PyObject (*)(struct __pyx_obj_8PyClical_clifford *, int
__pyx_skip_dispatch))__pyx_f_8PyClical_8clifford_copy;
21167 if (PyType_Ready(&__pyx_type_8PyClical_clifford) < 0) __PYX_ERR(0, 537, __pyx_L1_error)
21168 #if PY_VERSION_HEX < 0x030800B1
21169 __pyx_type_8PyClical_clifford.tp_print = 0;
21170 #endif
21171 if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
likely(!__pyx_type_8PyClical_clifford.tp_dictoffset && __pyx_type_8PyClical_clifford.tp_getattro ==
PyObject_GenericGetAttr)) {
21172     __pyx_type_8PyClical_clifford.tp_getattro = __Pyx_PyObject_GenericGetAttr;
21173 }
21174 #if CYTHON_COMPILING_IN_CPYTHON
21175 {
21176     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
"__contains__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21177     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21178         __pyx_wrapperbase_8PyClical_8clifford_6__contains__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21179         __pyx_wrapperbase_8PyClical_8clifford_6__contains__.doc =
__pyx_doc_8PyClical_8clifford_6__contains__;
21180         ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_8clifford_6__contains__;
21181     }
21182 }
21183 #endif
21184 #if CYTHON_COMPILING_IN_CPYTHON
21185 {
21186     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
"__iter__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21187     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21188         __pyx_wrapperbase_8PyClical_8clifford_8__iter__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21189         __pyx_wrapperbase_8PyClical_8clifford_8__iter__.doc = __pyx_doc_8PyClical_8clifford_8__iter__;
21190         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_8__iter__;
21191     }
21192 }
21193 #endif
21194 #if CYTHON_COMPILING_IN_CPYTHON
21195 {
21196     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
"__getitem__"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21197     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21198         __pyx_wrapperbase_8PyClical_8clifford_14__getitem__ = *((PyWrapperDescrObject
*)wrapper)->d_base;
21199         __pyx_wrapperbase_8PyClical_8clifford_14__getitem__.doc =
__pyx_doc_8PyClical_8clifford_14__getitem__;
21200         ((PyWrapperDescrObject *)wrapper)->d_base =
&__pyx_wrapperbase_8PyClical_8clifford_14__getitem__;
21201     }
21202 }
21203 #endif
21204 #if CYTHON_COMPILING_IN_CPYTHON
21205 {
21206     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__neg__");
if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21207     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21208         __pyx_wrapperbase_8PyClical_8clifford_16__neg__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21209         __pyx_wrapperbase_8PyClical_8clifford_16__neg__.doc = __pyx_doc_8PyClical_8clifford_16__neg__;
21210         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_16__neg__;
21211     }
21212 }
21213 #endif
21214 #if CYTHON_COMPILING_IN_CPYTHON
21215 {
21216     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__pos__");
if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21217     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21218         __pyx_wrapperbase_8PyClical_8clifford_18__pos__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21219         __pyx_wrapperbase_8PyClical_8clifford_18__pos__.doc = __pyx_doc_8PyClical_8clifford_18__pos__;
21220         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_18__pos__;
21221     }
21222 }
21223 #endif
21224 #if CYTHON_COMPILING_IN_CPYTHON
21225 {
21226     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__add__");
if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21227     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21228         __pyx_wrapperbase_8PyClical_8clifford_20__add__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21229         __pyx_wrapperbase_8PyClical_8clifford_20__add__.doc = __pyx_doc_8PyClical_8clifford_20__add__;
21230         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_20__add__;
21231     }
21232 }

```

```

21233     #endif
21234     #if CYTHON_COMPILING_IN_CPYTHON
21235     {
21236         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21237 "___iadd___"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21238         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21239             __pyx_wrapperbase_8PyClical_8clifford_22___iadd___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21240             __pyx_wrapperbase_8PyClical_8clifford_22___iadd___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_22___iadd___;
21241             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_22___iadd___;
21242         }
21243     #endif
21244     #if CYTHON_COMPILING_IN_CPYTHON
21245     {
21246         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "___sub___");
21247         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21248         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21249             __pyx_wrapperbase_8PyClical_8clifford_24___sub___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21250             __pyx_wrapperbase_8PyClical_8clifford_24___sub___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_24___sub___;
21251             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_24___sub___;
21252         }
21253     #endif
21254     #if CYTHON_COMPILING_IN_CPYTHON
21255     {
21256         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21257 "___isub___"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21258         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21259             __pyx_wrapperbase_8PyClical_8clifford_26___isub___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21260             __pyx_wrapperbase_8PyClical_8clifford_26___isub___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_26___isub___;
21261             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_26___isub___;
21262         }
21263     #endif
21264     #if CYTHON_COMPILING_IN_CPYTHON
21265     {
21266         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "___mul___");
21267         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21268         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21269             __pyx_wrapperbase_8PyClical_8clifford_28___mul___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21270             __pyx_wrapperbase_8PyClical_8clifford_28___mul___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_28___mul___;
21271             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_28___mul___;
21272         }
21273     #endif
21274     #if CYTHON_COMPILING_IN_CPYTHON
21275     {
21276         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21277 "___imul___"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21278         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21279             __pyx_wrapperbase_8PyClical_8clifford_30___imul___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21280             __pyx_wrapperbase_8PyClical_8clifford_30___imul___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_30___imul___;
21281             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_30___imul___;
21282         }
21283     #endif
21284     #if CYTHON_COMPILING_IN_CPYTHON
21285     {
21286         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "___mod___");
21287         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21288         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21289             __pyx_wrapperbase_8PyClical_8clifford_32___mod___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21290             __pyx_wrapperbase_8PyClical_8clifford_32___mod___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_32___mod___;
21291             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_32___mod___;
21292         }
21293     #endif
21294     #if CYTHON_COMPILING_IN_CPYTHON
21295     {
21296         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21297 "___imod___"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21298         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21299             __pyx_wrapperbase_8PyClical_8clifford_34___imod___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21300             __pyx_wrapperbase_8PyClical_8clifford_34___imod___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_34___imod___;
21301             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_34___imod___;
21302         }
21303     #endif
21304     #if CYTHON_COMPILING_IN_CPYTHON
21305     {
21306         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "___and___");
21307         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21308         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21309             __pyx_wrapperbase_8PyClical_8clifford_36___and___ = *((PyWrapperDescrObject *)wrapper)->d_base;
21310             __pyx_wrapperbase_8PyClical_8clifford_36___and___<u>.doc</u> = __pyx_doc_8PyClical_8clifford_36___and___;
21311             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_36___and___;

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21312     }
21313     #endif
21314     #if CYTHON_COMPILING_IN_CPYTHON
21315     {
21316         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21317 "iand"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21318         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21319             __pyx_wrapperbase_8PyClical_8clifford_38_iand__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21320             __pyx_wrapperbase_8PyClical_8clifford_38_iand__.doc = __pyx_doc_8PyClical_8clifford_38_iand__;
21321             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_38_iand__;
21322         }
21323     }
21324     #endif
21325     #if CYTHON_COMPILING_IN_CPYTHON
21326     {
21327         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__xor__");
21328         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21329         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21330             __pyx_wrapperbase_8PyClical_8clifford_40_xor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21331             __pyx_wrapperbase_8PyClical_8clifford_40_xor__.doc = __pyx_doc_8PyClical_8clifford_40_xor__;
21332             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_40_xor__;
21333         }
21334     }
21335     #endif
21336     #if CYTHON_COMPILING_IN_CPYTHON
21337     {
21338         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21339 "ixor"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21340         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21341             __pyx_wrapperbase_8PyClical_8clifford_42_ixor__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21342             __pyx_wrapperbase_8PyClical_8clifford_42_ixor__.doc = __pyx_doc_8PyClical_8clifford_42_ixor__;
21343             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_42_ixor__;
21344         }
21345     }
21346     #endif
21347     #if CYTHON_COMPILING_IN_CPYTHON
21348     {
21349         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21350 "truediv"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21351         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21352             __pyx_wrapperbase_8PyClical_8clifford_44_truediv__ = *((PyWrapperDescrObject
21353 *)wrapper)->d_base;
21354             __pyx_wrapperbase_8PyClical_8clifford_44_truediv__.doc =
21355             __pyx_doc_8PyClical_8clifford_44_truediv__;
21356             ((PyWrapperDescrObject *)wrapper)->d_base =
21357             &__pyx_wrapperbase_8PyClical_8clifford_44_truediv__;
21358         }
21359     }
21360     #endif
21361     #if PY_MAJOR_VERSION < 3 || (CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX < 0x03050000)
21362     #if CYTHON_COMPILING_IN_CPYTHON
21363     {
21364         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford,
21365 "idiv"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21366         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21367             __pyx_wrapperbase_8PyClical_8clifford_46_idiv__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21368             __pyx_wrapperbase_8PyClical_8clifford_46_idiv__.doc = __pyx_doc_8PyClical_8clifford_46_idiv__;
21369             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_46_idiv__;
21370         }
21371     }
21372     #endif
21373     #endif
21374     #if CYTHON_COMPILING_IN_CPYTHON
21375     {
21376         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__or__");
21377         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21378         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21379             __pyx_wrapperbase_8PyClical_8clifford_50_or__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21380             __pyx_wrapperbase_8PyClical_8clifford_50_or__.doc = __pyx_doc_8PyClical_8clifford_50_or__;
21381             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_50_or__;
21382         }
21383     }
21384     #endif
21385     #if CYTHON_COMPILING_IN_CPYTHON
21386     {
21387         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__ior__");
21388         if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_L1_error)
21389         if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21390             __pyx_wrapperbase_8PyClical_8clifford_52_ior__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21391             __pyx_wrapperbase_8PyClical_8clifford_52_ior__.doc = __pyx_doc_8PyClical_8clifford_52_ior__;
21392             ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_52_ior__;
21393         }
21394     }
21395     #endif
21396     #if CYTHON_COMPILING_IN_CPYTHON
21397     {
21398         PyObject *wrapper = PyObject_GetAttrString((PyObject *) &__pyx_type_8PyClical_clifford, "__pow__");

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    if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21389     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21390         __pyx_wrapperbase_8PyClical_8clifford_54_pow__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21391         __pyx_wrapperbase_8PyClical_8clifford_54_pow__.doc = __pyx_doc_8PyClical_8clifford_54_pow__;
21392         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_54_pow__;
21393     }
21394 }
21395 #endif
21396 #if CYTHON_COMPILING_IN_CPYTHON
21397 {
21398     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21399 "call"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21399     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21400         __pyx_wrapperbase_8PyClical_8clifford_60_call__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21401         __pyx_wrapperbase_8PyClical_8clifford_60_call__.doc = __pyx_doc_8PyClical_8clifford_60_call__;
21402         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_60_call__;
21403     }
21404 }
21405 #endif
21406 #if CYTHON_COMPILING_IN_CPYTHON
21407 {
21408     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford,
21409 "repr"); if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21409     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21410         __pyx_wrapperbase_8PyClical_8clifford_94_repr__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21411         __pyx_wrapperbase_8PyClical_8clifford_94_repr__.doc = __pyx_doc_8PyClical_8clifford_94_repr__;
21412         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_94_repr__;
21413     }
21414 }
21415 #endif
21416 #if CYTHON_COMPILING_IN_CPYTHON
21417 {
21418     PyObject *wrapper = PyObject_GetAttrString((PyObject *)&__pyx_type_8PyClical_clifford, "__str__");
21419     if (unlikely(!wrapper)) __PYX_ERR(0, 537, __pyx_l1_error)
21420     if (Py_TYPE(wrapper) == &PyWrapperDescr_Type) {
21421         __pyx_wrapperbase_8PyClical_8clifford_96_str__ = *((PyWrapperDescrObject *)wrapper)->d_base;
21422         __pyx_wrapperbase_8PyClical_8clifford_96_str__.doc = __pyx_doc_8PyClical_8clifford_96_str__;
21423         ((PyWrapperDescrObject *)wrapper)->d_base = &__pyx_wrapperbase_8PyClical_8clifford_96_str__;
21424     }
21425 #endif
21426 if (__Pyx_SetVtable(__pyx_type_8PyClical_clifford.tp_dict, __pyx_vtabptr_8PyClical_clifford) < 0)
__PYX_ERR(0, 537, __pyx_l1_error)
21427 if (PyObject_SetAttr(__pyx_m, __pyx_n_s_clifford, (PyObject *)&__pyx_type_8PyClical_clifford) < 0)
__PYX_ERR(0, 537, __pyx_l1_error)
21428 if (__Pyx_setup_reduce((PyObject *)&__pyx_type_8PyClical_clifford) < 0) __PYX_ERR(0, 537,
__pyx_l1_error)
21429 __pyx_ptype_8PyClical_clifford = &__pyx_type_8PyClical_clifford;
21430 if (PyType_Ready(&__pyx_type_8PyClical__pyx_scope_struct__iter__) < 0) __PYX_ERR(0, 229,
__pyx_l1_error)
21431 #if PY_VERSION_HEX < 0x030800B1
21432 __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_print = 0;
21433 #endif
21434 if ((CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP) &&
likely(!__pyx_type_8PyClical__pyx_scope_struct__iter__.tp_dictoffset &&
__pyx_type_8PyClical__pyx_scope_struct__iter__.tp_getattro == PyObject_GenericGetAttr)) {
21435     __pyx_type_8PyClical__pyx_scope_struct__iter__.tp_getattro =
__Pyx_PyObject_GenericGetAttrNoDict;
21436 }
21437 __pyx_ptype_8PyClical__pyx_scope_struct__iter__ =
&__pyx_type_8PyClical__pyx_scope_struct__iter__;
21438 __Pyx_RefNannyFinishContext();
21439 return 0;
21440 __pyx_l1_error:;
21441 __Pyx_RefNannyFinishContext();
21442 return -1;
21443 }
21444
21445 static int __Pyx_modinit_type_import_code(void) {
21446     __Pyx_RefNannyDeclarations
21447     __Pyx_RefNannySetupContext("__Pyx_modinit_type_import_code", 0);
21448     /*--- Type import code ---*/
21449     __Pyx_RefNannyFinishContext();
21450     return 0;
21451 }
21452
21453 static int __Pyx_modinit_variable_import_code(void) {
21454     __Pyx_RefNannyDeclarations
21455     __Pyx_RefNannySetupContext("__Pyx_modinit_variable_import_code", 0);
21456     /*--- Variable import code ---*/
21457     __Pyx_RefNannyFinishContext();
21458     return 0;
21459 }
21460
21461 static int __Pyx_modinit_function_import_code(void) {
21462     __Pyx_RefNannyDeclarations
21463     __Pyx_RefNannySetupContext("__Pyx_modinit_function_import_code", 0);

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```

21464  /*--- Function import code ---*/
21465  __Pyx_RefNannyFinishContext();
21466  return 0;
21467 }
21468
21469
21470 #ifndef CYTHON_NO_PYINIT_EXPORT
21471 #define __Pyx_PyMODINIT_FUNC PyMODINIT_FUNC
21472 #elif PY_MAJOR_VERSION < 3
21473 #ifdef __cplusplus
21474 #define __Pyx_PyMODINIT_FUNC extern "C" void
21475 #else
21476 #define __Pyx_PyMODINIT_FUNC void
21477 #endif
21478 #else
21479 #ifdef __cplusplus
21480 #define __Pyx_PyMODINIT_FUNC extern "C" PyObject *
21481 #else
21482 #define __Pyx_PyMODINIT_FUNC PyObject *
21483 #endif
21484 #endif
21485
21486
21487 #if PY_MAJOR_VERSION < 3
21488 __Pyx_PyMODINIT_FUNC initPyClical(void) CYTHON_SMALL_CODE; /*proto*/
21489 __Pyx_PyMODINIT_FUNC initPyClical(void)
21490 #else
21491 __Pyx_PyMODINIT_FUNC PyInit_PyClical(void) CYTHON_SMALL_CODE; /*proto*/
21492 __Pyx_PyMODINIT_FUNC PyInit_PyClical(void)
21493 #if CYTHON_PEP489_MULTI_PHASE_INIT
21494 {
21495     return PyModuleDef_Init(&__pyx_moduledef);
21496 }
21497 static CYTHON_SMALL_CODE int __Pyx_check_single_interpreter(void) {
21498     #if PY_VERSION_HEX >= 0x030700A1
21499     static PY_INT64_T main_interpreter_id = -1;
21500     PY_INT64_T current_id = PyInterpreterState_GetID(PyThreadState_Get()->interp);
21501     if (main_interpreter_id == -1) {
21502         main_interpreter_id = current_id;
21503         return (unlikely(current_id == -1)) ? -1 : 0;
21504     } else if (unlikely(main_interpreter_id != current_id))
21505     #else
21506     static PyInterpreterState *main_interpreter = NULL;
21507     PyInterpreterState *current_interpreter = PyThreadState_Get()->interp;
21508     if (!main_interpreter) {
21509         main_interpreter = current_interpreter;
21510     } else if (unlikely(main_interpreter != current_interpreter))
21511     #endif
21512     {
21513         PyErr_SetString(
21514             PyExc_ImportError,
21515             "Interpreter change detected - this module can only be loaded into one interpreter per
process.");
21516         return -1;
21517     }
21518     return 0;
21519 }
21520 static CYTHON_SMALL_CODE int __Pyx_copy_spec_to_module(PyObject *spec, PyObject *moddict, const char*
from_name, const char* to_name, int allow_none) {
21521     PyObject *value = PyObject_GetAttrString(spec, from_name);
21522     int result = 0;
21523     if (likely(value)) {
21524         if (allow_none || value != Py_None) {
21525             result = PyDict_SetItemString(moddict, to_name, value);
21526         }
21527         Py_DECREF(value);
21528     } else if (PyErr_ExceptionMatches(PyExc_AttributeError)) {
21529         PyErr_Clear();
21530     } else {
21531         result = -1;
21532     }
21533     return result;
21534 }
21535 static CYTHON_SMALL_CODE PyObject* __pyx_pymod_create(PyObject *spec, CYTHON_UNUSED PyModuleDef *def)
{
21536     PyObject *module = NULL, *moddict, *modname;
21537     if (__Pyx_check_single_interpreter())
21538         return NULL;
21539     if (__pyx_m)
21540         return __Pyx_NewRef(__pyx_m);
21541     modname = PyObject_GetAttrString(spec, "name");
21542     if (unlikely(!modname)) goto bad;
21543     module = PyModule_NewObject(modname);
21544     Py_DECREF(modname);
21545     if (unlikely(!module)) goto bad;
21546     moddict = PyModule_GetDict(module);
21547     if (unlikely(!moddict)) goto bad;

```



```

21548     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "loader", "__loader__", 1) < 0)) goto bad;
21549     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "origin", "__file__", 1) < 0)) goto bad;
21550     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "parent", "__package__", 1) < 0)) goto bad;
21551     if (unlikely(__Pyx_copy_spec_to_module(spec, moddict, "submodule_search_locations", "__path__", 0)
    < 0)) goto bad;
21552     return module;
21553 bad:
21554     Py_XDECREF(module);
21555     return NULL;
21556 }
21557
21558
21559 static CYTHON_SMALL_CODE int __pyx_pymod_exec_PyClical(PyObject *__pyx_pyinit_module)
21560 #endif
21561 #endif
21562 {
21563     PyObject *__pyx_t_1 = NULL;
21564     PyObject *__pyx_t_2 = NULL;
21565     PyObject *__pyx_t_3 = NULL;
21566     int __pyx_t_4;
21567     int __pyx_lineno = 0;
21568     const char *__pyx_filename = NULL;
21569     int __pyx_clineno = 0;
21570     __Pyx_RefNannyDeclarations
21571     #if CYTHON_PEP489_MULTI_PHASE_INIT
21572     if (__pyx_m) {
21573         if (__pyx_m == __pyx_pyinit_module) return 0;
21574         PyErr_SetString(PyExc_RuntimeError, "Module 'PyClical' has already been imported.
    Re-initialisation is not supported.");
21575         return -1;
21576     }
21577     #elif PY_MAJOR_VERSION >= 3
21578     if (__pyx_m) return __Pyx_NewRef(__pyx_m);
21579     #endif
21580     #if CYTHON_REFNANNY
21581     __Pyx_RefNanny = __Pyx_RefNannyImportAPI("refnanny");
21582     if (!__Pyx_RefNanny) {
21583         PyErr_Clear();
21584         __Pyx_RefNanny = __Pyx_RefNannyImportAPI("Cython.Runtime.refnanny");
21585         if (!__Pyx_RefNanny)
21586             Py_FatalError("failed to import 'refnanny' module");
21587     }
21588     #endif
21589     __Pyx_RefNannySetupContext("__Pyx_PyMODINIT_FUNC PyInit_PyClical(void)", 0);
21590     if (__Pyx_check_binary_version() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21591     #ifdef __Pxy_PyFrame_Initialize_Offsets
21592     __Pxy_PyFrame_Initialize_Offsets();
21593     #endif
21594     __pyx_empty_tuple = PyTuple_New(0); if (unlikely(!__pyx_empty_tuple)) __PYX_ERR(0, 1,
    __pyx_L1_error)
21595     __pyx_empty_bytes = PyBytes_FromStringAndSize("", 0); if (unlikely(!__pyx_empty_bytes)) __PYX_ERR(0,
    1, __pyx_L1_error)
21596     __pyx_empty_unicode = PyUnicode_FromStringAndSize("", 0); if (unlikely(!__pyx_empty_unicode))
    __PYX_ERR(0, 1, __pyx_L1_error)
21597     #ifdef __Pyx_CyFunction_USED
21598     if (__pyx_CyFunction_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21599     #endif
21600     #ifdef __Pyx_FusedFunction_USED
21601     if (__pyx_FusedFunction_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21602     #endif
21603     #ifdef __Pyx_Coroutine_USED
21604     if (__pyx_Coroutine_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21605     #endif
21606     #ifdef __Pyx_Generator_USED
21607     if (__pyx_Generator_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21608     #endif
21609     #ifdef __Pyx_AsyncGen_USED
21610     if (__pyx_AsyncGen_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21611     #endif
21612     #ifdef __Pyx_StopAsyncIteration_USED
21613     if (__pyx_StopAsyncIteration_init() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21614     #endif
21615     /*--- Library function declarations ---*/
21616     /*--- Threads initialization code ---*/
21617     #if defined(WITH_THREAD) && PY_VERSION_HEX < 0x030700F0 && defined(__PYX_FORCE_INIT_THREADS) &&
    __PYX_FORCE_INIT_THREADS
21618     PyEval_InitThreads();
21619     #endif
21620     /*--- Module creation code ---*/
21621     #if CYTHON_PEP489_MULTI_PHASE_INIT
21622     __pyx_m = __pyx_pyinit_module;
21623     Py_INCREF(__pyx_m);
21624     #else
21625     #if PY_MAJOR_VERSION < 3
21626     __pyx_m = Py_InitModule4("PyClical", __pyx_methods, 0, 0, PYTHON_API_VERSION); Py_XINCREF(__pyx_m);
21627     #else
21628     __pyx_m = PyModule_Create(&__pyx_moduledef);

```



```

21629 #endif
21630 if (unlikely(!__pyx_m)) __PYX_ERR(0, 1, __pyx_L1_error)
21631 #endif
21632 __pyx_d = PyModule_GetDict(__pyx_m); if (unlikely(!__pyx_d)) __PYX_ERR(0, 1, __pyx_L1_error)
21633 Py_INCREF(__pyx_d);
21634 __pyx_b = PyImport_AddModule(__Pyx_BUILTIN_MODULE_NAME); if (unlikely(!__pyx_b)) __PYX_ERR(0, 1,
__pyx_L1_error)
21635 Py_INCREF(__pyx_b);
21636 __pyx_cython_runtime = PyImport_AddModule((char *) "cython_runtime"); if
(unlikely(!__pyx_cython_runtime)) __PYX_ERR(0, 1, __pyx_L1_error)
21637 Py_INCREF(__pyx_cython_runtime);
21638 if (PyObject_SetAttrString(__pyx_m, "__builtins__", __pyx_b) < 0) __PYX_ERR(0, 1, __pyx_L1_error);
21639 /*--- Initialize various global constants etc. ---*/
21640 if (__Pyx_InitGlobals() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21641 #if PY_MAJOR_VERSION < 3 && (__PYX_DEFAULT_STRING_ENCODING_IS_ASCII ||
__PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT)
21642 if (__Pyx_init_sys_getdefaultencoding_params() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21643 #endif
21644 if (__pyx_module_is_main_PyClical) {
21645     if (PyObject_SetAttr(__pyx_m, __pyx_n_s_name, __pyx_n_s_main) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21646 }
21647 #if PY_MAJOR_VERSION >= 3
21648 {
21649     PyObject *modules = PyImport_GetModuleDict(); if (unlikely(!modules)) __PYX_ERR(0, 1,
__pyx_L1_error)
21650     if (!PyDict_GetItemString(modules, "PyClical")) {
21651         if (unlikely(PyDict_SetItemString(modules, "PyClical", __pyx_m) < 0)) __PYX_ERR(0, 1,
__pyx_L1_error)
21652     }
21653 }
21654 #endif
21655 /*--- Builtin init code ---*/
21656 if (__Pyx_InitCachedBuiltins() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21657 /*--- Constants init code ---*/
21658 if (__Pyx_InitCachedConstants() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21659 /*--- Global type/function init code ---*/
21660 (void)__Pyx_modinit_global_init_code();
21661 (void)__Pyx_modinit_variable_export_code();
21662 (void)__Pyx_modinit_function_export_code();
21663 if (unlikely(__Pyx_modinit_type_init_code() < 0)) __PYX_ERR(0, 1, __pyx_L1_error)
21664 (void)__Pyx_modinit_type_import_code();
21665 (void)__Pyx_modinit_variable_import_code();
21666 (void)__Pyx_modinit_function_import_code();
21667 /*--- Execution code ---*/
21668 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
21669 if (__Pyx_patch_abc() < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21670 #endif
21671
21672 /* "PyClical.pyx":29
21673 * # C. Doran and A. Lasenby, "Geometric algebra for physicists", Cambridge, 2003.
21674 *
21675 * import math                # ««««««««
21676 * import numbers
21677 * import collections
21678 */
21679 __pyx_t_1 = __Pyx_Import(__pyx_n_s_math, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 29,
__pyx_L1_error)
21680 __Pyx_GOTREF(__pyx_t_1);
21681 if (PyDict_SetItem(__pyx_d, __pyx_n_s_math, __pyx_t_1) < 0) __PYX_ERR(0, 29, __pyx_L1_error)
21682 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21683
21684 /* "PyClical.pyx":30
21685 *
21686 * import math
21687 * import numbers                # ««««««««
21688 * import collections
21689 *
21690 */
21691 __pyx_t_1 = __Pyx_Import(__pyx_n_s_numbers, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 30,
__pyx_L1_error)
21692 __Pyx_GOTREF(__pyx_t_1);
21693 if (PyDict_SetItem(__pyx_d, __pyx_n_s_numbers, __pyx_t_1) < 0) __PYX_ERR(0, 30, __pyx_L1_error)
21694 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21695
21696 /* "PyClical.pyx":31
21697 * import math
21698 * import numbers
21699 * import collections            # ««««««««
21700 *
21701 * from PyClical cimport *
21702 */
21703 __pyx_t_1 = __Pyx_Import(__pyx_n_s_collections, 0, 0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 31,
__pyx_L1_error)
21704 __Pyx_GOTREF(__pyx_t_1);
21705 if (PyDict_SetItem(__pyx_d, __pyx_n_s_collections, __pyx_t_1) < 0) __PYX_ERR(0, 31, __pyx_L1_error)
21706 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21707

```

```

21708  /* "PyClical.pyx":35
21709  * from PyClical cimport *
21710  *
21711  * __version__ = str(glucat_package_version,'utf-8')          # ««««««««
21712  *
21713  * # Forward reference
21714  */
21715  __pyx_t_1 = __pyx_convert_PyBytes_string_to_py_std_in_string(glucat_package_version); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 35, __pyx_L1_error)
21716  __Pyx_GOTREF(__pyx_t_1);
21717  __pyx_t_2 = PyTuple_New(2); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 35, __pyx_L1_error)
21718  __Pyx_GOTREF(__pyx_t_2);
21719  __Pyx_GIVEREF(__pyx_t_1);
21720  PyTuple_SET_ITEM(__pyx_t_2, 0, __pyx_t_1);
21721  __Pyx_INCREF(__pyx_kp_u_utf_8);
21722  __Pyx_GIVEREF(__pyx_kp_u_utf_8);
21723  PyTuple_SET_ITEM(__pyx_t_2, 1, __pyx_kp_u_utf_8);
21724  __pyx_t_1 = 0;
21725  __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)(&PyUnicode_Type)), __pyx_t_2, NULL); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 35, __pyx_L1_error)
21726  __Pyx_GOTREF(__pyx_t_1);
21727  __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21728  if (PyDict_SetItem(__pyx_d, __pyx_n_s_version, __pyx_t_1) < 0) __PYX_ERR(0, 35, __pyx_L1_error)
21729  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21730
21731  /* "PyClical.pyx":406
21732  *         return index_set_to_str( self.unwrap() ).decode()
21733  *
21734  * def index_set_hidden_doctests():          # ««««««««
21735  *     """
21736  *     Tests for functions that Doctest cannot see.
21737  */
21738  __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_1index_set_hidden_doctests, NULL,
__pyx_n_s_PyClical); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 406, __pyx_L1_error)
21739  __Pyx_GOTREF(__pyx_t_1);
21740  if (PyDict_SetItem(__pyx_d, __pyx_n_s_index_set_hidden_doctests, __pyx_t_1) < 0) __PYX_ERR(0, 406,
__pyx_L1_error)
21741  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21742
21743  /* "PyClical.pyx":1253
21744  *         return clifford_to_str( self.unwrap() ).decode()
21745  *
21746  * def clifford_hidden_doctests():          # ««««««««
21747  *     """
21748  *     Tests for functions that Doctest cannot see.
21749  */
21750  __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_9clifford_hidden_doctests, NULL,
__pyx_n_s_PyClical); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1253, __pyx_L1_error)
21751  __Pyx_GOTREF(__pyx_t_1);
21752  if (PyDict_SetItem(__pyx_d, __pyx_n_s_clifford_hidden_doctests, __pyx_t_1) < 0) __PYX_ERR(0, 1253,
__pyx_L1_error)
21753  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21754
21755  /* "PyClical.pyx":1905
21756  *
21757  * # Some abbreviations.
21758  * scalar_epsilon = epsilon          # ««««««««
21759  *
21760  * pi = atan(clifford(1.0)) * 4.0
21761  */
21762  __pyx_t_1 = PyFloat_FromDouble(epsilon); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1905,
__pyx_L1_error)
21763  __Pyx_GOTREF(__pyx_t_1);
21764  if (PyDict_SetItem(__pyx_d, __pyx_n_s_scalar_epsilon, __pyx_t_1) < 0) __PYX_ERR(0, 1905,
__pyx_L1_error)
21765  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21766
21767  /* "PyClical.pyx":1907
21768  * scalar_epsilon = epsilon
21769  *
21770  * pi = atan(clifford(1.0)) * 4.0          # ««««««««
21771  * tau = atan(clifford(1.0)) * 8.0
21772  *
21773  */
21774  __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)__pyx_ptype_8PyClical_clifford), __pyx_tuple__15,
NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1907, __pyx_L1_error)
21775  __Pyx_GOTREF(__pyx_t_1);
21776  __pyx_t_2 = __pyx_f_8PyClical_atan(__pyx_t_1, 0, NULL); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1907,
__pyx_L1_error)
21777  __Pyx_GOTREF(__pyx_t_2);
21778  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21779  __pyx_t_1 = PyNumber_Multiply(__pyx_t_2, __pyx_float_4_0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1907, __pyx_L1_error)
21780  __Pyx_GOTREF(__pyx_t_1);
21781  __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21782  if (PyDict_SetItem(__pyx_d, __pyx_n_s_pi, __pyx_t_1) < 0) __PYX_ERR(0, 1907, __pyx_L1_error)
21783  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;

```

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21784
21785  /* "PyClical.pyx":1908
21786  *
21787  * pi = atan(clifford(1.0)) * 4.0
21788  * tau = atan(clifford(1.0)) * 8.0          # ««««««««
21789  *
21790  * cl = clifford
21791  */
21792  __pyx_t_1 = __Pyx_PyObject_Call(((PyObject *)__pyx_ptype_8PyClical_clifford), __pyx_tuple__15,
NULL); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1908, __pyx_L1_error)
21793  __Pyx_GOTREF(__pyx_t_1);
21794  __pyx_t_2 = __pyx_f_8PyClical_atan(__pyx_t_1, 0, NULL); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1908,
__pyx_L1_error)
21795  __Pyx_GOTREF(__pyx_t_2);
21796  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21797  __pyx_t_1 = PyNumber_Multiply(__pyx_t_2, __pyx_float_8_0); if (unlikely(!__pyx_t_1)) __PYX_ERR(0,
1908, __pyx_L1_error)
21798  __Pyx_GOTREF(__pyx_t_1);
21799  __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21800  if (PyDict_SetItem(__pyx_d, __pyx_n_s_tau, __pyx_t_1) < 0) __PYX_ERR(0, 1908, __pyx_L1_error)
21801  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21802
21803  /* "PyClical.pyx":1910
21804  * tau = atan(clifford(1.0)) * 8.0
21805  *
21806  * cl = clifford          # ««««««««
21807  * """
21808  * Abbreviation for clifford.
21809  */
21810  if (PyDict_SetItem(__pyx_d, __pyx_n_s_cl, ((PyObject *)__pyx_ptype_8PyClical_clifford)) < 0)
__PYX_ERR(0, 1910, __pyx_L1_error)
21811
21812  /* "PyClical.pyx":1928
21813  * """
21814  *
21815  * ist = index_set          # ««««««««
21816  * """
21817  * Abbreviation for index_set.
21818  */
21819  if (PyDict_SetItem(__pyx_d, __pyx_n_s_ist, ((PyObject *)__pyx_ptype_8PyClical_index_set)) < 0)
__PYX_ERR(0, 1928, __pyx_L1_error)
21820
21821  /* "PyClical.pyx":1936
21822  * """
21823  *
21824  * def e(obj):          # ««««««««
21825  *     """
21826  *     Abbreviation for clifford(index_set(obj)).
21827  */
21828  __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_89e, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1936, __pyx_L1_error)
21829  __Pyx_GOTREF(__pyx_t_1);
21830  if (PyDict_SetItem(__pyx_d, __pyx_n_s_e, __pyx_t_1) < 0) __PYX_ERR(0, 1936, __pyx_L1_error)
21831  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21832
21833  /* "PyClical.pyx":1949
21834  *     return clifford(index_set(obj))
21835  *
21836  * def istpq(p, q):          # ««««««««
21837  *     """
21838  *     Abbreviation for index_set({-q,...p}).
21839  */
21840  __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClical_91istpq, NULL, __pyx_n_s_PyClical); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1949, __pyx_L1_error)
21841  __Pyx_GOTREF(__pyx_t_1);
21842  if (PyDict_SetItem(__pyx_d, __pyx_n_s_istpq, __pyx_t_1) < 0) __PYX_ERR(0, 1949, __pyx_L1_error)
21843  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21844
21845  /* "PyClical.pyx":1958
21846  *     return index_set(set(range(-q,p+1)))
21847  *
21848  * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].          #
21849  * ««««««««
21849  * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL].
21850  *
21851  */
21852  __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21853  __Pyx_GOTREF(__pyx_t_1);
21854  __pyx_t_2 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__20, NULL); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1958, __pyx_L1_error)
21855  __Pyx_GOTREF(__pyx_t_2);
21856  __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21857  __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_L1_error)
21858  __Pyx_GOTREF(__pyx_t_1);
21859  __pyx_t_3 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__21, NULL); if (unlikely(!__pyx_t_3))

```

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__PYX_ERR(0, 1958, __pyx_l1_error)
21860 __Pyx_GOTREF(__pyx_t_3);
21861 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21862 __pyx_t_1 = PyNumber_Add(__pyx_t_2, __pyx_t_3); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1958,
__pyx_l1_error)
21863 __Pyx_GOTREF(__pyx_t_1);
21864 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21865 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
21866 if (PyDict_SetItem(__pyx_d, __pyx_n_s_ninf3, __pyx_t_1) < 0) __PYX_ERR(0, 1958, __pyx_l1_error)
21867 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21868
21869 /* "PyClicl.pyx":1959
21870 *
21871 * ninf3 = e(4) + e(-1) # Null infinity point in 3D Conformal Geometric Algebra [DL].
21872 * nbar3 = e(4) - e(-1) # Null bar point in 3D Conformal Geometric Algebra [DL]. # ««««««««
21873 *
21874 * # Doctest interface.
21875 */
21876 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_l1_error)
21877 __Pyx_GOTREF(__pyx_t_1);
21878 __pyx_t_3 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__20, NULL); if (unlikely(!__pyx_t_3))
__PYX_ERR(0, 1959, __pyx_l1_error)
21879 __Pyx_GOTREF(__pyx_t_3);
21880 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21881 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_e); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_l1_error)
21882 __Pyx_GOTREF(__pyx_t_1);
21883 __pyx_t_2 = __Pyx_PyObject_Call(__pyx_t_1, __pyx_tuple__21, NULL); if (unlikely(!__pyx_t_2))
__PYX_ERR(0, 1959, __pyx_l1_error)
21884 __Pyx_GOTREF(__pyx_t_2);
21885 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21886 __pyx_t_1 = PyNumber_Subtract(__pyx_t_3, __pyx_t_2); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1959,
__pyx_l1_error)
21887 __Pyx_GOTREF(__pyx_t_1);
21888 __Pyx_DECREF(__pyx_t_3); __pyx_t_3 = 0;
21889 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21890 if (PyDict_SetItem(__pyx_d, __pyx_n_s_nbar3, __pyx_t_1) < 0) __PYX_ERR(0, 1959, __pyx_l1_error)
21891 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21892
21893 /* "PyClicl.pyx":1962
21894 *
21895 * # Doctest interface.
21896 * def _test(): # ««««««««
21897 *     import PyClicl, doctest
21898 *     return doctest.testmod(PyClicl)
21899 */
21900 __pyx_t_1 = PyCFunction_NewEx(&__pyx_mdef_8PyClicl_93_test, NULL, __pyx_n_s_PyClicl); if
(unlikely(!__pyx_t_1)) __PYX_ERR(0, 1962, __pyx_l1_error)
21901 __Pyx_GOTREF(__pyx_t_1);
21902 if (PyDict_SetItem(__pyx_d, __pyx_n_s_test, __pyx_t_1) < 0) __PYX_ERR(0, 1962, __pyx_l1_error)
21903 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21904
21905 /* "PyClicl.pyx":1966
21906 *     return doctest.testmod(PyClicl)
21907 *
21908 * if __name__ == "__main__": # ««««««««
21909 *     _test()
21910 */
21911 __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_name); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1966,
__pyx_l1_error)
21912 __Pyx_GOTREF(__pyx_t_1);
21913 __pyx_t_4 = (__Pyx_PyUnicode_Equals(__pyx_t_1, __pyx_n_u_main, Py_EQ)); if (unlikely(__pyx_t_4 < 0))
__PYX_ERR(0, 1966, __pyx_l1_error)
21914 __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21915 if (__pyx_t_4) {
21916
21917     /* "PyClicl.pyx":1967
21918     *
21919     * if __name__ == "__main__":
21920     *     _test() # ««««««««
21921     */
21922     __Pyx_GetModuleGlobalName(__pyx_t_1, __pyx_n_s_test); if (unlikely(!__pyx_t_1)) __PYX_ERR(0, 1967,
__pyx_l1_error)
21923     __Pyx_GOTREF(__pyx_t_1);
21924     __pyx_t_2 = __Pyx_PyObject_CallNoArg(__pyx_t_1); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1967,
__pyx_l1_error)
21925     __Pyx_GOTREF(__pyx_t_2);
21926     __Pyx_DECREF(__pyx_t_1); __pyx_t_1 = 0;
21927     __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
21928
21929     /* "PyClicl.pyx":1966
21930     *     return doctest.testmod(PyClicl)
21931     *
21932     * if __name__ == "__main__": # ««««««««
21933     *     _test()
21934     */

```

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21935     }
21936
21937     /* "PyClical.pyx":1
21938     * # -*- coding: utf-8 -*-
21939     * # cython: language_level=3
21940     * # distutils: language = c++
21941     */
21942     __pyx_t_2 = __Pyx_PyDict_NewPresized(111); if (unlikely(!__pyx_t_2)) __PYX_ERR(0, 1, __pyx_L1_error)
21943     __Pyx_GOTREF(__pyx_t_2);
21944     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_copy_line_65,
21945     __pyx_kp_u_Copy_this_index_set_object_s_in) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21946     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_setitem_line_179,
21947     __pyx_kp_u_Set_the_value_of_an_index_set_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21948     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_getitem_line_191,
21949     __pyx_kp_u_Get_the_value_of_an_index_set_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21950     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_iter_line_229,
21951     __pyx_kp_u_Iterate_over_the_indices_of_an) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21952     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_invert_line_240,
21953     __pyx_kp_u_Set_complement_not_print_index) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21954     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_xor_line_249,
21955     __pyx_kp_u_Symmetric_set_difference_exclus) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21956     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_ixor_line_260,
21957     __pyx_kp_u_Symmetric_set_difference_exclus_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21958     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_and_line_271,
21959     __pyx_kp_u_Set_intersection_and_print_inde) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21960     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_1and_line_282,
21961     __pyx_kp_u_Set_intersection_and_x_index_se) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21962     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_or_line_293,
21963     __pyx_kp_u_Set_union_or_print_index_set_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21964     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_ior_line_304,
21965     __pyx_kp_u_Set_union_or_x_index_set_1_x_in) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21966     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_line_315,
21967     __pyx_kp_u_Cardinality_Number_of_indices_i) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21968     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_neg_line_324,
21969     __pyx_kp_u_Number_of_negative_indices_incl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21970     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_count_pos_line_333,
21971     __pyx_kp_u_Number_of_positive_indices_incl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21972     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_min_line_342,
21973     __pyx_kp_u_Minimum_member_index_set_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21974     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_max_line_351,
21975     __pyx_kp_u_Maximum_member_index_set_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21976     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_sign_of_mult_line_366,
21977     __pyx_kp_u_Sign_of_geometric_product_of_tw) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21978     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_sign_of_square_line_37,
21979     __pyx_kp_u_Sign_of_geometric_square_of_a_C) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21980     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_repr_line_384,
21981     __pyx_kp_u_The_official_string_representat) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21982     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_str_line_395,
21983     __pyx_kp_u_The_informal_string_representat) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21984     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_index_set_hidden_doctests_line_4,
21985     __pyx_kp_u_Tests_for_functions_that_Doctes) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21986     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_compare_line_492,
21987     __pyx_kp_u_lexicographic_compare_eg_3_4_5) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21988     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_min_neg_line_504,
21989     __pyx_kp_u_Minimum_negative_index_or_0_if) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21990     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_max_pos_line_513,
21991     __pyx_kp_u_Maximum_positive_index_or_0_if) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21992     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_copy_line_556,
21993     __pyx_kp_u_Copy_this_clifford_object_x_cli) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21994     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iter_line_638,
21995     __pyx_kp_u_Not_applicable_for_a_in_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21996     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_reframe_line_649,
21997     __pyx_kp_u_Put_self_into_a_larger_frame_co) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21998     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_getitem_line_707,
21999     __pyx_kp_u_Subscripting_map_from_index_set) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22000     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_neg_line_722,
22001     __pyx_kp_u_Unary_print_clifford_1_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22002     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pos_line_731,
22003     __pyx_kp_u_Unary_print_clifford_1_1_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22004     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_add_line_740,
22005     __pyx_kp_u_Geometric_sum_print_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22006     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iadd_line_751,
22007     __pyx_kp_u_Geometric_sum_x_clifford_1_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22008     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_sub_line_760,
22009     __pyx_kp_u_Geometric_difference_print_clif) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22010     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_isub_line_771,
22011     __pyx_kp_u_Geometric_difference_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22012     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_mul_line_780,
22013     __pyx_kp_u_Geometric_product_print_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22014     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_imul_line_793,
22015     __pyx_kp_u_Geometric_product_x_clifford_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22016     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_mod_line_806,
22017     __pyx_kp_u_Contraction_print_clifford_1_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22018     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_imod_line_821,
22019     __pyx_kp_u_Contraction_x_clifford_1_x_clif) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22020     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_and_line_836,
22021     __pyx_kp_u_Inner_product_print_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)

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21983     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_iand_line_851,
21984         __pyx_kp_u_Inner_product_x_clifford_1_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21985     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_xor_line_866,
21986         __pyx_kp_u_Outer_product_print_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21987     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_ixor_line_881,
21988         __pyx_kp_u_Outer_product_x_clifford_1_x_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21989     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_truediv_line_896,
21990         __pyx_kp_u_Geometric_quotient_print_cliffo) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21991     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_idiv_line_911,
21992         __pyx_kp_u_Geometric_quotient_x_clifford_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21993     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_inv_line_926,
21994         __pyx_kp_u_Geometric_multiplicative_invers) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21995     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_or_line_939,
21996         __pyx_kp_u_Transform_left_hand_side_using) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21997     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_ior_line_950,
21998         __pyx_kp_u_Transform_left_hand_side_using_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
21999     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pow_line_961,
22000         __pyx_kp_u_Power_self_to_the_m_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22001     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pow_line_980,
22002         __pyx_kp_u_Power_self_to_the_m_x_clifford_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22003     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_outer_pow_line_1004,
22004         __pyx_kp_u_Outer_product_power_x_clifford) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22005     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_call_line_1020,
22006         __pyx_kp_u_Pure_grade_vector_part_print_cl) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22007     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_scalar_line_1039,
22008         __pyx_kp_u_Scalar_part_clifford_1_l_1_2_sc) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22009     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_pure_line_1050,
22010         __pyx_kp_u_Pure_part_print_clifford_1_l_1) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22011     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_even_line_1061,
22012         __pyx_kp_u_Even_part_of_multivector_sum_of) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22013     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_odd_line_1070,
22014         __pyx_kp_u_Odd_part_of_multivector_sum_of) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22015     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_vector_part_line_1079,
22016         __pyx_kp_u_Vector_part_of_multivector_as_a) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22017     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_involute_line_1107,
22018         __pyx_kp_u_Main_involution_each_i_is_repla) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22019     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_reverse_line_1123,
22020         __pyx_kp_u_Reversion_eg_clifford_1_cliffor) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22021     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_conj_line_1138,
22022         __pyx_kp_u_Conjugation_reverse_o_involute) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22023     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_quad_line_1153,
22024         __pyx_kp_u_Quadratic_form_rev_x_x_0_print) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22025     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_norm_line_1164,
22026         __pyx_kp_u_Norm_sum_of_squares_of_coordina) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22027     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_abs_line_1175,
22028         __pyx_kp_u_Absolute_value_square_root_of_n) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22029     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_max_abs_line_1184,
22030         __pyx_kp_u_Maximum_of_absolute_values_of_c) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22031     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_truncated_line_1195,
22032         __pyx_kp_u_Remove_all_terms_of_self_with_r) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22033     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_isinf_line_1206,
22034         __pyx_kp_u_Check_if_a_multivector_contains) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22035     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_isnan_line_1215,
22036         __pyx_kp_u_Check_if_a_multivector_contains_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22037     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_frame_line_1224,
22038         __pyx_kp_u_Subalgebra_generated_by_all_gen) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22039     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_repr_line_1235,
22040         __pyx_kp_u_The_official_string_representat_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22041     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_str_line_1244,
22042         __pyx_kp_u_The_informal_string_representat_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22043     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_clifford_hidden_doctests_line_12,
22044         __pyx_kp_u_Tests_for_functions_that_Doctes_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22045     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_error_squared_tol_line_1337,
22046         __pyx_kp_u_Quadratic_norm_error_tolerance) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22047     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_error_squared_line_1346,
22048         __pyx_kp_u_Relative_or_absolute_error_usin) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22049     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_approx_equal_line_1359,
22050         __pyx_kp_u_Test_for_approximate_equality_o) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22051     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_inv_line_1378,
22052         __pyx_kp_u_Geometric_multiplicative_invers_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22053     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_scalar_line_1393,
22054         __pyx_kp_u_Scalar_part_scalar_clifford_1_l) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22055     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_real_line_1404, __pyx_kp_u_Real_part_synonym_for_scalar_pa)
22056         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22057     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_imag_line_1415, __pyx_kp_u_Imaginary_part_deprecated_alway)
22058         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22059     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_pure_line_1426, __pyx_kp_u_Pure_part_print_pure_clifford_1)
22060         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22061     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_even_line_1437,
22062         __pyx_kp_u_Even_part_of_multivector_sum_of_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22063     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_odd_line_1446, __pyx_kp_u_Odd_part_of_multivector_sum_of_2)
22064         < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22065     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_involute_line_1455,
22066         __pyx_kp_u_Main_involution_each_i_is_repla_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22067     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_reverse_line_1470,
22068         __pyx_kp_u_Reversion_eg_l_2_2_1_print_reve) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22069     if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_conj_line_1485,

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__pyx_kp_u_Conjugation_reverse_o_involute_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22027 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_quad_line1500,
__pyx_kp_u_Quadratic_form_rev_x_x_0_print_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22028 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_norm_line1511, __pyx_kp_u_norm_sum_of_squares_of_coordina
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22029 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_abs_line1522, __pyx_kp_u_Absolute_value_of_multivector_m)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22030 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_max_abs_line1531,
__pyx_kp_u_Maximum_absolute_value_of_coord) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22031 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_pow_line1543, __pyx_kp_u_Integer_power_of_multivector_ob)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22032 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_outer_pow_line1567,
__pyx_kp_u_Outer_product_power_of_multivec) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22033 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_complexifier_line1576,
__pyx_kp_u_Square_root_of_1_which_commutates) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22034 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sqrt_line1591, __pyx_kp_u_Square_root_of_multivector_with)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22035 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_exp_line1614, __pyx_kp_u_Exponential_of_multivector_x_cl)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22036 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_log_line1628, __pyx_kp_u_Natural_logarithm_of_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22037 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cos_line1651, __pyx_kp_u_Cosine_of_multivector_with_opti)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22038 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_acos_line1668, __pyx_kp_u_Inverse_cosine_of_multivector_w)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22039 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cosh_line1689, __pyx_kp_u_Hyperbolic_cosine_of_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22040 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_acosh_line1705,
__pyx_kp_u_Inverse_hyperbolic_cosine_of_mu) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22041 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sin_line1728, __pyx_kp_u_Sine_of_multivector_with_option)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22042 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_asin_line1747, __pyx_kp_u_Inverse_sine_of_multivector_wit)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22043 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_sinh_line1768, __pyx_kp_u_Hyperbolic_sine_of_multivector)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22044 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_asinh_line1782,
__pyx_kp_u_Inverse_hyperbolic_sine_of_mult) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22045 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_tan_line1801, __pyx_kp_u_Tangent_of_multivector_with_opt)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22046 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_atan_line1818, __pyx_kp_u_Inverse_tangent_of_multivector)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22047 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_tanh_line1835, __pyx_kp_u_Hyperbolic_tangent_of_multivect)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22048 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_atanh_line1847,
__pyx_kp_u_Inverse_hyperbolic_tangent_of_m) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22049 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_random_clifford_line1864,
__pyx_kp_u_Random_multivector_within_a_fra) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22050 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cga3_line1873, __pyx_kp_u_Convert_Euclidean_3D_multivecto)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22051 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_cga3std_line1882,
__pyx_kp_u_Convert_CGA3_null_vector_to_sta) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22052 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_agc3_line1893, __pyx_kp_u_Convert_CGA3_null_vector_to_Euc)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22053 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_e_line1936, __pyx_kp_u_Abbreviation_for_clifford_index) <
0) __PYX_ERR(0, 1, __pyx_L1_error)
22054 if (PyDict_SetItem(__pyx_t_2, __pyx_kp_u_istpq_line1949, __pyx_kp_u_Abbreviation_for_index_set_q_p)
< 0) __PYX_ERR(0, 1, __pyx_L1_error)
22055 if (PyDict_SetItem(__pyx_d, __pyx_n_s_test_2, __pyx_t_2) < 0) __PYX_ERR(0, 1, __pyx_L1_error)
22056 __Pyx_DECREF(__pyx_t_2); __pyx_t_2 = 0;
22057
22058 /* "string.to_py":55
22059 *
22060 * @cname("__pyx_convert_PyByteArray_string_to_py_std_in_string")
22061 * cdef inline object __pyx_convert_PyByteArray_string_to_py_std_in_string(const string& s):
# «««««
22062 *     return __Pyx_PyByteArray_FromStringAndSize(s.data(), s.size())
22063 *
22064 */
22065
22066 /---- Wrapped vars code ----*/
22067
22068 goto __pyx_L0;
22069 __pyx_L1_error:;
22070 __Pyx_XDECREF(__pyx_t_1);
22071 __Pyx_XDECREF(__pyx_t_2);
22072 __Pyx_XDECREF(__pyx_t_3);
22073 if (__pyx_m) {
22074     if (__pyx_d) {
22075         __Pyx_AddTraceback("init PyClical", __pyx_clineno, __pyx_lineno, __pyx_filename);
22076     }
22077     Py_CLEAR(__pyx_m);
22078 } else if (!PyErr_Occurred()) {
22079     PyErr_SetString(PyExc_ImportError, "init PyClical");
22080 }
22081 __pyx_L0:;
22082 __Pyx_RefNannyFinishContext();
22083 #if CYTHON_PEP489_MULTI_PHASE_INIT

```



```

22084     return (__pyx_m != NULL) ? 0 : -1;
22085     #elif PY_MAJOR_VERSION >= 3
22086     return __pyx_m;
22087     #else
22088     return;
22089     #endif
22090 }
22091
22092 /* --- Runtime support code --- */
22093 /* Refnanny */
22094 #if CYTHON_REFNANNY
22095 static __Pyx_RefNannyAPIStruct *__Pyx_RefNannyImportAPI(const char *modname) {
22096     PyObject *m = NULL, *p = NULL;
22097     void *r = NULL;
22098     m = PyImport_ImportModule(modname);
22099     if (!m) goto end;
22100     p = PyObject_GetAttrString(m, "RefNannyAPI");
22101     if (!p) goto end;
22102     r = PyLong_AsVoidPtr(p);
22103 end:
22104     Py_XDECREF(p);
22105     Py_XDECREF(m);
22106     return (__Pyx_RefNannyAPIStruct *)r;
22107 }
22108 #endif
22109
22110 /* PyObjectGetAttrStr */
22111 #if CYTHON_USE_TYPE_SLOTS
22112 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStr(PyObject* obj, PyObject* attr_name) {
22113     PyTypeObject* tp = Py_TYPE(obj);
22114     if (likely(tp->tp_getattro))
22115         return tp->tp_getattro(obj, attr_name);
22116 #if PY_MAJOR_VERSION < 3
22117     if (likely(tp->tp_getattr))
22118         return tp->tp_getattr(obj, PyString_AS_STRING(attr_name));
22119 #endif
22120     return PyObject_GetAttr(obj, attr_name);
22121 }
22122 #endif
22123
22124 /* GetBuiltinName */
22125 static PyObject* __Pyx_GetBuiltinName(PyObject *name) {
22126     PyObject* result = __Pyx_PyObject_GetAttrStr(__pyx_b, name);
22127     if (unlikely(!result)) {
22128         PyErr_Format(PyExc_NameError,
22129 #if PY_MAJOR_VERSION >= 3
22130             "name '%U' is not defined", name);
22131 #else
22132             "name '%.200s' is not defined", PyString_AS_STRING(name));
22133 #endif
22134     }
22135     return result;
22136 }
22137
22138 /* PyCFunctionFastCall */
22139 #if CYTHON_FAST_PYCALL
22140 static CYTHON_INLINE PyObject* __Pyx_PyCFunction_FastCall(PyObject *func_obj, PyObject **args,
22141     Py_ssize_t nargs) {
22142     PyCFunctionObject *func = (PyCFunctionObject*)func_obj;
22143     PyCFunction meth = PyCFunction_GET_FUNCTION(func);
22144     PyObject *self = PyCFunction_GET_SELF(func);
22145     int flags = PyCFunction_GET_FLAGS(func);
22146     assert(PyCFunction_Check(func));
22147     assert(METH_FASTCALL == (flags & ~(METH_CLASS | METH_STATIC | METH_COEXIST | METH_KEYWORDS |
22148 METH_STACKLESS)));
22149     assert(nargs >= 0);
22150     assert(nargs == 0 || args != NULL);
22151     /* _PyCFunction_FastCallDict() must not be called with an exception set,
22152        because it may clear it (directly or indirectly) and so the
22153        caller loses its exception */
22154     assert(!PyErr_Occurred());
22155     if ((PY_VERSION_HEX < 0x030700A0) || unlikely(flags & METH_KEYWORDS)) {
22156         return ((*((__Pyx_PyCFunctionFastWithKeywords)(void*)meth)) (self, args, nargs, NULL));
22157     } else {
22158         return ((*((__Pyx_PyCFunctionFast)(void*)meth)) (self, args, nargs);
22159     }
22160 }
22161 #endif
22162
22163 /* PyFunctionFastCall */
22164 #if CYTHON_FAST_PYCALL
22165 static PyObject* __Pyx_PyFunction_FastCallNoKw(PyCodeObject *co, PyObject **args, Py_ssize_t na,
22166     PyObject *globals) {
22167     PyFrameObject *f;
22168     PyThreadState *tstate = __Pyx_PyThreadState_Current;
22169     PyObject **fastlocals;
22170     Py_ssize_t i;

```



```

22169     PyObject *result;
22170     assert(globals != NULL);
22171     /* XXX Perhaps we should create a specialized
22172        PyFrame_New() that doesn't take locals, but does
22173        take builtins without sanity checking them.
22174        */
22175     assert(tstate != NULL);
22176     f = PyFrame_New(tstate, co, globals, NULL);
22177     if (f == NULL) {
22178         return NULL;
22179     }
22180     fastlocals = __Pyx_PyFrame_GetLocalsplus(f);
22181     for (i = 0; i < na; i++) {
22182         Py_INCREF(*args);
22183         fastlocals[i] = *args++;
22184     }
22185     result = PyEval_EvalFrameEx(f, 0);
22186     ++tstate->recursion_depth;
22187     Py_DECREF(f);
22188     --tstate->recursion_depth;
22189     return result;
22190 }
22191 #if 1 || PY_VERSION_HEX < 0x030600B1
22192 static PyObject *__Pyx_PyFunction_FastCallDict(PyObject *func, PyObject **args, Py_ssize_t nargs,
22193 PyObject *kwargs) {
22194     PyCodeObject *co = (PyCodeObject *)PyFunction_GET_CODE(func);
22195     PyObject *globals = PyFunction_GET_GLOBALS(func);
22196     PyObject *argdefs = PyFunction_GET_DEFAULTS(func);
22197     PyObject *closure;
22198     #if PY_MAJOR_VERSION >= 3
22199     PyObject *kwdefs;
22200     #endif
22201     PyObject *kwtuple, **k;
22202     PyObject **d;
22203     Py_ssize_t nd;
22204     Py_ssize_t nk;
22205     PyObject *result;
22206     assert(kwargs == NULL || PyDict_Check(kwargs));
22207     nk = kwargs ? PyDict_Size(kwargs) : 0;
22208     if (Py_EnterRecursiveCall((char*)" while calling a Python object")) {
22209         return NULL;
22210     }
22211     if (
22212     #if PY_MAJOR_VERSION >= 3
22213         co->co_kwonlyargcount == 0 &&
22214     #endif
22215         likely(kwargs == NULL || nk == 0) &&
22216         co->co_flags == (CO_OPTIMIZED | CO_NEWLOCALS | CO_NOFREE)) {
22217         if (argdefs == NULL && co->co_argcount == nargs) {
22218             result = __Pyx_PyFunction_FastCallNoKw(co, args, nargs, globals);
22219             goto done;
22220         }
22221         else if (nargs == 0 && argdefs != NULL
22222                 && co->co_argcount == Py_SIZE(argdefs)) {
22223             /* function called with no arguments, but all parameters have
22224                a default value: use default values as arguments .*/
22225             args = &PyTuple_GET_ITEM(argdefs, 0);
22226             result = __Pyx_PyFunction_FastCallNoKw(co, args, Py_SIZE(argdefs), globals);
22227             goto done;
22228         }
22229     }
22230     if (kwargs != NULL) {
22231         Py_ssize_t pos, i;
22232         kwtuple = PyTuple_New(2 * nk);
22233         if (kwtuple == NULL) {
22234             result = NULL;
22235             goto done;
22236         }
22237         k = &PyTuple_GET_ITEM(kwtuple, 0);
22238         pos = i = 0;
22239         while (PyDict_Next(kwargs, &pos, &k[i], &k[i+1])) {
22240             Py_INCREF(k[i]);
22241             Py_INCREF(k[i+1]);
22242             i += 2;
22243         }
22244         nk = i / 2;
22245     }
22246     else {
22247         kwtuple = NULL;
22248         k = NULL;
22249     }
22250     closure = PyFunction_GET_CLOSURE(func);
22251     #if PY_MAJOR_VERSION >= 3
22252     kwdefs = PyFunction_GET_KW_DEFAULTS(func);
22253     #endif
22254     if (argdefs != NULL) {
22255         d = &PyTuple_GET_ITEM(argdefs, 0);

```

```

22255     nd = Py_SIZE(argdefs);
22256 }
22257 else {
22258     d = NULL;
22259     nd = 0;
22260 }
22261 #if PY_MAJOR_VERSION >= 3
22262     result = PyEval_EvalCodeEx((PyObject*)co, globals, (PyObject *)NULL,
22263                                args, (int)nargs,
22264                                k, (int)nk,
22265                                d, (int)nd, kwdefs, closure);
22266 #else
22267     result = PyEval_EvalCodeEx(co, globals, (PyObject *)NULL,
22268                                args, (int)nargs,
22269                                k, (int)nk,
22270                                d, (int)nd, closure);
22271 #endif
22272     Py_XDECREF(kwtuple);
22273 done:
22274     Py_LeaveRecursiveCall();
22275     return result;
22276 }
22277 #endif
22278 #endif
22279
22280 /* PyObjectCall */
22281 #if CYTHON_COMPILING_IN_CPYTHON
22282 static CYTHON_INLINE PyObject* __Pyx_PyObject_Call(PyObject *func, PyObject *arg, PyObject *kw) {
22283     PyObject *result;
22284     ternaryfunc call = Py_TYPE(func)->tp_call;
22285     if (unlikely(!call))
22286         return PyObject_Call(func, arg, kw);
22287     if (unlikely(Py_EnterRecursiveCall((char*)" while calling a Python object")))
22288         return NULL;
22289     result = (*call)(func, arg, kw);
22290     Py_LeaveRecursiveCall();
22291     if (unlikely(!result) && unlikely(!PyErr_Occurred())) {
22292         PyErr_SetString(
22293             PyExc_SystemError,
22294             "NULL result without error in PyObject_Call");
22295     }
22296     return result;
22297 }
22298 #endif
22299
22300 /* PyObjectCallMethO */
22301 #if CYTHON_COMPILING_IN_CPYTHON
22302 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallMethO(PyObject *func, PyObject *arg) {
22303     PyObject *self, *result;
22304     PyCFunction cfunc;
22305     cfunc = PyCFunction_GET_FUNCTION(func);
22306     self = PyCFunction_GET_SELF(func);
22307     if (unlikely(Py_EnterRecursiveCall((char*)" while calling a Python object")))
22308         return NULL;
22309     result = cfunc(self, arg);
22310     Py_LeaveRecursiveCall();
22311     if (unlikely(!result) && unlikely(!PyErr_Occurred())) {
22312         PyErr_SetString(
22313             PyExc_SystemError,
22314             "NULL result without error in PyObject_Call");
22315     }
22316     return result;
22317 }
22318 #endif
22319
22320 /* PyObjectCallOneArg */
22321 #if CYTHON_COMPILING_IN_CPYTHON
22322 static PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22323     PyObject *result;
22324     PyObject *args = PyTuple_New(1);
22325     if (unlikely(!args)) return NULL;
22326     Py_INCREF(arg);
22327     PyTuple_SET_ITEM(args, 0, arg);
22328     result = __Pyx_PyObject_Call(func, args, NULL);
22329     Py_DECREF(args);
22330     return result;
22331 }
22332 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22333 #if CYTHON_FAST_PYCALL
22334     if (PyFunction_Check(func)) {
22335         return __Pyx_PyFunction_FastCall(func, &arg, 1);
22336     }
22337 #endif
22338     if (likely(PyCFunction_Check(func))) {
22339         if (likely(PyCFunction_GET_FLAGS(func) & METH_O)) {
22340             return __Pyx_PyObject_CallMethO(func, arg);
22341 #if CYTHON_FAST_PYCCALL

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```

22342         } else if (__Pyx_PyFastCFunction_Check(func)) {
22343             return __Pyx_PyCFunction_FastCall(func, &arg, 1);
22344 #endif
22345     }
22346 }
22347 return __Pyx_PyObject_CallOneArg(func, arg);
22348 }
22349 #else
22350 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallOneArg(PyObject *func, PyObject *arg) {
22351     PyObject *result;
22352     PyObject *args = PyTuple_Pack(1, arg);
22353     if (unlikely(!args)) return NULL;
22354     result = __Pyx_PyObject_Call(func, args, NULL);
22355     Py_DECREF(args);
22356     return result;
22357 }
22358 #endif
22359
22360 /* PyErrFetchRestore */
22361 #if CYTHON_FAST_THREAD_STATE
22362 static CYTHON_INLINE void __Pyx_ErrRestoreInState(PyThreadState *tstate, PyObject *type, PyObject
*value, PyObject *tb) {
22363     PyObject *tmp_type, *tmp_value, *tmp_tb;
22364     tmp_type = tstate->curexc_type;
22365     tmp_value = tstate->curexc_value;
22366     tmp_tb = tstate->curexc_traceback;
22367     tstate->curexc_type = type;
22368     tstate->curexc_value = value;
22369     tstate->curexc_traceback = tb;
22370     Py_XDECREF(tmp_type);
22371     Py_XDECREF(tmp_value);
22372     Py_XDECREF(tmp_tb);
22373 }
22374 static CYTHON_INLINE void __Pyx_ErrFetchInState(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb) {
22375     *type = tstate->curexc_type;
22376     *value = tstate->curexc_value;
22377     *tb = tstate->curexc_traceback;
22378     tstate->curexc_type = 0;
22379     tstate->curexc_value = 0;
22380     tstate->curexc_traceback = 0;
22381 }
22382 #endif
22383
22384 /* WriteUnraisableException */
22385 static void __Pyx_WriteUnraisable(const char *name, CYTHON_UNUSED int clineno,
CYTHON_UNUSED int lineno, CYTHON_UNUSED const char *filename,
22386 int full_traceback, CYTHON_UNUSED int nogil) {
22387     PyObject *old_exc, *old_val, *old_tb;
22388     PyObject *ctx;
22389     __Pyx_PyThreadState_declare
22390 #ifdef WITH_THREAD
22391     PyGILState_STATE state;
22392     if (nogil)
22393         state = PyGILState_Ensure();
22394 #endif
22395     else state = (PyGILState_STATE)-1;
22396 #endif
22397 #endif
22398 #endif
22399     __Pyx_PyThreadState_assign
22400     __Pyx_ErrFetch(&old_exc, &old_val, &old_tb);
22401     if (full_traceback) {
22402         Py_XINCREf(old_exc);
22403         Py_XINCREf(old_val);
22404         Py_XINCREf(old_tb);
22405         __Pyx_ErrRestore(old_exc, old_val, old_tb);
22406         PyErr_PrintEx(1);
22407     }
22408     #if PY_MAJOR_VERSION < 3
22409     ctx = PyString_FromString(name);
22410     #else
22411     ctx = PyUnicode_FromString(name);
22412     #endif
22413     __Pyx_ErrRestore(old_exc, old_val, old_tb);
22414     if (!ctx) {
22415         PyErr_WriteUnraisable(Py_None);
22416     } else {
22417         PyErr_WriteUnraisable(ctx);
22418         Py_DECREF(ctx);
22419     }
22420 #ifdef WITH_THREAD
22421     if (nogil)
22422         PyGILState_Release(state);
22423 #endif
22424 }
22425
22426 /* PyDictVersioning */

```

```

22427 #if CYTHON_USE_DICT_VERSIONS && CYTHON_USE_TYPE_SLOTS
22428 static CYTHON_INLINE PY_UINT64_T __Pyx_get_tp_dict_version(PyObject *obj) {
22429     PyObject *dict = Py_TYPE(obj)->tp_dict;
22430     return likely(dict) ? __PYX_GET_DICT_VERSION(dict) : 0;
22431 }
22432 static CYTHON_INLINE PY_UINT64_T __Pyx_get_object_dict_version(PyObject *obj) {
22433     PyObject **dictptr = NULL;
22434     Py_ssize_t offset = Py_TYPE(obj)->tp_dictoffset;
22435     if (offset) {
22436 #if CYTHON_COMPILING_IN_CPYTHON
22437         dictptr = (likely(offset > 0)) ? (PyObject **) ((char *)obj + offset) :
                _PyObject_GetDictPtr(obj);
22438 #else
22439         dictptr = _PyObject_GetDictPtr(obj);
22440 #endif
22441     }
22442     return (dictptr && *dictptr) ? __PYX_GET_DICT_VERSION(*dictptr) : 0;
22443 }
22444 static CYTHON_INLINE int __Pyx_object_dict_version_matches(PyObject* obj, PY_UINT64_T tp_dict_version,
PY_UINT64_T obj_dict_version) {
22445     PyObject *dict = Py_TYPE(obj)->tp_dict;
22446     if (unlikely(!dict) || unlikely(tp_dict_version != __PYX_GET_DICT_VERSION(dict)))
22447         return 0;
22448     return obj_dict_version == __Pyx_get_object_dict_version(obj);
22449 }
22450 #endif
22451
22452 /* PyObjectCallNoArg */
22453 #if CYTHON_COMPILING_IN_CPYTHON
22454 static CYTHON_INLINE PyObject* __Pyx_PyObject_CallNoArg(PyObject *func) {
22455 #if CYTHON_FAST_PYCALL
22456     if (PyFunction_Check(func)) {
22457         return __Pyx_PyFunction_FastCall(func, NULL, 0);
22458     }
22459 #endif
22460 #ifdef __Pyx_CyFunction_USED
22461     if (likely(PyCFunction_Check(func) || __Pyx_CyFunction_Check(func)))
22462 #else
22463     if (likely(PyCFunction_Check(func)))
22464 #endif
22465     {
22466         if (likely(PyCFunction_GET_FLAGS(func) & METH_NOARGS)) {
22467             return __Pyx_PyObject_CallMethO(func, NULL);
22468         }
22469     }
22470     return __Pyx_PyObject_Call(func, __pyx_empty_tuple, NULL);
22471 }
22472 #endif
22473
22474 /* RaiseDoubleKeywords */
22475 static void __Pyx_RaiseDoubleKeywordsError(
22476     const char* func_name,
22477     PyObject* kw_name)
22478 {
22479     PyErr_Format(PyExc_TypeError,
22480         #if PY_MAJOR_VERSION >= 3
22481         "%s() got multiple values for keyword argument '%U'", func_name, kw_name);
22482         #else
22483         "%s() got multiple values for keyword argument '%s'", func_name,
22484         PyString_AsString(kw_name));
22485         #endif
22486 }
22487
22488 /* ParseKeywords */
22489 static int __Pyx_ParseOptionalKeywords(
22490     PyObject *kwds,
22491     PyObject **argnames[],
22492     PyObject *kwds2,
22493     PyObject *values[],
22494     Py_ssize_t num_pos_args,
22495     const char* function_name)
22496 {
22497     PyObject *key = 0, *value = 0;
22498     Py_ssize_t pos = 0;
22499     PyObject*** name;
22500     PyObject*** first_kw_arg = argnames + num_pos_args;
22501     while (PyDict_Next(kwds, &pos, &key, &value)) {
22502         name = first_kw_arg;
22503         while (*name && (**name != key)) name++;
22504         if (*name) {
22505             values[name-argnames] = value;
22506             continue;
22507         }
22508         name = first_kw_arg;
22509         #if PY_MAJOR_VERSION < 3
22510         if (likely(PyString_Check(key))) {
22511             while (*name) {

```

```

22512         if ((CYTHON_COMPILING_IN_PYPY || PyString_GET_SIZE(**name) == PyString_GET_SIZE(key))
22513             && _PyString_Eq(**name, key)) {
22514             values[name-argnames] = value;
22515             break;
22516         }
22517         name++;
22518     }
22519     if (*name) continue;
22520     else {
22521         PyObject*** argname = argnames;
22522         while (argname != first_kw_arg) {
22523             if (**argname == key) || (
22524                 (CYTHON_COMPILING_IN_PYPY || PyString_GET_SIZE(**argname) ==
PyString_GET_SIZE(key))
22525                 && _PyString_Eq(**argname, key))) {
22526                 goto arg_passed_twice;
22527             }
22528             argname++;
22529         }
22530     }
22531 } else
22532 #endif
22533 if (likely(PyUnicode_Check(key))) {
22534     while (*name) {
22535         int cmp = (**name == key) ? 0 :
22536             #if !CYTHON_COMPILING_IN_PYPY && PY_MAJOR_VERSION >= 3
22537             (__Pyx_PyUnicode_GET_LENGTH(**name) != __Pyx_PyUnicode_GET_LENGTH(key)) ? 1 :
22538             #endif
22539             PyUnicode_Compare(**name, key);
22540         if (cmp < 0 && unlikely(PyErr_Occurred())) goto bad;
22541         if (cmp == 0) {
22542             values[name-argnames] = value;
22543             break;
22544         }
22545         name++;
22546     }
22547     if (*name) continue;
22548     else {
22549         PyObject*** argname = argnames;
22550         while (argname != first_kw_arg) {
22551             int cmp = (**argname == key) ? 0 :
22552             #if !CYTHON_COMPILING_IN_PYPY && PY_MAJOR_VERSION >= 3
22553             (__Pyx_PyUnicode_GET_LENGTH(**argname) != __Pyx_PyUnicode_GET_LENGTH(key)) ? 1
22554             :
22555             #endif
22556             PyUnicode_Compare(**argname, key);
22557             if (cmp < 0 && unlikely(PyErr_Occurred())) goto bad;
22558             if (cmp == 0) goto arg_passed_twice;
22559             argname++;
22560         }
22561     } else
22562         goto invalid_keyword_type;
22563     if (kwds2) {
22564         if (unlikely(PyDict_SetItem(kwds2, key, value))) goto bad;
22565     } else {
22566         goto invalid_keyword;
22567     }
22568 }
22569 return 0;
22570 arg_passed_twice:
22571     __Pyx_RaiseDoubleKeywordsError(function_name, key);
22572     goto bad;
22573 invalid_keyword_type:
22574     PyErr_Format(PyExc_TypeError,
22575         "%.200s() keywords must be strings", function_name);
22576     goto bad;
22577 invalid_keyword:
22578     PyErr_Format(PyExc_TypeError,
22579         #if PY_MAJOR_VERSION < 3
22580         "%.200s() got an unexpected keyword argument '%.200s'",
22581         function_name, PyString_AsString(key));
22582         #else
22583         "%s() got an unexpected keyword argument '%U'",
22584         function_name, key);
22585         #endif
22586 bad:
22587     return -1;
22588 }
22589
22590 /* RaiseArgTupleInvalid */
22591 static void __Pyx_RaiseArgtupleInvalid(
22592     const char* func_name,
22593     int exact,
22594     Py_ssize_t num_min,
22595     Py_ssize_t num_max,
22596     Py_ssize_t num_found)

```

```

22597 {
22598     Py_ssize_t num_expected;
22599     const char *more_or_less;
22600     if (num_found < num_min) {
22601         num_expected = num_min;
22602         more_or_less = "at least";
22603     } else {
22604         num_expected = num_max;
22605         more_or_less = "at most";
22606     }
22607     if (exact) {
22608         more_or_less = "exactly";
22609     }
22610     PyErr_Format(PyExc_TypeError,
22611                 "%.200s() takes %.8s %" CYTHON_FORMAT_SSIZE_T "d positional argument%.1s (%"
CYTHON_FORMAT_SSIZE_T "d given)",
22612                 func_name, more_or_less, num_expected,
22613                 (num_expected == 1) ? "" : "s", num_found);
22614 }
22615
22616 /* GetModuleGlobalName */
22617 #if CYTHON_USE_DICT_VERSIONS
22618 static PyObject *__Pyx_GetModuleGlobalName(PyObject *name, PY_UINT64_T *dict_version, PyObject
**dict_cached_value)
22619 #else
22620 static CYTHON_INLINE PyObject *__Pyx_GetModuleGlobalName(PyObject *name)
22621 #endif
22622 {
22623     PyObject *result;
22624     #if !CYTHON_AVOID_BORROWED_REFS
22625     #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x030500A1
22626         result = _PyDict_GetItem_KnownHash(__pyx_d, name, ((PyASCIIObject *) name)->hash);
22627         __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22628         if (likely(result)) {
22629             return __Pyx_NewRef(result);
22630         } else if (unlikely(PyErr_Occurred())) {
22631             return NULL;
22632         }
22633     #else
22634         result = PyDict_GetItem(__pyx_d, name);
22635         __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22636         if (likely(result)) {
22637             return __Pyx_NewRef(result);
22638         }
22639     #endif
22640     #else
22641         result = PyObject_GetItem(__pyx_d, name);
22642         __PYX_UPDATE_DICT_CACHE(__pyx_d, result, *dict_cached_value, *dict_version)
22643         if (likely(result)) {
22644             return __Pyx_NewRef(result);
22645         }
22646         PyErr_Clear();
22647     #endif
22648     return __Pyx_GetBuiltinName(name);
22649 }
22650
22651 /* GetTopmostException */
22652 #if CYTHON_USE_EXC_INFO_STACK
22653 static _PyErr_StackItem *
22654 __Pyx_PyErr_GetTopmostException(PyThreadState *tstate)
22655 {
22656     _PyErr_StackItem *exc_info = tstate->exc_info;
22657     while ((exc_info->exc_type == NULL || exc_info->exc_type == Py_None) &&
22658           exc_info->previous_item != NULL)
22659     {
22660         exc_info = exc_info->previous_item;
22661     }
22662     return exc_info;
22663 }
22664 #endif
22665
22666 /* SaveResetException */
22667 #if CYTHON_FAST_THREAD_STATE
22668 static CYTHON_INLINE void __Pyx_ExceptionSave(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb) {
22669     #if CYTHON_USE_EXC_INFO_STACK
22670         _PyErr_StackItem *exc_info = __Pyx_PyErr_GetTopmostException(tstate);
22671         *type = exc_info->exc_type;
22672         *value = exc_info->exc_value;
22673         *tb = exc_info->exc_traceback;
22674     #else
22675         *type = tstate->exc_type;
22676         *value = tstate->exc_value;
22677         *tb = tstate->exc_traceback;
22678     #endif
22679     Py_XINCREF(*type);
22680     Py_XINCREF(*value);

```

```

22681     Py_XINCREf(*tb);
22682 }
22683 static CYTHON_INLINE void __Pyx__ExceptionReset(PyThreadState *tstate, PyObject *type, PyObject
*value, PyObject *tb) {
22684     PyObject *tmp_type, *tmp_value, *tmp_tb;
22685     #if CYTHON_USE_EXC_INFO_STACK
22686     _PyErr_StackItem *exc_info = tstate->exc_info;
22687     tmp_type = exc_info->exc_type;
22688     tmp_value = exc_info->exc_value;
22689     tmp_tb = exc_info->exc_traceback;
22690     exc_info->exc_type = type;
22691     exc_info->exc_value = value;
22692     exc_info->exc_traceback = tb;
22693     #else
22694     tmp_type = tstate->exc_type;
22695     tmp_value = tstate->exc_value;
22696     tmp_tb = tstate->exc_traceback;
22697     tstate->exc_type = type;
22698     tstate->exc_value = value;
22699     tstate->exc_traceback = tb;
22700     #endif
22701     Py_XDECREF(tmp_type);
22702     Py_XDECREF(tmp_value);
22703     Py_XDECREF(tmp_tb);
22704 }
22705 #endif
22706
22707 /* PyErrExceptionMatches */
22708 #if CYTHON_FAST_THREAD_STATE
22709 static int __Pyx_PyErr_ExceptionMatchesTuple(PyObject *exc_type, PyObject *tuple) {
22710     Py_ssize_t i, n;
22711     n = PyTuple_GET_SIZE(tuple);
22712     #if PY_MAJOR_VERSION >= 3
22713     for (i=0; i<n; i++) {
22714         if (exc_type == PyTuple_GET_ITEM(tuple, i)) return 1;
22715     }
22716     #endif
22717     for (i=0; i<n; i++) {
22718         if (__Pyx_PyErr_GivenExceptionMatches(exc_type, PyTuple_GET_ITEM(tuple, i))) return 1;
22719     }
22720     return 0;
22721 }
22722 static CYTHON_INLINE int __Pyx_PyErr_ExceptionMatchesInState(PyThreadState* tstate, PyObject* err) {
22723     PyObject *exc_type = tstate->curexc_type;
22724     if (exc_type == err) return 1;
22725     if (unlikely(!exc_type)) return 0;
22726     if (unlikely(PyTuple_Check(err)))
22727         return __Pyx_PyErr_ExceptionMatchesTuple(exc_type, err);
22728     return __Pyx_PyErr_GivenExceptionMatches(exc_type, err);
22729 }
22730 #endif
22731
22732 /* GetException */
22733 #if CYTHON_FAST_THREAD_STATE
22734 static int __Pyx_GetException(PyThreadState *tstate, PyObject **type, PyObject **value, PyObject
**tb)
22735 #else
22736 static int __Pyx_GetException(PyObject **type, PyObject **value, PyObject **tb)
22737 #endif
22738 {
22739     PyObject *local_type, *local_value, *local_tb;
22740     #if CYTHON_FAST_THREAD_STATE
22741     PyObject *tmp_type, *tmp_value, *tmp_tb;
22742     local_type = tstate->curexc_type;
22743     local_value = tstate->curexc_value;
22744     local_tb = tstate->curexc_traceback;
22745     tstate->curexc_type = 0;
22746     tstate->curexc_value = 0;
22747     tstate->curexc_traceback = 0;
22748     #else
22749     PyErr_Fetch(&local_type, &local_value, &local_tb);
22750     #endif
22751     PyErr_NormalizeException(&local_type, &local_value, &local_tb);
22752     #if CYTHON_FAST_THREAD_STATE
22753     if (unlikely(tstate->curexc_type))
22754     #else
22755     if (unlikely(PyErr_Occurred()))
22756     #endif
22757         goto bad;
22758     #if PY_MAJOR_VERSION >= 3
22759     if (local_tb) {
22760         if (unlikely(PyException_SetTraceback(local_value, local_tb) < 0))
22761             goto bad;
22762     }
22763     #endif
22764     Py_XINCREf(local_tb);
22765     Py_XINCREf(local_type);

```

```

22766     Py_XINCREf(local_value);
22767     *type = local_type;
22768     *value = local_value;
22769     *tb = local_tb;
22770 #if CYTHON_FAST_THREAD_STATE
22771     #if CYTHON_USE_EXC_INFO_STACK
22772     {
22773         _PyErr_StackItem *exc_info = tstate->exc_info;
22774         tmp_type = exc_info->exc_type;
22775         tmp_value = exc_info->exc_value;
22776         tmp_tb = exc_info->exc_traceback;
22777         exc_info->exc_type = local_type;
22778         exc_info->exc_value = local_value;
22779         exc_info->exc_traceback = local_tb;
22780     }
22781     #else
22782     tmp_type = tstate->exc_type;
22783     tmp_value = tstate->exc_value;
22784     tmp_tb = tstate->exc_traceback;
22785     tstate->exc_type = local_type;
22786     tstate->exc_value = local_value;
22787     tstate->exc_traceback = local_tb;
22788     #endif
22789     Py_XDECREF(tmp_type);
22790     Py_XDECREF(tmp_value);
22791     Py_XDECREF(tmp_tb);
22792 #else
22793     PyErr_SetExcInfo(local_type, local_value, local_tb);
22794 #endif
22795     return 0;
22796 bad:
22797     *type = 0;
22798     *value = 0;
22799     *tb = 0;
22800     Py_XDECREF(local_type);
22801     Py_XDECREF(local_value);
22802     Py_XDECREF(local_tb);
22803     return -1;
22804 }
22805
22806 /* RaiseException */
22807 #if PY_MAJOR_VERSION < 3
22808 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb,
22809                      CYTHON_UNUSED PyObject *cause) {
22810     __Pyx_PyThreadState_declare
22811     Py_XINCREf(type);
22812     if (!value || value == Py_None)
22813         value = NULL;
22814     else
22815         Py_INCREF(value);
22816     if (!tb || tb == Py_None)
22817         tb = NULL;
22818     else {
22819         Py_INCREF(tb);
22820         if (!PyTraceBack_Check(tb)) {
22821             PyErr_SetString(PyExc_TypeError,
22822                           "raise: arg 3 must be a traceback or None");
22823             goto raise_error;
22824         }
22825     }
22826     if (PyType_Check(type)) {
22827 #if CYTHON_COMPILING_IN_PYPY
22828         if (!value) {
22829             Py_INCREF(Py_None);
22830             value = Py_None;
22831         }
22832 #endif
22833         PyErr_NormalizeException(&type, &value, &tb);
22834     } else {
22835         if (value) {
22836             PyErr_SetString(PyExc_TypeError,
22837                           "instance exception may not have a separate value");
22838             goto raise_error;
22839         }
22840         value = type;
22841         type = (PyObject*) Py_TYPE(type);
22842         Py_INCREF(type);
22843         if (!PyType_IsSubtype((PyTypeObject *)type, (PyTypeObject *)PyExc_BaseException)) {
22844             PyErr_SetString(PyExc_TypeError,
22845                           "raise: exception class must be a subclass of BaseException");
22846             goto raise_error;
22847         }
22848     }
22849     __Pyx_PyThreadState_assign
22850     __Pyx_ErrRestore(type, value, tb);
22851     return;
22852 raise_error:

```



```

22853     Py_XDECREF(value);
22854     Py_XDECREF(type);
22855     Py_XDECREF(tb);
22856     return;
22857 }
22858 #else
22859 static void __Pyx_Raise(PyObject *type, PyObject *value, PyObject *tb, PyObject *cause) {
22860     PyObject* owned_instance = NULL;
22861     if (tb == Py_None) {
22862         tb = 0;
22863     } else if (tb && !PyTraceBack_Check(tb)) {
22864         PyErr_SetString(PyExc_TypeError,
22865             "raise: arg 3 must be a traceback or None");
22866         goto bad;
22867     }
22868     if (value == Py_None)
22869         value = 0;
22870     if (PyExceptionInstance_Check(type)) {
22871         if (value) {
22872             PyErr_SetString(PyExc_TypeError,
22873                 "instance exception may not have a separate value");
22874             goto bad;
22875         }
22876         value = type;
22877         type = (PyObject*) Py_TYPE(value);
22878     } else if (PyExceptionClass_Check(type)) {
22879         PyObject *instance_class = NULL;
22880         if (value && PyExceptionInstance_Check(value)) {
22881             instance_class = (PyObject*) Py_TYPE(value);
22882             if (instance_class != type) {
22883                 int is_subclass = PyObject_IsSubclass(instance_class, type);
22884                 if (!is_subclass) {
22885                     instance_class = NULL;
22886                 } else if (unlikely(is_subclass == -1)) {
22887                     goto bad;
22888                 } else {
22889                     type = instance_class;
22890                 }
22891             }
22892         }
22893         if (!instance_class) {
22894             PyObject *args;
22895             if (!value)
22896                 args = PyTuple_New(0);
22897             else if (PyTuple_Check(value)) {
22898                 Py_INCREF(value);
22899                 args = value;
22900             } else
22901                 args = PyTuple_Pack(1, value);
22902             if (!args)
22903                 goto bad;
22904             owned_instance = PyObject_Call(type, args, NULL);
22905             Py_DECREF(args);
22906             if (!owned_instance)
22907                 goto bad;
22908             value = owned_instance;
22909             if (!PyExceptionInstance_Check(value)) {
22910                 PyErr_Format(PyExc_TypeError,
22911                     "calling %R should have returned an instance of "
22912                     "BaseException, not %R",
22913                     type, Py_TYPE(value));
22914                 goto bad;
22915             }
22916         }
22917     } else {
22918         PyErr_SetString(PyExc_TypeError,
22919             "raise: exception class must be a subclass of BaseException");
22920         goto bad;
22921     }
22922     if (cause) {
22923         PyObject *fixed_cause;
22924         if (cause == Py_None) {
22925             fixed_cause = NULL;
22926         } else if (PyExceptionClass_Check(cause)) {
22927             fixed_cause = PyObject_CallObject(cause, NULL);
22928             if (fixed_cause == NULL)
22929                 goto bad;
22930         } else if (PyExceptionInstance_Check(cause)) {
22931             fixed_cause = cause;
22932             Py_INCREF(fixed_cause);
22933         } else {
22934             PyErr_SetString(PyExc_TypeError,
22935                 "exception causes must derive from "
22936                 "BaseException");
22937             goto bad;
22938         }
22939         PyException_SetCause(value, fixed_cause);

```

```

22940     }
22941     PyErr_SetObject(type, value);
22942     if (tb) {
22943 #if CYTHON_COMPILING_IN_PYPY
22944         PyObject *tmp_type, *tmp_value, *tmp_tb;
22945         PyErr_Fetch(&tmp_type, &tmp_value, &tmp_tb);
22946         Py_INCREF(tb);
22947         PyErr_Restore(tmp_type, tmp_value, tb);
22948         Py_XDECREF(tmp_tb);
22949 #else
22950         PyThreadState *tstate = __Pyx_PyThreadState_Current;
22951         PyObject* tmp_tb = tstate->curexc_traceback;
22952         if (tb != tmp_tb) {
22953             Py_INCREF(tb);
22954             tstate->curexc_traceback = tb;
22955             Py_XDECREF(tmp_tb);
22956         }
22957 #endif
22958     }
22959 bad:
22960     Py_XDECREF(owned_instance);
22961     return;
22962 }
22963 #endif
22964
22965 /* PyObjectCall2Args */
22966 static CYTHON_UNUSED PyObject* __Pyx_PyObject_Call2Args(PyObject* function, PyObject* arg1, PyObject*
arg2) {
22967     PyObject *args, *result = NULL;
22968     #if CYTHON_FAST_PYCALL
22969     if (PyFunction_Check(function)) {
22970         PyObject *args[2] = {arg1, arg2};
22971         return __Pyx_PyFunction_FastCall(function, args, 2);
22972     }
22973     #endif
22974     #if CYTHON_FAST_PYCCALL
22975     if (__Pyx_PyFastCFunction_Check(function)) {
22976         PyObject *args[2] = {arg1, arg2};
22977         return __Pyx_PyCFunction_FastCall(function, args, 2);
22978     }
22979     #endif
22980     args = PyTuple_New(2);
22981     if (unlikely(!args)) goto done;
22982     Py_INCREF(arg1);
22983     PyTuple_SET_ITEM(args, 0, arg1);
22984     Py_INCREF(arg2);
22985     PyTuple_SET_ITEM(args, 1, arg2);
22986     Py_INCREF(function);
22987     result = __Pyx_PyObject_Call(function, args, NULL);
22988     Py_DECREF(args);
22989     Py_DECREF(function);
22990 done:
22991     return result;
22992 }
22993
22994 /* PyIntBinop */
22995 #if !CYTHON_COMPILING_IN_PYPY
22996 static PyObject* __Pyx_PyInt_AddObjC(PyObject *op1, PyObject *op2, CYTHON_UNUSED long intval, int
inplace, int zerodivision_check) {
22997     (void)inplace;
22998     (void)zerodivision_check;
22999     #if PY_MAJOR_VERSION < 3
23000     if (likely(PyInt_CheckExact(op1))) {
23001         const long b = intval;
23002         long x;
23003         long a = PyInt_AS_LONG(op1);
23004         x = (long)((unsigned long)a + b);
23005         if (likely((x^a) >= 0 || (x^b) >= 0))
23006             return PyInt_FromLong(x);
23007         return PyLong_Type.tp_as_number->nb_add(op1, op2);
23008     }
23009     #endif
23010     #if CYTHON_USE_PYLONG_INTERNALS
23011     if (likely(PyLong_CheckExact(op1))) {
23012         const long b = intval;
23013         long a, x;
23014 #ifdef HAVE_LONG_LONG
23015         const PY_LONG_LONG llb = intval;
23016         PY_LONG_LONG lla, llx;
23017 #endif
23018         const digit* digits = ((PyLongObject*)op1)->ob_digit;
23019         const Py_ssize_t size = Py_SIZE(op1);
23020         if (likely(__Pyx_sst_abs(size) <= 1)) {
23021             a = likely(size) ? digits[0] : 0;
23022             if (size == -1) a = -a;
23023         } else {
23024             switch (size) {

```

```

23025         case -2:
23026             if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
23027                 a = -(long) (((((unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]));
23028                 break;
23029             #ifdef HAVE_LONG_LONG
23030             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 2 * PyLong_SHIFT) {
23031                 lla = -(PY_LONG_LONG) (((((unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) |
(unsigned PY_LONG_LONG)digits[0]));
23032                 goto long_long;
23033             #endif
23034             }
23035             CYTHON_FALLTHROUGH;
23036         case 2:
23037             if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
23038                 a = (long) (((((unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]));
23039                 break;
23040             #ifdef HAVE_LONG_LONG
23041             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 2 * PyLong_SHIFT) {
23042                 lla = (PY_LONG_LONG) (((((unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) |
(unsigned PY_LONG_LONG)digits[0]));
23043                 goto long_long;
23044             #endif
23045             }
23046             CYTHON_FALLTHROUGH;
23047         case -3:
23048             if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
23049                 a = -(long) ((((((unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]));
23050                 break;
23051             #ifdef HAVE_LONG_LONG
23052             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 3 * PyLong_SHIFT) {
23053                 lla = -(PY_LONG_LONG) ((((((unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT)
| (unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]));
23054                 goto long_long;
23055             #endif
23056             }
23057             CYTHON_FALLTHROUGH;
23058         case 3:
23059             if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
23060                 a = (long) ((((((unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]));
23061                 break;
23062             #ifdef HAVE_LONG_LONG
23063             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 3 * PyLong_SHIFT) {
23064                 lla = (PY_LONG_LONG) ((((((unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT) |
(unsigned PY_LONG_LONG)digits[1]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]));
23065                 goto long_long;
23066             #endif
23067             }
23068             CYTHON_FALLTHROUGH;
23069         case -4:
23070             if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
23071                 a = -(long) (((((((((unsigned long)digits[3]) « PyLong_SHIFT) | (unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]));
23072                 break;
23073             #ifdef HAVE_LONG_LONG
23074             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 4 * PyLong_SHIFT) {
23075                 lla = -(PY_LONG_LONG) (((((((((unsigned PY_LONG_LONG)digits[3]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[1]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]));
23076                 goto long_long;
23077             #endif
23078             }
23079             CYTHON_FALLTHROUGH;
23080         case 4:
23081             if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
23082                 a = (long) (((((((((unsigned long)digits[3]) « PyLong_SHIFT) | (unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0]));
23083                 break;
23084             #ifdef HAVE_LONG_LONG
23085             } else if (8 * sizeof(PY_LONG_LONG) - 1 > 4 * PyLong_SHIFT) {
23086                 lla = (PY_LONG_LONG) (((((((((unsigned PY_LONG_LONG)digits[3]) « PyLong_SHIFT)
| (unsigned PY_LONG_LONG)digits[2]) « PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[1]) «
PyLong_SHIFT) | (unsigned PY_LONG_LONG)digits[0]));
23087                 goto long_long;
23088             #endif
23089             }
23090             CYTHON_FALLTHROUGH;
23091         default: return PyLong_Type.tp_as_number->nb_add(op1, op2);
23092     }
23093 }
23094     x = a + b;
23095     return PyLong_FromLong(x);

```

```

23096 #ifdef HAVE_LONG_LONG
23097     long_long:
23098         llx = lla + llb;
23099         return PyLong_FromLongLong(llx);
23100 #endif
23101
23102
23103 }
23104 #endif
23105 if (PyFloat_CheckExact(op1)) {
23106     const long b = intval;
23107     double a = PyFloat_AS_DOUBLE(op1);
23108     double result;
23109     PyFPE_START_PROTECT("add", return NULL)
23110     result = ((double)a) + (double)b;
23111     PyFPE_END_PROTECT(result)
23112     return PyFloat_FromDouble(result);
23113 }
23114 return (inplace ? PyNumber_InPlaceAdd : PyNumber_Add)(op1, op2);
23115 }
23116 #endif
23117
23118 /* decode_c_bytes */
23119 static CYTHON_INLINE PyObject* __Pyx_decode_c_bytes(
23120     const char* cstring, Py_ssize_t length, Py_ssize_t start, Py_ssize_t stop,
23121     const char* encoding, const char* errors,
23122     PyObject* (*decode_func)(const char *s, Py_ssize_t size, const char *errors)) {
23123     if (unlikely((start < 0) | (stop < 0))) {
23124         if (start < 0) {
23125             start += length;
23126             if (start < 0)
23127                 start = 0;
23128         }
23129         if (stop < 0)
23130             stop += length;
23131     }
23132     if (stop > length)
23133         stop = length;
23134     if (unlikely(stop <= start))
23135         return __Pyx_NewRef(__pyx_empty_unicode);
23136     length = stop - start;
23137     cstring += start;
23138     if (decode_func) {
23139         return decode_func(cstring, length, errors);
23140     } else {
23141         return PyUnicode_Decode(cstring, length, encoding, errors);
23142     }
23143 }
23144
23145 /* SwapException */
23146 #if CYTHON_FAST_THREAD_STATE
23147 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyThreadState *tstate, PyObject **type, PyObject
**value, PyObject **tb) {
23148     PyObject *tmp_type, *tmp_value, *tmp_tb;
23149     #if CYTHON_USE_EXC_INFO_STACK
23150     _PyErr_StackItem *exc_info = tstate->exc_info;
23151     tmp_type = exc_info->exc_type;
23152     tmp_value = exc_info->exc_value;
23153     tmp_tb = exc_info->exc_traceback;
23154     exc_info->exc_type = *type;
23155     exc_info->exc_value = *value;
23156     exc_info->exc_traceback = *tb;
23157     #else
23158     tmp_type = tstate->exc_type;
23159     tmp_value = tstate->exc_value;
23160     tmp_tb = tstate->exc_traceback;
23161     tstate->exc_type = *type;
23162     tstate->exc_value = *value;
23163     tstate->exc_traceback = *tb;
23164     #endif
23165     *type = tmp_type;
23166     *value = tmp_value;
23167     *tb = tmp_tb;
23168 }
23169 #else
23170 static CYTHON_INLINE void __Pyx_ExceptionSwap(PyObject **type, PyObject **value, PyObject **tb) {
23171     PyObject *tmp_type, *tmp_value, *tmp_tb;
23172     PyErr_GetExcInfo(&tmp_type, &tmp_value, &tmp_tb);
23173     PyErr_SetExcInfo(*type, *value, *tb);
23174     *type = tmp_type;
23175     *value = tmp_value;
23176     *tb = tmp_tb;
23177 }
23178 #endif
23179
23180 /* SetItemInt */
23181 static int __Pyx_SetItemInt_Generic(PyObject *o, PyObject *j, PyObject *v) {

```

```

23182     int r;
23183     if (!j) return -1;
23184     r = PyObject_SetItem(o, j, v);
23185     Py_DECREF(j);
23186     return r;
23187 }
23188 static CYTHON_INLINE int __Pyx_SetItemInt_Fast(PyObject *o, Py_ssize_t i, PyObject *v, int is_list,
23189   CYTHON_NCP_UNUSED int wraparound, CYTHON_NCP_UNUSED int
23190   boundscheck) {
23191     #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS && CYTHON_USE_TYPE_SLOTS
23192     if (is_list || PyList_CheckExact(o)) {
23193         Py_ssize_t n = (!wraparound) ? i : ((likely(i >= 0)) ? i : i + PyList_GET_SIZE(o));
23194         if ((!boundscheck) || likely(__Pyx_is_valid_index(n, PyList_GET_SIZE(o)))) {
23195             PyObject* old = PyList_GET_ITEM(o, n);
23196             Py_INCREF(v);
23197             PyList_SET_ITEM(o, n, v);
23198             Py_DECREF(old);
23199             return 1;
23200         } else {
23201             PySequenceMethods *m = Py_TYPE(o)->tp_as_sequence;
23202             if (likely(m && m->sq_ass_item)) {
23203                 if (wraparound && unlikely(i < 0) && likely(m->sq_length)) {
23204                     Py_ssize_t l = m->sq_length(o);
23205                     if (likely(l >= 0)) {
23206                         i += l;
23207                     } else {
23208                         if (!PyErr_ExceptionMatches(PyExc_OverflowError))
23209                             return -1;
23210                         PyErr_Clear();
23211                     }
23212                 }
23213                 return m->sq_ass_item(o, i, v);
23214             }
23215         }
23216     #else
23217     #if CYTHON_COMPILING_IN_PYPY
23218     if (is_list || (PySequence_Check(o) && !PyDict_Check(o)))
23219     #else
23220     if (is_list || PySequence_Check(o))
23221     #endif
23222     {
23223         return PySequence_SetItem(o, i, v);
23224     }
23225     #endif
23226     return __Pyx_SetItemInt_Generic(o, PyInt_FromSsize_t(i), v);
23227 }
23228
23229 /* ArgTypeTest */
23230 static int __Pyx_ArgTypeTest(PyObject *obj, PyTypeObject *type, const char *name, int exact)
23231 {
23232     if (unlikely(!type)) {
23233         PyErr_SetString(PyExc_SystemError, "Missing type object");
23234         return 0;
23235     }
23236     else if (exact) {
23237         #if PY_MAJOR_VERSION == 2
23238         if ((type == &PyBaseString_Type) && likely(__Pyx_PyBaseString_CheckExact(obj))) return 1;
23239         #endif
23240     }
23241     else {
23242         if (likely(__Pyx_TypeCheck(obj, type))) return 1;
23243     }
23244     PyErr_Format(PyExc_TypeError,
23245                 "Argument '%.200s' has incorrect type (expected %.200s, got %.200s)",
23246                 name, type->tp_name, Py_TYPE(obj)->tp_name);
23247     return 0;
23248 }
23249
23250 /* Import */
23251 static PyObject *__Pyx_Import(PyObject *name, PyObject *from_list, int level) {
23252     PyObject *empty_list = 0;
23253     PyObject *module = 0;
23254     PyObject *global_dict = 0;
23255     PyObject *empty_dict = 0;
23256     PyObject *list;
23257     #if PY_MAJOR_VERSION < 3
23258     PyObject *py_import;
23259     py_import = __Pyx_PyObject_GetAttrStr(__pyx_b, __pyx_n_s_import);
23260     if (!py_import)
23261         goto bad;
23262     #endif
23263     if (from_list)
23264         list = from_list;
23265     else {
23266         empty_list = PyList_New(0);
23267         if (!empty_list)

```

```

23268         goto bad;
23269         list = empty_list;
23270     }
23271     global_dict = PyModule_GetDict(__pyx_m);
23272     if (!global_dict)
23273         goto bad;
23274     empty_dict = PyDict_New();
23275     if (!empty_dict)
23276         goto bad;
23277     {
23278         #if PY_MAJOR_VERSION >= 3
23279         if (level == -1) {
23280             if ((1) && (strchr(__Pyx_MODULE_NAME, '.'))) {
23281                 module = PyImport_ImportModuleLevelObject(
23282                     name, global_dict, empty_dict, list, 1);
23283                 if (!module) {
23284                     if (!PyErr_ExceptionMatches(PyExc_ImportError))
23285                         goto bad;
23286                     PyErr_Clear();
23287                 }
23288             }
23289             level = 0;
23290         }
23291         #endif
23292         if (!module) {
23293             #if PY_MAJOR_VERSION < 3
23294             PyObject *py_level = PyInt_FromLong(level);
23295             if (!py_level)
23296                 goto bad;
23297             module = PyObject_CallFunctionObjArgs(py_import,
23298                 name, global_dict, empty_dict, list, py_level, (PyObject *)NULL);
23299             Py_DECREF(py_level);
23300             #else
23301             module = PyImport_ImportModuleLevelObject(
23302                 name, global_dict, empty_dict, list, level);
23303             #endif
23304         }
23305     }
23306 bad:
23307     #if PY_MAJOR_VERSION < 3
23308     Py_XDECREF(py_import);
23309     #endif
23310     Py_XDECREF(empty_list);
23311     Py_XDECREF(empty_dict);
23312     return module;
23313 }
23314
23315 /* PyObject_GenericGetAttrNoDict */
23316 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
23317 static PyObject * __Pyx_RaiseGenericGetAttributeError(PyTypeObject *tp, PyObject *attr_name) {
23318     PyErr_Format(PyExc_AttributeError,
23319         #if PY_MAJOR_VERSION >= 3
23320         "%50s' object has no attribute '%U'",
23321         tp->tp_name, attr_name);
23322     #else
23323         "%50s' object has no attribute '%.400s'",
23324         tp->tp_name, PyString_AS_STRING(attr_name));
23325     #endif
23326     return NULL;
23327 }
23328 static CYTHON_INLINE PyObject* __Pyx_PyObject_GenericGetAttrNoDict(PyObject* obj, PyObject* attr_name)
23329 {
23330     PyObject *descr;
23331     PyTypeObject *tp = Py_TYPE(obj);
23332     if (unlikely(!PyString_Check(attr_name))) {
23333         return PyObject_GenericGetAttr(obj, attr_name);
23334     }
23335     assert(!tp->tp_dictoffset);
23336     descr = _PyType_Lookup(tp, attr_name);
23337     if (unlikely(!descr)) {
23338         return __Pyx_RaiseGenericGetAttributeError(tp, attr_name);
23339     }
23340     Py_INCREF(descr);
23341     #if PY_MAJOR_VERSION < 3
23342     if (likely(PyType_HasFeature(Py_TYPE(descr), Py_TPFLAGS_HAVE_CLASS)))
23343     #endif
23344     {
23345         descrgetfunc f = Py_TYPE(descr)->tp_descr_get;
23346         if (unlikely(!f)) {
23347             PyObject *res = f(descr, obj, (PyObject *)tp);
23348             Py_DECREF(descr);
23349             return res;
23350         }
23351     }
23352     return descr;
23353 }
23354 #endif

```

```

23354
23355 /* PyObject_GenericGetAttr */
23356 #if CYTHON_USE_TYPE_SLOTS && CYTHON_USE_PYTYPE_LOOKUP && PY_VERSION_HEX < 0x03070000
23357 static PyObject* __Pyx_PyObject_GenericGetAttr(PyObject* obj, PyObject* attr_name) {
23358     if (unlikely(Py_TYPE(obj)->tp_dictoffset)) {
23359         return PyObject_GenericGetAttr(obj, attr_name);
23360     }
23361     return __Pyx_PyObject_GenericGetAttrNoDict(obj, attr_name);
23362 }
23363 #endif
23364
23365 /* SetVTable */
23366 static int __Pyx_SetVtable(PyObject *dict, void *vtable) {
23367     #if PY_VERSION_HEX >= 0x02070000
23368         PyObject *ob = PyCapsule_New(vtable, 0, 0);
23369     #else
23370         PyObject *ob = PyCObject_FromVoidPtr(vtable, 0);
23371     #endif
23372     if (!ob)
23373         goto bad;
23374     if (PyDict_SetItem(dict, __pyx_n_s_pyx_vtable, ob) < 0)
23375         goto bad;
23376     Py_DECREF(ob);
23377     return 0;
23378 bad:
23379     Py_XDECREF(ob);
23380     return -1;
23381 }
23382
23383 /* PyObject_GetAttrStrNoError */
23384 static void __Pyx_PyObject_GetAttrStr_ClearAttributeError(void) {
23385     __Pyx_PyThreadState_declare
23386     __Pyx_PyThreadState_assign
23387     if (likely(__Pyx_PyErr_ExceptionMatches(PyExc_AttributeError)))
23388         __Pyx_PyErr_Clear();
23389 }
23390 static CYTHON_INLINE PyObject* __Pyx_PyObject_GetAttrStrNoError(PyObject* obj, PyObject* attr_name) {
23391     PyObject *result;
23392     #if CYTHON_COMPILING_IN_CPYTHON && CYTHON_USE_TYPE_SLOTS && PY_VERSION_HEX >= 0x030700B1
23393     PyTypeObject* tp = Py_TYPE(obj);
23394     if (likely(tp->tp_getattro == PyObject_GenericGetAttr)) {
23395         return PyObject_GenericGetAttrWithDict(obj, attr_name, NULL, 1);
23396     }
23397 #endif
23398     result = __Pyx_PyObject_GetAttrStr(obj, attr_name);
23399     if (unlikely(!result)) {
23400         __Pyx_PyObject_GetAttrStr_ClearAttributeError();
23401     }
23402     return result;
23403 }
23404
23405 /* SetupReduce */
23406 static int __Pyx_setup_reduce_is_named(PyObject* meth, PyObject* name) {
23407     int ret;
23408     PyObject *name_attr;
23409     name_attr = __Pyx_PyObject_GetAttrStr(meth, __pyx_n_s_name);
23410     if (likely(name_attr)) {
23411         ret = PyObject_RichCompareBool(name_attr, name, Py_EQ);
23412     } else {
23413         ret = -1;
23414     }
23415     if (unlikely(ret < 0)) {
23416         PyErr_Clear();
23417         ret = 0;
23418     }
23419     Py_XDECREF(name_attr);
23420     return ret;
23421 }
23422 static int __Pyx_setup_reduce(PyObject* type_obj) {
23423     int ret = 0;
23424     PyObject *object_reduce = NULL;
23425     PyObject *object_reduce_ex = NULL;
23426     PyObject *reduce = NULL;
23427     PyObject *reduce_ex = NULL;
23428     PyObject *reduce_cython = NULL;
23429     PyObject *setstate = NULL;
23430     PyObject *setstate_cython = NULL;
23431     #if CYTHON_USE_PYTYPE_LOOKUP
23432     if (_PyType_Lookup((PyTypeObject*)type_obj, __pyx_n_s_getstate)) goto __PYX_GOOD;
23433     #else
23434     if (PyObject_HasAttr(type_obj, __pyx_n_s_getstate)) goto __PYX_GOOD;
23435     #endif
23436     #if CYTHON_USE_PYTYPE_LOOKUP
23437     object_reduce_ex = _PyType_Lookup(&PyBaseObject_Type, __pyx_n_s_reduce_ex); if (!object_reduce_ex)
23438         goto __PYX_BAD;
23439     #else
23439     object_reduce_ex = __Pyx_PyObject_GetAttrStr((PyObject*)&PyBaseObject_Type, __pyx_n_s_reduce_ex);

```

```

    if (!object_reduce_ex) goto __PYX_BAD;
23440 #endif
23441     reduce_ex = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_reduce_ex); if (unlikely(!reduce_ex))
    goto __PYX_BAD;
23442     if (reduce_ex == object_reduce_ex) {
23443     #if CYTHON_USE_PYTYPE_LOOKUP
23444         object_reduce = _PyType_Lookup(&PyBaseObject_Type, __pyx_n_s_reduce); if (!object_reduce) goto
        __PYX_BAD;
23445     #else
23446         object_reduce = __Pyx_PyObject_GetAttrStr((PyObject*)&PyBaseObject_Type, __pyx_n_s_reduce); if
        (!object_reduce) goto __PYX_BAD;
23447     #endif
23448         reduce = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_reduce); if (unlikely(!reduce)) goto
        __PYX_BAD;
23449         if (reduce == object_reduce || __Pyx_setup_reduce_is_named(reduce, __pyx_n_s_reduce_cython)) {
23450             reduce_cython = __Pyx_PyObject_GetAttrStrNoError(type_obj, __pyx_n_s_reduce_cython);
23451             if (likely(reduce_cython)) {
23452                 ret = PyDict_SetItem(((PyTypeObject*)type_obj)->tp_dict, __pyx_n_s_reduce,
reduce_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23453                 ret = PyDict_DelItem(((PyTypeObject*)type_obj)->tp_dict, __pyx_n_s_reduce_cython); if
        (unlikely(ret < 0)) goto __PYX_BAD;
23454             } else if (reduce == object_reduce || PyErr_Occurred()) {
23455                 goto __PYX_BAD;
23456             }
23457             setstate = __Pyx_PyObject_GetAttrStr(type_obj, __pyx_n_s_setstate);
23458             if (!setstate) PyErr_Clear();
23459             if (!setstate || __Pyx_setup_reduce_is_named(setstate, __pyx_n_s_setstate_cython)) {
23460                 setstate_cython = __Pyx_PyObject_GetAttrStrNoError(type_obj,
        __pyx_n_s_setstate_cython);
23461                 if (likely(setstate_cython)) {
23462                     ret = PyDict_SetItem(((PyTypeObject*)type_obj)->tp_dict, __pyx_n_s_setstate,
        setstate_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23463                     ret = PyDict_DelItem(((PyTypeObject*)type_obj)->tp_dict,
        __pyx_n_s_setstate_cython); if (unlikely(ret < 0)) goto __PYX_BAD;
23464                 } else if (!setstate || PyErr_Occurred()) {
23465                     goto __PYX_BAD;
23466                 }
23467             }
23468             PyType_Modified((PyTypeObject*)type_obj);
23469         }
23470     }
23471     goto __PYX_GOOD;
23472 __PYX_BAD:
23473     if (!PyErr_Occurred())
23474         PyErr_Format(PyExc_RuntimeError, "Unable to initialize pickling for %s",
        ((PyTypeObject*)type_obj)->tp_name);
23475     ret = -1;
23476 __PYX_GOOD:
23477     #if !CYTHON_USE_PYTYPE_LOOKUP
23478         Py_XDECREF(object_reduce);
23479         Py_XDECREF(object_reduce_ex);
23480     #endif
23481     Py_XDECREF(reduce);
23482     Py_XDECREF(reduce_ex);
23483     Py_XDECREF(reduce_cython);
23484     Py_XDECREF(setstate);
23485     Py_XDECREF(setstate_cython);
23486     return ret;
23487 }
23488
23489 /* BytesEquals */
23490 static CYTHON_INLINE int __Pyx_PyBytes_Equals(PyObject* s1, PyObject* s2, int equals) {
23491     #if CYTHON_COMPILING_IN_PYPY
23492     return PyObject_RichCompareBool(s1, s2, equals);
23493     #else
23494     if (s1 == s2) {
23495         return (equals == Py_EQ);
23496     } else if (PyBytes_CheckExact(s1) & PyBytes_CheckExact(s2)) {
23497         const char *ps1, *ps2;
23498         Py_ssize_t length = PyBytes_GET_SIZE(s1);
23499         if (length != PyBytes_GET_SIZE(s2))
23500             return (equals == Py_NE);
23501         ps1 = PyBytes_AS_STRING(s1);
23502         ps2 = PyBytes_AS_STRING(s2);
23503         if (ps1[0] != ps2[0]) {
23504             return (equals == Py_NE);
23505         } else if (length == 1) {
23506             return (equals == Py_EQ);
23507         } else {
23508             int result;
23509             #if CYTHON_USE_UNICODE_INTERNALS
23510                 Py_hash_t hash1, hash2;
23511                 hash1 = ((PyBytesObject*)s1)->ob_shash;
23512                 hash2 = ((PyBytesObject*)s2)->ob_shash;
23513                 if (hash1 != hash2 && hash1 != -1 && hash2 != -1) {
23514                     return (equals == Py_NE);
23515                 }

```



```

23516 #endif
23517         result = memcmp(ps1, ps2, (size_t)length);
23518         return (equals == Py_EQ) ? (result == 0) : (result != 0);
23519     }
23520 } else if ((s1 == Py_None) & PyBytes_CheckExact(s2)) {
23521     return (equals == Py_NE);
23522 } else if ((s2 == Py_None) & PyBytes_CheckExact(s1)) {
23523     return (equals == Py_NE);
23524 } else {
23525     int result;
23526     PyObject* py_result = PyObject_RichCompare(s1, s2, equals);
23527     if (!py_result)
23528         return -1;
23529     result = __Pyx_PyObject_IsTrue(py_result);
23530     Py_DECREF(py_result);
23531     return result;
23532 }
23533 #endif
23534 }
23535
23536 /* UnicodeEquals */
23537 static CYTHON_INLINE int __Pyx_PyUnicode_Equals(PyObject* s1, PyObject* s2, int equals) {
23538     #if CYTHON_COMPILING_IN_PYPY
23539         return PyObject_RichCompareBool(s1, s2, equals);
23540     #else
23541     #if PY_MAJOR_VERSION < 3
23542         PyObject* owned_ref = NULL;
23543     #endif
23544     int s1_is_unicode, s2_is_unicode;
23545     if (s1 == s2) {
23546         goto return_eq;
23547     }
23548     s1_is_unicode = PyUnicode_CheckExact(s1);
23549     s2_is_unicode = PyUnicode_CheckExact(s2);
23550     #if PY_MAJOR_VERSION < 3
23551     if ((s1_is_unicode & (!s2_is_unicode)) && PyString_CheckExact(s2)) {
23552         owned_ref = PyUnicode_FromObject(s2);
23553         if (unlikely(!owned_ref))
23554             return -1;
23555         s2 = owned_ref;
23556         s2_is_unicode = 1;
23557     } else if ((s2_is_unicode & (!s1_is_unicode)) && PyString_CheckExact(s1)) {
23558         owned_ref = PyUnicode_FromObject(s1);
23559         if (unlikely(!owned_ref))
23560             return -1;
23561         s1 = owned_ref;
23562         s1_is_unicode = 1;
23563     } else if (((!s2_is_unicode) & (!s1_is_unicode))) {
23564         return __Pyx_PyBytes_Equals(s1, s2, equals);
23565     }
23566     #endif
23567     if (s1_is_unicode & s2_is_unicode) {
23568         Py_ssize_t length;
23569         int kind;
23570         void *data1, *data2;
23571         if (unlikely(__Pyx_PyUnicode_READY(s1) < 0) || unlikely(__Pyx_PyUnicode_READY(s2) < 0))
23572             return -1;
23573         length = __Pyx_PyUnicode_GET_LENGTH(s1);
23574         if (length != __Pyx_PyUnicode_GET_LENGTH(s2)) {
23575             goto return_ne;
23576         }
23577         #if CYTHON_USE_UNICODE_INTERNALS
23578         {
23579             Py_hash_t hash1, hash2;
23580             #if CYTHON_PEP393_ENABLED
23581                 hash1 = ((PyASCIIObject*)s1)->hash;
23582                 hash2 = ((PyASCIIObject*)s2)->hash;
23583             #else
23584                 hash1 = ((PyUnicodeObject*)s1)->hash;
23585                 hash2 = ((PyUnicodeObject*)s2)->hash;
23586             #endif
23587             if (hash1 != hash2 && hash1 != -1 && hash2 != -1) {
23588                 goto return_ne;
23589             }
23590         }
23591     #endif
23592     kind = __Pyx_PyUnicode_KIND(s1);
23593     if (kind != __Pyx_PyUnicode_KIND(s2)) {
23594         goto return_ne;
23595     }
23596     data1 = __Pyx_PyUnicode_DATA(s1);
23597     data2 = __Pyx_PyUnicode_DATA(s2);
23598     if (__Pyx_PyUnicode_READ(kind, data1, 0) != __Pyx_PyUnicode_READ(kind, data2, 0)) {
23599         goto return_ne;
23600     } else if (length == 1) {
23601         goto return_eq;
23602     } else {

```

```

23603         int result = memcmp(data1, data2, (size_t)(length * kind));
23604         #if PY_MAJOR_VERSION < 3
23605         Py_XDECREF(owned_ref);
23606         #endif
23607         return (equals == Py_EQ) ? (result == 0) : (result != 0);
23608     }
23609     } else if ((s1 == Py_None) & s2_is_unicode) {
23610         goto return_ne;
23611     } else if ((s2 == Py_None) & s1_is_unicode) {
23612         goto return_ne;
23613     } else {
23614         int result;
23615         PyObject* py_result = PyObject_RichCompare(s1, s2, equals);
23616         #if PY_MAJOR_VERSION < 3
23617         Py_XDECREF(owned_ref);
23618         #endif
23619         if (!py_result)
23620             return -1;
23621         result = __Pyx_PyObject_IsTrue(py_result);
23622         Py_DECREF(py_result);
23623         return result;
23624     }
23625 return_eq:
23626     #if PY_MAJOR_VERSION < 3
23627     Py_XDECREF(owned_ref);
23628     #endif
23629     return (equals == Py_EQ);
23630 return_ne:
23631     #if PY_MAJOR_VERSION < 3
23632     Py_XDECREF(owned_ref);
23633     #endif
23634     return (equals == Py_NE);
23635 #endif
23636 }
23637
23638 /* CLineInTraceback */
23639 #ifndef CYTHON_CLINE_IN_TRACEBACK
23640 static int __Pyx_CLineForTraceback(CYTHON_NCP_UNUSED PyThreadState *tstate, int c_line) {
23641     PyObject *use_cline;
23642     PyObject *ptype, *pvalue, *ptraceback;
23643     #if CYTHON_COMPILING_IN_CPYTHON
23644     PyObject **cython_runtime_dict;
23645     #endif
23646     if (unlikely(!__pyx_cython_runtime)) {
23647         return c_line;
23648     }
23649     __Pyx_ErrFetchInState(tstate, &ptype, &pvalue, &ptraceback);
23650     #if CYTHON_COMPILING_IN_CPYTHON
23651     cython_runtime_dict = _PyObject_GetDictPtr(__pyx_cython_runtime);
23652     if (likely(cython_runtime_dict)) {
23653         __PYX_PY_DICT_LOOKUP_IF_MODIFIED(
23654             use_cline, *cython_runtime_dict,
23655             __Pyx_PyDict_GetItemStr(*cython_runtime_dict, __pyx_n_s_cline_in_traceback))
23656     } else
23657     #endif
23658     {
23659         PyObject *use_cline_obj = __Pyx_PyObject_GetAttrStr(__pyx_cython_runtime,
23660             __pyx_n_s_cline_in_traceback);
23661         if (use_cline_obj) {
23662             use_cline = PyObject_Not(use_cline_obj) ? Py_False : Py_True;
23663             Py_DECREF(use_cline_obj);
23664         } else {
23665             PyErr_Clear();
23666             use_cline = NULL;
23667         }
23668     }
23669     if (!use_cline) {
23670         c_line = 0;
23671         (void) PyObject_SetAttr(__pyx_cython_runtime, __pyx_n_s_cline_in_traceback, Py_False);
23672     }
23673     else if (use_cline == Py_False || (use_cline != Py_True && PyObject_Not(use_cline) != 0)) {
23674         c_line = 0;
23675     }
23676     __Pyx_ErrRestoreInState(tstate, ptype, pvalue, ptraceback);
23677     return c_line;
23678 #endif
23679
23680 /* CodeObjectCache */
23681 static int __pyx_bisect_code_objects(__Pyx_CodeObjectCacheEntry* entries, int count, int code_line) {
23682     int start = 0, mid = 0, end = count - 1;
23683     if (end >= 0 && code_line > entries[end].code_line) {
23684         return count;
23685     }
23686     while (start < end) {
23687         mid = start + (end - start) / 2;
23688         if (code_line < entries[mid].code_line) {

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```

23689         end = mid;
23690     } else if (code_line > entries[mid].code_line) {
23691         start = mid + 1;
23692     } else {
23693         return mid;
23694     }
23695 }
23696 if (code_line <= entries[mid].code_line) {
23697     return mid;
23698 } else {
23699     return mid + 1;
23700 }
23701 }
23702 static PyCodeObject* __pyx_find_code_object(int code_line) {
23703     PyCodeObject* code_object;
23704     int pos;
23705     if (unlikely(!code_line) || unlikely(!__pyx_code_cache.entries)) {
23706         return NULL;
23707     }
23708     pos = __pyx_bisect_code_objects(__pyx_code_cache.entries, __pyx_code_cache.count, code_line);
23709     if (unlikely(pos >= __pyx_code_cache.count) || unlikely(__pyx_code_cache.entries[pos].code_line !=
code_line)) {
23710         return NULL;
23711     }
23712     code_object = __pyx_code_cache.entries[pos].code_object;
23713     Py_INCREF(code_object);
23714     return code_object;
23715 }
23716 static void __pyx_insert_code_object(int code_line, PyCodeObject* code_object) {
23717     int pos, i;
23718     __Pyx_CodeObjectCacheEntry* entries = __pyx_code_cache.entries;
23719     if (unlikely(!code_line)) {
23720         return;
23721     }
23722     if (unlikely(!entries)) {
23723         entries = (__Pyx_CodeObjectCacheEntry*)PyMem_Malloc(64*sizeof(__Pyx_CodeObjectCacheEntry));
23724         if (likely(entries)) {
23725             __pyx_code_cache.entries = entries;
23726             __pyx_code_cache.max_count = 64;
23727             __pyx_code_cache.count = 1;
23728             entries[0].code_line = code_line;
23729             entries[0].code_object = code_object;
23730             Py_INCREF(code_object);
23731         }
23732         return;
23733     }
23734     pos = __pyx_bisect_code_objects(__pyx_code_cache.entries, __pyx_code_cache.count, code_line);
23735     if ((pos < __pyx_code_cache.count) && unlikely(__pyx_code_cache.entries[pos].code_line ==
code_line)) {
23736         PyCodeObject* tmp = entries[pos].code_object;
23737         entries[pos].code_object = code_object;
23738         Py_DECREF(tmp);
23739         return;
23740     }
23741     if (__pyx_code_cache.count == __pyx_code_cache.max_count) {
23742         int new_max = __pyx_code_cache.max_count + 64;
23743         entries = (__Pyx_CodeObjectCacheEntry*)PyMem_Realloc(
23744             __pyx_code_cache.entries, ((size_t)new_max) * sizeof(__Pyx_CodeObjectCacheEntry));
23745         if (unlikely(!entries)) {
23746             return;
23747         }
23748         __pyx_code_cache.entries = entries;
23749         __pyx_code_cache.max_count = new_max;
23750     }
23751     for (i=__pyx_code_cache.count; i>pos; i--) {
23752         entries[i] = entries[i-1];
23753     }
23754     entries[pos].code_line = code_line;
23755     entries[pos].code_object = code_object;
23756     __pyx_code_cache.count++;
23757     Py_INCREF(code_object);
23758 }
23759
23760 /* AddTraceback */
23761 #include "compile.h"
23762 #include "frameobject.h"
23763 #include "traceback.h"
23764 static PyCodeObject* __Pyx_CreateCodeObjectForTraceback(
23765     const char *funcname, int c_line,
23766     int py_line, const char *filename) {
23767     PyCodeObject *py_code = NULL;
23768     PyObject *py_funcname = NULL;
23769     #if PY_MAJOR_VERSION < 3
23770     PyObject *py_srcfile = NULL;
23771     py_srcfile = PyString_FromString(filename);
23772     if (!py_srcfile) goto bad;
23773     #endif

```

```

23774     if (c_line) {
23775         #if PY_MAJOR_VERSION < 3
23776         py_funcname = PyString_FromFormat( "%s (%s:%d)", funcname, __pyx_cfilenm, c_line);
23777         if (!py_funcname) goto bad;
23778         #else
23779         py_funcname = PyUnicode_FromFormat( "%s (%s:%d)", funcname, __pyx_cfilenm, c_line);
23780         if (!py_funcname) goto bad;
23781         funcname = PyUnicode_AsUTF8(py_funcname);
23782         if (!funcname) goto bad;
23783         #endif
23784     }
23785     else {
23786         #if PY_MAJOR_VERSION < 3
23787         py_funcname = PyString_FromString(funcname);
23788         if (!py_funcname) goto bad;
23789         #endif
23790     }
23791     #if PY_MAJOR_VERSION < 3
23792     py_code = __Pyx_PyCode_New(
23793         0,
23794         0,
23795         0,
23796         0,
23797         0,
23798         __pyx_empty_bytes, /*PyObject *code,*/
23799         __pyx_empty_tuple, /*PyObject *consts,*/
23800         __pyx_empty_tuple, /*PyObject *names,*/
23801         __pyx_empty_tuple, /*PyObject *varnames,*/
23802         __pyx_empty_tuple, /*PyObject *freevars,*/
23803         __pyx_empty_tuple, /*PyObject *cellvars,*/
23804         py_srcfile, /*PyObject *filename,*/
23805         py_funcname, /*PyObject *name,*/
23806         py_line,
23807         __pyx_empty_bytes /*PyObject *notab*/
23808     );
23809     Py_DECREF(py_srcfile);
23810     #else
23811     py_code = PyCode_NewEmpty(filename, funcname, py_line);
23812     #endif
23813     Py_XDECREF(py_funcname); // XDECREF since it's only set on Py3 if cline
23814     return py_code;
23815 bad:
23816     Py_XDECREF(py_funcname);
23817     #if PY_MAJOR_VERSION < 3
23818     Py_XDECREF(py_srcfile);
23819     #endif
23820     return NULL;
23821 }
23822 static void __Pyx_AddTraceback(const char *funcname, int c_line,
23823                               int py_line, const char *filename) {
23824     PyCodeObject *py_code = 0;
23825     PyFrameObject *py_frame = 0;
23826     PyThreadState *tstate = __Pyx_PyThreadState_Current;
23827     if (c_line) {
23828         c_line = __Pyx_CLineForTraceback(tstate, c_line);
23829     }
23830     py_code = __pyx_find_code_object(c_line ? -c_line : py_line);
23831     if (!py_code) {
23832         py_code = __Pyx_CreateCodeObjectForTraceback(
23833             funcname, c_line, py_line, filename);
23834         if (!py_code) goto bad;
23835         __pyx_insert_code_object(c_line ? -c_line : py_line, py_code);
23836     }
23837     py_frame = PyFrame_New(
23838         tstate, /*PyThreadState *tstate,*/
23839         py_code, /*PyObject *code,*/
23840         __pyx_d, /*PyObject *globals,*/
23841         0 /*PyObject *locals*/
23842     );
23843     if (!py_frame) goto bad;
23844     __Pyx_PyFrame_SetLineNumber(py_frame, py_line);
23845     PyTraceBack_Here(py_frame);
23846 bad:
23847     Py_XDECREF(py_code);
23848     Py_XDECREF(py_frame);
23849 }
23850
23851 /* CIntFromPyVerify */
23852 #define __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value)\
23853     __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value, 0)
23854 #define __PYX_VERIFY_RETURN_INT_EXC(target_type, func_type, func_value)\
23855     __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value, 1)
23856 #define __PYX_VERIFY_RETURN_INT(target_type, func_type, func_value, exc)\
23857     {\
23858         func_type value = func_value;\
23859         if (sizeof(target_type) < sizeof(func_type)) {\
23860             if (unlikely(value != (func_type) (target_type) value)) {\

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23861         func_type zero = 0;\
23862         if (exc && unlikely(value == (func_type)-1 && PyErr_Occurred()))\
23863             return (target_type) -1;\
23864         if (is_unsigned && unlikely(value < zero))\
23865             goto raise_neg_overflow;\
23866         else\
23867             goto raise_overflow;\
23868     }\
23869 }\
23870     return (target_type) value;\
23871 }
23872
23873 /* CIntFromPy */
23874 static CYTHON_INLINE int __Pyx_PyInt_As_int(PyObject *x) {
23875 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
23876 #pragma GCC diagnostic push
23877 #pragma GCC diagnostic ignored "-Wconversion"
23878 #endif
23879     const int neg_one = (int) -1, const_zero = (int) 0;
23880 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
23881 #pragma GCC diagnostic pop
23882 #endif
23883     const int is_unsigned = neg_one > const_zero;
23884 #if PY_MAJOR_VERSION < 3
23885     if (likely(PyInt_Check(x))) {
23886         if (sizeof(int) < sizeof(long)) {
23887             __PYX_VERIFY_RETURN_INT(int, long, PyInt_AS_LONG(x))
23888         } else {
23889             long val = PyInt_AS_LONG(x);
23890             if (is_unsigned && unlikely(val < 0)) {
23891                 goto raise_neg_overflow;
23892             }
23893             return (int) val;
23894         }
23895     } else
23896 #endif
23897     if (likely(PyLong_Check(x))) {
23898         if (is_unsigned) {
23899 #if CYTHON_USE_PYLONG_INTERNALS
23900             const digit* digits = ((PyLongObject*)x)->ob_digit;
23901             switch (Py_SIZE(x)) {
23902                 case 0: return (int) 0;
23903                 case 1: __PYX_VERIFY_RETURN_INT(int, digit, digits[0])
23904                 case 2:
23905                     if (8 * sizeof(int) > 1 * PyLong_SHIFT) {
23906                         if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23907                             __PYX_VERIFY_RETURN_INT(int, unsigned long, (((unsigned long)digits[1])
23908 « PyLong_SHIFT) | (unsigned long)digits[0]))
23909                         } else if (8 * sizeof(int) >= 2 * PyLong_SHIFT) {
23910                             return (int) (((int)digits[1]) « PyLong_SHIFT) | (int)digits[0]);
23911                         }
23912                     } break;
23913                 case 3:
23914                     if (8 * sizeof(int) > 2 * PyLong_SHIFT) {
23915                         if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23916                             __PYX_VERIFY_RETURN_INT(int, unsigned long, ((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
23917                         } else if (8 * sizeof(int) >= 3 * PyLong_SHIFT) {
23918                             return (int) ((((((int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) «
PyLong_SHIFT) | (int)digits[0]));
23919                         }
23920                     } break;
23921                 case 4:
23922                     if (8 * sizeof(int) > 3 * PyLong_SHIFT) {
23923                         if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
23924                             __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
23925                         } else if (8 * sizeof(int) >= 4 * PyLong_SHIFT) {
23926                             return (int) (((((((int)digits[3]) « PyLong_SHIFT) | (int)digits[2]) «
PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]));
23927                         }
23928                     } break;
23929                 }
23930             } break;
23931         }
23932 #endif
23933 #if CYTHON_COMPILING_IN_CPYTHON
23934         if (unlikely(Py_SIZE(x) < 0)) {
23935             goto raise_neg_overflow;
23936         }
23937     } else
23938     {
23939         int result = PyObject_RichCompareBool(x, Py_False, Py_LT);
23940         if (unlikely(result < 0))

```

```

23941         return (int) -1;
23942         if (unlikely(result == 1))
23943             goto raise_neg_overflow;
23944     }
23945 #endif
23946     if (sizeof(int) <= sizeof(unsigned long)) {
23947         __PYX_VERIFY_RETURN_INT_EXC(int, unsigned long, PyLong_AsUnsignedLong(x))
23948 #ifdef HAVE_LONG_LONG
23949     } else if (sizeof(int) <= sizeof(unsigned PY_LONG_LONG)) {
23950         __PYX_VERIFY_RETURN_INT_EXC(int, unsigned PY_LONG_LONG, PyLong_AsUnsignedLongLong(x))
23951 #endif
23952     }
23953     } else {
23954 #if CYTHON_USE_PYLONG_INTERNALS
23955         const digit* digits = ((PyLongObject*)x)->ob_digit;
23956         switch (Py_SIZE(x)) {
23957             case 0: return (int) 0;
23958             case -1: __PYX_VERIFY_RETURN_INT(int, sdigit, (sdigit) -(sdigit)digits[0]))
23959             case 1: __PYX_VERIFY_RETURN_INT(int, digit, +digits[0])
23960             case -2:
23961                 if (8 * sizeof(int) - 1 > 1 * PyLong_SHIFT) {
23962                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23963                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((unsigned long)digits[1]) «
23964 PyLong_SHIFT) | (unsigned long)digits[0]))
23965                     } else if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23966                         return (int) (((int)-1)*((((int)digits[1]) « PyLong_SHIFT) |
23967 (int)digits[0]]));
23968                     }
23969                     break;
23970             case 2:
23971                 if (8 * sizeof(int) > 1 * PyLong_SHIFT) {
23972                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
23973                         __PYX_VERIFY_RETURN_INT(int, unsigned long, (((unsigned long)digits[1])
23974 « PyLong_SHIFT) | (unsigned long)digits[0]))
23975                     } else if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23976                         return (int) (((((int)digits[1]) « PyLong_SHIFT) | (int)digits[0]]));
23977                     }
23978                     break;
23979             case -3:
23980                 if (8 * sizeof(int) - 1 > 2 * PyLong_SHIFT) {
23981                     if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23982                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((unsigned long)digits[2])
23983 « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))
23984                     } else if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23985                         return (int) (((int)-1)*((((int)digits[2]) « PyLong_SHIFT) |
23986 (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]]));
23987                     }
23988                     break;
23989             case 3:
23990                 if (8 * sizeof(int) > 2 * PyLong_SHIFT) {
23991                     if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
23992                         __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((unsigned
23993 long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
23994 long)digits[0]))
23995                     } else if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
23996                         return (int) (((((((int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) «
23997 PyLong_SHIFT) | (int)digits[0]]));
23998                     }
23999                     break;
3000             case -4:
3001                 if (8 * sizeof(int) - 1 > 3 * PyLong_SHIFT) {
3002                     if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
3003                         __PYX_VERIFY_RETURN_INT(int, long, -(long) (((((((unsigned
3004 long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
3005 long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))
3006                     } else if (8 * sizeof(int) - 1 > 4 * PyLong_SHIFT) {
3007                         return (int) (((((((int)digits[3]) « PyLong_SHIFT) |
3008 (int)digits[2]) « PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]]));
3009                     }
3010                     break;
3011             case 4:
3012                 if (8 * sizeof(int) > 3 * PyLong_SHIFT) {
3013                     if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
3014                         __PYX_VERIFY_RETURN_INT(int, unsigned long, (((((((unsigned
3015 long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
3016 long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0]))
3017                     } else if (8 * sizeof(int) - 1 > 4 * PyLong_SHIFT) {
3018                         return (int) (((((((int)digits[3]) « PyLong_SHIFT) | (int)digits[2]) «
3019 PyLong_SHIFT) | (int)digits[1]) « PyLong_SHIFT) | (int)digits[0]]));
3020                     }
3021                     break;

```

```

24014     }
24015 #endif
24016     if (sizeof(int) <= sizeof(long)) {
24017         __PYX_VERIFY_RETURN_INT_EXC(int, long, PyLong_AsLong(x))
24018 #ifdef HAVE_LONG_LONG
24019     } else if (sizeof(int) <= sizeof(PY_LONG_LONG)) {
24020         __PYX_VERIFY_RETURN_INT_EXC(int, PY_LONG_LONG, PyLong_AsLongLong(x))
24021 #endif
24022     }
24023 }
24024 {
24025 #if CYTHON_COMPILING_IN_PYPY && !defined(_PyLong_AsByteArray)
24026     PyErr_SetString(PyExc_RuntimeError,
24027         "_PyLong_AsByteArray() not available in PyPy, cannot convert large
numbers");
24028 #else
24029     int val;
24030     PyObject *v = __Pyx_PyNumber_IntOrLong(x);
24031     #if PY_MAJOR_VERSION < 3
24032     if (likely(v) && !PyLong_Check(v)) {
24033         PyObject *tmp = v;
24034         v = PyNumber_Long(tmp);
24035         Py_DECREF(tmp);
24036     }
24037 #endif
24038     if (likely(v)) {
24039         int one = 1; int is_little = (int)*(unsigned char *)&one;
24040         unsigned char *bytes = (unsigned char *)&val;
24041         int ret = _PyLong_AsByteArray((PyLongObject *)v,
24042             bytes, sizeof(val),
24043             is_little, !is_unsigned);
24044         Py_DECREF(v);
24045         if (likely(!ret))
24046             return val;
24047     }
24048 #endif
24049     return (int) -1;
24050 }
24051 } else {
24052     int val;
24053     PyObject *tmp = __Pyx_PyNumber_IntOrLong(x);
24054     if (!tmp) return (int) -1;
24055     val = __Pyx_PyInt_As_int(tmp);
24056     Py_DECREF(tmp);
24057     return val;
24058 }
24059 raise_overflow:
24060     PyErr_SetString(PyExc_OverflowError,
24061         "value too large to convert to int");
24062     return (int) -1;
24063 raise_neg_overflow:
24064     PyErr_SetString(PyExc_OverflowError,
24065         "can't convert negative value to int");
24066     return (int) -1;
24067 }
24068
24069 /* CIntToPy */
24070 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_int(int value) {
24071 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24072 #pragma GCC diagnostic push
24073 #pragma GCC diagnostic ignored "-Wconversion"
24074 #endif
24075     const int neg_one = (int) -1, const_zero = (int) 0;
24076 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24077 #pragma GCC diagnostic pop
24078 #endif
24079     const int is_unsigned = neg_one > const_zero;
24080     if (is_unsigned) {
24081         if (sizeof(int) < sizeof(long)) {
24082             return PyInt_FromLong((long) value);
24083         } else if (sizeof(int) <= sizeof(unsigned long)) {
24084             return PyLong_FromUnsignedLong((unsigned long) value);
24085 #ifdef HAVE_LONG_LONG
24086         } else if (sizeof(int) <= sizeof(unsigned PY_LONG_LONG)) {
24087             return PyLong_FromUnsignedLongLong((unsigned PY_LONG_LONG) value);
24088 #endif
24089     }
24090     } else {
24091         if (sizeof(int) <= sizeof(long)) {
24092             return PyInt_FromLong((long) value);
24093 #ifdef HAVE_LONG_LONG
24094         } else if (sizeof(int) <= sizeof(PY_LONG_LONG)) {
24095             return PyLong_FromLongLong((PY_LONG_LONG) value);
24096 #endif
24097     }
24098 }
24099 {

```

```

24100         int one = 1; int little = (int)*(unsigned char *)&one;
24101         unsigned char *bytes = (unsigned char *)&value;
24102         return _PyLong_FromByteArray(bytes, sizeof(int),
24103                                     little, !is_unsigned);
24104     }
24105 }
24106
24107 /* CIntToPy */
24108 static CYTHON_INLINE PyObject* __Pyx_PyInt_From_long(long value) {
24109 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24110 #pragma GCC diagnostic push
24111 #pragma GCC diagnostic ignored "-Wconversion"
24112 #endif
24113     const long neg_one = (long) -1, const_zero = (long) 0;
24114 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24115 #pragma GCC diagnostic pop
24116 #endif
24117     const int is_unsigned = neg_one > const_zero;
24118     if (is_unsigned) {
24119         if (sizeof(long) < sizeof(unsigned long)) {
24120             return PyInt_FromLong((long) value);
24121         } else if (sizeof(long) <= sizeof(unsigned long)) {
24122             return PyLong_FromUnsignedLong((unsigned long) value);
24123 #ifdef HAVE_LONG_LONG
24124         } else if (sizeof(long) <= sizeof(unsigned PY_LONG_LONG)) {
24125             return PyLong_FromUnsignedLongLong((unsigned PY_LONG_LONG) value);
24126 #endif
24127         }
24128     } else {
24129         if (sizeof(long) <= sizeof(long)) {
24130             return PyInt_FromLong((long) value);
24131 #ifdef HAVE_LONG_LONG
24132         } else if (sizeof(long) <= sizeof(PY_LONG_LONG)) {
24133             return PyLong_FromLongLong((PY_LONG_LONG) value);
24134 #endif
24135         }
24136     }
24137     {
24138         int one = 1; int little = (int)*(unsigned char *)&one;
24139         unsigned char *bytes = (unsigned char *)&value;
24140         return _PyLong_FromByteArray(bytes, sizeof(long),
24141                                     little, !is_unsigned);
24142     }
24143 }
24144
24145 /* CIntFromPy */
24146 static CYTHON_INLINE long __Pyx_PyInt_As_long(PyObject *x) {
24147 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24148 #pragma GCC diagnostic push
24149 #pragma GCC diagnostic ignored "-Wconversion"
24150 #endif
24151     const long neg_one = (long) -1, const_zero = (long) 0;
24152 #ifdef __Pyx_HAS_GCC_DIAGNOSTIC
24153 #pragma GCC diagnostic pop
24154 #endif
24155     const int is_unsigned = neg_one > const_zero;
24156 #if PY_MAJOR_VERSION < 3
24157     if (likely(PyInt_Check(x))) {
24158         if (sizeof(long) < sizeof(long)) {
24159             __PYX_VERIFY_RETURN_INT(long, long, PyInt_AS_LONG(x))
24160         } else {
24161             long val = PyInt_AS_LONG(x);
24162             if (is_unsigned && unlikely(val < 0)) {
24163                 goto raise_neg_overflow;
24164             }
24165             return (long) val;
24166         }
24167     } else
24168 #endif
24169     if (likely(PyLong_Check(x))) {
24170         if (is_unsigned) {
24171 #if CYTHON_USE_PYLONG_INTERNALS
24172             const digit* digits = ((PyLongObject*)x)->ob_digit;
24173             switch (Py_SIZE(x)) {
24174                 case 0: return (long) 0;
24175                 case 1: __PYX_VERIFY_RETURN_INT(long, digit, digits[0])
24176                 case 2:
24177                     if (8 * sizeof(long) > 1 * PyLong_SHIFT) {
24178                         if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24179                             __PYX_VERIFY_RETURN_INT(long, unsigned long, (((unsigned long)digits[1])
24180                                     « PyLong_SHIFT) | (unsigned long)digits[0]))
24181                         } else if (8 * sizeof(long) >= 2 * PyLong_SHIFT) {
24182                             return (long) (((((long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24183                         }
24184                     }
24185                     break;
24186                 case 3:

```



```

24186         if (8 * sizeof(long) > 2 * PyLong_SHIFT) {
24187             if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24188                 __PYX_VERIFY_RETURN_INT(long, unsigned long, ((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
24189             } else if (8 * sizeof(long) >= 3 * PyLong_SHIFT) {
24190                 return (long) ((((((long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) «
PyLong_SHIFT) | (long)digits[0]));
24191             }
24192         }
24193         break;
24194     case 4:
24195         if (8 * sizeof(long) > 3 * PyLong_SHIFT) {
24196             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24197                 __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24198             } else if (8 * sizeof(long) >= 4 * PyLong_SHIFT) {
24199                 return (long) (((((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
« PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24200             }
24201         }
24202         break;
24203     }
24204 #endif
24205 #if CYTHON_COMPILING_IN_CPYTHON
24206     if (unlikely(Py_SIZE(x) < 0)) {
24207         goto raise_neg_overflow;
24208     }
24209 #else
24210     {
24211         int result = PyObject_RichCompareBool(x, Py_False, Py_LT);
24212         if (unlikely(result < 0))
24213             return (long) -1;
24214         if (unlikely(result == 1))
24215             goto raise_neg_overflow;
24216     }
24217 #endif
24218     if (sizeof(long) <= sizeof(unsigned long)) {
24219         __PYX_VERIFY_RETURN_INT_EXC(long, unsigned long, PyLong_AsUnsignedLong(x))
24220 #ifdef HAVE_LONG_LONG
24221     } else if (sizeof(long) <= sizeof(unsigned PY_LONG_LONG)) {
24222         __PYX_VERIFY_RETURN_INT_EXC(long, unsigned PY_LONG_LONG, PyLong_AsUnsignedLongLong(x))
24223 #endif
24224     }
24225     } else {
24226 #if CYTHON_USE_PYLONG_INTERNALS
24227         const digit* digits = ((PyLongObject*)x)->ob_digit;
24228         switch (Py_SIZE(x)) {
24229             case 0: return (long) 0;
24230             case -1: __PYX_VERIFY_RETURN_INT(long, sdigit, (sdigit) -(sdigit)digits[0]))
24231             case 1: __PYX_VERIFY_RETURN_INT(long, digit, +digits[0])
24232             case -2:
24233                 if (8 * sizeof(long) - 1 > 1 * PyLong_SHIFT) {
24234                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24235                         __PYX_VERIFY_RETURN_INT(long, long, -(long) (((((unsigned long)digits[1])
« PyLong_SHIFT) | (unsigned long)digits[0])))
24236                     } else if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24237                         return (long) (((long)-1)*(((long)digits[1]) « PyLong_SHIFT) |
(long)digits[0]));
24238                     }
24239                 }
24240                 break;
24241             case 2:
24242                 if (8 * sizeof(long) > 1 * PyLong_SHIFT) {
24243                     if (8 * sizeof(unsigned long) > 2 * PyLong_SHIFT) {
24244                         __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((unsigned long)digits[1])
« PyLong_SHIFT) | (unsigned long)digits[0])))
24245                     } else if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24246                         return (long) (((((long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24247                     }
24248                 }
24249                 break;
24250             case -3:
24251                 if (8 * sizeof(long) - 1 > 2 * PyLong_SHIFT) {
24252                     if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24253                         __PYX_VERIFY_RETURN_INT(long, long, -(long) (((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
24254                     } else if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24255                         return (long) (((long)-1)*((((((long)digits[2]) « PyLong_SHIFT) |
(long)digits[1]) « PyLong_SHIFT) | (long)digits[0]));
24256                     }
24257                 }
24258                 break;
24259             case 3:
24260                 if (8 * sizeof(long) > 2 * PyLong_SHIFT) {

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```

24261         if (8 * sizeof(unsigned long) > 3 * PyLong_SHIFT) {
24262             __PYX_VERIFY_RETURN_INT(long, unsigned long, ((((((unsigned
long)digits[2]) « PyLong_SHIFT) | (unsigned long)digits[1]) « PyLong_SHIFT) | (unsigned
long)digits[0])))
24263         } else if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24264             return (long) ((((((long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) «
PyLong_SHIFT) | (long)digits[0]));
24265         }
24266     }
24267     break;
24268     case -4:
24269         if (8 * sizeof(long) - 1 > 3 * PyLong_SHIFT) {
24270             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24271                 __PYX_VERIFY_RETURN_INT(long, long, -(long) (((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24272             } else if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
24273                 return (long) (((((long)-1)*(((((((long)digits[3]) « PyLong_SHIFT) |
(long)digits[2]) « PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0])));
24274             }
24275         }
24276     break;
24277     case 4:
24278         if (8 * sizeof(long) > 3 * PyLong_SHIFT) {
24279             if (8 * sizeof(unsigned long) > 4 * PyLong_SHIFT) {
24280                 __PYX_VERIFY_RETURN_INT(long, unsigned long, (((((((unsigned
long)digits[3]) « PyLong_SHIFT) | (unsigned long)digits[2]) « PyLong_SHIFT) | (unsigned
long)digits[1]) « PyLong_SHIFT) | (unsigned long)digits[0])))
24281             } else if (8 * sizeof(long) - 1 > 4 * PyLong_SHIFT) {
24282                 return (long) ((((((((((long)digits[3]) « PyLong_SHIFT) | (long)digits[2])
« PyLong_SHIFT) | (long)digits[1]) « PyLong_SHIFT) | (long)digits[0])));
24283             }
24284         }
24285     break;
24286 }
24287 #endif
24288     if (sizeof(long) <= sizeof(long)) {
24289         __PYX_VERIFY_RETURN_INT_EXC(long, long, PyLong_AsLong(x))
24290 #ifdef HAVE_LONG_LONG
24291     } else if (sizeof(long) <= sizeof(PY_LONG_LONG)) {
24292         __PYX_VERIFY_RETURN_INT_EXC(long, PY_LONG_LONG, PyLong_AsLongLong(x))
24293 #endif
24294     }
24295 }
24296 {
24297 #if CYTHON_COMPILING_IN_PYPY && !defined(_PyLong_AsByteArray)
24298     PyErr_SetString(PyExc_RuntimeError,
24299         "_PyLong_AsByteArray() not available in PyPy, cannot convert large
numbers");
24300 #else
24301     long val;
24302     PyObject *v = __Pyx_PyNumber_IntOrLong(x);
24303     #if PY_MAJOR_VERSION < 3
24304         if (likely(v) && !PyLong_Check(v)) {
24305             PyObject *tmp = v;
24306             v = PyNumber_Long(tmp);
24307             Py_DECREF(tmp);
24308         }
24309     #endif
24310     if (likely(v)) {
24311         int one = 1; int is_little = (int)*(unsigned char *)&one;
24312         unsigned char *bytes = (unsigned char *)&val;
24313         int ret = _PyLong_AsByteArray((PyLongObject *)v,
24314             bytes, sizeof(val),
24315             is_little, !is_unsigned);
24316         Py_DECREF(v);
24317         if (likely(!ret))
24318             return val;
24319     }
24320 #endif
24321     return (long) -1;
24322 }
24323 } else {
24324     long val;
24325     PyObject *tmp = __Pyx_PyNumber_IntOrLong(x);
24326     if (!tmp) return (long) -1;
24327     val = __Pyx_PyInt_As_long(tmp);
24328     Py_DECREF(tmp);
24329     return val;
24330 }
24331 raise_overflow:
24332     PyErr_SetString(PyExc_OverflowError,
24333         "value too large to convert to long");
24334     return (long) -1;
24335 raise_neg_overflow:
24336     PyErr_SetString(PyExc_OverflowError,
24337         "can't convert negative value to long");

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24338     return (long) -1;
24339 }
24340
24341 /* FastTypeChecks */
24342 #if CYTHON_COMPILING_IN_CPYTHON
24343 static int __Pyx_InBases(PyTypeObject *a, PyTypeObject *b) {
24344     while (a) {
24345         a = a->tp_base;
24346         if (a == b)
24347             return 1;
24348     }
24349     return b == &PyBaseObject_Type;
24350 }
24351 static CYTHON_INLINE int __Pyx_IsSubtype(PyTypeObject *a, PyTypeObject *b) {
24352     PyObject *mro;
24353     if (a == b) return 1;
24354     mro = a->tp_mro;
24355     if (likely(mro)) {
24356         Py_ssize_t i, n;
24357         n = PyTuple_GET_SIZE(mro);
24358         for (i = 0; i < n; i++) {
24359             if (PyTuple_GET_ITEM(mro, i) == (PyObject *)b)
24360                 return 1;
24361         }
24362         return 0;
24363     }
24364     return __Pyx_InBases(a, b);
24365 }
24366 #if PY_MAJOR_VERSION == 2
24367 static int __Pyx_inner_PyErr_GivenExceptionMatches2(PyObject *err, PyObject* exc_type1, PyObject*
exc_type2) {
24368     PyObject *exception, *value, *tb;
24369     int res;
24370     __Pyx_PyThreadState_declare
24371     __Pyx_PyThreadState_assign
24372     __Pyx_ErrFetch(&exception, &value, &tb);
24373     res = exc_type1 ? PyObject_IsSubclass(err, exc_type1) : 0;
24374     if (unlikely(res == -1)) {
24375         PyErr_WriteUnraisable(err);
24376         res = 0;
24377     }
24378     if (!res) {
24379         res = PyObject_IsSubclass(err, exc_type2);
24380         if (unlikely(res == -1)) {
24381             PyErr_WriteUnraisable(err);
24382             res = 0;
24383         }
24384     }
24385     __Pyx_ErrRestore(exception, value, tb);
24386     return res;
24387 }
24388 #else
24389 static CYTHON_INLINE int __Pyx_inner_PyErr_GivenExceptionMatches2(PyObject *err, PyObject* exc_type1,
PyObject* exc_type2) {
24390     int res = exc_type1 ? __Pyx_IsSubtype((PyTypeObject*)err, (PyTypeObject*)exc_type1) : 0;
24391     if (!res) {
24392         res = __Pyx_IsSubtype((PyTypeObject*)err, (PyTypeObject*)exc_type2);
24393     }
24394     return res;
24395 }
24396 #endif
24397 static int __Pyx_PyErr_GivenExceptionMatchesTuple(PyObject *exc_type, PyObject *tuple) {
24398     Py_ssize_t i, n;
24399     assert(PyExceptionClass_Check(exc_type));
24400     n = PyTuple_GET_SIZE(tuple);
24401     #if PY_MAJOR_VERSION >= 3
24402     for (i=0; i<n; i++) {
24403         if (exc_type == PyTuple_GET_ITEM(tuple, i)) return 1;
24404     }
24405     #endif
24406     for (i=0; i<n; i++) {
24407         PyObject *t = PyTuple_GET_ITEM(tuple, i);
24408         #if PY_MAJOR_VERSION < 3
24409         if (likely(exc_type == t)) return 1;
24410         #endif
24411         if (likely(PyExceptionClass_Check(t))) {
24412             if (__Pyx_inner_PyErr_GivenExceptionMatches2(exc_type, NULL, t)) return 1;
24413         } else {
24414             }
24415     }
24416     return 0;
24417 }
24418 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches(PyObject *err, PyObject* exc_type) {
24419     if (likely(err == exc_type)) return 1;
24420     if (likely(PyExceptionClass_Check(err))) {
24421         if (likely(PyExceptionClass_Check(exc_type))) {
24422             return __Pyx_inner_PyErr_GivenExceptionMatches2(err, NULL, exc_type);

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```

24423         } else if (likely(PyTuple_Check(exc_type))) {
24424             return __Pyx_PyErr_GivenExceptionMatchesTuple(err, exc_type);
24425         } else {
24426         }
24427     }
24428     return PyErr_GivenExceptionMatches(err, exc_type);
24429 }
24430 static CYTHON_INLINE int __Pyx_PyErr_GivenExceptionMatches2(PyObject *err, PyObject *exc_type1,
24431     PyObject *exc_type2) {
24432     assert(PyExceptionClass_Check(exc_type1));
24433     assert(PyExceptionClass_Check(exc_type2));
24434     if (likely(err == exc_type1 || err == exc_type2)) return 1;
24435     if (likely(PyExceptionClass_Check(err))) {
24436         return __Pyx_inner_PyErr_GivenExceptionMatches2(err, exc_type1, exc_type2);
24437     }
24438     return (PyErr_GivenExceptionMatches(err, exc_type1) || PyErr_GivenExceptionMatches(err,
24439         exc_type2));
24440 }
24441 #endif
24442 /* FetchCommonType */
24443 static PyTypeObject* __Pyx_FetchCommonType(PyTypeObject* type) {
24444     PyObject* fake_module;
24445     PyTypeObject* cached_type = NULL;
24446     fake_module = PyImport_AddModule((char*) "_cython_" CYTHON_ABI);
24447     if (!fake_module) return NULL;
24448     Py_INCREF(fake_module);
24449     cached_type = (PyTypeObject*) PyObject_GetAttrString(fake_module, type->tp_name);
24450     if (cached_type) {
24451         if (!PyType_Check((PyObject*)cached_type)) {
24452             PyErr_Format(PyExc_TypeError,
24453                 "Shared Cython type %.200s is not a type object",
24454                 type->tp_name);
24455             goto bad;
24456         }
24457         if (cached_type->tp_basicsize != type->tp_basicsize) {
24458             PyErr_Format(PyExc_TypeError,
24459                 "Shared Cython type %.200s has the wrong size, try recompiling",
24460                 type->tp_name);
24461             goto bad;
24462         }
24463     } else {
24464         if (!PyErr_ExceptionMatches(PyExc_AttributeError)) goto bad;
24465         PyErr_Clear();
24466         if (PyType_Ready(type) < 0) goto bad;
24467         if (PyObject_SetAttrString(fake_module, type->tp_name, (PyObject*) type) < 0)
24468             goto bad;
24469         Py_INCREF(type);
24470         cached_type = type;
24471     }
24472 done:
24473     Py_DECREF(fake_module);
24474     return cached_type;
24475 bad:
24476     Py_XDECREF(cached_type);
24477     cached_type = NULL;
24478     goto done;
24479 }
24480 /* PyObjectGetMethod */
24481 static int __Pyx_PyObject_GetMethod(PyObject *obj, PyObject *name, PyObject **method) {
24482     PyObject *attr;
24483     #if CYTHON_UNPACK_METHODS && CYTHON_COMPILING_IN_CPYTHON && CYTHON_USE_PYTYPE_LOOKUP
24484     PyTypeObject *tp = Py_TYPE(obj);
24485     PyObject *descr;
24486     descrgetfunc f = NULL;
24487     PyObject **dictptr, *dict;
24488     int meth_found = 0;
24489     assert (*method == NULL);
24490     if (unlikely(tp->tp_getattro != PyObject_GenericGetAttr)) {
24491         attr = __Pyx_PyObject_GetAttrStr(obj, name);
24492         goto try_unpack;
24493     }
24494     if (unlikely(tp->tp_dict == NULL) && unlikely(PyType_Ready(tp) < 0)) {
24495         return 0;
24496     }
24497     descr = _PyType_Lookup(tp, name);
24498     if (likely(descr != NULL)) {
24499         Py_INCREF(descr);
24500     #if PY_MAJOR_VERSION >= 3
24501         #ifdef __Pyx_CyFunction_USED
24502         if (likely(PyFunction_Check(descr) || (Py_TYPE(descr) == &PyMethodDescr_Type) ||
24503             __Pyx_CyFunction_Check(descr)))
24504             #else
24505             if (likely(PyFunction_Check(descr) || (Py_TYPE(descr) == &PyMethodDescr_Type)))
24506             #endif
24507         #else

```

```

24507     #ifdef __Pyx_CyFunction_USED
24508     if (likely(PyFunction_Check(descr) || __Pyx_CyFunction_Check(descr)))
24509     #else
24510     if (likely(PyFunction_Check(descr)))
24511     #endif
24512 #endif
24513 {
24514     meth_found = 1;
24515 } else {
24516     f = Py_TYPE(descr)->tp_descr_get;
24517     if (f != NULL && PyDescr_IsData(descr)) {
24518         attr = f(descr, obj, (PyObject *)Py_TYPE(obj));
24519         Py_DECREF(descr);
24520         goto try_unpack;
24521     }
24522 }
24523 }
24524 dictptr = _PyObject_GetDictPtr(obj);
24525 if (dictptr != NULL && (dict = *dictptr) != NULL) {
24526     Py_INCREF(dict);
24527     attr = __Pyx_PyDict_GetItemStr(dict, name);
24528     if (attr != NULL) {
24529         Py_INCREF(attr);
24530         Py_DECREF(dict);
24531         Py_XDECREF(descr);
24532         goto try_unpack;
24533     }
24534     Py_DECREF(dict);
24535 }
24536 if (meth_found) {
24537     *method = descr;
24538     return 1;
24539 }
24540 if (f != NULL) {
24541     attr = f(descr, obj, (PyObject *)Py_TYPE(obj));
24542     Py_DECREF(descr);
24543     goto try_unpack;
24544 }
24545 if (descr != NULL) {
24546     *method = descr;
24547     return 0;
24548 }
24549 PyErr_Format(PyExc_AttributeError,
24550 #if PY_MAJOR_VERSION >= 3
24551     "%s object has no attribute '%U'",
24552     tp->tp_name, name);
24553 #else
24554     "%s object has no attribute '%.400s'",
24555     tp->tp_name, PyString_AS_STRING(name));
24556 #endif
24557 return 0;
24558 #else
24559     attr = __Pyx_PyObject_GetAttrStr(obj, name);
24560     goto try_unpack;
24561 #endif
24562 try_unpack:
24563 #if CYTHON_UNPACK_METHODS
24564     if (likely(attr) && PyMethod_Check(attr) && likely(PyMethod_GET_SELF(attr) == obj)) {
24565         PyObject *function = PyMethod_GET_FUNCTION(attr);
24566         Py_INCREF(function);
24567         Py_DECREF(attr);
24568         *method = function;
24569         return 1;
24570     }
24571 #endif
24572 *method = attr;
24573 return 0;
24574 }
24575
24576 /* PyObjectCallMethod1 */
24577 static PyObject* __Pyx__PyObject_CallMethod1(PyObject* method, PyObject* arg) {
24578     PyObject *result = __Pyx_PyObject_CallOneArg(method, arg);
24579     Py_DECREF(method);
24580     return result;
24581 }
24582 static PyObject* __Pyx_PyObject_CallMethod1(PyObject* obj, PyObject* method_name, PyObject* arg) {
24583     PyObject *method = NULL, *result;
24584     int is_method = __Pyx_PyObject_GetMethod(obj, method_name, &method);
24585     if (likely(is_method)) {
24586         result = __Pyx_PyObject_Call2Args(method, obj, arg);
24587         Py_DECREF(method);
24588         return result;
24589     }
24590     if (unlikely(!method)) return NULL;
24591     return __Pyx__PyObject_CallMethod1(method, arg);
24592 }
24593

```

```

24594 /* CoroutineBase */
24595 #include <structmember.h>
24596 #include <frameobject.h>
24597 #define __Pyx_Coroutine_Undelegate(gen) Py_CLEAR((gen)->yieldfrom)
24598 static int __Pyx_PyGen_FetchStopIterationValue(CYTHON_UNUSED PyThreadState *__pyx_tstate, PyObject
**pvalue) {
24599     PyObject *et, *ev, *tb;
24600     PyObject *value = NULL;
24601     __Pyx_ErrFetch(&et, &ev, &tb);
24602     if (!et) {
24603         Py_XDECREF(tb);
24604         Py_XDECREF(ev);
24605         Py_INCREF(Py_None);
24606         *pvalue = Py_None;
24607         return 0;
24608     }
24609     if (likely(et == PyExc_StopIteration)) {
24610         if (!ev) {
24611             Py_INCREF(Py_None);
24612             value = Py_None;
24613         }
24614         #if PY_VERSION_HEX >= 0x030300A0
24615         else if (Py_TYPE(ev) == (PyTypeObject*)PyExc_StopIteration) {
24616             value = ((PyStopIterationObject *)ev)->value;
24617             Py_INCREF(value);
24618             Py_DECREF(ev);
24619         }
24620         #endif
24621         else if (unlikely(PyTuple_Check(ev))) {
24622             if (PyTuple_GET_SIZE(ev) >= 1) {
24623                 #if CYTHON_ASSUME_SAFE_MACROS && !CYTHON_AVOID_BORROWED_REFS
24624                 value = PyTuple_GET_ITEM(ev, 0);
24625                 Py_INCREF(value);
24626             #else
24627                 value = PySequence_ITEM(ev, 0);
24628             #endif
24629             } else {
24630                 Py_INCREF(Py_None);
24631                 value = Py_None;
24632             }
24633             Py_DECREF(ev);
24634         }
24635         else if (!__Pyx_TypeCheck(ev, (PyTypeObject*)PyExc_StopIteration)) {
24636             value = ev;
24637         }
24638         if (likely(value)) {
24639             Py_XDECREF(tb);
24640             Py_DECREF(et);
24641             *pvalue = value;
24642             return 0;
24643         }
24644     } else if (!__Pyx_PyErr_GivenExceptionMatches(et, PyExc_StopIteration)) {
24645         __Pyx_ErrRestore(et, ev, tb);
24646         return -1;
24647     }
24648     PyErr_NormalizeException(&et, &ev, &tb);
24649     if (unlikely(!PyObject_TypeCheck(ev, (PyTypeObject*)PyExc_StopIteration))) {
24650         __Pyx_ErrRestore(et, ev, tb);
24651         return -1;
24652     }
24653     Py_XDECREF(tb);
24654     Py_DECREF(et);
24655     #if PY_VERSION_HEX >= 0x030300A0
24656     value = ((PyStopIterationObject *)ev)->value;
24657     Py_INCREF(value);
24658     Py_DECREF(ev);
24659     #else
24660     {
24661         PyObject* args = __Pyx_PyObject_GetAttrStr(ev, __pyx_n_s_args);
24662         Py_DECREF(ev);
24663         if (likely(args)) {
24664             value = PySequence_GetItem(args, 0);
24665             Py_DECREF(args);
24666         }
24667         if (unlikely(!value)) {
24668             __Pyx_ErrRestore(NULL, NULL, NULL);
24669             Py_INCREF(Py_None);
24670             value = Py_None;
24671         }
24672     }
24673     #endif
24674     *pvalue = value;
24675     return 0;
24676 }
24677 static CYTHON_INLINE
24678 void __Pyx_Coroutine_ExceptionClear(__Pyx_ExcInfoStruct *exc_state) {
24679     PyObject *t, *v, *tb;

```

```

24680     t = exc_state->exc_type;
24681     v = exc_state->exc_value;
24682     tb = exc_state->exc_traceback;
24683     exc_state->exc_type = NULL;
24684     exc_state->exc_value = NULL;
24685     exc_state->exc_traceback = NULL;
24686     Py_XDECREF(t);
24687     Py_XDECREF(v);
24688     Py_XDECREF(tb);
24689 }
24690 #define __Pyx_Coroutine_AlreadyRunningError(gen)  (__Pyx_Coroutine_AlreadyRunningError(gen),
(PyObject*)NULL)
24691 static void __Pyx_Coroutine_AlreadyRunningError(CYTHON_UNUSED __pyx_CoroutineObject *gen) {
24692     const char *msg;
24693     if ((0)) {
24694         #ifdef __Pyx_Coroutine_USED
24695     } else if (__Pyx_Coroutine_Check((PyObject*)gen)) {
24696         msg = "coroutine already executing";
24697     #endif
24698     #ifdef __Pyx_AsyncGen_USED
24699     } else if (__Pyx_AsyncGen_CheckExact((PyObject*)gen)) {
24700         msg = "async generator already executing";
24701     #endif
24702     } else {
24703         msg = "generator already executing";
24704     }
24705     PyErr_SetString(PyExc_ValueError, msg);
24706 }
24707 #define __Pyx_Coroutine_NotStartedError(gen)  (__Pyx_Coroutine_NotStartedError(gen), (PyObject*)NULL)
24708 static void __Pyx_Coroutine_NotStartedError(CYTHON_UNUSED PyObject *gen) {
24709     const char *msg;
24710     if ((0)) {
24711         #ifdef __Pyx_Coroutine_USED
24712     } else if (__Pyx_Coroutine_Check(gen)) {
24713         msg = "can't send non-None value to a just-started coroutine";
24714     #endif
24715     #ifdef __Pyx_AsyncGen_USED
24716     } else if (__Pyx_AsyncGen_CheckExact(gen)) {
24717         msg = "can't send non-None value to a just-started async generator";
24718     #endif
24719     } else {
24720         msg = "can't send non-None value to a just-started generator";
24721     }
24722     PyErr_SetString(PyExc_TypeError, msg);
24723 }
24724 #define __Pyx_Coroutine_AlreadyTerminatedError(gen, value, closing)
(__Pyx_Coroutine_AlreadyTerminatedError(gen, value, closing), (PyObject*)NULL)
24725 static void __Pyx_Coroutine_AlreadyTerminatedError(CYTHON_UNUSED PyObject *gen, PyObject *value,
CYTHON_UNUSED int closing) {
24726     #ifdef __Pyx_Coroutine_USED
24727     if (!closing && __Pyx_Coroutine_Check(gen)) {
24728         PyErr_SetString(PyExc_RuntimeError, "cannot reuse already awaited coroutine");
24729     } else
24730     #endif
24731     if (value) {
24732         #ifdef __Pyx_AsyncGen_USED
24733         if (__Pyx_AsyncGen_CheckExact(gen))
24734             PyErr_SetNone(__Pyx_PyExc_StopAsyncIteration);
24735         else
24736         #endif
24737         PyErr_SetNone(PyExc_StopIteration);
24738     }
24739 }
24740 static
24741 PyObject __Pyx_Coroutine_SendEx(__pyx_CoroutineObject *self, PyObject *value, int closing) {
24742     __Pyx_PyThreadState_declare
24743     PyThreadState *tstate;
24744     __Pyx_ExcInfoStruct *exc_state;
24745     PyObject *retval;
24746     assert(!self->is_running);
24747     if (unlikely(self->resume_label == 0)) {
24748         if (unlikely(value && value != Py_None)) {
24749             return __Pyx_Coroutine_NotStartedError((PyObject*)self);
24750         }
24751     }
24752     if (unlikely(self->resume_label == -1)) {
24753         return __Pyx_Coroutine_AlreadyTerminatedError((PyObject*)self, value, closing);
24754     }
24755     #if CYTHON_FAST_THREAD_STATE
24756     __Pyx_PyThreadState_assign
24757     tstate = __pyx_tstate;
24758     #else
24759     tstate = __Pyx_PyThreadState_Current;
24760     #endif
24761     exc_state = &self->gi_exc_state;
24762     if (exc_state->exc_type) {
24763         #if CYTHON_COMPILING_IN_PYPY || CYTHON_COMPILING_IN_PYSTON

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```

24764         #else
24765         if (exc_state->exc_traceback) {
24766             PyTracebackObject *tb = (PyTracebackObject *) exc_state->exc_traceback;
24767             PyFrameObject *f = tb->tb_frame;
24768             assert(f->f_back == NULL);
24769             #if PY_VERSION_HEX >= 0x030B00A1
24770             f->f_back = PyThreadState_GetFrame(tstate);
24771             #else
24772             Py_XINCREf(tstate->frame);
24773             f->f_back = tstate->frame;
24774             #endif
24775         }
24776         #endif
24777     }
24778     #if CYTHON_USE_EXC_INFO_STACK
24779     exc_state->previous_item = tstate->exc_info;
24780     tstate->exc_info = exc_state;
24781     #else
24782     if (exc_state->exc_type) {
24783         __Pyx_ExceptionSwap(&exc_state->exc_type, &exc_state->exc_value, &exc_state->exc_traceback);
24784     } else {
24785         __Pyx_Coroutine_ExceptionClear(exc_state);
24786         __Pyx_ExceptionSave(&exc_state->exc_type, &exc_state->exc_value, &exc_state->exc_traceback);
24787     }
24788     #endif
24789     self->is_running = 1;
24790     retval = self->body((PyObject *) self, tstate, value);
24791     self->is_running = 0;
24792     #if CYTHON_USE_EXC_INFO_STACK
24793     exc_state = &self->gi_exc_state;
24794     tstate->exc_info = exc_state->previous_item;
24795     exc_state->previous_item = NULL;
24796     __Pyx_Coroutine_ResetFrameBackpointer(exc_state);
24797     #endif
24798     return retval;
24799 }
24800 static CYTHON_INLINE void __Pyx_Coroutine_ResetFrameBackpointer(__Pyx_ExcInfoStruct *exc_state) {
24801     PyObject *exc_tb = exc_state->exc_traceback;
24802     if (likely(exc_tb)) {
24803         #if CYTHON_COMPILING_IN_PYPY || CYTHON_COMPILING_IN_PYSTON
24804         #else
24805             PyTracebackObject *tb = (PyTracebackObject *) exc_tb;
24806             PyFrameObject *f = tb->tb_frame;
24807             Py_CLEAR(f->f_back);
24808         #endif
24809     }
24810 }
24811 static CYTHON_INLINE
24812 PyObject * __Pyx_Coroutine_MethodReturn(CYTHON_UNUSED PyObject* gen, PyObject *retval) {
24813     if (unlikely(!retval)) {
24814         __Pyx_PyThreadState_declare
24815         __Pyx_PyThreadState_assign
24816         if (!__Pyx_PyErr_Occurred()) {
24817             PyObject *exc = PyExc_StopIteration;
24818             #ifdef __Pyx_AsyncGen_USED
24819             if (__Pyx_AsyncGen_CheckExact(gen))
24820                 exc = __Pyx_PyExc_StopAsyncIteration;
24821             #endif
24822             __Pyx_PyErr_SetNone(exc);
24823         }
24824     }
24825     return retval;
24826 }
24827 #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
PY_VERSION_HEX >= 0x030600B3)
24828 static CYTHON_INLINE
24829 PyObject * __Pyx_PyGen_Send(PyGenObject *gen, PyObject *arg) {
24830     #if PY_VERSION_HEX <= 0x030A00A1
24831     return _PyGen_Send(gen, arg);
24832     #else
24833     PyObject *result;
24834     if (PyIter_Send((PyObject*)gen, arg ? arg : Py_None, &result) == PYGEN_RETURN) {
24835         if (PyAsyncGen_CheckExact(gen)) {
24836             assert(result == Py_None);
24837             PyErr_SetNone(PyExc_StopAsyncIteration);
24838         }
24839         else if (result == Py_None) {
24840             PyErr_SetNone(PyExc_StopIteration);
24841         }
24842         else {
24843             _PyGen_SetStopIterationValue(result);
24844         }
24845         Py_CLEAR(result);
24846     }
24847     return result;
24848     #endif
24849 }

```



```

24850 #endif
24851 static CYTHON_INLINE
24852 PyObject *__Pyx_Coroutine_FinishDelegation(__pyx_CoroutineObject *gen) {
24853     PyObject *ret;
24854     PyObject *val = NULL;
24855     __Pyx_Coroutine_Undelegate(gen);
24856     __Pyx_PyGen__FetchStopIterationValue(__Pyx_PyThreadState_Current, &val);
24857     ret = __Pyx_Coroutine_SendEx(gen, val, 0);
24858     Py_XDECREF(val);
24859     return ret;
24860 }
24861 static PyObject *__Pyx_Coroutine_Send(PyObject *self, PyObject *value) {
24862     PyObject *retval;
24863     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject*) self;
24864     PyObject *yf = gen->yieldfrom;
24865     if (unlikely(gen->is_running))
24866         return __Pyx_Coroutine_AlreadyRunningError(gen);
24867     if (yf) {
24868         PyObject *ret;
24869         gen->is_running = 1;
24870         #ifdef __Pyx_Generator_USED
24871         if (__Pyx_Generator_CheckExact(yf)) {
24872             ret = __Pyx_Coroutine_Send(yf, value);
24873         } else
24874         #endif
24875         #ifdef __Pyx_Coroutine_USED
24876         if (__Pyx_Coroutine_Check(yf)) {
24877             ret = __Pyx_Coroutine_Send(yf, value);
24878         } else
24879         #endif
24880         #ifdef __Pyx_AsyncGen_USED
24881         if (__pyx_PyAsyncGenASend_CheckExact(yf)) {
24882             ret = __Pyx_async_gen_asend_send(yf, value);
24883         } else
24884         #endif
24885         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
PY_VERSION_HEX >= 0x030600B3)
24886         if (PyGen_CheckExact(yf)) {
24887             ret = __Pyx_PyGen_Send((PyGenObject*)yf, value == Py_None ? NULL : value);
24888         } else
24889         #endif
24890         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03050000 && defined(PyCoro_CheckExact)
&& (defined(__linux__) || PY_VERSION_HEX >= 0x030600B3)
24891         if (PyCoro_CheckExact(yf)) {
24892             ret = __Pyx_PyGen_Send((PyGenObject*)yf, value == Py_None ? NULL : value);
24893         } else
24894         #endif
24895         {
24896             if (value == Py_None)
24897                 ret = Py_TYPE(yf)->tp_iternext(yf);
24898             else
24899                 ret = __Pyx_PyObject_CallMethod1(yf, __pyx_n_s_send, value);
24900         }
24901         gen->is_running = 0;
24902         if (likely(ret)) {
24903             return ret;
24904         }
24905         retval = __Pyx_Coroutine_FinishDelegation(gen);
24906     } else {
24907         retval = __Pyx_Coroutine_SendEx(gen, value, 0);
24908     }
24909     return __Pyx_Coroutine_MethodReturn(self, retval);
24910 }
24911 static int __Pyx_Coroutine_CloseIter(__pyx_CoroutineObject *gen, PyObject *yf) {
24912     PyObject *retval = NULL;
24913     int err = 0;
24914     #ifdef __Pyx_Generator_USED
24915     if (__Pyx_Generator_CheckExact(yf)) {
24916         retval = __Pyx_Coroutine_Close(yf);
24917         if (!retval)
24918             return -1;
24919     } else
24920     #endif
24921     #ifdef __Pyx_Coroutine_USED
24922     if (__Pyx_Coroutine_Check(yf)) {
24923         retval = __Pyx_Coroutine_Close(yf);
24924         if (!retval)
24925             return -1;
24926     } else
24927     #endif
24928     if (__Pyx_CoroutineAwait_CheckExact(yf)) {
24929         retval = __Pyx_CoroutineAwait_Close((__pyx_CoroutineAwaitObject*)yf, NULL);
24930         if (!retval)
24931             return -1;
24932     } else
24933     #endif
24934     #ifdef __Pyx_AsyncGen_USED
24935     if (__pyx_PyAsyncGenASend_CheckExact(yf)) {

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```

24935     retval = __Pyx_async_gen_asend_close(yf, NULL);
24936 } else
24937 if (__pyx_PyAsyncGenAThrow_CheckExact(yf)) {
24938     retval = __Pyx_async_gen_athrow_close(yf, NULL);
24939 } else
24940 #endif
24941 {
24942     PyObject *meth;
24943     gen->is_running = 1;
24944     meth = __Pyx_PyObject_GetAttrStr(yf, __pyx_n_s_close);
24945     if (unlikely(!meth)) {
24946         if (!PyErr_ExceptionMatches(PyExc_AttributeError)) {
24947             PyErr_WriteUnraisable(yf);
24948         }
24949         PyErr_Clear();
24950     } else {
24951         retval = PyObject_CallFunction(meth, NULL);
24952         Py_DECREF(meth);
24953         if (!retval)
24954             err = -1;
24955     }
24956     gen->is_running = 0;
24957 }
24958 Py_XDECREF(retval);
24959 return err;
24960 }
24961 static PyObject *__Pyx_Generator_Next(PyObject *self) {
24962     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject*) self;
24963     PyObject *yf = gen->yieldfrom;
24964     if (unlikely(gen->is_running))
24965         return __Pyx_Coroutine_AlreadyRunningError(gen);
24966     if (yf) {
24967         PyObject *ret;
24968         gen->is_running = 1;
24969         #ifdef __Pyx_Generator_USED
24970         if (__Pyx_Generator_CheckExact(yf)) {
24971             ret = __Pyx_Generator_Next(yf);
24972         } else
24973         #endif
24974         #if CYTHON_COMPILING_IN_CPYTHON && PY_VERSION_HEX >= 0x03030000 && (defined(__linux__) ||
24975             PY_VERSION_HEX >= 0x030600B3)
24976             if (PyGen_CheckExact(yf)) {
24977                 ret = __Pyx_PyGen_Send((PyGenObject*)yf, NULL);
24978             } else
24979             #endif
24980             #ifdef __Pyx_Coroutine_USED
24981             if (__Pyx_Coroutine_Check(yf)) {
24982                 ret = __Pyx_Coroutine_Send(yf, Py_None);
24983             } else
24984             #endif
24985             ret = Py_TYPE(yf)->tp_iternext(yf);
24986         gen->is_running = 0;
24987         if (likely(ret)) {
24988             return ret;
24989         }
24990         return __Pyx_Coroutine_FinishDelegation(gen);
24991     }
24992     return __Pyx_Coroutine_SendEx(gen, Py_None, 0);
24993 }
24994 static PyObject *__Pyx_Coroutine_Close_Method(PyObject *self, CYTHON_UNUSED PyObject *arg) {
24995     return __Pyx_Coroutine_Close(self);
24996 }
24997 static PyObject *__Pyx_Coroutine_Close(PyObject *self) {
24998     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
24999     PyObject *retval, *raised_exception;
25000     PyObject *yf = gen->yieldfrom;
25001     int err = 0;
25002     if (unlikely(gen->is_running))
25003         return __Pyx_Coroutine_AlreadyRunningError(gen);
25004     if (yf) {
25005         Py_INCREF(yf);
25006         err = __Pyx_Coroutine_CloseIter(gen, yf);
25007         __Pyx_Coroutine_Undelegate(gen);
25008         Py_DECREF(yf);
25009     }
25010     if (err == 0)
25011         PyErr_SetNone(PyExc_GeneratorExit);
25012     retval = __Pyx_Coroutine_SendEx(gen, NULL, 1);
25013     if (unlikely(retval)) {
25014         const char *msg;
25015         Py_DECREF(retval);
25016         if ((0)) {
25017             #ifdef __Pyx_Coroutine_USED
25018             } else if (__Pyx_Coroutine_Check(self)) {
25019                 msg = "coroutine ignored GeneratorExit";
25020             #endif
25021             #ifdef __Pyx_AsyncGen_USED

```

```

25021         } else if (__Pyx_AsyncGen_CheckExact(self)) {
25022             #if PY_VERSION_HEX < 0x03060000
25023                 msg = "async generator ignored GeneratorExit - might require Python 3.6+ finalisation (PEP
525)";
25024             #else
25025                 msg = "async generator ignored GeneratorExit";
25026             #endif
25027         #endif
25028         } else {
25029             msg = "generator ignored GeneratorExit";
25030         }
25031         PyErr_SetString(PyExc_RuntimeError, msg);
25032         return NULL;
25033     }
25034     raised_exception = PyErr_Occurred();
25035     if (likely(!raised_exception || __Pyx_PyErr_GivenExceptionMatches2(raised_exception,
PyExc_GeneratorExit, PyExc_StopIteration))) {
25036         if (raised_exception) PyErr_Clear();
25037         Py_INCREF(Py_None);
25038         return Py_None;
25039     }
25040     return NULL;
25041 }
25042 static PyObject *__Pyx_Coroutine_Throw(PyObject *self, PyObject *typ, PyObject *val, PyObject *tb,
25043 PyObject *args, int close_on_genexit) {
25044     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25045     PyObject *yf = gen->yieldfrom;
25046     if (unlikely(gen->is_running))
25047         return __Pyx_Coroutine_AlreadyRunningError(gen);
25048     if (yf) {
25049         PyObject *ret;
25050         Py_INCREF(yf);
25051         if (__Pyx_PyErr_GivenExceptionMatches(typ, PyExc_GeneratorExit) && close_on_genexit) {
25052             int err = __Pyx_Coroutine_CloseIter(gen, yf);
25053             Py_DECREF(yf);
25054             __Pyx_Coroutine_Undelegate(gen);
25055             if (err < 0)
25056                 return __Pyx_Coroutine_MethodReturn(self, __Pyx_Coroutine_SendEx(gen, NULL, 0));
25057             goto throw_here;
25058         }
25059         gen->is_running = 1;
25060         if (0
25061             #ifdef __Pyx_Generator_USED
25062             || __Pyx_Generator_CheckExact(yf)
25063             #endif
25064             #ifdef __Pyx_Coroutine_USED
25065             || __Pyx_Coroutine_Check(yf)
25066             #endif
25067         ) {
25068             ret = __Pyx_Coroutine_Throw(yf, typ, val, tb, args, close_on_genexit);
25069             #ifdef __Pyx_Coroutine_USED
25070         } else if (__Pyx_CoroutineAwait_CheckExact(yf)) {
25071                 ret = __Pyx_Coroutine_Throw(((__pyx_CoroutineAwaitObject*)yf)->coroutine, typ, val, tb,
args, close_on_genexit);
25072             #endif
25073         } else {
25074             PyObject *meth = __Pyx_PyObject_GetAttrStr(yf, __pyx_n_s_throw);
25075             if (unlikely(!meth)) {
25076                 Py_DECREF(yf);
25077                 if (!PyErr_ExceptionMatches(PyExc_AttributeError)) {
25078                     gen->is_running = 0;
25079                     return NULL;
25080                 }
25081                 PyErr_Clear();
25082                 __Pyx_Coroutine_Undelegate(gen);
25083                 gen->is_running = 0;
25084                 goto throw_here;
25085             }
25086             if (likely(args)) {
25087                 ret = PyObject_CallObject(meth, args);
25088             } else {
25089                 ret = PyObject_CallFunctionObjArgs(meth, typ, val, tb, NULL);
25090             }
25091             Py_DECREF(meth);
25092         }
25093         gen->is_running = 0;
25094         Py_DECREF(yf);
25095         if (!ret) {
25096             ret = __Pyx_Coroutine_FinishDelegation(gen);
25097         }
25098         return __Pyx_Coroutine_MethodReturn(self, ret);
25099     }
25100 throw_here:
25101     __Pyx_Raise(typ, val, tb, NULL);
25102     return __Pyx_Coroutine_MethodReturn(self, __Pyx_Coroutine_SendEx(gen, NULL, 0));
25103 }
25104 static PyObject *__Pyx_Coroutine_Throw(PyObject *self, PyObject *args) {

```

```

25105     PyObject *typ;
25106     PyObject *val = NULL;
25107     PyObject *tb = NULL;
25108     if (!PyArg_UnpackTuple(args, (char *)"throw", 1, 3, &typ, &val, &tb))
25109         return NULL;
25110     return __Pyx_Coroutine_Throw(self, typ, val, tb, args, 1);
25111 }
25112 static CYTHON_INLINE int __Pyx_Coroutine_traverse_excstate(__Pyx_ExcInfoStruct *exc_state, visitproc
    visit, void *arg) {
25113     Py_VISIT(exc_state->exc_type);
25114     Py_VISIT(exc_state->exc_value);
25115     Py_VISIT(exc_state->exc_traceback);
25116     return 0;
25117 }
25118 static int __Pyx_Coroutine_traverse(__pyx_CoroutineObject *gen, visitproc visit, void *arg) {
25119     Py_VISIT(gen->closure);
25120     Py_VISIT(gen->classobj);
25121     Py_VISIT(gen->yieldfrom);
25122     return __Pyx_Coroutine_traverse_excstate(&gen->gi_exc_state, visit, arg);
25123 }
25124 static int __Pyx_Coroutine_clear(PyObject *self) {
25125     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25126     Py_CLEAR(gen->closure);
25127     Py_CLEAR(gen->classobj);
25128     Py_CLEAR(gen->yieldfrom);
25129     __Pyx_Coroutine_ExceptionClear(&gen->gi_exc_state);
25130 #ifdef __Pyx_AsyncGen_USED
25131     if (__Pyx_AsyncGen_CheckExact(self)) {
25132         Py_CLEAR(((__pyx_PyAsyncGenObject*)gen)->ag_finalizer);
25133     }
25134 #endif
25135     Py_CLEAR(gen->gi_code);
25136     Py_CLEAR(gen->gi_frame);
25137     Py_CLEAR(gen->gi_name);
25138     Py_CLEAR(gen->gi_qualname);
25139     Py_CLEAR(gen->gi_modulename);
25140     return 0;
25141 }
25142 static void __Pyx_Coroutine_dealloc(PyObject *self) {
25143     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25144     PyObject_GC_UnTrack(gen);
25145     if (gen->gi_weakreflist != NULL)
25146         PyObject_ClearWeakRefs(self);
25147     if (gen->resume_label >= 0) {
25148         PyObject_GC_Track(self);
25149 #if PY_VERSION_HEX >= 0x030400a1 && CYTHON_USE_TP_FINALIZE
25150         if (PyObject_CallFinalizerFromDealloc(self))
25151 #else
25152         Py_TYPE(gen)->tp_del(self);
25153         if (Py_REFCNT(self) > 0)
25154 #endif
25155         {
25156             return;
25157         }
25158         PyObject_GC_UnTrack(self);
25159     }
25160 #ifdef __Pyx_AsyncGen_USED
25161     if (__Pyx_AsyncGen_CheckExact(self)) {
25162         /* We have to handle this case for asynchronous generators
25163            right here, because this code has to be between UNTRACK
25164            and GC_Del. */
25165         Py_CLEAR(((__pyx_PyAsyncGenObject*)self)->ag_finalizer);
25166     }
25167 #endif
25168     __Pyx_Coroutine_clear(self);
25169     PyObject_GC_Del(gen);
25170 }
25171 static void __Pyx_Coroutine_del(PyObject *self) {
25172     PyObject *error_type, *error_value, *error_traceback;
25173     __pyx_CoroutineObject *gen = (__pyx_CoroutineObject *) self;
25174     __Pyx_PyThreadState_declare
25175     if (gen->resume_label < 0) {
25176         return;
25177     }
25178 #if !CYTHON_USE_TP_FINALIZE
25179     assert(self->ob_refcnt == 0);
25180     __Pyx_SET_REFCNT(self, 1);
25181 #endif
25182     __Pyx_PyThreadState_assign
25183     __Pyx_ErrFetch(&error_type, &error_value, &error_traceback);
25184 #ifdef __Pyx_AsyncGen_USED
25185     if (__Pyx_AsyncGen_CheckExact(self)) {
25186         __pyx_PyAsyncGenObject *agen = (__pyx_PyAsyncGenObject*)self;
25187         PyObject *finalizer = agen->ag_finalizer;
25188         if (finalizer && !agen->ag_closed) {
25189             PyObject *res = __Pyx_PyObject_CallOneArg(finalizer, self);
25190             if (unlikely(!res)) {

```

```

25191         PyErr_WriteUnraisable(self);
25192     } else {
25193         Py_DECREF(res);
25194     }
25195     __Pyx_ErrRestore(error_type, error_value, error_traceback);
25196     return;
25197 }
25198 }
25199 #endif
25200 if (unlikely(gen->resume_label == 0 && !error_value)) {
25201 #ifdef __Pyx_Coroutine_USED
25202 #ifdef __Pyx_Generator_USED
25203     if (!__Pyx_Generator_CheckExact(self))
25204 #endif
25205     {
25206         PyObject_GC_UnTrack(self);
25207 #if PY_MAJOR_VERSION >= 3 || defined(PyErr_WarnFormat)
25208         if (unlikely(PyErr_WarnFormat(PyExc_RuntimeWarning, 1, "coroutine '%.50S' was never awaited",
gen->gi_qualname) < 0))
25209             PyErr_WriteUnraisable(self);
25210 #else
25211         {PyObject *msg;
25212          char *cmsg;
25213          #if CYTHON_COMPILING_IN_PYPY
25214          msg = NULL;
25215          cmsg = (char*) "coroutine was never awaited";
25216          #else
25217          char *cname;
25218          PyObject *qualname;
25219          qualname = gen->gi_qualname;
25220          cname = PyString_AS_STRING(qualname);
25221          msg = PyString_FromFormat("coroutine '%.50s' was never awaited", cname);
25222          if (unlikely(!msg)) {
25223              PyErr_Clear();
25224              cmsg = (char*) "coroutine was never awaited";
25225          } else {
25226              cmsg = PyString_AS_STRING(msg);
25227          }
25228          #endif
25229          if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning, cmsg, 1) < 0))
25230              PyErr_WriteUnraisable(self);
25231          Py_XDECREF(msg);}
25232 #endif
25233         PyObject_GC_Track(self);
25234     }
25235 #endif
25236     } else {
25237         PyObject *res = __Pyx_Coroutine_Close(self);
25238         if (unlikely(!res)) {
25239             if (PyErr_Occurred())
25240                 PyErr_WriteUnraisable(self);
25241         } else {
25242             Py_DECREF(res);
25243         }
25244     }
25245     __Pyx_ErrRestore(error_type, error_value, error_traceback);
25246 #if !CYTHON_USE_TP_FINALIZE
25247     assert(Py_REFCNT(self) > 0);
25248     if (--self->ob_refcnt == 0) {
25249         return;
25250     }
25251     {
25252         Py_ssize_t refcnt = Py_REFCNT(self);
25253         __Pyx_NewReference(self);
25254         __Pyx_SET_REFCNT(self, refcnt);
25255     }
25256 #if CYTHON_COMPILING_IN_CPYTHON
25257     assert(PyType_IS_GC(Py_TYPE(self)) &&
25258            __Pyx_AS_GC(self)->gc.gc_refs != __PyGC_REFS_UNTRACKED);
25259     __Pyx_DEC_REFTOTAL;
25260 #endif
25261 #ifdef COUNT_ALLOCS
25262     --Py_TYPE(self)->tp_frees;
25263     --Py_TYPE(self)->tp_allocs;
25264 #endif
25265 #endif
25266 }
25267 static PyObject *
25268 __Pyx_Coroutine_get_name(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25269 {
25270     PyObject *name = self->gi_name;
25271     if (unlikely(!name)) name = Py_None;
25272     Py_INCREF(name);
25273     return name;
25274 }
25275 static int
25276 __Pyx_Coroutine_set_name(__pyx_CoroutineObject *self, PyObject *value, CYTHON_UNUSED void *context)

```

```

25277 {
25278     PyObject *tmp;
25279     #if PY_MAJOR_VERSION >= 3
25280     if (unlikely(value == NULL || !PyUnicode_Check(value)))
25281     #else
25282     if (unlikely(value == NULL || !PyString_Check(value)))
25283     #endif
25284     {
25285         PyErr_SetString(PyExc_TypeError,
25286             "__name__ must be set to a string object");
25287         return -1;
25288     }
25289     tmp = self->gi_name;
25290     Py_INCREF(value);
25291     self->gi_name = value;
25292     Py_XDECREF(tmp);
25293     return 0;
25294 }
25295 static PyObject *
25296 __Pyx_Coroutine_get_qualname(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25297 {
25298     PyObject *name = self->gi_qualname;
25299     if (unlikely(!name)) name = Py_None;
25300     Py_INCREF(name);
25301     return name;
25302 }
25303 static int
25304 __Pyx_Coroutine_set_qualname(__pyx_CoroutineObject *self, PyObject *value, CYTHON_UNUSED void
25305 *context)
25306 {
25307     PyObject *tmp;
25308     #if PY_MAJOR_VERSION >= 3
25309     if (unlikely(value == NULL || !PyUnicode_Check(value)))
25310     #else
25311     if (unlikely(value == NULL || !PyString_Check(value)))
25312     #endif
25313     {
25314         PyErr_SetString(PyExc_TypeError,
25315             "__qualname__ must be set to a string object");
25316         return -1;
25317     }
25318     tmp = self->gi_qualname;
25319     Py_INCREF(value);
25320     self->gi_qualname = value;
25321     Py_XDECREF(tmp);
25322     return 0;
25323 }
25324 static PyObject *
25325 __Pyx_Coroutine_get_frame(__pyx_CoroutineObject *self, CYTHON_UNUSED void *context)
25326 {
25327     PyObject *frame = self->gi_frame;
25328     if (!frame) {
25329         if (unlikely(!self->gi_code)) {
25330             Py_RETURN_NONE;
25331         }
25332         frame = (PyObject *) PyFrame_New(
25333             PyThreadState_Get(), /*PyThreadState *tstate,*/
25334             (PyCodeObject*) self->gi_code, /*PyCodeObject *code,*/
25335             __pyx_d, /*PyObject *globals,*/
25336             0 /*PyObject *locals*/
25337         );
25338         if (unlikely(!frame))
25339             return NULL;
25340         self->gi_frame = frame;
25341     }
25342     Py_INCREF(frame);
25343     return frame;
25344 }
25345 static __pyx_CoroutineObject * __Pyx__Coroutine_New(
25346     PyTypeObject* type, __pyx_coroutine_body_t body, PyObject *code, PyObject *closure,
25347     PyObject *name, PyObject *qualname, PyObject *module_name) {
25348     __pyx_CoroutineObject *gen = PyObject_GC_New(__pyx_CoroutineObject, type);
25349     if (unlikely(!gen))
25350         return NULL;
25351     return __Pyx__Coroutine_NewInit(gen, body, code, closure, name, qualname, module_name);
25352 }
25353 static __pyx_CoroutineObject * __Pyx__Coroutine_NewInit(
25354     __pyx_CoroutineObject *gen, __pyx_coroutine_body_t body, PyObject *code, PyObject
25355 *closure,
25356     PyObject *name, PyObject *qualname, PyObject *module_name) {
25357     gen->body = body;
25358     gen->closure = closure;
25359     Py_XINCREF(closure);
25360     gen->is_running = 0;
25361     gen->resume_label = 0;
25362     gen->classobj = NULL;
25363     gen->yieldfrom = NULL;

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```

25362     gen->gi_exc_state.exc_type = NULL;
25363     gen->gi_exc_state.exc_value = NULL;
25364     gen->gi_exc_state.exc_traceback = NULL;
25365     #if CYTHON_USE_EXC_INFO_STACK
25366     gen->gi_exc_state.previous_item = NULL;
25367     #endif
25368     gen->gi_weakreflist = NULL;
25369     Py_XINCREF(qualname);
25370     gen->gi_qualname = qualname;
25371     Py_XINCREF(name);
25372     gen->gi_name = name;
25373     Py_XINCREF(module_name);
25374     gen->gi_modulename = module_name;
25375     Py_XINCREF(code);
25376     gen->gi_code = code;
25377     gen->gi_frame = NULL;
25378     PyObject_GC_Track(gen);
25379     return gen;
25380 }
25381
25382 /* PatchModuleWithCoroutine */
25383 static PyObject* __Pyx_Coroutine_patch_module(PyObject* module, const char* py_code) {
25384     #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25385     int result;
25386     PyObject *globals, *result_obj;
25387     globals = PyDict_New(); if (unlikely(!globals)) goto ignore;
25388     result = PyDict_SetItemString(globals, "_cython_coroutine_type",
25389     #ifdef __Pyx_Coroutine_USED
25390         (PyObject*)__pyx_CoroutineType);
25391     #else
25392         Py_None);
25393     #endif
25394     if (unlikely(result < 0)) goto ignore;
25395     result = PyDict_SetItemString(globals, "_cython_generator_type",
25396     #ifdef __Pyx_Generator_USED
25397         (PyObject*)__pyx_GeneratorType);
25398     #else
25399         Py_None);
25400     #endif
25401     if (unlikely(result < 0)) goto ignore;
25402     if (unlikely(PyDict_SetItemString(globals, "_module", module) < 0)) goto ignore;
25403     if (unlikely(PyDict_SetItemString(globals, "__builtins__", __pyx_b) < 0)) goto ignore;
25404     result_obj = PyRun_String(py_code, Py_file_input, globals, globals);
25405     if (unlikely(!result_obj)) goto ignore;
25406     Py_DECREF(result_obj);
25407     Py_DECREF(globals);
25408     return module;
25409 ignore:
25410     Py_XDECREF(globals);
25411     PyErr_WriteUnraisable(module);
25412     if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning, "Cython module failed to patch module with custom
type", 1) < 0)) {
25413         Py_DECREF(module);
25414         module = NULL;
25415     }
25416     #else
25417     py_code++;
25418     #endif
25419     return module;
25420 }
25421
25422 /* PatchGeneratorABC */
25423 #ifndef CYTHON_REGISTER_ABCS
25424 #define CYTHON_REGISTER_ABCS 1
25425 #endif
25426 #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25427 static PyObject* __Pyx_patch_abc_module(PyObject *module);
25428 static PyObject* __Pyx_patch_abc_module(PyObject *module) {
25429     module = __Pyx_Coroutine_patch_module(
25430         module, ""
25431     "if _cython_generator_type is not None:\n"
25432     "    try: Generator = _module.Generator\n"
25433     "    except AttributeError: pass\n"
25434     "    else: Generator.register(_cython_generator_type)\n"
25435     "if _cython_coroutine_type is not None:\n"
25436     "    try: Coroutine = _module.Coroutine\n"
25437     "    except AttributeError: pass\n"
25438     "    else: Coroutine.register(_cython_coroutine_type)\n"
25439     );
25440     return module;
25441 }
25442 #endif
25443 static int __Pyx_patch_abc(void) {
25444     #if defined(__Pyx_Generator_USED) || defined(__Pyx_Coroutine_USED)
25445     static int abc_patched = 0;
25446     if (CYTHON_REGISTER_ABCS && !abc_patched) {
25447         PyObject *module;

```

```

25448     module = PyImport_ImportModule((PY_MAJOR_VERSION >= 3) ? "collections.abc" : "collections");
25449     if (!module) {
25450         PyErr_WriteUnraisable(NULL);
25451         if (unlikely(PyErr_WarnEx(PyExc_RuntimeWarning,
25452             ((PY_MAJOR_VERSION >= 3) ?
25453                 "Cython module failed to register with collections.abc module" :
25454                 "Cython module failed to register with collections module"), 1) < 0)) {
25455             return -1;
25456         }
25457     } else {
25458         module = __Pyx_patch_abc_module(module);
25459         abc_patched = 1;
25460         if (unlikely(!module))
25461             return -1;
25462         Py_DECREF(module);
25463     }
25464     module = PyImport_ImportModule("backports_abc");
25465     if (module) {
25466         module = __Pyx_patch_abc_module(module);
25467         Py_XDECREF(module);
25468     }
25469     if (!module) {
25470         PyErr_Clear();
25471     }
25472 }
25473 #else
25474 if ((0)) __Pyx_Coroutine_patch_module(NULL, NULL);
25475 #endif
25476 return 0;
25477 }
25478
25479 /* Generator */
25480 static PyMethodDef __pyx_Generator_methods[] = {
25481     {"send", (PyCFunction) __Pyx_Coroutine_Send, METH_O,
25482      (char*) PyDoc_STR("send(arg) -> send 'arg' into generator,\nreturn next yielded value or raise
StopIteration.")},
25483     {"throw", (PyCFunction) __Pyx_Coroutine_Throw, METH_VARARGS,
25484      (char*) PyDoc_STR("throw(typ[,val[,tb]]) -> raise exception in generator,\nreturn next yielded
value or raise StopIteration.")},
25485     {"close", (PyCFunction) __Pyx_Coroutine_Close_Method, METH_NOARGS,
25486      (char*) PyDoc_STR("close() -> raise GeneratorExit inside generator.")},
25487     {0, 0, 0, 0}
25488 };
25489 static PyMemberDef __pyx_Generator_memberlist[] = {
25490     {(char *) "gi_running", T_BOOL, offsetof(__pyx_CoroutineObject, is_running), READONLY, NULL},
25491     {(char *) "gi_yieldfrom", T_OBJECT, offsetof(__pyx_CoroutineObject, yieldfrom), READONLY,
25492      (char*) PyDoc_STR("object being iterated by 'yield from', or None")},
25493     {(char *) "gi_code", T_OBJECT, offsetof(__pyx_CoroutineObject, gi_code), READONLY, NULL},
25494     {0, 0, 0, 0, 0}
25495 };
25496 static PyGetSetDef __pyx_Generator_getsets[] = {
25497     {(char *) "__name__", (getter) __Pyx_Coroutine_get_name, (setter) __Pyx_Coroutine_set_name,
25498      (char*) PyDoc_STR("name of the generator"), 0},
25499     {(char *) "__qualname__", (getter) __Pyx_Coroutine_get_qualname,
25500      (setter) __Pyx_Coroutine_set_qualname,
25501      (char*) PyDoc_STR("qualified name of the generator"), 0},
25502     {(char *) "gi_frame", (getter) __Pyx_Coroutine_get_frame, NULL,
25503      (char*) PyDoc_STR("Frame of the generator"), 0},
25504 };
25505 static PyTypeObject __pyx_GeneratorType_type = {
25506     PyVarObject_HEAD_INIT(0, 0)
25507     "generator",
25508     sizeof(__pyx_CoroutineObject),
25509     0,
25510     (destructor) __Pyx_Coroutine_dealloc,
25511     0,
25512     0,
25513     0,
25514     0,
25515     0,
25516     0,
25517     0,
25518     0,
25519     0,
25520     0,
25521     0,
25522     0,
25523     0,
25524     0,
25525     Py_TPFLAGS_DEFAULT | Py_TPFLAGS_HAVE_GC | Py_TPFLAGS_HAVE_FINALIZE,
25526     0,
25527     (traverseproc) __Pyx_Coroutine_traverse,
25528     0,
25529     0,
25530     offsetof(__pyx_CoroutineObject, gi_weakreflist),
25531     0,

```



```

25532     (iternextfunc) __Pyx_Generator_Next,
25533     __pyx_Generator_methods,
25534     __pyx_Generator_memberlist,
25535     __pyx_Generator_getsets,
25536     0,
25537     0,
25538     0,
25539     0,
25540     0,
25541     0,
25542     0,
25543     0,
25544     0,
25545     0,
25546     0,
25547     0,
25548     0,
25549     0,
25550     0,
25551     #if CYTHON_USE_TP_FINALIZE
25552     0,
25553     #else
25554     __Pyx_Coroutine_del,
25555     #endif
25556     0,
25557     #if CYTHON_USE_TP_FINALIZE
25558     __Pyx_Coroutine_del,
25559     #elif PY_VERSION_HEX >= 0x030400a1
25560     0,
25561     #endif
25562     #if PY_VERSION_HEX >= 0x030800b1 && (!CYTHON_COMPILING_IN_PYPY || PYPY_VERSION_NUM >= 0x07030800)
25563     0,
25564     #endif
25565     #if PY_VERSION_HEX >= 0x030800b4 && PY_VERSION_HEX < 0x03090000
25566     0,
25567     #endif
25568     #if CYTHON_COMPILING_IN_PYPY && PY_VERSION_HEX >= 0x03090000
25569     0,
25570     #endif
25571     };
25572     static int __pyx_Generator_init(void) {
25573         __pyx_GeneratorType_type.tp_getattro = __Pyx_PyObject_GenericGetAttrNoDict;
25574         __pyx_GeneratorType_type.tp_iter = PyObject_SelfIter;
25575         __pyx_GeneratorType = __Pyx_FetchCommonType(&__pyx_GeneratorType_type);
25576         if (unlikely(!__pyx_GeneratorType)) {
25577             return -1;
25578         }
25579         return 0;
25580     }
25581
25582     /* CheckBinaryVersion */
25583     static int __Pyx_check_binary_version(void) {
25584         char ctversion[4], rtversion[4];
25585         PyOS_snprintf(ctversion, 4, "%d.%d", PY_MAJOR_VERSION, PY_MINOR_VERSION);
25586         PyOS_snprintf(rtversion, 4, "%s", Py_GetVersion());
25587         if (ctversion[0] != rtversion[0] || ctversion[2] != rtversion[2]) {
25588             char message[200];
25589             PyOS_snprintf(message, sizeof(message),
25590                 "compiletime version %s of module '%.100s' "
25591                 "does not match runtime version %s",
25592                 ctversion, __Pyx_MODULE_NAME, rtversion);
25593             return PyErr_WarnEx(NULL, message, 1);
25594         }
25595         return 0;
25596     }
25597
25598     /* InitStrings */
25599     static int __Pyx_InitStrings(__Pyx_StringTabEntry *t) {
25600         while (t->p) {
25601             #if PY_MAJOR_VERSION < 3
25602                 if (t->is_unicode) {
25603                     *t->p = PyUnicode_DecodeUTF8(t->s, t->n - 1, NULL);
25604                 } else if (t->intern) {
25605                     *t->p = PyString_InternFromString(t->s);
25606                 } else {
25607                     *t->p = PyString_FromStringAndSize(t->s, t->n - 1);
25608                 }
25609             } else
25610             if (t->is_unicode | t->is_str) {
25611                 if (t->intern) {
25612                     *t->p = PyUnicode_InternFromString(t->s);
25613                 } else if (t->encoding) {
25614                     *t->p = PyUnicode_Decode(t->s, t->n - 1, t->encoding, NULL);
25615                 } else {
25616                     *t->p = PyUnicode_FromStringAndSize(t->s, t->n - 1);
25617                 }
25618             } else {

```

```

25619         *t->p = PyBytes_FromStringAndSize(t->s, t->n - 1);
25620     }
25621     #endif
25622     if (!*t->p)
25623         return -1;
25624     if (PyObject_Hash(*t->p) == -1)
25625         return -1;
25626     ++t;
25627 }
25628 return 0;
25629 }
25630
25631 static CYTHON_INLINE PyObject* __Pyx_PyUnicode_FromString(const char* c_str) {
25632     return __Pyx_PyUnicode_FromStringAndSize(c_str, (Py_ssize_t)strlen(c_str));
25633 }
25634 static CYTHON_INLINE const char* __Pyx_PyObject_AsString(PyObject* o) {
25635     Py_ssize_t ignore;
25636     return __Pyx_PyObject_AsStringAndSize(o, &ignore);
25637 }
25638 #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII || __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
25639 #if !CYTHON_PEP393_ENABLED
25640 static const char* __Pyx_PyUnicode_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25641     char* defenc_c;
25642     PyObject* defenc = _PyUnicode_AsDefaultEncodedString(o, NULL);
25643     if (!defenc) return NULL;
25644     defenc_c = PyBytes_AS_STRING(defenc);
25645     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25646     {
25647         char* end = defenc_c + PyBytes_GET_SIZE(defenc);
25648         char* c;
25649         for (c = defenc_c; c < end; c++) {
25650             if ((unsigned char) (*c) >= 128) {
25651                 PyUnicode_AsASCIIString(o);
25652                 return NULL;
25653             }
25654         }
25655     }
25656     #endif
25657     *length = PyBytes_GET_SIZE(defenc);
25658     return defenc_c;
25659 }
25660 #else
25661 static CYTHON_INLINE const char* __Pyx_PyUnicode_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25662     if (unlikely(__Pyx_PyUnicode_READY(o) == -1)) return NULL;
25663     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25664     if (likely(PyUnicode_IS_ASCII(o))) {
25665         *length = PyUnicode_GET_LENGTH(o);
25666         return PyUnicode_AsUTF8(o);
25667     } else {
25668         PyUnicode_AsASCIIString(o);
25669         return NULL;
25670     }
25671     #else
25672     return PyUnicode_AsUTF8AndSize(o, length);
25673     #endif
25674 }
25675 #endif
25676 #endif
25677 static CYTHON_INLINE const char* __Pyx_PyObject_AsStringAndSize(PyObject* o, Py_ssize_t *length) {
25678     #if __PYX_DEFAULT_STRING_ENCODING_IS_ASCII || __PYX_DEFAULT_STRING_ENCODING_IS_DEFAULT
25679     if (
25680     #if PY_MAJOR_VERSION < 3 && __PYX_DEFAULT_STRING_ENCODING_IS_ASCII
25681         __Pyx_sys_getdefaultencoding_not_ascii &&
25682     #endif
25683         PyUnicode_Check(o)) {
25684         return __Pyx_PyUnicode_AsStringAndSize(o, length);
25685     } else
25686     #endif
25687     #if (!CYTHON_COMPILING_IN_PYPY) || (defined(PyByteArray_AS_STRING) && defined(PyByteArray_GET_SIZE))
25688     if (PyByteArray_Check(o)) {
25689         *length = PyByteArray_GET_SIZE(o);
25690         return PyByteArray_AS_STRING(o);
25691     } else
25692     #endif
25693     {
25694         char* result;
25695         int r = PyBytes_AsStringAndSize(o, &result, length);
25696         if (unlikely(r < 0)) {
25697             return NULL;
25698         } else {
25699             return result;
25700         }
25701     }
25702 }
25703 static CYTHON_INLINE int __Pyx_PyObject_IsTrue(PyObject* x) {
25704     int is_true = x == Py_True;
25705     if (is_true | (x == Py_False) | (x == Py_None)) return is_true;

```

```

25706     else return PyObject_IsTrue(x);
25707 }
25708 static CYTHON_INLINE int __Pyx_PyObject_IsTrueAndDecref(PyObject* x) {
25709     int retval;
25710     if (unlikely(!x)) return -1;
25711     retval = __Pyx_PyObject_IsTrue(x);
25712     Py_DECREF(x);
25713     return retval;
25714 }
25715 static PyObject* __Pyx_PyNumber_IntOrLongWrongResultType(PyObject* result, const char* type_name) {
25716 #if PY_MAJOR_VERSION >= 3
25717     if (PyLong_Check(result)) {
25718         if (PyErr_WarnFormat(PyExc_DeprecationWarning, 1,
25719             "__int__ returned non-int (type %.200s). "
25720             "The ability to return an instance of a strict subclass of int "
25721             "is deprecated, and may be removed in a future version of Python.",
25722             Py_TYPE(result)->tp_name)) {
25723             Py_DECREF(result);
25724             return NULL;
25725         }
25726         return result;
25727     }
25728 #endif
25729     PyErr_Format(PyExc_TypeError,
25730         "\"%.4s__ returned non-%.4s (type %.200s)",
25731         type_name, type_name, Py_TYPE(result)->tp_name);
25732     Py_DECREF(result);
25733     return NULL;
25734 }
25735 static CYTHON_INLINE PyObject* __Pyx_PyNumber_IntOrLong(PyObject* x) {
25736 #if CYTHON_USE_TYPE_SLOTS
25737     PyNumberMethods *m;
25738 #endif
25739     const char *name = NULL;
25740     PyObject *res = NULL;
25741 #if PY_MAJOR_VERSION < 3
25742     if (likely(PyInt_Check(x) || PyLong_Check(x)))
25743     #else
25744     if (likely(PyLong_Check(x)))
25745 #endif
25746         return __Pyx_NewRef(x);
25747 #if CYTHON_USE_TYPE_SLOTS
25748     m = Py_TYPE(x)->tp_as_number;
25749     #if PY_MAJOR_VERSION < 3
25750     if (m && m->nb_int) {
25751         name = "int";
25752         res = m->nb_int(x);
25753     }
25754     else if (m && m->nb_long) {
25755         name = "long";
25756         res = m->nb_long(x);
25757     }
25758     #else
25759     if (likely(m && m->nb_int)) {
25760         name = "int";
25761         res = m->nb_int(x);
25762     }
25763     #endif
25764 #else
25765     if (!PyBytes_CheckExact(x) && !PyUnicode_CheckExact(x)) {
25766         res = PyNumber_Int(x);
25767     }
25768 #endif
25769     if (likely(res)) {
25770 #if PY_MAJOR_VERSION < 3
25771         if (unlikely(!PyInt_Check(res) && !PyLong_Check(res))) {
25772             #else
25773             if (unlikely(!PyLong_CheckExact(res))) {
25774                 #endif
25775                 return __Pyx_PyNumber_IntOrLongWrongResultType(res, name);
25776             }
25777         }
25778     else if (!PyErr_Occurred()) {
25779         PyErr_SetString(PyExc_TypeError,
25780             "an integer is required");
25781     }
25782     return res;
25783 }
25784 static CYTHON_INLINE Py_ssize_t __Pyx_PyIndex_AsSsize_t(PyObject* b) {
25785     Py_ssize_t ival;
25786     PyObject *x;
25787 #if PY_MAJOR_VERSION < 3
25788     if (likely(PyInt_CheckExact(b))) {
25789         if (sizeof(Py_ssize_t) >= sizeof(long))
25790             return PyInt_AS_LONG(b);
25791         else
25792             return PyInt_AsSsize_t(b);

```

```

25793     }
25794 #endif
25795     if (likely(PyLong_CheckExact(b))) {
25796         #if CYTHON_USE_PYLONG_INTERNALS
25797         const digit* digits = ((PyLongObject*)b)->ob_digit;
25798         const Py_ssize_t size = Py_SIZE(b);
25799         if (likely(__Pyx_sst_abs(size) <= 1)) {
25800             ival = likely(size) ? digits[0] : 0;
25801             if (size == -1) ival = -ival;
25802             return ival;
25803         } else {
25804             switch (size) {
25805                 case 2:
25806                     if (8 * sizeof(Py_ssize_t) > 2 * PyLong_SHIFT) {
25807                         return (Py_ssize_t) (((size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25808                     }
25809                     break;
25810                 case -2:
25811                     if (8 * sizeof(Py_ssize_t) > 2 * PyLong_SHIFT) {
25812                         return -(Py_ssize_t) (((size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25813                     }
25814                     break;
25815                 case 3:
25816                     if (8 * sizeof(Py_ssize_t) > 3 * PyLong_SHIFT) {
25817                         return (Py_ssize_t) (((size_t)digits[2]) << PyLong_SHIFT) | (size_t)digits[1]) <<
PyLong_SHIFT) | (size_t)digits[0]);
25818                     }
25819                     break;
25820                 case -3:
25821                     if (8 * sizeof(Py_ssize_t) > 3 * PyLong_SHIFT) {
25822                         return -(Py_ssize_t) (((size_t)digits[2]) << PyLong_SHIFT) | (size_t)digits[1]) <<
PyLong_SHIFT) | (size_t)digits[0]);
25823                     }
25824                     break;
25825                 case 4:
25826                     if (8 * sizeof(Py_ssize_t) > 4 * PyLong_SHIFT) {
25827                         return (Py_ssize_t) (((size_t)digits[3]) << PyLong_SHIFT) | (size_t)digits[2]) <<
PyLong_SHIFT) | (size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25828                     }
25829                     break;
25830                 case -4:
25831                     if (8 * sizeof(Py_ssize_t) > 4 * PyLong_SHIFT) {
25832                         return -(Py_ssize_t) (((size_t)digits[3]) << PyLong_SHIFT) | (size_t)digits[2]) <<
PyLong_SHIFT) | (size_t)digits[1]) << PyLong_SHIFT) | (size_t)digits[0]);
25833                     }
25834                     break;
25835             }
25836         }
25837     #endif
25838     return PyLong_AsSsize_t(b);
25839 }
25840 x = PyNumber_Index(b);
25841 if (!x) return -1;
25842 ival = PyInt_AsSsize_t(x);
25843 Py_DECREF(x);
25844 return ival;
25845 }
25846 static CYTHON_INLINE Py_hash_t __Pyx_PyIndex_AsHash_t(PyObject* o) {
25847     if (sizeof(Py_hash_t) == sizeof(Py_ssize_t)) {
25848         return (Py_hash_t) __Pyx_PyIndex_AsSsize_t(o);
25849     #if PY_MAJOR_VERSION < 3
25850     } else if (likely(PyInt_CheckExact(o))) {
25851         return PyInt_AS_LONG(o);
25852     #endif
25853     } else {
25854         Py_ssize_t ival;
25855         PyObject *x;
25856         x = PyNumber_Index(o);
25857         if (!x) return -1;
25858         ival = PyInt_AsLong(x);
25859         Py_DECREF(x);
25860         return ival;
25861     }
25862 }
25863 static CYTHON_INLINE PyObject * __Pyx_PyBool_FromLong(long b) {
25864     return b ? __Pyx_NewRef(Py_True) : __Pyx_NewRef(Py_False);
25865 }
25866 static CYTHON_INLINE PyObject * __Pyx_PyInt_FromSize_t(size_t ival) {
25867     return PyInt_FromSize_t(ival);
25868 }
25869
25870
25871 #endif /* Py_PYTHON_H */

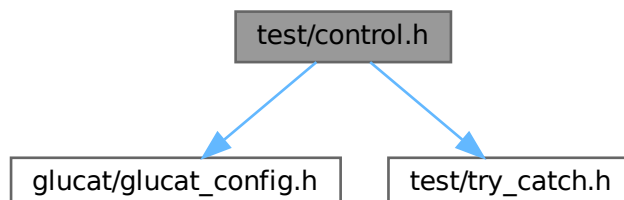
```

## 7.65 test/control.h File Reference

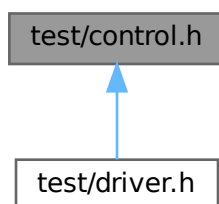
```
#include "glucat/glucat_config.h"
```

```
#include "test/try_catch.h"
```

Include dependency graph for control.h:



This graph shows which files directly or indirectly include this file:



### Classes

- class `glucat::control_t`  
*Parameters to control tests.*

### Namespaces

- namespace `glucat`

## 7.66 control.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_CONTROL_H
00002 #define _GLUCAT_CONTROL_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     control.h : Define and set parameters to control tests
00006
00007     begin                : 2010-04-21
00008     copyright            : (C) 2010-2016 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033 #include "glucat/glucat_config.h"
00034 #include "test/try_catch.h"
00035
00036 namespace glucat
00037 {
00038     class control_t
00039     {
00040     private:
00041         bool m_valid;
00042         bool valid() const
00043         { return m_valid; }
00044
00045         bool m_catch_exceptions;
00046         bool catch_exceptions() const
00047         { return m_catch_exceptions; }
00048
00049         static bool m_verbose_output;
00050
00051         control_t(int argc, char ** argv);
00052         // Enforce singleton
00053         // Reference: A. Alexandrescu, "Modern C++ Design", Chapter 6
00054         control_t() = default;
00055         ~control_t() = default;
00056         control_t(const control_t&) = delete;
00057         control_t& operator= (const control_t&) = delete;
00058
00059         friend class friend_for_private_destructor;
00060     public:
00061         static const control_t& control(int argc, char ** argv)
00062         { static const control_t c(argc, argv); return c; }
00063
00064         int call(intfn f) const;
00065         int call(intintfn f, int arg) const;
00066
00067         static bool verbose()
00068         { return m_verbose_output; }
00069     };
00070
00071     bool control_t::m_verbose_output = false;
00072
00073     control_t::
00074     control_t(int argc, char ** argv)
00075     : m_valid(true), m_catch_exceptions(true)
00076     {
00077         bool print_help = false;
00078         const std::string& arg_0_str = argv[0];
00079         const std::string program_name = arg_0_str.substr(arg_0_str.find_last_of('/')+1);
00080         for (int arg_ndx = 1; arg_ndx < argc; ++arg_ndx)
00081         {
00082             const std::string& arg_str = argv[arg_ndx];

```

```

00098     bool valid = false;
00099     if (arg_str.substr(0,2) == "--")
00100     {
00101         valid = true;
00102         const std::string& arg_name = arg_str.substr(2);
00103         if (arg_name == "help")
00104         {
00105             this->m_valid = false;
00106             print_help = true;
00107         }
00108         else if (arg_name == "verbose")
00109             this->m_verbose_output = true;
00110         else if (arg_name == "no-catch")
00111             this->m_catch_exceptions = false;
00112         else
00113             valid = false;
00114     }
00115     if (!valid)
00116     {
00117         std::cout << "Invalid argument: " << arg_str << std::endl;
00118         this->m_valid = false;
00119         print_help = true;
00120     }
00121 }
00122 if (print_help)
00123 {
00124     std::cout << program_name << " for " << GLUCAT_PACKAGE_NAME << " version " << GLUCAT_VERSION << ":" <<
std::endl;
00125     std::cout << "Usage: " << program_name << " [option ...]" << std::endl;
00126     std::cout << "Options:" << std::endl;
00127     std::cout << "  --help      : Print this summary." << std::endl;
00128     std::cout << "  --no-catch  : Do not catch exceptions." << std::endl;
00129     std::cout << "  --verbose   : Produce more detailed test output." << std::endl;
00130 }
00131 }
00132
00133 inline
00134 int
00135 control_t::
00136 call(intfn f) const
00137 {
00138     if (valid())
00139         return (catch_exceptions())
00140             ? try_catch(f)
00141             : (*f)();
00142     else
00143         return 1;
00144 }
00145
00146 inline
00147 int
00148 control_t::
00149 call(intintfn f, int arg) const
00150 {
00151     if (valid())
00152         return (catch_exceptions())
00153             ? try_catch(f, arg)
00154             : (*f)(arg);
00155     else
00156         return 1;
00157 }
00158 }
00159 }
00160 }
00161 #endif // _GLUCAT_CONTROL_H

```

## 7.67 test/driver.h File Reference

```

#include "glucat/glucat.h"
#include "glucat/glucat_imp.h"
#include "test/tuning.h"
#include "test/try_catch.h"
#include "test/control.h"
#include <stdio>

```

Include dependency graph for driver.h:



## 7.68 driver.h

[Go to the documentation of this file.](#)

```

00001 #ifndef GLUCAT_TEST_DRIVER_H
00002 #define GLUCAT_TEST_DRIVER_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     driver.h : Header for example and timing test driver
00006     -----
00007     begin                : Sun 2001-12-09
00008     copyright             : (C) 2001-2021 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 #include "glucat/glucat.h"
00035 #include "glucat/glucat_imp.h"
00036 #include "test/tuning.h"
00037 #include "test/try_catch.h"
00038 #include "test/control.h"
00039 #include <cstdio>
00040
00041 #endif // GLUCAT_TEST_DRIVER_H

```

## 7.69 test/timing.h File Reference

### Namespaces

- namespace [glucat](#)
- namespace [glucat::timing](#)

### Functions

- static double [glucat::timing::elapsed](#) (clock\_t cpu\_time)  
*Elapsed time in milliseconds.*



## Variables

- const double `glucat::timing::MS_PER_SEC` = 1000.0  
*Timing constant: milliseconds per second.*
- const double `glucat::timing::MS_PER_CLOCK` = `MS_PER_SEC` / `double(CLOCKS_PER_SEC)`  
*Timing constant: milliseconds per clock.*
- const int `glucat::timing::EXTRA_TRIALS` = 2  
*Timing constant: trial expansion factor.*

## 7.70 timing.h

Go to the documentation of this file.

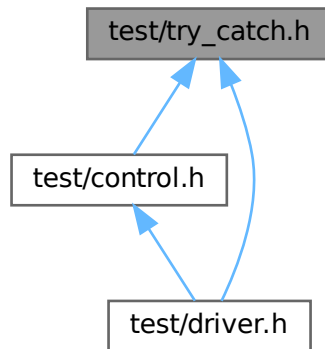
```

00001 #ifndef GLUCAT_TEST_TIMING_H
00002 #define GLUCAT_TEST_TIMING_H
00003 /*****
00004     GluCat : Generic library of universal Clifford algebra templates
00005     timing.h : Common definitions for timing tests
00006     -----
00007     begin                : Tue 2012-03-27
00008     copyright            : (C) 2012 by Paul C. Leopardi
00009     *****/
00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****/
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****/
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 namespace glucat
00035 {
00036     namespace timing
00037     {
00038     {
00039         const double MS_PER_SEC = 1000.0;
00040
00042         const double MS_PER_CLOCK = MS_PER_SEC / double(CLOCKS_PER_SEC);
00043
00045         const int EXTRA_TRIALS = 2;
00046
00048         inline
00049         static
00050         double
00051         elapsed(clock_t cpu_time)
00052         { return double(clock() - cpu_time) * MS_PER_CLOCK; }
00053
00054     }
00055     }
00056 #endif // GLUCAT_TEST_TIMING_H

```

## 7.71 test/try\_catch.h File Reference

This graph shows which files directly or indirectly include this file:



### Namespaces

- namespace [glucat](#)

### Typedefs

- typedef int(\* [glucat::intfn](#)) ()  
*For exception catching: pointer to function returning int.*
- typedef int(\* [glucat::intintfn](#)) (int)  
*For exception catching: pointer to function of int returning int.*

### Functions

- int [glucat::try\\_catch](#) ([intfn](#) f)  
*Exception catching for functions returning int.*
- int [glucat::try\\_catch](#) ([intintfn](#) f, int arg)  
*Exception catching for functions of int returning int.*

## 7.72 try\_catch.h

[Go to the documentation of this file.](#)

```

00001 #ifndef _GLUCAT_TRY_CATCH_H
00002 #define _GLUCAT_TRY_CATCH_H
00003 /*****
00004   GluCat : Generic library of universal Clifford algebra templates
00005   try_catch.h : Catch exceptions
00006   -----
00007   begin                : Sun 2001-12-20
00008   copyright             : (C) 2001-2010 by Paul C. Leopardi
00009   *****/

```

```

00010
00011     This library is free software: you can redistribute it and/or modify
00012     it under the terms of the GNU Lesser General Public License as published
00013     by the Free Software Foundation, either version 3 of the License, or
00014     (at your option) any later version.
00015
00016     This library is distributed in the hope that it will be useful,
00017     but WITHOUT ANY WARRANTY; without even the implied warranty of
00018     MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the
00019     GNU Lesser General Public License for more details.
00020
00021     You should have received a copy of the GNU Lesser General Public License
00022     along with this library. If not, see <http://www.gnu.org/licenses/>.
00023
00024     *****
00025     This library is based on a prototype written by Arvind Raja and was
00026     licensed under the LGPL with permission of the author. See Arvind Raja,
00027     "Object-oriented implementations of Clifford algebras in C++: a prototype",
00028     in Ablamowicz, Lounesto and Parra (eds.)
00029     "Clifford algebras with numeric and symbolic computations", Birkhauser, 1996.
00030     *****
00031     See also Arvind Raja's original header comments in glucat.h
00032     *****/
00033
00034 namespace glucat
00035 {
00036     typedef int (*intfn)();
00037
00038     typedef int (*intintfn)(int);
00039
00040     int try_catch(intfn f);
00041
00042     int try_catch(intintfn f, int arg);
00043
00044     int try_catch(intfn f)
00045     {
00046         int result = 0;
00047         try
00048         {
00049             result = (*f)();
00050         }
00051         catch (const glucat_error& e)
00052         {
00053             e.print_error_msg();
00054         }
00055         catch (const std::bad_alloc& e)
00056         {
00057             std::cerr << "bad_alloc" << std::endl;
00058         }
00059         catch (...)
00060         {
00061             std::cerr << "unexpected exception" << std::endl;
00062         }
00063         return result;
00064     }
00065
00066     int try_catch(intintfn f, int arg)
00067     {
00068         int result = 0;
00069         try
00070         {
00071             result = (*f)(arg);
00072         }
00073         catch (const glucat_error& e)
00074         {
00075             e.print_error_msg();
00076         }
00077         catch (const std::bad_alloc& e)
00078         {
00079             std::cerr << "bad_alloc" << std::endl;
00080         }
00081         catch (...)
00082         {
00083             std::cerr << "unexpected exception" << std::endl;
00084         }
00085         return result;
00086     }
00087 }
00088 #endif // _GLUCAT_TRY_CATCH_H

```



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